Breaking the Bound: Rate-2, Full Diversity, Orthogonal MIMO-STBC Transceiver Design

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***Abstract*—Space Time Block Codes (STBCs) from orthogonal designs have attracted significant interest in recent years. However, with the growing demand for higher capacity schemes, the multiantenna transmission techniques must support and achieve higher symbol transmission rates. In this article, we focus on three and four transmit antenna schemes. For over two decades, STBC schemes for three and four transmit antennas that achieve a very high symbol transmission rate while being orthogonal, fully diverse, delay-efficient, and with very low decoding complexity have not been achieved. The schemes proposed so far trades off orthogonality, delay, diversity, or decoding complexity while achieving rate or vice-versa. This work is first of its kind to solve this problem while fulfilling all the desired properties.**

In this work, we carefully study the various aspects that must be considered in designing higher symbol transmission rate STBCs. The proposed designs, hereby referred to as, Jagannath schemes - for 4×3 and 4×4 configurations are orthogonal, achieve full diversity, and support a symbol transmission rate of 2 symbols/s/Hz. The design methodology follows a coding gain maximization approach. The low decoding complexity procedure to decode the proposed transmission schemes are proposed as well as extensively evaluated in simulations under diverse settings. The performance of proposed schemes are empirically compared to two state-of-the-art schemes; ACIOD and Jafarkhani. Jagannath 4×3 was shown to outperform ACIOD by $\sim$12 dB while Jagannath 4×4 exhibited a $\sim$7 dB superior performance in contrast to Jafarkhani at 4 bpcu. This motivates the adoption of Jagannath schemes in tactical and commercial communication systems to effectively double the throughput or to extend the communication range.

***Index Terms*—high rate STBC, spectral efficiency, orthogonality, low decoding complexity, rate-2 STBC**

I. Introduction

Multiple antenna technologies - multiple-input multiple-output (MIMO) - are playing a key role in fifth generation (5G) networks and will continue to be vital for beyond 5G networks. Multiple antenna technologies ranging from small scale to massive MIMO techniques is an active research area to meet the growing demand for higher data rate wireless communications. Consumer demands are on the spike and commercial service providers are striving to fulfill the consumer demands for high data rate applications such as ultra HD video streaming, multiplayer VR/AR gaming, smart home, smart vehicle IoT applications, etc. Henceforth, MIMO forms the fuel that ignites the 5G and beyond 5G networks. 2019 marks the official year of 5G roll out with Verizon launching the worlds first commercial millimeter wave (mmWave) 5G service in Chicago and Minneapolis (400 MHz channel at 28 GHz) [1]. More such commercial vendors are rolling out 5G services to mark their footprints across the world. Higher spectral efficiency is highly desired in tactical communications to accommodate high rate transmissions in a limited spectrum [2], [3]. The race for higher throughput continues and the research community is actively looking at ways to *scale* the achievable data rate by two-fold or a whopping 100-fold. MIMO holds the key to these goals by numerous techniques such as beamforming, spatial modulation, space-time block coding, space-frequency block coding, and so on.

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In this work, we achieve *very high* symbol transmission rate pertaining to 3 and 4 transmission antenna while being orthogonal, fully diverse, and with a very low decoding complexity in contrast to other space-time block codes [4]–[8] for 3 and 4 antenna configurations. The remainder of this article is organized as follows. In Section II, we describe the problem statement which ponders upon the current state-of-the-art, challenges and motivates this work. Section III introduces the code construction, establishes orthogonality, and defines the received signal model. The diversity order analysis is elaborated in Section IV. Section V describes the design criteria for the novel designs by maximizing the coding gain (diversity product). The conditional maximum likelihood decoding procedure for the two designs are extensively shown in Section VI. The performance analysis of the codewords based on simulations are reported in Section VII. Finally, the concluding remarks of this work is presented in Section VIII.

II. Problem Statement

Space-time block codes (STBC) offer immense potential to maximize the achievable capacity of multiple antenna systems. Consequently, STBC is a subject of great interest among MIMO researchers [4]–[7], [9]–[19]. We define a perfect STBC as the one that meets the following properties to offer remarkable system performance:

- Orthogonality - ensures that the symbols transmitted from multiple transmit antennas are uncorrelated and separable at the receiver.
- Full diversity - the rank of the difference matrix between any two distinct codewords must be full rank to ensure full diversity and hence the maximum diversity order. The diversity order dictates the slope of the error curve [8], [15].
- High symbol transmission rate - implies transmission of as many symbols in a given time slot or epoch. Symbol

rate effects the spectral efficiency by injecting as many useful symbols per channel use [19].
- Low decoding complexity - is required to ensure adoption of computationally lighter receivers.

The most simplistic STBC that satisfies the orthogonality, full diversity and low decoding complexity was introduced by Alamouti [10] in 1998. Alamouti scheme was a 2×2 STBC proposed for two transmit antennas and of rate 1. Alamouti's scheme opened doors to further research to derive similar analogous orthogonal designs [11]. Tarokh et. al presented real orthogonal designs for $2, 4,$ and 8 transmit antennas and introduced the notion of generalized orthogonal and generalized linear processing orthogonal designs.

For the sake of clarity, we reinstate the definition of complex linear processing orthogonal design (CLOD) and generalized complex orthogonal design (GCOD) as proposed by Tarokh et.al. A $\mathcal{N}_t \times \mathcal{N}_t$ STBC $\mathbf{X}$ in information symbols $x_1, x_2, \cdots, x_{\mathcal{N}_t}$ is said to be CLOD if $\mathbf{X}^H\mathbf{X}$ is a diagonal matrix with (i,i)th diagonal element of the form $\Big(l_1^i|x_1|^2 + l_2^i|x_2|^2 + \cdots + l_k^i|x_k|^2\Big)$ where the coefficients $l_1^i, l_2^i, \cdots, l_k^i$ are strictly positive integers. A $\mathcal{T} \times \mathcal{N}_t$ STBC $\mathbf{X}$ formed of symbols $\pm x_1, \pm x_1^*, \pm x_2, \pm x_2^*, \cdots, \pm x_k, \pm x_k^*$ is GCOD if it yields a diagonal matrix $\mathbf{X}^H\mathbf{X}$ with (i,i)th diagonal element of the form $\Big(l_1^i|x_1|^2 + l_2^i|x_2|^2 + \cdots + l_k^i|x_k|^2\Big)$ where the coefficients $l_1^i, l_2^i, \cdots, l_k^i$ are strictly positive integers. Hence, both CLOD and GCOD yield diagonal correlation matrix with unequal norms. They further concluded the paper with a presentation of $1/2$ rate complex generalized orthogonal designs for 3 and 4 transmit antennas. But these designs were spectrally inefficient as they required transmission over 8 epochs. As an alternative for even higher rate, they proposed GCODs for both antenna configurations to achieve a rate of $3/4$ over 4 epochs.

Ganesan and Stoica [17] followed a maximum signal-to-noise ratio (SNR) approach to design complex orthogonal codewords of rate $3/4$ for 3 and 4 transmit antennas but with a delay of 4 and posed an open question whether the rates can go higher than $3/4$ for 3 and 4 transmit antennas. In [14], Su et. al. proposed systematic design method to generate high rate complex orthogonal STBCs for any number of transmit antennas. However, for 3 and 4 transmit antennas, the maximum achievable rate with their design was $3/4$ and required transmission over 4 and 8 epochs respectively.

Further, Theorem 1 in [16] states the maximum achievable rate for a $\mathcal{T} \times \mathcal{N}_t$ square matrix embeddable code as

$$\mathcal{R} \leq \frac{\lceil \log_2 \mathcal{N}_t \rceil + 1}{2^{\lceil \log_2 \mathcal{N}_t \rceil}}. \tag{1}$$

where $\lceil . \rceil$ denotes the ceil operator. This implies for 3 and 4 transmit antennas the maximum achievable rate is bounded by $3/4$. Additionally, in [20] the upper bound for GCOD was established to be $4/5$. More generally speaking, the rate achievable by 3 and 4 transmit antenna STBCs presently are limited by the following bound

$$\mathcal{R} \leq 4/5 \tag{2}$$

Although these unitary (complex orthogonal) designs for 3 and 4 transmit antennas allow minimal complexity decoding, they are not rate-optimal or in other words, are spectrally inefficient and are limited by being able to achieve a symbol transmission rate of only $3/4$ (complex orthogonal) or $4/5$ (GCOD). This preexisting challenge of the lack of rate-optimal STBC designs for 3 and 4 transmit antennas forms the key motivation for this work. Jafarkhani [21] proposed rate 1 design for 4 transmit antennas by relaxing the orthogonality constraint. But these codes can only achieve partial diversity. Tirkkonen et. al [22] introduced full rate codes for more than 3 transmit antennas. However, these codes are non-orthogonal causing symbol interference. Such partial diversity schemes cause poorer performance in the high SNR region. Mitigating the partial diversity challenge, quasi-orthogonal designs which for 4 transmit antennas achieved a rate 1 and full diversity was proposed in [8].

The quest for higher rate codes is growing and new designs are proposed to surpass such previously established bounds on achievable rate [6]–[8], [19]. The authors of [6] proposed rectangular single-symbol decodable STBC that are derived from CLOD and achieved a rate 1 for 3 transmit antennas. Khan and Rajan [7] proposed rate 1 coordinate interleaved STBC constructed from CLOD for 4 transmit antennas. STBC designs for 3 and 4 transmit antennas were proposed in [5] which achieves a symbol transmission rate of 1 under both configurations. But the designs are quasi-orthogonal. The work in [9] proposed STBC designs for 4 transmit antennas that achieves a rate of 1.5 and 1.625. However, it is quasi-orthogonal causing the transmitted symbols to interfere with each other and no analysis on diversity or decoding complexity is presented. Finally, the work in [4] proposed 4×4 STBC that achieves a symbol transmission rate of 2. But here again, the design is quasi-orthogonal and has a very high decoding complexity that grows exponentially with the constellation size as $\mathcal{O}(2|Q|^7)$.

Table I shows a concise summary of the state-of-the-art STBCs with the proposed Jagannath designs. The table demonstrates the highest achieved . The table demonstrates highest symbol transmission rate of 2 with both configurations of Jagannath design while being orthogonal, fully diverse as well as very low decoding complexity. The quasi-orthogonality and non-orthogonality with other schemes will significantly cause symbol interference as quantified by the off-diagonal entries of the corresponding coding schemes. The table showcases the trade-offs where some codes are fully orthogonal with linear decoding at the receiver but does not offer high symbol transmission rate or delay efficiency.

In this work, we aim at achieving a symbol transmission rate of 2 while being orthogonal, fully diverse, and a very low decoding complexity for 3 and 4 transmit antennas. This work, therefore, opens door to very high rate code design that satisfies several design criteria. Therefore, the main contributions of this work are as follows:

- We introduce *Jagannath designs* of 4×3 and 4×4 configurations which achieves the highest symbol transmission rate.

TABLE I
COMPARISON OF RATE AND DELAY OF KNOWN STBCS WITH THE PROPOSED DESIGNS

Design	TX antennas	Rate	Delay	Comments
Jagannath 4×3	**3**	**2**	**4**	Orthogonal, Full Diversity, and joint double symbol decodable with complexity of $\mathcal{O}(\|\mathbf{Q}\|)$
Jagannath 4×4	**4**	**2**	**4**	Orthogonal, Full Diversity, and joint double symbol decodable with complexity of $\mathcal{O}(\|\mathbf{Q}\|)$
4×4 [9]	4	2	4	Quasi-orthogonal, Exponential decoding complexity of $\mathcal{O}(\mathbf{2}\|\mathbf{Q}\|^{\mathbf{7}})$
ACIOD [6]	3	1	4	Orthogonal, Full Diversity, and Linear decoding at receiver
CIOD [6]	4	1	4	Orthogonal, Full Diversity, and Linear decoding at receiver
Jafarkhani [21]	4	1	4	Quasi-orthogonal, Partial Diversity, and Linear decoding at receiver
Ozbek.et.al [23]	3	1	4	Non-orthogonal, Partial Diversity, and Zero-Forcing decoder
Tarokh et. al [11]	3	3/4	4	Orthogonal, Full Diversity, and Linear processing at the receiver
Tarokh et. al [11]	4	3/4	4	Orthogonal, Full Diversity, and Linear processing at the receiver
Grover et. al [15]	4	1	8	Quasi-orthogonal, Full Diversity, and joint double symbol decodable
Ganesan et. al [17]	3	3/4	4	Orthogonal, unknown diversity, and linear processing at the receiver
Ganesan et. al [17]	4	3/4	4	Orthogonal, unknown diversity, and linear processing at the receiver
Tirkkonen et. al [22]	3	1	4	Non-orthogonal, Full Diversity, and linear decoding based on iterative interference cancellation
Tirkkonen et. al [22]	4	1	4	Non-orthogonal, Full Diversity, and linear decoding based on iterative interference cancellation

- An elaborate insight into the code construction and the decoding methodology is presented. We present a very low decoding complexity procedure.
- The code design based on diversity product maximization is elaborated. We present the diversity order analysis and also prove the orthogonality of the proposed designs.
- The proposed designs are compared and contrasted with several state of the art schemes in terms of symbol transmission rate, delay, and other design criteria in Table I.
- We show extensive simulation studies to demonstrate the performance of the novel Jagannath designs with state of the art schemes for 3 and 4 transmit antennas [6], [21].

III. CODE CONSTRUCTION AND CHANNEL MODEL

Consider a MIMO system with $\mathcal{N}_t$ transmit antennas, $\mathcal{N}_r$ receive antennas, and let $\mathcal{T}$ be the number of time slots (epochs/channel uses) over which the codeword is transmitted. The information symbols $q_i \in \mathcal{Q}$ are drawn from a complex signal constellation such as quadrature amplitude modulation (QAM). We define the code rate as the number of symbols transmitted per epoch and is expressed as,

$$\mathcal{R} = \frac{K}{\mathcal{T}} \text{ symbols/s} \tag{3}$$

where K is the number of information symbols. The novel codeword designs introduced in [19] are formed by GCOD submatrices with codeword elements $c^j_{mn} = q_m \sin\alpha_j - q^*_n \cos\alpha_j$ where $n = m+1$ and the angles (α_j for $j = 1, 2$) maintains the transmit energy.

Jagannath 4 × 3 definition: A rate 2 STBC $\mathbf{C}_3$ of size $4{\times}3$ designed using information symbols $q_1, q_2, q_3, q_4, q_5, q_6, q_7, q_8$ is constructed as in equation (4) such that it is transmitted over $\mathcal{N}_t = 3$ transmit antennas and $\mathcal{T} = 4$ time slots.

$$\mathbf{C}_3 = \begin{bmatrix} 0 & c^1_{12} & c^2_{34} \\ 0 & -{c^2_{34}}^* & {c^1_{12}}^* \\ c^1_{56} & c^2_{78} & 0 \\ -{c^2_{78}}^* & {c^1_{56}}^* & 0 \end{bmatrix} \tag{4}$$

Jagannath 4 × 4 definition: A rate 2 STBC $\mathbf{C}_4$ of size 4×4 formed using the fundamental information symbols $q_1, q_2, q_3, q_4, q_5, q_6, q_7, q_8$ can be obtained as in equation (5) and requires transmission over $\mathcal{N}_t = 4$ transmit antennas and $\mathcal{T} = 4$ time slots.

$$\mathbf{C}_4 = \begin{bmatrix} c^1_{12} & c^2_{34} & 0 & 0 \\ -{c^2_{34}}^* & {c^1_{12}}^* & 0 & 0 \\ 0 & 0 & c^1_{56} & c^2_{78} \\ 0 & 0 & -{c^2_{78}}^* & {c^1_{56}}^* \end{bmatrix} \tag{5}$$

The orthogonality of the codeword matrices is significant in determining how each symbols transmitted from the different transmit antennas interfere with each other and effects their separability. The orthogonality property of both designs where the columns which correspond to transmission from the different antennas are orthogonal to each other can be shown as,

$$\mathbf{C}^H_3\mathbf{C}_3 = \text{diag}\Big\{ \left(\left|c^1_{12}\right|^2 + \left|c^2_{34}\right|^2\right), \left(\left|c^1_{12}\right|^2 + \left|c^2_{34}\right|^2 + \left|c^1_{56}\right|^2 + \left|c^2_{78}\right|^2\right), \left(\left|c^1_{56}\right|^2 + \left|c^2_{78}\right|^2\right) \Big\} \tag{6}$$

$$\mathbf{C}^H_4\mathbf{C}_4 = \text{diag}\Big\{ \left(\left|c^1_{12}\right|^2 + \left|c^2_{34}\right|^2\right), \left(\left|c^1_{12}\right|^2 + \left|c^2_{34}\right|^2\right) \left(\left|c^1_{56}\right|^2 + \left|c^2_{78}\right|^2\right), \left(\left|c^1_{56}\right|^2 + \left|c^2_{78}\right|^2\right) \Big\} \tag{7}$$

where $\text{diag}\Big\{x_1, x_2, \cdots, x_k\Big\}$ is the $k \times k$ diagonal matrix with diagonal elements $x_1, x_2, \cdots, x_k$ and zeros in the off-diagonal entries. Hence, both configurations of the proposed schemes satisfy mutual column orthogonality. Similar to the GCOD design, the correlation matrix is diagonal with unequal norms.

Remark 1: *Rate 2 codewords $\mathbf{C}_3$ and $\mathbf{C}_4$ comprising of GCOD submatrices and eight complex information symbols satisfy mutual column orthogonality.*

The received signal can be expressed as

$$\mathbf{Y} = \sqrt{\frac{\rho}{\mathcal{N}_t}}\mathbf{C}_i\mathbf{H} + \mathbf{N} \tag{8}$$

where $\mathbf{Y}$ is the $4 \times \mathcal{N}_r$ received signal, ρ is the signal-to-noise ratio (SNR) at each receive antenna, $\mathbf{C}_i, i = 3, 4$ is the transmitted codeword, $\mathbf{H}$ is the flat-fading Rayleigh channel matrix of size $4 \times \mathcal{N}_r$, and $\mathbf{N}$ is the $4 \times \mathcal{N}_r$ noise matrix. The samples of $\mathbf{H}$ and $\mathbf{N}$ are independent identically distributed (iid) zero-mean complex Gaussian random variables with variance 1.

IV. Diversity Order Analysis

STBC must be designed to ensure maximum transmit diversity and low decoding complexity. It is well known that to achieve a maximum diversity order, the difference matrix between any two distinct codewords must be of full-rank [24]. If the rank of the difference codeword matrix is denoted by r, then the minimum value of r over all possible codeword pairs is called the diversity order of the STBC [15]. The full diversity analyses of the novel 4×3 and 4×4 designs are presented in this section.

Let $\mathbf{C}_3$ and $\tilde{\mathbf{C}}_3$ be two distinct (*i.e.,* $\mathbf{D}_3 = \mathbf{C}_3 - \tilde{\mathbf{C}}_3 \neq 0$) 4×3 codewords. To evaluate the diversity order, we evaluate the eigenvalues of $\mathbf{D}_3^H \mathbf{D}_3$. Set the codeword element $\delta_{mn}^j = (q_m - \tilde{q}_m) \sin \alpha_j - (q_n^* - \tilde{q}_n^*) \cos \alpha_j$ with $n = m + 1$. Now the difference matrix $\mathbf{D}_3$ is given by,

$$\mathbf{D}_3 = \mathbf{C}_3 - \tilde{\mathbf{C}}_3 = \begin{bmatrix} 0 & \delta_{12}^1 & \delta_{34}^2 \\ 0 & -\delta_{34}^{2\,*} & \delta_{12}^{1\,*} \\ \delta_{56}^1 & \delta_{78}^2 & 0 \\ -\delta_{78}^{2\,*} & \delta_{56}^{1\,*} & 0 \end{bmatrix}. \quad (9)$$

The diagonal nature of the matrix $\mathbf{D}_3^H \mathbf{D}_3$ as shown in equation (10) eases the computation of eigenvalues.

$$\mathbf{D}_3^H \mathbf{D}_3 = \mathrm{diag}\Big\{ \left(\left|\delta_{12}^1\right|^2 + \left|\delta_{34}^2\right|^2 \right), \left(\left|\delta_{12}^1\right|^2 + \left|\delta_{34}^2\right|^2 + \left|\delta_{56}^1\right|^2 + \left|\delta_{78}^2\right|^2 \right), \left(\left|\delta_{56}^1\right|^2 + \left|\delta_{78}^2\right|^2 \right) \Big\} \quad (10)$$

The eigenvalues of $\mathbf{D}_3^H \mathbf{D}_3$ are $\Big\{ \left(\left|\delta_{12}^1\right|^2 + \left|\delta_{34}^2\right|^2 \right), \left(\left|\delta_{12}^1\right|^2 + \left|\delta_{34}^2\right|^2 + \left|\delta_{56}^1\right|^2 + \left|\delta_{78}^2\right|^2 \right), \left(\left|\delta_{56}^1\right|^2 + \left|\delta_{78}^2\right|^2 \right) \Big\}$. It can be easily verified that each of the eigenvalues are the sum of squares and are non-zero scalars. *Hence, the matrix $\mathbf{D}_3^H \mathbf{D}_3$ is full-rank with diversity order* $\omega_{\{4\times3\}} = 3$.

It easily follows that the determinant of matrix $\mathbf{D}_3^H \mathbf{D}_3$ is non-zero from equation (11).

$$\det \left[\mathbf{D}_3^H \mathbf{D}_3 \right] = \left(|\delta_{12}|^2 + |\delta_{34}|^2 \right) \left(|\delta_{12}|^2 + |\delta_{34}|^2 + |\delta_{56}|^2 + |\delta_{78}|^2 \right) \left(|\delta_{56}|^2 + |\delta_{78}|^2 \right) \neq 0 \quad (11)$$

Similarly, let the two distinct 4×4 codewords be $\mathbf{C}_4$, $\tilde{\mathbf{C}}_4$, the difference matrix $\mathbf{D}_4 = \mathbf{C}_4 - \tilde{\mathbf{C}}_4$, and the codeword element δ_{mn}^j (same as in 4×3 case). The difference matrix $\mathbf{D}_4$ can be obtained as,

$$\mathbf{D}_4 = \mathbf{C}_4 - \tilde{\mathbf{C}}_4 = \begin{bmatrix} \delta_{12}^1 & \delta_{34}^2 & 0 & 0 \\ -\delta_{34}^{2\,*} & \delta_{12}^{1\,*} & 0 & 0 \\ 0 & 0 & \delta_{56}^1 & \delta_{78}^2 \\ 0 & 0 & -\delta_{78}^{2\,*} & \delta_{56}^{1\,*} \end{bmatrix} \quad (12)$$

with the product matrix $\mathbf{D}_4^H \mathbf{D}_4$ as

$$\mathbf{D}_4^H \mathbf{D}_4 = \begin{bmatrix} \left(|\delta_{12}|^2 + |\delta_{34}|^2 \right) \mathbf{I}_2 & \mathbf{0}_2 \\ \mathbf{0}_2 & \left(|\delta_{56}|^2 + |\delta_{78}|^2 \right) \mathbf{I}_2 \end{bmatrix}. \quad (13)$$

The block diagonal product matrix yields eigenvalues $\Big\{ \left(|\delta_{12}|^2 + |\delta_{34}|^2 \right), \left(|\delta_{56}|^2 + |\delta_{78}|^2 \right) \Big\}$ with multiplicity of 2. Therefore, the product matrix is full-rank with diversity order $\omega_{\{4\times4\}} = 4$. The determinant of product matrix can be expressed as

$$det \left[\mathbf{D}_4^H \mathbf{D}_4 \right] = \left(|\delta_{12}|^2 + |\delta_{34}|^2 \right)^2 \left(|\delta_{56}|^2 + |\delta_{78}|^2 \right)^2 \neq 0. \quad (14)$$

It must be noted that the full diversity is achieved when the angles satisfy $\alpha_1 + \alpha_2 = 90°$.

Once full-diversity is achieved, the next objective is to maximize the diversity product/coding gain of the codewords. Further, the diversity order of the STBC dictates the slope of error-rate curves in the high SNR region (*i.e.,* the steepness of the error-rate curve in the high SNR region).

Remark 2: *Rate 2 codewords $\mathbf{C}_3$ and $\mathbf{C}_4$ offer full rank - full diversity iff angles satisfy $\alpha_1 + \alpha_2 = 90°$ to yield non-zero eigenvalues.*

V. Coding Gain/Diversity Product Maximization

For a fully diverse STBC, the coding gain (diversity product) is expressed in terms of minimum determinant as

$$\zeta = (\gamma_{min})^{1/\mathcal{N}_t} \quad (15)$$

Since the full diversity of both designs has been explicitly stated in the previous section, the subsequent step is to obtain the angle that will maximize the coding gain ζ in both cases.

A. *Maximize Coding Gain of* 4×3 *design*

The minimum determinant of the 4×3 design can be expressed as

$$\gamma_{min} = \min_{\mathbf{D}_3 \neq 0} det \left[\mathbf{D}_3^H \mathbf{D}_3 \right]. \quad (16)$$

The codeword $\mathbf{C}_3$ must be designed to maximize the minimum determinant such that the coding gain can be expressed as

$$\begin{aligned} \zeta_3 &= \max \left(\gamma_{min} \right)^{1/\mathcal{N}_t} \\ &= \max \min_{\mathbf{D}_3 \neq 0} \left\{ det \left[\mathbf{D}_3^H \mathbf{D}_3 \right] \right\}^{1/\mathcal{N}_t} \\ &= \max \min_{\mathbf{D}_3 \neq 0} \Big\{ \left(|\delta_{12}|^2 + |\delta_{34}|^2 \right) . \\ &\quad \left(|\delta_{12}|^2 + |\delta_{34}|^2 + |\delta_{56}|^2 + |\delta_{78}|^2 \right) \left(|\delta_{56}|^2 + |\delta_{78}|^2 \right) \Big\}^{1/\mathcal{N}_t} \\ &= \max \min_{\mathbf{D}_3 \neq 0} \left(|\delta_{12}|^2 + |\delta_{34}|^2 \right)^{1/\mathcal{N}_t} \times \\ &\quad \max \min_{\mathbf{D}_3 \neq 0} \left(|\delta_{12}|^2 + |\delta_{34}|^2 + |\delta_{56}|^2 + |\delta_{78}|^2 \right)^{1/\mathcal{N}_t} \times \\ &\quad \max \min_{\mathbf{D}_3 \neq 0} \left(|\delta_{56}|^2 + |\delta_{78}|^2 \right)^{1/\mathcal{N}_t} . \end{aligned} \quad (17)$$

The optimum angle corresponds to the one that maximizes the minimum determinant and can be obtained as

$$\begin{aligned}\alpha^3_{opt} &= \arg \zeta_3 \\ &= \text{argmax} \min_{\mathbf{D}_3 \neq 0} \left(|\delta_{12}|^2 + |\delta_{34}|^2\right)^{1/\mathcal{N}_t} \times \\ &\quad \text{argmax} \min_{\mathbf{D}_3 \neq 0} \left(|\delta_{12}|^2 + |\delta_{34}|^2 + |\delta_{56}|^2 + |\delta_{78}|^2\right)^{1/\mathcal{N}_t} \times \\ &\quad \text{argmax} \min_{\mathbf{D}_3 \neq 0} \left(|\delta_{56}|^2 + |\delta_{78}|^2\right)^{1/\mathcal{N}_t} \\ &= \text{argmax}\, opt_1 \times \text{argmax}\, opt_2 \times \text{argmax}\, opt_3\end{aligned} \tag{18}$$

The optimization problem in equation (18) is the product of three equivalent subproblems. It can be easily verified that the optimizations opt_1 and opt_3 are the same. The angle that maximizes opt_1 will essentially maximize opt_2 and opt_3.

$$\begin{aligned}\alpha^3_{opt} &= \underset{\alpha_i}{\text{argmax}}\, opt_1 \\ &= \underset{\alpha_i}{\text{argmax}} \min_{\mathbf{D}_3 \neq 0} \left(|\delta_{12}|^2 + |\delta_{34}|^2\right)^{1/\mathcal{N}_t} \\ &= \underset{\alpha_1}{\text{argmax}} \min_{\mathbf{D}_3 \neq 0} \Big\{ |\delta_1|^2 \sin^2 \alpha_1 + |\delta_2|^2 \cos^2 \alpha_1 - \\ &\quad 2|\delta_1 \delta_2| \sin \alpha_1 \cos \alpha_1 + |\delta_3|^2 \sin^2 \alpha_2 + |\delta_4|^2 \cos^2 \alpha_2 - \\ &\quad 2|\delta_3 \delta_4| \sin \alpha_2 \cos \alpha_2 \Big\}^{1/\mathcal{N}_t} \\ &= \underset{\alpha_1}{\text{argmax}} \min_{\mathbf{D}_3 \neq 0} \Big\{ |\delta_1|^2 \sin^2 \alpha_1 + |\delta_2|^2 \cos^2 \alpha_1 - \\ &\quad |\delta_1 \delta_2| \sin 2\alpha_1 + |\delta_3|^2 \cos^2 \alpha_1 + |\delta_4|^2 \sin^2 \alpha_1 - \\ &\quad |\delta_3 \delta_4| \sin 2\alpha_1 \Big\}^{1/\mathcal{N}_t} \\ &= \underset{\alpha_1}{\text{argmax}} \min_{\mathbf{D}_3 \neq 0} \Gamma(\alpha_1)\end{aligned} \tag{19}$$

The optimum angle can now be obtained by setting the first-order derivative of $\Gamma(\alpha_1)$ with respect to α_1 to zero as shown below.

$$\begin{aligned}&\frac{d\,\Gamma(\alpha_1)}{d\alpha_1} = 0 \implies \\ &\quad \left(|\delta_1|^2 - |\delta_2|^2 + |\delta_4|^2 - |\delta_3|^2\right) \sin 2\alpha_1 = \\ &\quad \left(2|\delta_1\delta_2| + 2|\delta_3\delta_4|\right) \cos 2\alpha_1 \\ &\alpha^3_{opt} = \alpha_1 = 0.5 \arctan \frac{2|\delta_1\delta_2| + 2|\delta_3\delta_4|}{|\delta_1|^2 - |\delta_2|^2 + |\delta_4|^2 - |\delta_3|^2}\end{aligned} \tag{20}$$

The expression in (20) gives the optimum angle α^3_{opt} that maximizes the coding gain.

*B. **Maximize Coding Gain of*** 4×4 ***design***

Without loss of rigor, the same method can be applied to derive the optimum angle that maximizes the coding gain of codeword $\mathbf{C}_4$. The minimum determinant of the codeword is given by

$$\gamma_{min} = \min_{\mathbf{C}_4 \neq \tilde{\mathbf{C}}_4} det\left[\mathbf{D}_4^H \mathbf{D}_4\right] \tag{21}$$

The coding gain is given by

$$\begin{aligned}\zeta_4 &= \max \left(\gamma_{min}\right)^{1/\mathcal{N}_t} \\ &= \max \min_{\mathbf{D}_4 \neq 0} \left\{ det\left[\mathbf{D}_4^H \mathbf{D}_4\right] \right\}^{1/\mathcal{N}_t} \\ &= \max \min_{\mathbf{D}_4 \neq 0} \left\{ \left(|\delta_{12}|^2 + |\delta_{34}|^2\right)^2 \left(|\delta_{56}|^2 + |\delta_{78}|^2\right)^2 \right\}^{1/\mathcal{N}_t} \\ &= \max \min_{\mathbf{D}_4 \neq 0} \left(|\delta_{12}|^2 + |\delta_{34}|^2\right)^{2/\mathcal{N}_t} \times \\ &\quad \max \min_{\mathbf{D}_4 \neq 0} \left(|\delta_{56}|^2 + |\delta_{78}|^2\right)^{2/\mathcal{N}_t}\end{aligned} \tag{22}$$

The optimum angle corresponds to the one that maximizes the coding gain ζ_4, i.e.,

$$\begin{aligned}\alpha^4_{opt} &= \arg \zeta_4 \\ &= \text{argmax}\, opt_1^2 \times \text{argmax}\, opt_3^2 \\ &\equiv \underset{\alpha_1}{\text{argmax}}\, opt_1 \\ &= 0.5 \arctan \frac{2|\delta_1\delta_2| + 2|\delta_3\delta_4|}{|\delta_1|^2 - |\delta_2|^2 + |\delta_4|^2 - |\delta_3|^2}\end{aligned} \tag{23}$$

Thus, yielding the same optimum angle expression for both designs.

Remark 3: *Rate-2 codewords* $\mathbf{C}_3$ *and* $\mathbf{C}_4$ *achieve maximum coding gain when the optimum angle is* $0.5 \arctan\left(\frac{2|\delta_1\delta_2| + 2|\delta_3\delta_4|}{|\delta_1|^2 - |\delta_2|^2 + |\delta_4|^2 - |\delta_3|^2}\right)$.

VI. Decoding Methodology

In this section, we will present the complete conditional ML decoding methodology of both schemes. To achieve this, let us consider a $3 \times \mathcal{N}_r$ coherent MIMO system with the received signal model following the same expression as in equation (8) where the codeword $\mathbf{C}_i = \mathbf{C}_3$. Each column i of the channel matrix corresponds to the channel between the 3 transmitter antennas and i^{th} receive antenna such that $\mathbf{H} = \left[\mathbf{h}_1, \mathbf{h}_2, \cdots, \mathbf{h}_{\mathcal{N}_r}\right]$. Here, $\mathbf{h}_i = \left[h_{1i}, h_{2i}, h_{3i}\right]^T$ is a 3×1 channel vector.

The signals received at the i^{th} receive antenna at time slots $\{1, 2, 3, 4\}$ are written in equivalent virtual channel matrix (EVCM) form as,

$$\begin{bmatrix} y_i^1 \\ y_i^{2*} \end{bmatrix} = \sqrt{\frac{\rho}{3}} \begin{bmatrix} h_{2i} & h_{3i} \\ h_{3i}^* & -h_{2i}^* \end{bmatrix} \begin{bmatrix} q_1 \sin \alpha_1 - q_2^* \cos \alpha_1 \\ q_3 \sin \alpha_2 - q_4^* \cos \alpha_2 \end{bmatrix} + \begin{bmatrix} n_i^1 \\ n_i^{2*} \end{bmatrix} \tag{24}$$

$$\begin{bmatrix} y_i^3 \\ y_i^{4*} \end{bmatrix} = \sqrt{\frac{\rho}{3}} \begin{bmatrix} h_{1i} & h_{2i} \\ h_{2i}^* & -h_{1i}^* \end{bmatrix} \begin{bmatrix} q_5 \sin \alpha_1 - q_6^* \cos \alpha_1 \\ q_7 \sin \alpha_2 - q_8^* \cos \alpha_2 \end{bmatrix} + \begin{bmatrix} n_i^3 \\ n_i^{4*} \end{bmatrix} \tag{25}$$

$$\begin{bmatrix} y_i^1 \\ y_i^{2*} \\ y_i^3 \\ y_i^{4*} \end{bmatrix} = \sqrt{\frac{\rho}{4}} \begin{bmatrix} h_{1i} & h_{2i} & 0 & 0 \\ h_{2i}^* & -h_{1i}^* & 0 & 0 \\ 0 & 0 & h_{3i} & h_{4i} \\ 0 & 0 & h_{4i}^* & -h_{3i}^* \end{bmatrix} \begin{bmatrix} q_1 \sin\alpha_1 - q_2^* \cos\alpha_1 \\ q_3 \sin\alpha_2 - q_4^* \cos\alpha_2 \\ q_5 \sin\alpha_1 - q_6^* \cos\alpha_1 \\ q_7 \sin\alpha_2 - q_8^* \cos\alpha_2 \end{bmatrix} + \begin{bmatrix} n_i^1 \\ n_i^{2*} \\ n_i^3 \\ n_i^{4*} \end{bmatrix} \tag{33}$$

$$\begin{bmatrix} z_i^1 \\ z_i^2 \\ z_i^3 \\ z_i^4 \end{bmatrix} = \sqrt{\frac{\rho}{4}} \begin{bmatrix} (|h_{1i}|^2 + |h_{2i}|^2)\mathbf{I}_2 & \mathbf{0}_2 \\ \mathbf{0}_2 & (|h_{3i}|^2 + |h_{4i}|^2)\mathbf{I}_2 \end{bmatrix} \begin{bmatrix} q_1 \sin\alpha_1 - q_2^* \cos\alpha_1 \\ q_3 \sin\alpha_2 - q_4^* \cos\alpha_2 \\ q_5 \sin\alpha_1 - q_6^* \cos\alpha_1 \\ q_7 \sin\alpha_2 - q_8^* \cos\alpha_2 \end{bmatrix} + \begin{bmatrix} g_i^1 \\ g_i^2 \\ g_i^3 \\ g_i^4 \end{bmatrix} \tag{34}$$

Channel equalization would result in

$$\begin{bmatrix} z_i^1 \\ z_i^2 \end{bmatrix} = \sqrt{\frac{\rho}{3}} (|h_{2i}|^2 + |h_{3i}|^2) \begin{bmatrix} q_1 \sin\alpha_1 - q_2^* \cos\alpha_1 \\ q_3 \sin\alpha_2 - q_4^* \cos\alpha_2 \end{bmatrix} + \begin{bmatrix} g_i^1 \\ g_i^{2*} \end{bmatrix} \tag{26}$$

$$\begin{bmatrix} z_i^3 \\ z_i^4 \end{bmatrix} = \sqrt{\frac{\rho}{3}} (|h_{1i}|^2 + |h_{2i}|^2) \begin{bmatrix} q_5 \sin\alpha_1 - q_6^* \cos\alpha_1 \\ q_7 \sin\alpha_2 - q_8^* \cos\alpha_2 \end{bmatrix} + \begin{bmatrix} g_i^3 \\ g_i^{4*} \end{bmatrix} \tag{27}$$

The sufficient statistic that yields the symbols $q_m, q_{m+1}, \forall m = \{1, 3, 5, 7\}$ is

$$\beta^t = \frac{1}{\mathcal{N}_r} \sum_i z_i^t. \tag{28}$$

where t refers to the time slot. The index m takes values such that

$$m = \begin{cases} 1 & \text{if } t = 1, \\ 3 & \text{if } t = 2, \\ 5 & \text{if } t = 3, \\ 7 & \text{if } t = 4. \end{cases} \tag{29}$$

The intermediate signal to obtain the conditional estimate of symbol pairs q_m, q_{m+1} is

$$\tilde{\beta}^t = \beta^t - \sqrt{\frac{\rho}{3}} \Psi_n [-q_{2t}^* \cos\alpha_j] \tag{30}$$

such that $\Psi_1 = \sum_i (|h_{2i}|^2 + |h_{3i}|^2)$ when $t = \{1, 2\}$ and $\Psi_2 = \sum_i (|h_{1i}|^2 + |h_{2i}|^2)$ when $t = 3, 4$ respectively. The angle index j takes values as

$$j = \begin{cases} 1 & \text{if } t = \{1, 3\} \text{ and} \\ 2 & \text{if } t = \{2, 4\}. \end{cases} \tag{31}$$

For all the QAM constellation points $q_{2t} \in \mathcal{Q}$, the symbol $q_{2t-1|2t}$ that minimizes the following cost function corresponds to the correctly decoded entity,

$$\tau^t = \left| \beta^t - \sqrt{\frac{\rho}{3}} \Psi_n \left[q_{2t-1|2t} \sin\alpha_j - q_{2t}^* \cos\alpha_j \right] \right|^2 \tag{32}$$

This concludes the formal presentation of the decoding procedure for a coherent MIMO system that use the Jagannath 4×3 design.

The decoding methodology for the 4×4 Jagannath STBC can be derived in a similar manner. This will correspond to a $4 \times \mathcal{N}_r$ coherent MIMO setup which will yield the EVCM representation as in equation (33). Applying channel equalization would yield expression (34).

The sufficient statistics β^t and intermediate signal $\tilde{\beta}^t$ can be obtained in a manner similar to the $\mathbf{C}_3$ case with the final cost function of the form

$$\tau^t = \left| \beta^t - \sqrt{\frac{\rho}{4}} \Psi_m \left[q_{2t-1|2t} \sin\alpha_j - q_{2t}^* \cos\alpha_j \right] \right|^2 \tag{35}$$

where $\Psi_1 = \sum_i (|h_{1i}|^2 + |h_{2i}|^2)$ and $\Psi_2 = \sum_i (|h_{3i}|^2 + |h_{4i}|^2)$. The decoding complexities of both designs are $\mathcal{O}(|Q|)$ which is significantly lower compared to the exponential complexity of [9] as shown in Table I.

VII. Simulation Studies

In this section, we present the simulation studies of the proposed designs. The multiantenna transmission strategy and the corresponding decoding methodology was tested extensively under varying SNR.

In Fig.1, the receiver diversity is analyzed, i.e., the advantage of having multiple receiver antennas is studied for the Jagannath 4×3 STBC. The number of receiver antennas are varied from $\mathcal{N}_r = 1$ to $\mathcal{N}_r = \mathcal{N}_t$ under spectral efficiencies of 4 and 8 bits/channel use (bpcu). The appropriate modulation orders (Q) are used to achieve the specific spectral efficiencies, i.e., 4 bpcu is achieved with QAM-4 for rate-2 STBC and QAM-16 for rate-1 codes. The same way 8 bpcu can be attained by QAM-16 for rate-2 STBC. The receiver diversity gain is estimated by comparing the BER performance of $\mathcal{N}_r = 3$ to that of $\mathcal{N}_r = 1$ under both spectral efficiency scenarios. The antenna diversity gain amounts to ∼8 dB and ∼7 dB for the 4 bpcu and 8 bpcu scenarios respectively.

Similarly, the antenna diversity gain for the 4×4 Jagannath STBC was analyzed under 4 bpcu and 8 bpcu scenarios. Here, again the number of receiver antennas are increased from $\mathcal{N}_r = 1$ to $\mathcal{N}_r = \mathcal{N}_t$ as shown in Fig.2. The antenna diversity gain of the $\mathcal{N}_r = \mathcal{N}_t$ with respect to $\mathcal{N}_r = 1$ corresponds to 10 dB under both 4 bpcu and 8 bpcu scenarios. The analysis of both Figures 1 and 2 considers the optimum angle as derived in section V which is $\alpha_1 = 63.4°$ and $\alpha_1 = 76°$ for 4 bpcu and 8 bpcu respectively. In Fig.3, the performance of Jagannath 4×3 STBC is compared to 4×3 design proposed in [6] (Asymmetric Coordinate Interleaved Orthogonal Design - ACIOD) at 4 bpcu. The number of receiver antennas is same as that of the transmitter antennas. Fig.3 demonstrates a

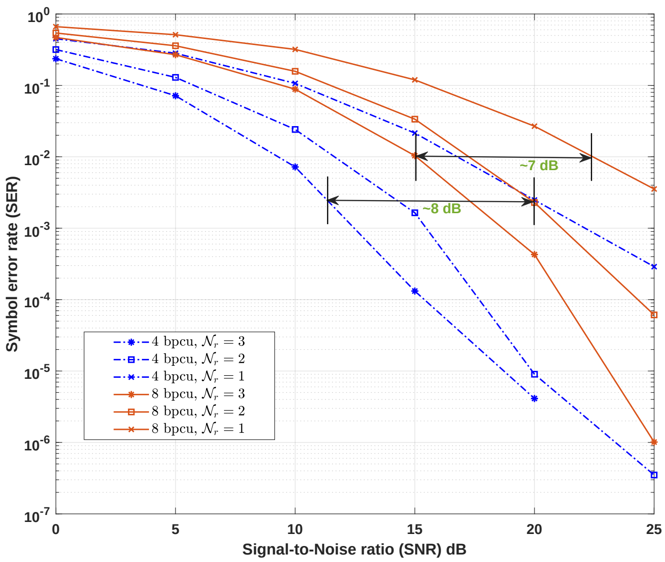

Fig. 1. Receiver diversity gain of 4×3 Jagannath STBC

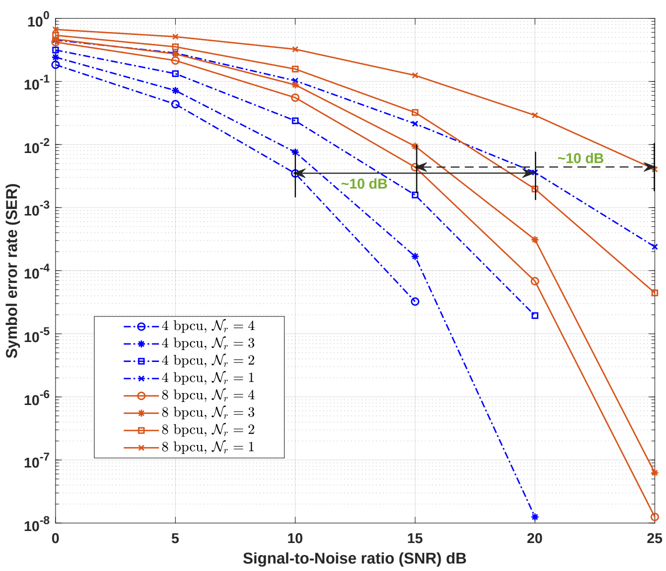

Fig. 2. Receiver diversity gain of 4×4 Jagannath STBC

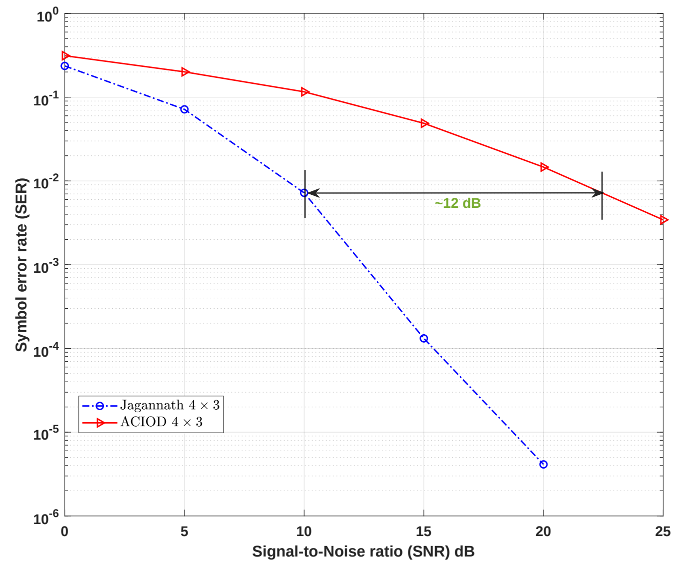

Fig. 3. Jagannath 4×3 STBC versus ACIOD [6] at 4 bpcu

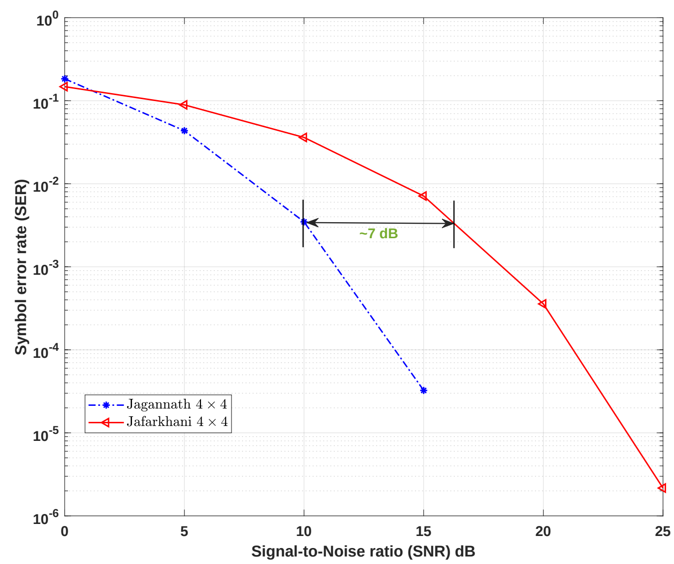

Fig. 4. Jagannath 4×4 STBC versus Jafarkhani [21] at 4 bpcu

$\sim$12 dB SNR gain with Jagannath 4×3 as compared to that of ACIOD. Similarly, the performance of 4×4 transmission configuration at 4 bpcu is demonstrated in Fig.4. Jagannath scheme outperforms Jafarkhani STBC [21] by $\sim$7 dB. This can be attributed to the full diversity achieved with Jagannath scheme. Further, the quasi-orthogonality of Jafarkhani design introduces cross-terms (i.e., symbol interferences) which is a significant performance deterrent.

VIII. Conclusion

We proposed rate-2 orthogonal STBCs for three and four transmit antennas that are orthogonal and achieves full diversity. The achieved rate and delay of Jagannath designs were compared and contrasted with several state-of-the-art schemes in Table I. It has been shown that the proposed schemes achieves the highest symbol transmission rate of 2 and with a delay of 4.

The orthogonality of the proposed schemes were elaborated and the diversity orders were established in this work. The design methodology followed a coding gain maximization approach. The decoding methodology for both transmission schemes were derived and presented for a $4 \times \mathcal{N}_r$ MIMO system. The receiver antenna diversity gain for both schemes at varying spectral efficiencies were demonstrated. The Jagannath 4×3 scheme showed a $\sim$12 dB SNR gain in contrast to the ACIOD approach while the 4×4 design outperformed Jafarkhani scheme by $\sim$7 dB. The SNR gains depict the communication range extension or equivalent transmit power saving with the proposed schemes. Hence, motivating their adoption in tactical and commercial MIMO systems to double the throughput with the proposed rate-2 schemes.

We believe these designs will open doors for future high data rate applications. Future work will entail higher antenna configurations for massive MIMO systems.

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