**Intensity-Only Localization and Inverse Scattering**
**URSI - CNC/USNC**

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This work reports a generalization of the time-reversal MUSIC method for localization and imaging of scatterers that, unlike previous work in this area [see (E.A. Marengo and F.K. Gruber, EURASIP J. Advances in Signal Processing, doi:10.1155/2007/17342, 16 pages, 2007) and the references therein], employs *intensity-only field data*, i.e., *without field phase information* (e.g., optical applications). The data are the intensities of the scattered fields, gathered at a receiver aperture or array, corresponding to a set of different incident fields, corresponding to different excitations or "experiments" which are labeled as $t = 1, 2, \cdots, T$, and the problem under study is the estimation of the unknown support of the scatterers from the data. Under this limited data, the mapping from the sources induced in the scatterers to the data is nonlinear. Yet by considering not the induced sources but instead another source function, which still carries the information about the scatterer support, the mapping becomes artificially linearized so that super-resolution signal-subspace imaging approaches become applicable. The results apply within exact scattering theory. In the remainder of this abstract, particular attention is given to the special version of the theory for $M$ point-like scatterers whose dimensions are small relative to the probing wavelength(s) so that the unknown support is the set of scatterer positions $\mathbf{X}_m, m = 1, 2, \cdots, M$; but the respective generalization for extended scatterers is already available and will be given at the talk. Consider a homogeneous background medium wherein wave radiation is governed, for scalar sources and fields, $\rho_t$ and $\psi_t^{\rm inc}$, respectively, by the scalar Helmholtz equation $(\nabla^2 + k^2)\psi_t^{\rm inc}(\mathbf{r}) = \rho_t(\mathbf{r})$ where $\mathbf{r} \in R^3$ denotes position, the frequency ($\omega$) dependence has been suppressed, and $k = \omega/c$ (where $c$ is the speed of light) is the free space wavenumber. The label $t$ denotes the $t$'th experiment. For example, if the sources $\rho_t$ correspond to an array of $N_T$ point sources located at positions $\mathbf{Y}_n, n = 1, 2, \cdots, N_T$ and having complex-valued excitation strengths $\alpha_{n,t} \in C, t = 1, 2, \cdots, T; n = 1, 2, \cdots, N_T$, then the incident fields $\psi_t^{\rm inc}(\mathbf{r}) = \sum_{n=1}^{N_T} \alpha_{n,t} G(\mathbf{r}, \mathbf{Y}_n)$ where $G$ denotes the Green function for the relevant boundary conditions. The intensity $I_t(\mathbf{r})$ of the scattered field $\psi_t^s(\mathbf{r})$ is given by $I_t(\mathbf{r}) = |\psi_t^{\rm s}(\mathbf{r})|^2 = \int d\mathbf{r}' \int d\mathbf{r}'' \mathcal{H}(\mathbf{r}; \mathbf{r}', \mathbf{r}'') \mathcal{Q}_t(\mathbf{r}', \mathbf{r}'')$ where: a) $\mathcal{Q}_t(\mathbf{r}', \mathbf{r}'') = Q_t^*(\mathbf{r}') Q_t(\mathbf{r}'')$, where $*$ denotes complex conjugation and $Q_t$ represents the source induced over the scatterer due to excitation with probing field $\psi_t^{\rm inc}$; and b) $\mathcal{H}(\mathbf{r}; \mathbf{r}', \mathbf{r}'') = G^*(\mathbf{r}, \mathbf{r}') G(\mathbf{r}, \mathbf{r}'') I_D(\mathbf{r}') I_D(\mathbf{r}'')$ where $I_D(\mathbf{r})$ is an indicator function whose value is $I_D(\mathbf{r}) = 1$ if $\mathbf{r} \in D$ and $I_D(\mathbf{r}) = 0$ otherwise. If the data are gathered at a number of receiver points $\mathbf{Z}_n, n = 1, 2, \cdots, N_R$, which corresponds to a total of $TN_R$ measured field intensities $I_t(\mathbf{Z}_n), t = 1, 2, \cdots, T; n = 1, 2, \cdots, N_R$, then for each $t$, one measures the $N_R \times 1$ intensity vector $\mathbf{I}_t = [I_t(\mathbf{Z}_1), I_t(\mathbf{Z}_2), \cdots, I_t(\mathbf{Z}_{N_R})] \in R^{N_R}$. Introducing $N_R \times 1$ vectors defined by $\mathbf{U}^{(\mathbf{R},\mathbf{R}')}(n) = \Re\left[G^*(\mathbf{Z}_n, \mathbf{R}) G(\mathbf{Z}_n, \mathbf{R}')\right]; \mathbf{V}^{(\mathbf{R},\mathbf{R}')}(n) = \Im\left[G^*(\mathbf{Z}_n, \mathbf{R}) G(\mathbf{Z}_n, \mathbf{R}')\right]; \quad n = 1, 2, \cdots, N_R$, and using the results above, and signal subspace analysis, we arrive at our two main results: 1) The data vectors $\mathbf{I}_t$ lie in the signal space $\mathcal{S}_y = \text{Span}\left[\mathbf{U}^{(\mathbf{X}_m, \mathbf{X}_{m'})}, \mathbf{V}^{(\mathbf{X}_m, \mathbf{X}_{m'})}, m = 1, 2, \cdots, M; m' \leq m\right]$ orthogonal to the space spanned by "noise subspace vectors" $W_p, p = M^2 + 1, \cdots, N_R$ which can be calculated from the data (via SVD). 2) If $M^2 < N_R$, then the pseudospectrum defined by $A(\mathbf{R}, \mathbf{R}') = \left[\sum_{p>M^2} \left| < W_p | \mathbf{U}^{(\mathbf{R},\mathbf{R}')} > \right|^2 + \sum_{p>M^2} \left| < W_p | \mathbf{V}^{(\mathbf{R},\mathbf{R}')} > \right|^2 \right]^{-1}$, where the Dirac bra-ket $< \cdot | \cdot >$ denotes Euclidean inner product, peaks if $(\mathbf{R}, \mathbf{R}') \in \{(\mathbf{X}_m, \mathbf{X}_{m'})\}$. This principle renders a number of imaging functions, which work despite the nonlinearity of the intensity-only forward mapping, as desired. They will be presented at the talk along with numerical illustrations.