

Better Understanding and Synthesis of Integrated Magnetics With Simplified Gyrator Model Method

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Abstract—A simplified modeling method is presented for integrated magnetics in DC/DC converters. The method extends gyrator-capacitor modeling techniques and is feasible in complex core and winding structures. Fundamental concepts of flux utilization are derived. Design and synthesis rules are established for the properties of magnetic cores.

I. INTRODUCTION

DC/DC converter topologies with integrated magnetics have seen widespread applications [1], due to their potential size reduction from combining inductors and transformers into one magnetic core. Often, the traditional inductive model is used for their analysis and design [2]. However, this model has difficulty predicting the flux change within the magnetic path [3]. Furthermore, it uses three interfaces to model magnetic components. In the magnetic circuit, the resistor is analogous to the reluctance. In the electrical circuit, inductors and ideal transformers express the inductance and the isolation. When integrating flux paths, the derivative of flux, or the flux rate is used [4]. The three different models show different partial magnetic properties respectively.

The traditional method has several deficiencies for integrated magnetics analysis. The voltages on windings are not expressed directly in the model. So, the energy interchange between windings and cores is lost. When deriving the inductor model from the resistor model, the duality transformation is used, which can cause confusion. Additionally, since discrete magnetic components fulfill only a simple function, it is straightforward to apply the traditional modeling approach to get their properties. For converters with integrated magnetics, however, this approach shows some limitations.

The gyrator-capacitor approach has been introduced to model magnetic cores [3, 5]. This approach avoids the problem above, because it describes the interaction between the electrical and the magnetic circuit, including the dynamic flux change. In practice, this modeling method begins with the establishment of gyrators and capacitors. The gyrator links the magnetic path and the electrical circuit by representations of analogs from windings. Capacitors represent the permeance of core, gap and leakage inductance as energy storage devices in the magnetic path. This model is generally used as a SPICE simulation tool. For analysis, it becomes complicated when there are multiple windings,

which requires the insertion of multiple gyrators. Intuition and quantitative understanding are often lost.

In this paper a simplified modeling method for integrated magnetics in DC/DC converters is presented. Section II presents the new method. In Section III, some fundamental properties of integrated magnetics are derived. In Section IV, synthesis rules are discussed and illustrated. Section V gives conclusions. Two detailed examples demonstrate the usage of the model.

II. PROPOSED MODELING METHOD

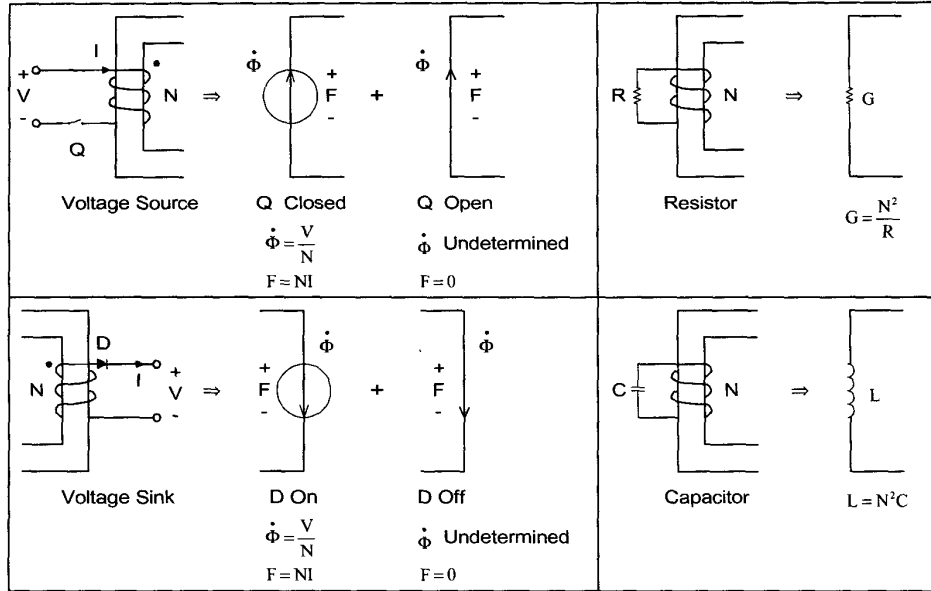
The new approach includes the basic gyrator-capacitor modeling principles, but eliminates the gyrator altogether and replaces it with a source, a sink, or an open circuit, etc. Hence, we refer to it as the capacitive modeling approach.

Table I represents the electrical actions on windings and their simplified models. In the voltage source case, the voltage V applied on the N -turn winding will generate the flux rate, $\dot{\phi}$. The current, I , generates the magnetomotive force F . The flux rate and the magnetomotive force are the variables in the magnetic path and represent the result of the electrical actions on the winding. They can be used simultaneously in a simple format to model the magnetic core. As shown in the model, the current source $\dot{\phi}$ is analogous to the flux rate. When the switch Q is closed, $\dot{\phi} = V/N$. When Q is open, $\dot{\phi}$ is determined by the flux rate distribution within the core. The magnetomotive force F of the winding is always an inactive variable, which is determined by the load or the magnetomotive force difference in the core. If considering the winding itself, F is defined as $F = NI$.

The direction of the flux rate is determined by the voltage V . Suppose a current is flowing into the winding from the positive end of V . By the right-hand rule with this fictitious current, the flux rate is flowing in the direction of the thumb. The direction of the magnetomotive force is determined by the current I . By the right-hand rule with this actual current in the winding, the thumb points towards the positive end of F . From the energy point of view, if the resulting model is a source, the electrical action stores energy into the core. If it is a sink, the energy goes out of the core.

In PWM DC/DC converters, including both the voltage-fed

TABLE I
ELECTRICAL ANALOGS FOR PROPOSED MODELING METHOD



and current-fed topologies, the voltages across the windings can be assumed as constants at a given steady state. So, the magnetic circuit operation can be simply modeled by the structures in Table I with constant sources or sinks in the magnetic path.

The resistor analog is useful when considering the short or open winding in many applications. The capacitor analog is used when the winding is eventually connected with a capacitor and the voltage on it is changing.

The transformations can be derived directly. They incorporate all the electrical actions on windings and provide sufficient information to calculate core and winding parameters. The energy interchange between the magnetic and the electrical circuit is clear with the power expression $P = F \dot{\Phi}$. Thus, this model provides a method to analyze the magnetic-electrical interaction within the magnetic path. This is especially beneficial for converters with integrated magnetics.

III. INTEGRATED MAGNETIC TOPOLOGIES ANALYSIS RULES

The elegance of the model proposed is that it leads to simple analysis and design rules. In this section, we discuss analysis rules that give practical insight to the properties of integrated magnetic cores. These useful concepts will lead to the design method in the following examples and the synthesis methods in Section IV.

A. *KCL and KVL laws are obeyed in the magnetic circuit.*

Since we model the magnetic core with the equivalent quantities from the electrical circuit, the KCL and KVL laws are obeyed. For example, by KCL, at any time t , the sum of the flux rate at one node must be equal to zero, $\sum \dot{\Phi}(t) = 0$. By KVL, in any closed loop, the sum of the magnetomotive force is equal to zero, $\sum F(t) = 0$. All flux utilization in the core is based on the two laws.

B. *AC flux is balanced in each leg.*

This ensures that the core is not saturated. In every cycle, the net flux change in each leg should be zero for steady state operation. This property is different from the normal core, which is considered as a single piece.

C. *DC flux bias exists in the core.*

In PWM DC/DC converters, a stable, unidirectional DC (or average) bias exists in the core to sustain a continuous flux flow for the continuous operation of circuits. In the integrated magnetic transformer, there is a DC flux bias in the whole core [6]. So, every leg contains a DC flux bias. This is because by KCL, at any moment we have $\sum \dot{\Phi}(t) = 0$. In any time interval ΔT , the flux change in the core is equal to zero, i.e. $\sum \Delta \Phi = \int_t^{t+\Delta T} \sum \dot{\Phi}(\tau) d\tau = 0$. So, if the DC flux accumulates within one leg, it also accumulates within the other legs in the reverse direction. When calculating the

maximum flux allowable in the design, the DC bias in each leg should be considered.

D. Current in the winding may change after integration.

A prominent phenomenon in the integrated magnetic circuit in comparison to its discrete core counterpart is that the current in the winding may change after integration. This is because, generally, DC/DC converters are voltage sources. In the integration, only the flux rate relation is necessary to be kept unchanged. This property makes the determination of the operating time interval of each winding more difficult.

Example 1. An Integrated Forward Converter

Fig. 1 is a frequently referenced integrated Forward converter [4, 6]. This example will show how to analyze the circuit with the new modeling method.

- Core Structure Establishment

The first step is to obtain the core structure as in Fig. 2 (a). P_I , P_{II} and P_{III} are the permeances of the core legs and P_g is the permeance of the gap. The operating modes of the windings are determined by the electrical activations

- Circuit Operation and Model Establishment

When Q_1 is closed, V_i is applied on the winding N_p . It generates the flux rate $\dot{\phi}_p = V_i/N_p$, which forces D_s to be conducting. D_L and D_r are off. $\dot{\phi}_p$ and $\dot{\phi}_s$ charge permeances P_I , P_{II} . Their difference $\dot{\phi}_{III} = \dot{\phi}_p - \dot{\phi}_s$ charges P_{III} and P_g . This state is depicted in Fig. 2 (b) with time interval T_1 . In this state, $\dot{\phi}_p$ provides energy for the core. The power is $P_{in} = F_p \dot{\phi}_p$. It is divided into two portions, the direct output

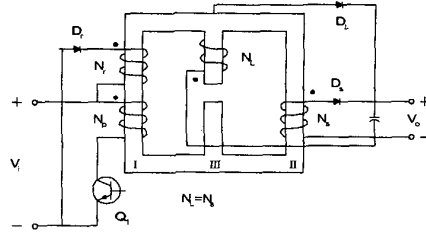


Fig. 1. An Integrated Forward Converter

power $P_o = F_s \dot{\phi}_s$, and the stored energy, which is mainly located in the gap.

When Q_1 is open, $\dot{\phi}_p$ disappears. The core tends to balance the flux between the legs. The flux rates have the directions as in Fig. 2 (c) with time interval T_2 . The directions make current sinks $\dot{\phi}_r$ and $\dot{\phi}_L$ work. They balance the magnetomotive force difference between the legs and discharge the permeances linearly. When *Leg I* and *Leg II* are balanced, a third state with time interval T_3 exists as in Fig. 2 (d). This is because in the continuous mode, F_g is large. F_{II} must reach F_I first. Otherwise, the accumulated flux will saturate the core. The flux rates in *Leg I* and *Leg II* are now equal to $V_o/(2N_L)$. The voltages on the windings they generate are not large enough to conduct the diodes.

From the above operation analysis, the time interval can now be determined. T_1 is equal to the time DT when Q_1 is closed, where D is the duty ratio and T is the switching period. T_2 is calculated according to the flux balance in *Leg I*. Assume $N_p = N_r$, we have (omitting lengthy calculations)

$$T_1 = DT, \quad T_2 = DT(1+D)/(2-D), \quad T_3 = T - T_1 - T_2.$$

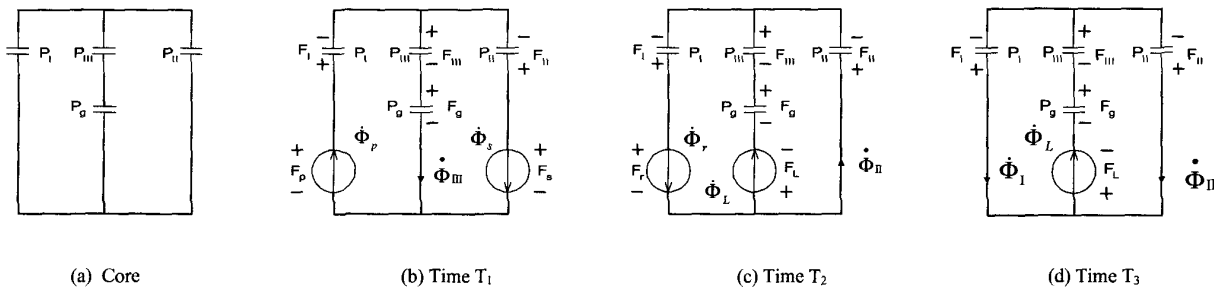


Fig. 2. Model of Integrated Forward Converter

• Design

1) *Voltage transfer ratio.* In steady state the flux is balanced in the gap. So,

$$DT(V_i/N_p - V_o/N_s) = (1-D)TV_o/N_L$$

When $N_L = N_s$, we have $V_o/V_i = DN_s/N_p$.

2) *Cross-sectional area A_c and gap length l_g design in the center leg.* Considering the magnetic path loop generated from *Leg II* and *Leg III*, the total capacitive magnetomotive force in the core can be expressed as $F = F_{II} + F_{III} + F_g$. It is always equal to the magnetomotive force F_s (in time T_1) or F_L (in time T_2 and T_3), which are generated from the output current I_o . Assume $N_L = N_s$. We have $F_s = F_L$. Assume that the continuous mode average (av) and ripple (Δ) output current specifications are given as I_{oav} and ΔI_o . Then, the average and ripple magnetomotive forces in the core are $F_{oav} = I_{oav}N_L$ and $\Delta F = \Delta I_o N_L$. Usually, the permeances P_{II} and P_{III} are very high and the magnetomotive forces on them can be ignored, i.e. $F_{II} \approx 0$ and $F_{III} \approx 0$. So, the average and ripple magnetomotive force in the gap is determined as $F_{gav} = I_{oav}N_L$ and $\Delta F_g = \Delta I_o N_L$. The permeance can be calculated as the variation of the flux divided by the variation of the magnetomotive force, or $P_g = \Delta\Phi_g / \Delta F_g$ (This is analogous to the relation in the electrical circuit that the capacitance is equal to the variation of the charge divided by the variation of the voltage). Considering the time interval when the switch Q_1 is off, the flux change in the gap is $\Delta\Phi_g = (1-D)TV_o/N_L$. So, the permeance of the gap can be calculated as $P_g = (1-D)TV_o / (N_L \Delta F_g)$. If a maximum flux density B_m is selected, the cross-sectional area A_c can be

determined by the relation $F_{gav} + \Delta F_g / 2 \leq B_m A_c / P_g$. The minimum gap length l_g is, therefore, derived from $l_g = \mu_0 A_c / P_g$.

3) *Cross-sectional area A_1 in the outer leg.* Assume initially the core is not magnetized, i.e. the flux in all the legs is zero. Considering the DC flux bias within *Leg I* and *Leg II* are symmetric, we have the average value $|\Phi_I| = |\Phi_{II}| = |\Phi_g|/2$. The directions are shown in the model (the same as the directions of F).

AC flux is derived with respect to each leg itself. For example in *Leg I*, the flux rate is $\dot{\Phi}_p = v_i/N_p$ in time DT . The flux change of P_1 is $\Delta\Phi_p = DTv_i/N_p$. Using the same maximum flux density B_m , the cross-sectional area A_1 can be determined by the relation $A_1 = (\Phi_I + \Delta\Phi_p/2)/B_m$.

IV. INTEGRATED MAGNETIC TOPOLOGIES SYNTHESIS RULES

The proposed model can help synthesis of integrated magnetics because it contains flux rate, the decisive factor of integration. Furthermore, this model corresponds to the physical structure directly. In the transformation from discrete cores to integrated ones, the physical meaning is retained. So, it is easy to manipulate and check. Some synthesis rules are found and presented here.

Rule 1. Flux rate: Add

In Fig. 3, *Leg 1* and *Leg 2* have no windings on them. They provide free paths for flux rate flowing. In this condition, *Leg 1* and *Leg 2* can be combined as *Leg B* in the integrated core M_3 , and vice versa.

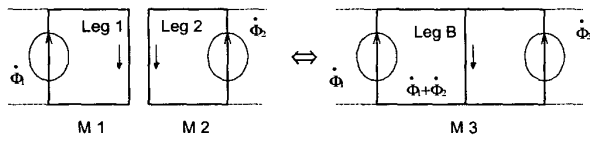


Fig. 3. Magnetic Integration Rule 1: Add

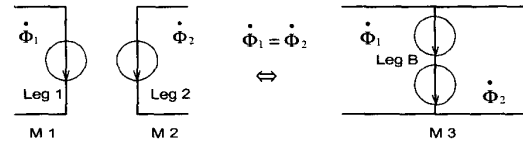


Fig. 4. Magnetic Integration Rule 2: Merge (1)

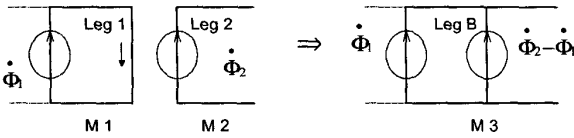


Fig. 5. Magnetic Integration Rule 2: Merge (2)

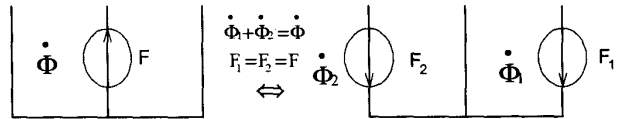


Fig. 6. Magnetic Integration Rule 3: Source Shifting

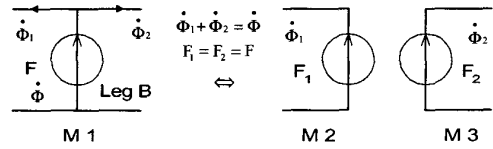


Fig. 7. Magnetic Integration Rule 4: Split

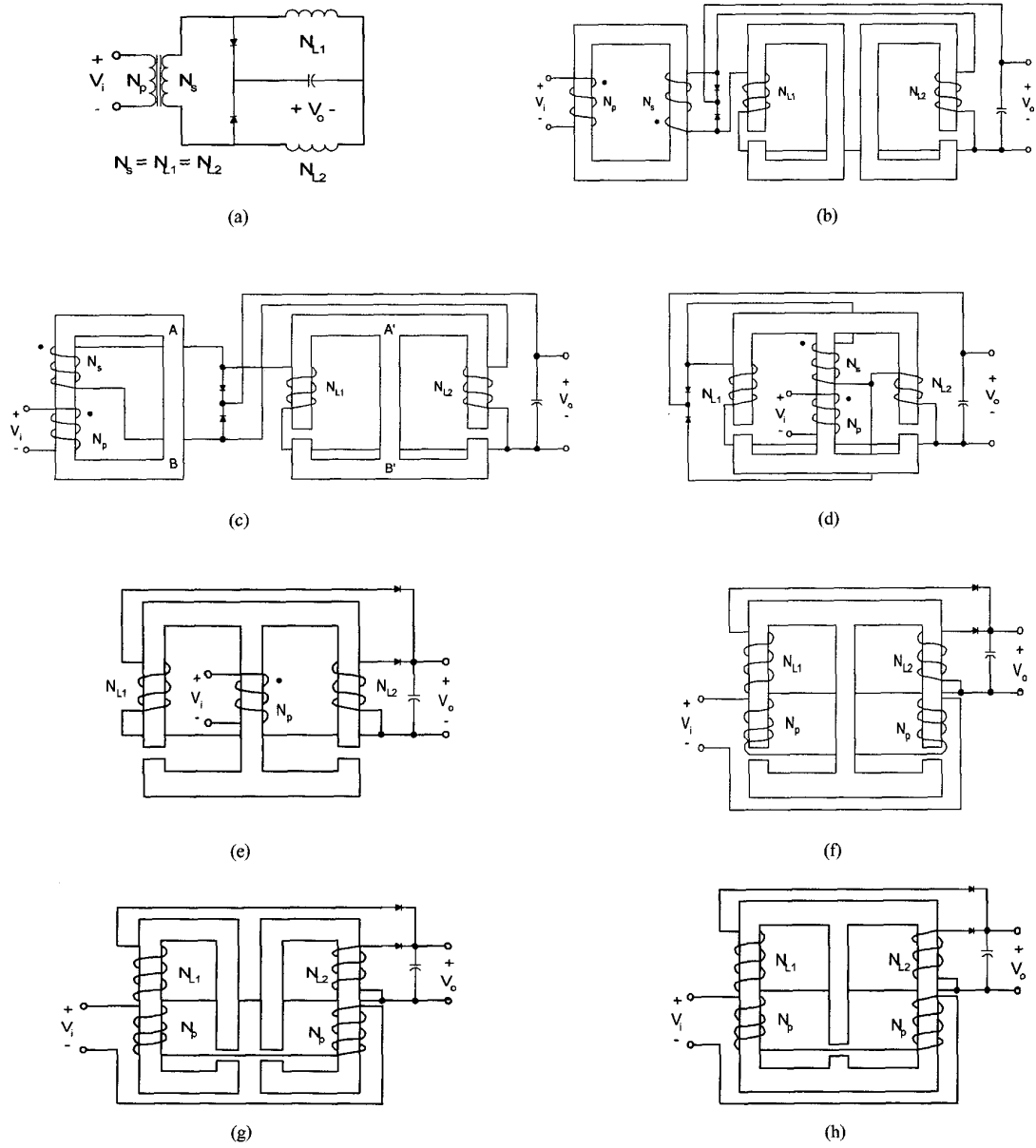


Fig. 8. Synthesis and Transformation of a Current Doubler Circuit

Rule 2. Flux rate: Merge

A. Two windings

In Fig. 4, if $\dot{\phi}_1 = \dot{\phi}_2$ holds for all time, M_1 and M_2 can be combined into M_3 , and vice versa. This concept is the same as the coupled inductor.

B. One winding

In Fig. 5, when *Leg 1* and *Leg 2* are merged, a new flux rate $\dot{\phi}_2 - \dot{\phi}_1$ appears. It must stand for a meaningful winding connection. Generally, the result of this integration is the indirect energy storage.

Rule 3. Source shifting

Source shifting is introduced in paper [1]. In Fig. 6, if $F_1 = F_2 = F$ and $\dot{\phi} = \dot{\phi}_1 + \dot{\phi}_2$, the two circuits are equivalent. This can be used even in a single operating state.

Rule 4. Flux rate: Split

In Fig. 7, M_1 can be split into M_2 and M_3 and vice versa. In order to maintain the magnetomotive force in M_2 and M_3 , the two windings should be in series with the same turns. This rule can be derived from the source shifting and the free-leg combination technique.

In the synthesis process, there are some practical considerations:

1. The synthesis is the integration of the flux rate. The original current in the winding may change.
2. All legs in a core cannot contain independent sources simultaneously. Otherwise, KCL is difficult to maintain.
3. At any time, there must exist a current source or sink to restrict the flux rate between two paralleled unbalanced legs. Otherwise, an extremely large voltage in the electrical circuit will appear.
4. These techniques can be used individually to create different integrated structures for a single circuit or transform one integrated form to another.
5. The integration of magnetic cores is indeed an operation on "time and space". The physical structure should be consistent in each operating state.
6. The gap does not influence the integration of the flux rate. However, it introduces flux bias into the core. In PWM DC/DC converters, when two gaps are combined, they are arranged to have the same bias directions (clockwise or counter-clockwise) to maintain the uniform flux bias in the core.

Example 2. Synthesis and Transformation of a Current Doubler Circuit.

The properties of the integrated current doubler circuit have been presented in papers [7, 8, 9]. This example shows how to combine the cores directly using Rules 1-4.

Fig. 8 (a) is a Full-Bridge current doubler circuit. Fig. 8 (b) presents the core arrangement of the transformer and the two inductors. The two inductors have free legs. So, they are combined as Fig. 8 (c) using Rule 1. It is important to check the flux directions in the cores before integration. Usually, it is desirable to generate a unidirectional flux bias. At the same time, the winding position in the transformer is also changed to obtain a free leg. Then, it is possible to combine the transformer and the inductor core using Rule 1 again. By a simple calculation of the flux rate in these two cores, it is easy to find that if A-A' and B-B' are combined together, the net flux rate in the center leg will be equal to zero after combination. As shown in Fig. 8 (d), the free leg disappears. To simplify the structure, assuming N_L is equal to N_s , Fig. 8 (e) can be derived using Rule 3. Using the same rule for N_p , Fig. 8 (f) is derived.

However, in manufacturing, it is not desirable to contain two gaps in a transformer. Fig. 8 (f) can be transformed into a single-gap structure. Since the center leg is free, it can be split into two cores. To obtain a unidirectional flux bias, the right side core is turned upside down as in Fig. 8 (g). By Rule 1, a single-gap Full-Bridge current doubler circuit is obtained as in Fig. 8 (h).

V. CONCLUSIONS

The proposed capacitive modeling method presented in this paper has been used as a tool to understand the integrated magnetics in PWM DC/DC converters. The approach provides direct intuition for synthesis and design rules. The method retains the magnetic circuit structure, and uses new notations to simplify the analysis. This method can be extended to analyze coupled inductors with complicated winding structure.

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