

Performance Prediction of Distributed Power Systems Based on Small-Scale Prototypes

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Abstract: A method to quantitatively predict performance of Distributed Power Systems (DPS) is presented. A small-scale prototype system is proposed that can be used to predict the performance of the large full-scale DPS. Two experimental implementations of the design procedure are discussed, each with different accuracy and complexity. Experiments verify that the small-scale prototypes provide performance benchmarks for the full-scale DPS.

I. INTRODUCTION

In distributed power systems (DPS) it is common for one source DC-DC converter to be connected to several load DC-DC converters in parallel. From the point of view of the source converter, the paralleled load converters can be viewed as one, single load. However, the input impedances of the load converters, which are non-resistive, greatly influence the relative stability properties and performance of the DPS. At low frequency, the load converters work as a negative resistor, and their deteriorating effect on the source converter can be addressed at the stand-alone module design phase [1]. However, the interaction among power modules at high frequency mainly depends on the load converters themselves. In the worst case, this interaction may even cause the DPS to become unstable. Therefore, it is important to carefully evaluate the DPS performance in the system design phase.

Stability of DPS is often guaranteed by ensuring that the magnitude of the output impedance of the power source is much lower than the input impedance of the load [2]. The main criticism of this basic impedance specification result is that it is overly conservative [3,4]. Design rules are based on sufficient conditions that attempt to prevent any noticeable interaction between source and load modules. The conditions are very general and therefore, can be applied to all DPS systems. However, by keeping the stability conditions so general, the results lose information on the specific structure of a DPS that may be helpful in reducing conservativeness. The work of [3,4,5] focuses on how to reduce restrictions on general impedance stability methods of DPS.

Recently, a method to ensure the performance of a DC-DC converter with a capacitive load has been proposed [6,7,8]. It

is assumed that the user has access to loop gain measurement of each individual nominal converter with a resistive load. With this additional information available, a formula is utilized to obtain quantitative results such as phase margin and gain margin of the converter when the load is capacitive. This leads to design rules that can guarantee precise performance requirements, e.g., phase margin of the outer loop gain above 60 degrees. Hence, design conditions are not conservative when the method is used. This technique can also be used to evaluate the small signal performance of an existing (full-scale) DPS [9]: The outer loop gain of the source converter in the DPS is calculated without breaking the control loop of the source converter. However, the evaluation result only indicates the performance of the source converter and does not pinpoint the reasons that design specifications are not satisfied.

On the other hand, performance of the DPS needs to be guaranteed at the beginning of the design phase rather than after it has already been built. It is costly and time-consuming to build a full-scale DPS, and it would be disastrous to build a DPS only to have it be unstable. As a result, the conservative design rules [2-5] for DPS are used when building the full-scale DPS. Preliminary testing of the design is then usually simulated to verify stability (time-consuming). The end result is usually a stable (yet conservative, sub-optimally performing) DPS. There is, clearly, a need for a more accurate, less time consuming method to predict DPS performance. With such a method, it will be possible to reduce conservativeness of the design rules and, therefore, improve DPS performance.

Specifically, this research presents the following:

A modeling method is proposed to predict the outer loop gain of a source converter in a DPS. With the loop gain, it is possible to calculate performance criteria such as phase margin, gain margin and crossover frequency.

Design procedures to build small-scale prototypes of a DPS are presented. Performance characteristics of the prototypes are shown to accurately predict performance of characteristics of the full-scale DPS.

Experimental results of the proposed methods are presented that verify the accuracy of the new approach to predict performance of a DPS.

Section II of this paper introduces the background and basic experimental setup that is being considered. Section III discusses the performance prediction of a general DPS, based on a new theoretical method. The method can be performed solely by software, but for increased accuracy, scaled prototypes should be built to reflect full-scale performance of the DPS. Moreover, a method is proposed to build small-scale prototypes of the DPS under consideration. Step-by-step procedures are also given for engineers to follow to predict the outer loop gain of the source converter in the full-scale DPS. In Section IV, the simplification for DPS with n identical load converters is given, which is a common application in a telecommunication power distribution system. Experimental results verify the accuracy of the method in Section V.

II. BACKGROUND

Because DC-DC converters are, in general, feedback control systems, the relationship between the output impedance of a converter and its outer loop gain can be addressed by control theory. For DC-DC converters, given the operating point, the closed loop output impedance is given as

$$Z_O = \frac{Z_{O_i}}{1 + T_O} \quad (1)$$

where T_O is the outer loop gain for the specific operating point, Z_O is the output impedance of the individual converter (not in a DPS), Z_{O_i} is the output impedance with the current loop closed and the voltage loop open in current mode control converters. For voltage mode control converters, Z_{O_i} is the open loop output impedance.

Fig. 1 shows a DPS with one source converter and n load converters in parallel. The output impedance of the converter is Z_O . The input impedance of the loads is Z_L .

Viewing the loads as part of the source converter, a new, total source output impedance is given as $Z_{total} = Z_O // Z_L$. On the other hand, if the source converter has its voltage loop open, then its new output impedance is $Z_{O_i} = Z_O // Z_L$. Therefore, when the voltage loop of the source converter is closed, there is

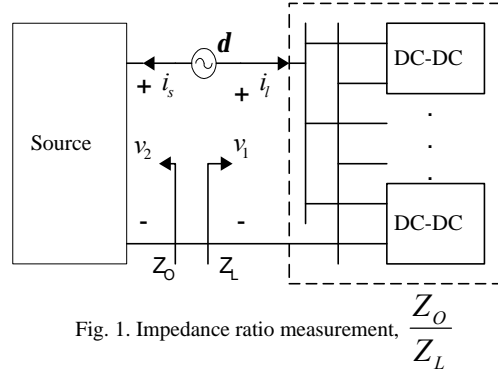


Fig. 1. Impedance ratio measurement, $\frac{Z_O}{Z_L}$

$$\begin{aligned} Z_O // Z_L = Z_{total} &= \frac{Z_{O_i}}{1 + T_O} = \frac{Z_{O_i} // Z_L}{1 + T_O} \\ &= \frac{Z_{O_i}}{(1 + T_O)(1 + \frac{Z_{O_i}}{Z_L})} \end{aligned} \quad (2)$$

where T_O is the outer loop gain with the influence of the load, and Z_{O_i} represents the output impedance of the source converter with load Z_L and with its voltage loop open. Further details of (2) can be found in [6].

Combining (1) and (2) leads to [6]

$$T_O = \frac{T_O}{(1 + T_O) \frac{Z_O}{Z_L} + 1} \quad (3)$$

T_O is the outer loop gain of the converter when the load is Z_L . The phase margin, gain margin, crossover frequency, etc can be obtained from the outer loop gain, and we can determine the performance of the converter.

Therefore, if output impedance Z_O and outer loop gain T_O of a DC-DC converter at an operating point are known, the influence of Z_L on the outer loop gain can be predicted without breaking the outer loop in the measurement. This provides a useful tool to predict a range of Z_L to guarantee certain performance criteria such as phase margin and crossover frequency of the outer loop gain T_O . (Under a given operating point, Z_O and T_O are usually termed as nominal output impedance and nominal outer loop gain, respectively. Therefore, when (3) is used, the operation point should be known in advance. This is not difficult because the

DC part of the load is usually known. On the other hand, the nominal value of output impedance and outer loop gain does not change greatly for a range of operating points [9], which means a single nominal value may be used for many operating points with acceptable errors.)

Notice that, in (3), the new loop gain depends on the impedance ratio $\frac{Z_O}{Z_L}$. Hence, it is not actually necessary to

measure both Z_O and Z_L separately, especially if it is possible to measure the ratio directly [9].

Fig. 1 shows a way to measure this impedance ratio directly. A small signal perturbation, \mathbf{d} , is injected into the connection cables. \mathbf{d} can be either a small signal voltage source or a small signal current source. Therefore, the output impedance of the source, Z_O , and the input impedance of the load, Z_L , are

$$Z_O = \frac{v_2}{i_s};$$

$$Z_L = \frac{v_1}{i_l}$$

Meanwhile, because

$$i_s = i_l,$$

there is

$$\frac{Z_O}{Z_L} = \frac{v_2}{v_1} \quad \text{impedance ratio}$$

Then (3) is simplified as [9]

$$T_O = \frac{T_O}{(1+T_O)\left(\frac{v_2}{v_1}\right)+1} \quad (4)$$

This method to obtain T_O in (4) is simple and can be performed by users or designers, since it does not require making connections to internal circuitry in any of the converters. (On the other hand, the DPS must be completely built before the measurement can be taken.)

In Fig. 1, the small signal perturbation \mathbf{d} is in series with the distribution line. However, it can also be put across the output of the source converter, which also appears in [10]. In this case, because there is

$$v_2 = v_1$$

Then (3) is simplified as

$$T_O = \frac{T_O}{(1+T_O)\frac{i_l}{i_s}+1}$$

In this paper, impedance ratio measurement by voltage is used, as in (4).

III. PERFORMANCE PREDICTION OF SOURCE CONVERTER IN A DPS

This section discusses methods to predict the outer loop gain T_O of the source converter in a DPS without entirely building the DPS. Either simulation or experiments (on a small-scale benchmark) can be used for prediction. The methods proposed are simple and can be performed by either the designer of the converters or the customer who buys converters and then uses them to build a DPS.

In the general case, if there are n load converters, each with input impedance $Z_{L1}, Z_{L2}, \dots, Z_{Ln}$ respectively, there is

$$Z_L = \frac{1}{\frac{1}{Z_{L1}} + \frac{1}{Z_{L2}} + \dots + \frac{1}{Z_{Ln}}}$$

The impedance ratio will be

$$\frac{Z_O}{Z_L} = \frac{Z_O}{Z_{L1}} + \frac{Z_O}{Z_{L2}} + \dots + \frac{Z_O}{Z_{Ln}}$$

Therefore, in Fig. 1, (3) can be rewritten as

$$T_O = \frac{T_O}{(1+T_O)\left(\frac{Z_O}{Z_{L1}} + \frac{Z_O}{Z_{L2}} + \dots + \frac{Z_O}{Z_{Ln}}\right)+1} \quad (5)$$

The output/input impedance of a converter is uniquely characterized by its operating points. That is, given the operating points of all the converters in a DPS, we can obtain their output/input impedances in stand-alone mode without connecting the entire network. With such information known, if outer loop gain T_O of the source converter is also known,

the changed outer loop of the source converter, T_O , can be predicted.

In the design phase, the loads of each load converter are usually known. Therefore, it is straightforward to obtain the operating points of the load converters. With the efficiency of

load converters known, the output current of the source converter is easily calculated, and so is the operating point of the source converter. Of course, there may be inaccuracies of the calculated operating point of the source converter because of tolerances of the efficiencies of each module. Fortunately, as mentioned in Section II, the nominal values often change little for a large range of operating points [9], and the error introduced is normally small.

Therefore, in the design phase, all data needed in (5) can be obtained from the stand-alone modules. An entire DPS does not need to be built to predict T_O . This reduces the workload greatly. More importantly, because the impact of each load converter on the outer loop gain of the source converter is known, we can manipulated the combination of converters and achieve a closer to optimally performing DPS instead of conservatively designed one.

A. Simulation

One possible method to predict the loop gain of the source converter in the DPS is to use (5) entirely in simulation. Because there are known small-signal models available to a designer, the input/output impedance and outer loop gain can be obtained by theoretical prediction. Then, simulation is convenient and easy to carry out. However, small signal stability simulation results tend to be suspect because they rely on averaging, which usually assumes ideal switching and ignores modeling opto-couplers, drivers, switch commutation, etc. Therefore, an experimental method is necessary to increase the accuracy [6]. The remaining part of the paper focuses on experimental prototypes.

B. Small Scale Prototypes

In order to predict T_O of a full-scale DPS, it is possible to build small-scale experiments and then scale-up the answer using (5). Based on simple measurements of benchmark experiments, it is possible to accurately predict the performance of a complicated large-scale DPS. Sometimes the benchmark experiments can be as simple as cascading two converters together.

Each impedance ratio $\frac{Z_O}{Z_{Lk}}$ can be uniquely characterized

for all operating points without being connected to a full-scale DPS. That is, given the operating point of each converter, it is possible to measure the (individual) impedance ratio of the source converter and one load converter instead, without building the full-scale DPS. Each individual impedance ratio measured can be added together

and be input into (5) to predict T_O .

This implies that scaled benchmark prototypes can be built, with one source converter and one load converter, in order to predict the performance when there are n load converters in a

DPS, where n is arbitrary. Although n sets of benchmark experiments may be necessary, it is still convenient to connect two modules instead of more. Moreover, with the impedance ratio known, the impact of certain load converter is known, and an optimal DPS can be designed. Another benefit is that, if the benchmark experiment is not stable, a correction to the problem can be made directly.

Fig. 2 shows implementations of the impedance ratio measurement for one load converter. Fig. 2(a) is similar to Fig. 1 except that a DC current sink I is used. A DC current sink is preferred because it ensures that the operating point of the source converter does not change when n load converters are applied. When the operation points of the converters are known, the input current of the k th load converter, I_{Lk} , and the output current of the source converter, I_O , are known. There is,

$$I = I_O - I_{Lk}$$

A DC current sink has theoretically infinite output impedance. Therefore, in Fig. 2(a), for the k th load converter,

$$b_{kA} \frac{Z_O}{Z_{Lk}} = \frac{v_2}{v_1} \quad \text{individual impedance ratio A}$$

$$k = 1, 2, \dots, n$$

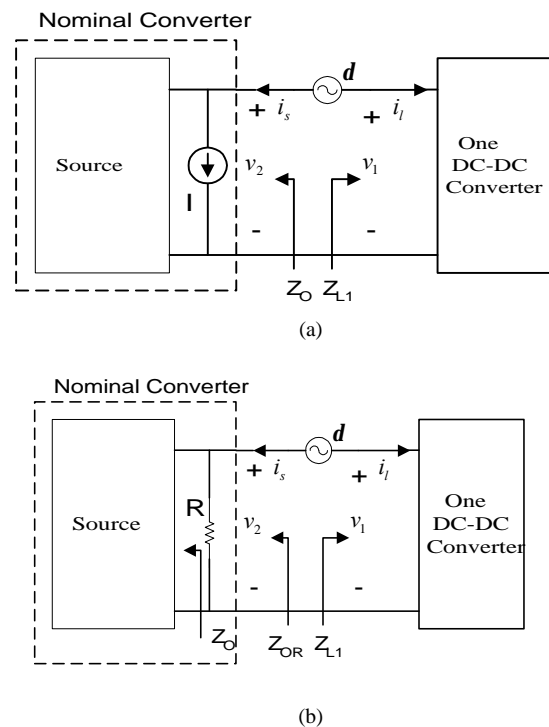


Fig. 2
 (a) Impedance ratio measurement for DPS with one load converter
 (b) Alternative individual impedance ratio measurement

It is possible to repeat the measurement n times until all \mathbf{b}_{kA} 's are obtained.

Then, assuming T_O known, T_O can be calculated with the formula (we use MATLAB to perform this calculation.)

$$T_O = \frac{T_O}{(1+T_O)(\mathbf{b}_{1A} + \mathbf{b}_{2A} + \dots + \mathbf{b}_{nA}) + 1} \quad (6)$$

Fig. 2(b) shows an alternative implementation. A resistor, R , is used to adjust the operating point. In this case, the measured impedance ratio is

$$\mathbf{b}_{kB} \frac{Z_{OR}}{Z_{Lk}} = \frac{v_2}{v_1} \quad \text{individual impedance ratio B}$$

$$k = 1, 2, \dots, n$$

where

$$Z_{OR} = Z_O // R.$$

If the output voltage of the source converter is V_O ,

$$R = \frac{V_O}{I_O} \frac{I_{Lk}}{I_{Lk}}$$

In many cases, $|Z_O| < 0.1R$. So, under this circumstance, we have

$$\frac{Z_O}{Z_{Ll}} \approx \frac{v_2}{v_1}$$

Because it is easier and cheaper to use a resistor rather than a DC current sink, this approximation simplifies the implementation. However, it brings some errors, which we discuss in Section V.

Hence, we are able to propose the following procedure to predict the outer loop gain of the source converter in a DPS:

Step 1: Determine the operating points of the prototype based on the operating points of the load converters and the source converter in the full-scale DPS;

Step 2: Measure or obtain nominal outer loop gain T_O of the source converter: Import the data into a simulation environment such as MATLAB.

Step 3: Build a prototype with the source converter and a load converter shown in Fig. 2, as previously described in the section;

Step 4: Measure individual impedance ratio of the prototype and record the data;

Step 5: Repeat *Step 3* until all individual impedance ratios of the load converters are obtained;

Step 6: Input the individual impedance ratios into simulation environment such as MATLAB. Use (6) to predict

the loop gain of the source converter in designed DPS (MATLAB easily performs these functions).

Step 7: Analyze the performance of the DPS based on predicted outer loop gain of the source converter such as phase margin and crossover frequency.

IV. PERFORMANCE PREDICTION OF DPS WITH N IDENTICAL CONVERTERS AS LOAD

In this section, the case of having n identical load converters in Fig. 1 is examined. (This is common in practice.) A simplified prediction formula and experimental procedure is given. Experimental verifications of the method are presented in Section V.

For the case of n identical load converters,

$$Z_L = \frac{Z_{Ll}}{n}$$

where Z_{Ll} is the input impedance of one load converter.

Then (5) can be rewritten as

$$T_O = \frac{T_O}{(1+T_O)n \frac{Z_O}{Z_{Ll}} + 1}$$

and (6) is simplified as

$$T_O = \frac{T_O}{(1+T_O)n\mathbf{b}_{1A} + 1} \quad (7)$$

Therefore, the performance of the source converter can be predicted if the impedance ratio of the output impedance of the source converter and that of one load converter and T_O , the outer loop gain of source converter, are known. The multiplication of $\frac{Z_O}{Z_{Ll}}$ by n can be performed in software.

This implies that when there are n identical load converters, the benchmark DPS can be further simplified to build only one small-scale prototype.

In this case, Step 5 in the above algorithm is eliminated since only one impedance ratio measurement is needed.

V. EXPERIMENTAL VERIFICATION

A system has been built to verify the proposed approach. The full-scale DPS consists of a forward source converter and 4 buck load converters. The source converter has 34V-75V input voltage and 3.3V output voltage. The 4 identical load converters have 3.3V input voltage, 1.9V output voltage, efficiency 90%, and each has resistive load 0.8 Ω .

The operating point of the load converter is input voltage 3.3V and output current 2.375A. Considering the efficiency of the load converter, the output current of the source converter is 6 A. First, the nominal outer loop gain of the

source converter under the operating point was measured and recorded on disk. Then, a prototype with one load converter and the source converter was built, as shown in Fig. 2, where the current sink is set to 4.5 A. The individual impedance ratio was measured and recorded. The data was imported into

MATLAB and manipulated according to (7) to give T_O .

From T_O , the phase margin and crossover frequency of the source converter in the full-scale DPS were easily obtained.

The magnitude and phase of the predicted loop gain T_O are plotted in Fig. 3. The crossover frequency and the phase margin of the outer loop gain of the source converter in the DPS both decrease because of the influence of the input impedance of load converters.

As Fig. 3 shows, the nominal outer loop gain has phase margin 69.9 degrees and crossover frequency 3.47 kHz. For the full-scale DPS, the measured source converter has phase margin 58.6 degrees and crossover frequency 3.17 kHz. Using individual impedance ratio A and (7), the predicted phase margin is 58.9 degrees, and the predicted crossover frequency is 3.12 kHz. Using individual impedance ratio B and (7), the predicted phase margin is 59.9 degrees, and the predicted crossover frequency is 3.32 kHz. In both cases, the predicted phase margin closely matches the measured phase margin.

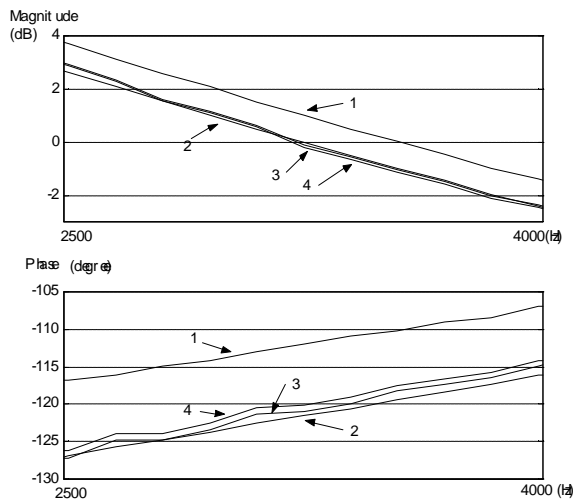


Fig. 3. Curve (1) is the nominal outer loop gain of source converter. Curves (2) – (4) consider four identical buck converters as the load. Curve (2) is the measured outer loop gain of source converter. Curve (3) is the predicted outer loop gain using individual impedance ratio A. Curve (4) is the predicted outer loop gain using individual impedance ratio B.

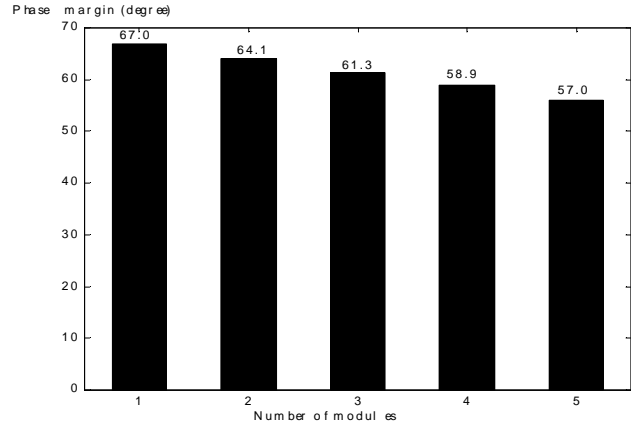


Fig. 4. Prediction of phase margin of source converter with n load converters

Often, a customer desires to know how many load converters can be connected to the source converter before performance begins to degrade. A common indicating standard is to maintain around a 60 degrees phase margin in the outer loop gain of the source converter. For this example, therefore, it is possible to safely have 3-4 buck converters as the load converters.

Fig. 4 shows a typical graph that can be created using the data from the simple prototype with one source converter and one load converter when the operating point of the load converter is known. Using measurements of the prototype and MATLAB, Fig. 4 shows how it is possible to predict outer loop gain performance of the source converter for an arbitrary number of converters, simply by manipulating (7).

VI. CONCLUSION

This paper discusses performance of a DPS by predicting the outer loop gain of the source converter. With the small-scale prototypes proposed in the paper, it is possible to accurately predict the outer loop gain of the source module in a DPS without building the full-scale DPS. Experiments verify the predictions

This paper only discusses the performance of DPS based on the outer loop gain of the source converter. The instant intuition is that, as the source converter is stable and so is the output voltage, the performance of DPS is guaranteed with the load converters having a stable input voltage. However, from the point of view of a load converter, other subsystems can be viewed as a source. The output impedance of the source will be a part of the input filter of the load converter, which may influence the stability of the load converter. Moreover, for current mode control, the change of the input filter will change its inner loop gain. Therefore, the outer loop gain of the load converter may also change. Future research may investigate the influence on the performance of the load converters in DPS, as well as the influence of changing operating points.

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