general is much smaller than the carrier frequency f_c. The waves arriving from ahead of the vehicle experience a positive Doppler shift, whereas those coming from behind the vehicle have a negative Doppler shift. Thus, each component of the received signal is shifted by different values of Doppler frequency. For example, a vehicle traveling at 50 mph and receiving signals at a carrier frequency of 880 MHz will introduce a maximum Doppler shift of $v/\lambda = (50 \times 0.447 \times 880 \times 10^6)/(3 \times 10^8) = 65.56$ Hz.

4.3.1 Short-Term Fading

By applying the central limit theorem and observing that α_i and ϕ_i are independent, it follows that E_z, H_x, and H_y are complex Gaussian random variables for large N.

We consider the RF version of Eq. (4.2) for the field intensity E_z:

$$E_z = E_o \sum_{i=1}^{N} e^{j(\omega_c t + \phi_i)} \tag{4.8}$$

The real part of E_z is given as:

$$Re[E_z] = E_o \sum_{i=1}^{N} \cos \omega_c t \cos \phi_i - E_o \sum_{i=1}^{N} \sin \omega_c t \sin \phi_i \tag{4.9}$$

Let $A_c = E_o \sum_{i=1}^{N} \cos \phi_i$ and $A_s = E_o \sum_{i=1}^{N} \sin \phi_i$, then Eq. (4.9) can be written as:

$$Re[E_z] = A_c \cos \omega_c t - A_s \sin \omega_c t \tag{4.10}$$

Since ϕ_i is uniformly distributed between 0 to 2π, therefore the mean values of A_c and A_s are zero. The mean square values of A_c and A_s are $E\left(A_c^2\right) = E\left(A_s^2\right) = \frac{E_o^2 N}{2} = P_o$, the mean received power at the mobile.

Since A_c and A_s are uncorrelated, and therefore independent, we can write $E[A_c A_s] = 0$. Thus, the density of A_c and A_s follows a normal distribution, and the envelope of A_c and A_s is given by:

$$r = \left(A_c^2 + A_s^2\right)^{1/2} \tag{4.11}$$

and the phase, θ, is given as:

$$\theta = \operatorname{atan} \frac{A_s}{A_c} \tag{4.12}$$

The square root of the sum of the square of two Gaussian functions is a Rayleigh distribution. Therefore, the probability density function for short-term or multipath fading is given by the Rayleigh distribution (refer to Figure 4.3):

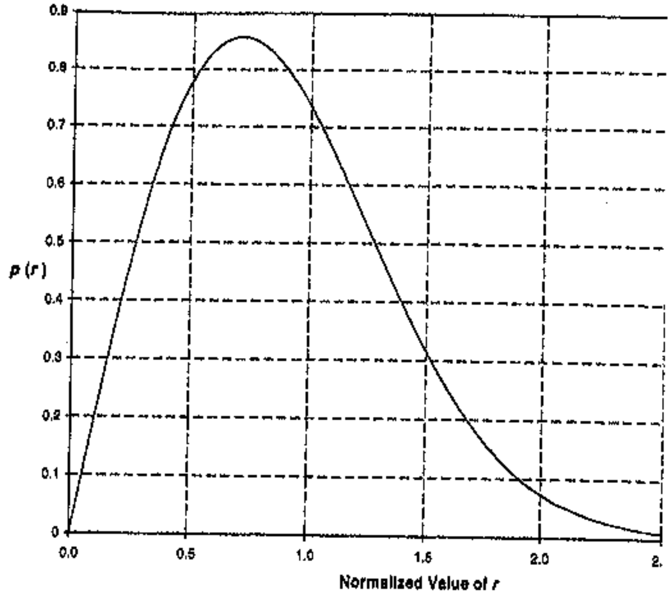

Fig. 4.3 Rayleigh Distribution—Short-Term Fading

$$p(r) = \frac{r}{P_o} e^{(-r^2)/(2P_o)}$$

where:
$2P_o = 2\sigma^2$ is the mean square power of the component subject to short-te fading and
r^2 is the instantaneous power.

The corresponding cumulative distribution function is

$$prob(r \le R) = P(R) = \int_o^R \frac{r}{P_o} e^{(-r^2)/(2P_o)} dr$$

$$P(R) = -e^{(-r^2)/(2P_o)} \Big|_0^R$$

$$P(R) = 1 - e^{-R^2/2P_o}$$

$$r_{mean} = E[r] = \int_0^\infty r p(r) dr = 1.2533\sqrt{P_o} = 1.2533$$

$$\text{Mean Square:} = E[r^2] = \int_0^\infty r^2 p(r) dr = 2P_o = 2\sigma^2$$

$$\text{Variance: } \sigma_r^2 = E[r^2] - (E[r])^2 = 0.4292 P_o = 0.429$$

: than the carrier frequency f_c. The waves arriving from
›erience a positive Doppler shift, whereas those coming
have a negative Doppler shift. Thus, each component of
ifted by different values of Doppler frequency. For exam-
ıt 50 mph and receiving signals at a carrier frequency of
ː a maximum Doppler shift of $v/\lambda = (50 \times 0.447 \times 880 \times$
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F version of Eq. (4.2) for the field intensity E_z:

$$E_z = E_o \sum_{i=1}^{N} e^{j(\omega_c t + \phi_i)} \tag{4.8}$$

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$$E_o \sum_{i=1}^{N} \cos \omega_c t \cos \phi_i - E_o \sum_{i=1}^{N} \sin \omega_c t \sin \phi_i \tag{4.9}$$

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$$[E_z] = A_c \cos \omega_c t - A_s \sin \omega_c t \tag{4.10}$$

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Therefore, the probability density function for short-term

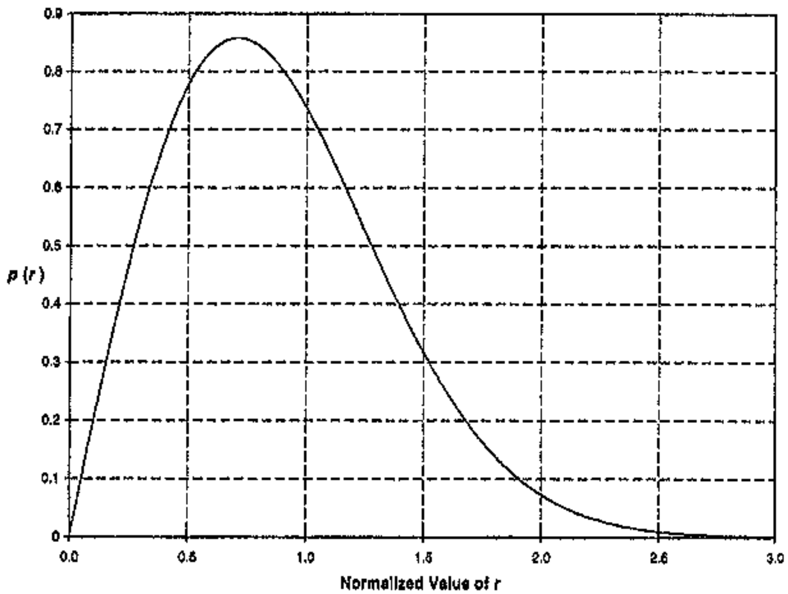

Fig. 4.3 Rayleigh Distribution—Short-Term Fading

$$p(r) = \frac{r}{P_o} e^{(-r^2)/(2P_o)} \tag{4.13}$$

where:
$2P_o = 2\sigma^2$ is the mean square power of the component subject to short-term fading and
r^2 is the instantaneous power.

The corresponding cumulative distribution function is

$$prob(r \le R) = P(R) = \int_o^R \frac{r}{P_o} e^{(-r^2)/(2P_o)} dr \tag{4.14a}$$

$$P(R) = -e^{(-r^2)/(2P_o)} \Big|_0^R \tag{4.14b}$$

$$P(R) = 1 - e^{-R^2/2P_o} \tag{4.14c}$$

$$r_{mean} = E[r] = \int_0^\infty r p(r)\, dr = 1.2533\sqrt{P_o} = 1.2533\sigma \tag{4.15}$$

$$\text{Mean Square:} = E[r^2] = \int_0^\infty r^2 p(r)\, dr = 2P_o = 2\sigma^2 \tag{4.16}$$

$$\text{Variance: } \sigma_r^2 = E[r^2] - (E[r])^2 = 0.4292 P_o = 0.4292\sigma^2 \tag{4.17}$$

The mean value r_M, is defined as that for which $P(r_M) = 0.5$,

$$\therefore 1 - e^{\left(-r_m^2\right)/(2P_o)} = 0.5 \tag{4.18a}$$

and

$$r_M = 1.1774\sqrt{P_o} = 1.774\sigma \tag{4.18b}$$

It is often convenient to write Eqs (4.13) and (4.14b) in terms of mean, mean-square value, or median rather than in terms of P_o.

Let $E[r] = \bar{r}$ and $E[r^2] = \overline{r^2}$.
In terms of the mean-square value

$$p(r) = \frac{2r}{\overline{r^2}} e^{\left[-\frac{r^2}{\overline{r^2}}\right]} \tag{4.19a}$$

$$P(R) = 1 - e^{\left[-\frac{R^2}{\overline{r^2}}\right]} \tag{4.19b}$$

In terms of the mean

$$p(r) = \frac{\pi r}{2\bar{r}^2} e^{\left[-\frac{\pi r^2}{(2\bar{r})^2}\right]} \tag{4.20a}$$

$$P(R) = 1 - e^{\left[-\frac{\pi R^2}{(2\bar{r})^2}\right]} \tag{4.20b}$$

In terms of the median

$$p(r) = \frac{2r\ln 2}{r_M^2} e^{\left(-\frac{r^2\ln 2}{2r_M^2}\right)} \tag{4.21a}$$

$$P(R) = 1 - 2^{-\left(\frac{R}{r_M}\right)^2} \tag{4.21b}$$

The Rayleigh probability density function describes the first-order statistics of the signal envelope over distances short enough for the mean level to be regarded as constant. First-order statistics are those for which distance is not a factor, and the Rayleigh distribution gives information such as the overall percentage of locations (or time) for which the envelope lies below a specified value.

System engineers are interested in a quantitative description of the rate at which fades of any depth occur and the average duration of a fade below any given depth. This provides a valuable aid in selecting transmission bit rates, word lengths, and coding schemes in digital radio systems and allows an assessment of system performance. The required information is pro level crossing rate and average fade duration below a specified

4.3.2 Level Crossing Rate

The level-crossing rate, $N(R)$, at a specified signal level I average number of times per second that the signal envelope c a positive-going direction $(\dot{r} > 0)$.

$$N(R) = \int_0^\infty \dot{r}\, p(R, \dot{r})\, d\dot{r}$$

where $p(R, \dot{r})$ is the joint Probability Density Function (PDF) of R and and a dot indicates the time derivative.

Using derivations given in references [5] and [6], the ave rate at a level R can be shown to be:

$$N(R) = \sqrt{\frac{\pi}{\sigma^2}} R f_m e^{\left(\frac{-R^2}{2\sigma^2}\right)}$$

Since $2\sigma^2$= mean-square value, therefore $\sqrt{2}\sigma$ is the root me value. The level crossing rate for a vertical monopole antenna as:

$$N(R) = \sqrt{2\pi} f_m \rho e^{-\rho^2} = n_o n_R$$

where:
$\rho = \frac{R}{\sqrt{2}\sigma} = \frac{R}{R_{RMS}}$ = the ratio between the specified level and the RMS tude of the fading envelope,
$f_m = \frac{v}{\lambda}$,
$n_o = \sqrt{2\pi} f_m$,
$n_R = \rho e^{-\rho^2}$,
n_R is the normalized level crossing that is independent of wavelength a vehicle speed,
v = speed of vehicle, and
λ = carrier wavelength.

Figure 4.4 gives the n_R at a level R.

An approximate expression for $N(R)$ below a given level, given as:

$$N(R) \approx \sqrt{2\pi} \frac{v}{\lambda} \rho$$

The above equations neglect the effect of the motion of th their motion is taken into account, the effect is to increase the

r_M, is defined as that for which $P(r_M) = 0.5$,

$$\therefore 1 - e^{(-r_m^2)/(2P_o)} = 0.5 \quad (4.18a)$$

$$r_M = 1.1774\sqrt{P_o} = 1.774\sigma \quad (4.18b)$$

enient to write Eqs (4.13) and (4.14b) in terms of mean, r median rather than in terms of P_o.

$E[r^2] = \overline{r^2}$.

mean-square value

$$p(r) = \frac{2r}{\overline{r^2}} e^{\left[-\frac{r^2}{\overline{r^2}}\right]} \quad (4.19a)$$

$$P(R) = 1 - e^{\left[-\frac{R^2}{\overline{r^2}}\right]} \quad (4.19b)$$

mean

$$p(r) = \frac{\pi r}{2\overline{r}^2} e^{\left[-\frac{\pi r^2}{(2\overline{r})^2}\right]} \quad (4.20a)$$

$$P(R) = 1 - e^{\left[-\frac{\pi R^2}{(2\overline{r})^2}\right]} \quad (4.20b)$$

e median

$$p(r) = \frac{2r\ln 2}{r_M^2} e^{\left(-\frac{r^2 \ln 2}{2 r_M^2}\right)} \quad (4.21a)$$

$$P(R) = 1 - 2^{-\left(\frac{R}{r_M}\right)^2} \quad (4.21b)$$

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ment of system performance. The required information is provided in terms of level crossing rate and average fade duration below a specified level.

4.3.2 Level Crossing Rate

The level-crossing rate, $N(R)$, at a specified signal level R is defined as the average number of times per second that the signal envelope crosses the level in a positive-going direction $(\dot{r} > 0)$.

$$N(R) = \int_0^\infty \dot{r}\, p(R, \dot{r})\, d\dot{r} \quad (4.22)$$

where $p(R, \dot{r})$ is the joint Probability Density Function (PDF) of R and $\dot{r}$, and a dot indicates the time derivative.

Using derivations given in references [5] and [6], the average level crossing rate at a level R can be shown to be:

$$N(R) = \sqrt{\frac{\pi}{\sigma^2}} R f_m e^{\left(\frac{-R^2}{2\sigma^2}\right)} \quad (4.23)$$

Since $2\sigma^2$= mean-square value, therefore $\sqrt{2}\sigma$ is the root mean square (RMS) value. The level crossing rate for a vertical monopole antenna can then be given as:

$$N(R) = \sqrt{2\pi} f_m \rho e^{-\rho^2} = n_o n_R \quad (4.24)$$

where:

$\rho = \frac{R}{\sqrt{2}\sigma} = \frac{R}{R_{RMS}}$ = the ratio between the specified level and the RMS amplitude of the fading envelope,

$f_m = \frac{v}{\lambda}$,

$n_o = \sqrt{2\pi} f_m$,

$n_R = \rho e^{-\rho^2}$,

n_R is the normalized level crossing that is independent of wavelength and vehicle speed,

v = speed of vehicle, and

λ = carrier wavelength.

Figure 4.4 gives the n_R at a level R.

An approximate expression for $N(R)$ below a given level, $\rho = R/R_{RMS}$, can be given as:

$$N(R) \approx \sqrt{2\pi} \frac{v}{\lambda} \rho \quad (4.25)$$

The above equations neglect the effect of the motion of the scatterers. When their motion is taken into account, the effect is to increase the fade rate.

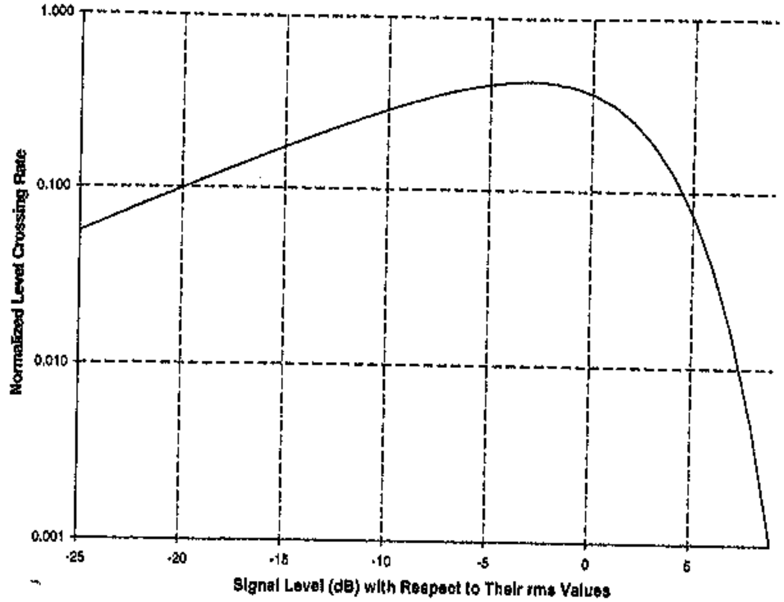

Fig. 4.4 n_R vs. Signal Level

4.3.3 Average Fade Duration

The average fade duration is the average of $\tau_1, \tau_2, \ldots, \tau_n$. The average duration of fades below the specified level R can be found from

$$E[\tau_R] = \tau(R) = \frac{prob[r \le R]}{N(R)} \qquad (4.26)$$

$$\tau(R) = \frac{e^{\rho^2}-1}{\sqrt{2\pi} f_m \rho} = \frac{e^{\rho^2}-1}{n_o \rho} \qquad (4.27)$$

An approximate expression for $\tau(R)$ can be given as:

$$\tau(R) \approx \frac{\lambda}{\nu} \frac{\rho}{\sqrt{2\pi}} \qquad (4.28)$$

EXAMPLE 4-1

Problem Statement

Calculate the level-crossing rate at a level of −10 dB and average duration of fade for a cellular system of 900 MHz and a vehicle speed of 24 km/h. Assume the free-space speed of propagation for electromagnetic waves = 3×10^8 m/s. Neglect the effects of the motion of the scatterers. Compare the results obtained using the approximate expressions.

Solution

At 900 MHz, $\lambda = \frac{3 \times 10^8}{900 \times 10^6} = \frac{1}{3}\, m$, $\nu = 6.67$ m/s, $f_m = \frac{6.6}{\frac{1}{3}}$

$$n_o = \sqrt{2\pi} f_m = 50.$$

From Fig. 4.3, $n_R = 0.32$ at −10 dB.

$$N(R) = 0.32 \times 50 = 16.0 \text{ fades/se}$$

$$\rho e^{\rho^2} = n_R = 0.32$$

$$\rho = 0.294$$

$$\tau(R) = \frac{(1.09-1)}{50 \times 0.294} = 0.0061 \text{ sec} = 6.1$$

Using the approximate expressions we get:

$$\text{Fading level} = \rho = -10 \text{ dB}$$

$$20 \log \rho = -10$$

$$\rho = 10^{-10/20} = 0.3162$$

$$N(R) \approx \sqrt{2\pi} \times \frac{6.67}{\frac{1}{3}} \times 0.3162 = 15.85 \text{ fa}$$

$$\tau(R) \approx \frac{1}{3 \times 6.67} \frac{0.3162}{\sqrt{2\pi}} = 0.0063 = 6.3$$

These results are quite close to those obtained using the exa

4.3.4 Long-Term Fading

The probability density function for long-term fadin normal distribution (refer to Figure 4.5).

$$p(m) = \frac{1}{m \sigma_m \sqrt{2\pi}} e^{\left[-(\log m - \bar{m})^2/\left(2\sigma_m^2\right)\right]}, \quad m > 0$$

where:
$\bar{m}$ is the mean of log m and
σ_m is the standard deviation.

Using $z = (\log m - \bar{m})/\sigma_m$, the cumulative distribution 1

$$prob(z \le Z) = P(z \le Z) = \frac{1}{2} + \frac{1}{2} erf\left(\frac{Z}{\sqrt{2}}\right) \approx 1 - \frac{1}{\sqrt{2\pi} Z}$$