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STABILITY OF A CONTINUOUS STIRRED REACTOR WITH DELAY IN THE RECYCLE STREAM

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ABSTRACT: This paper discusses the local stability properties of a first order, exothermic, irreversible reaction carried out in a well mixed continuously stirred tank reactor [CSTR] with a delayed recycle stream.

KEY WORDS: Delay, stability, chemical reaction.

I. INTRODUCTION

In the last forty years, there has been a great deal of literature published discussing the dynamics of continuous stirred chemical reactors (CSTR) [1-7]. The breakthrough work by Bilous and Amundson [1] developed methods of calculating criteria for stability and instability of a first order irreversible, exothermic reaction in a CSTR.

Since Bilous and Amundson's work, many authors have written on similar topics using much of the same techniques that Bilous and Amundson used. Control schemes [2,4,5] were developed using both the model and the stability/instability criteria of [1].

Several papers which discuss stability of a CSTR also include discussions on the effects of a recycle stream [1,6,7]. Reactor recycling not only increases the overall conversion, but also reduces the cost of a reaction, and therefore, is very popular in industry. Recycling is generally used in adjunct to various control schemes.

A problem with the literature written on stability of a CSTR with recycle is that the model almost always assumes no time delay in the recycle line [1,6,7]. While this assumption may make theoretical analysis simpler, it is highly unrealistic. In order to recycle, the output must be separated from the input, then travel through pipes after separation. This process often requires a great deal of time.

In [8], methods of controlling reactions in CSTR's with recycle delay are thoroughly discussed. However, [8] includes no discussion on stability of these delayed systems either. Since often times control algorithms for systems with delays are both complicated and expensive, it would seem reasonable to ask whether it is always necessary to implement feedback for a desired response. In order to answer this question, local stability properties of the reaction need to be understood.

II. THE MODEL

Consider the first order, exothermic, irreversible reaction $A \rightarrow B$ carried out in a well mixed continuously stirred tank reactor. Suppose that fresh feed of pure A is to be mixed with a recycled stream of unreacted A with recycle flow rate $(1-\lambda)q$. If there is a delay, α, in the recycle stream, then the material and energy balances become

$$V\frac{dA}{dt} = \lambda q A_o + q(1-\lambda)A(t-\alpha) - qA(t) - VK_o\exp\left\{\frac{-Q}{T}\right\}A(t)$$

$$VC\rho\frac{dT}{dt} = qC\rho\left[\lambda T_o + (1-\lambda)T(t-\alpha) - T(t)\right] + V(-\Delta H)K_o\exp\left\{\frac{-Q}{T}\right\}A(t) - U(T(t)-T_w) \quad (1)$$

where A(t) is the concentration of A, T(t) is temperature, λ is the coefficient of recirculation with $0 \le \lambda \le 1$, and descriptions of all other constants can be found in [1].

The local stability properties of the steady states can be analyzed by stability analysis of the linear delayed variational equation about each equilibrium. More specifically, it is known that if the linear variational equation corresponding to Eq. (1) is asymptotically stable, then the steady state is locally stable. The linear variational equation of Eq. (1) about steady states $A(t) = A(t-r) \triangleq A_s$ and $T(t) = T(t-r) \triangleq T_s$ is given as

$$\dot{x}(t) = Cx(t) + bx(t-\alpha) \quad (2)$$

where $x(t) = [A(t),T(t)]^T$, $C \in \mathbf{R}^{2x2}$, $b \in \mathbf{R}$, and

$$c_{11} = \frac{-q}{V} - K_o\exp\left\{\frac{-Q}{T_s}\right\},\ c_{12} = \frac{-K_oQA_s}{T_s^2}\exp\left\{\frac{-Q}{T_s}\right\},$$

$$c_{21} = \frac{(-\Delta H)K_o}{C\rho}\exp\left\{\frac{-Q}{T_s}\right\},$$

$$c_{22} = \frac{-q}{V} + \frac{(-\Delta H)QK_oA_s}{T_s^2C\rho}\exp\left\{\frac{-Q}{T_s}\right\} - \frac{U}{VC\rho}, \text{ and}$$

$$b = \frac{q(1-\lambda)}{V}.$$

III. LOCAL STABILITY

This section develops conditions under which Eq. (2) is asymptotically stable. Consider, first, the more general delay differential equation

$$\dot{z}(t) = Nz(t) + bz(t-r) \quad (3)$$

where $N \in \mathbf{R}^{nxn}$, $b \in \mathbf{R}$, and $r \ge 0$. Define $\lambda(N)$ as the eigenvalues of matrix N and $Re[\lambda(N)]$ as the real part of those eigenvalues. Let Det(N) denote the determinant of matrix N.

Theorem: Suppose

i. $Det(N) \neq 0$.
ii. $Re[\lambda(N)] + |b| < 0$.

Then all solutions to Eq. (3) are asymptotically stable for any $r \geq 0$.

Proof: Assume conditions i. and ii. of the theorem are true. Since $Det(N) \neq 0$, matrix N has full rank and there always exists a similarity transformation $z(t) = Py(t)$, with $P \in \mathbf{R}^{nxn}$, such that

$$\dot{y}(t) = J_n y(t) + by(t-r),$$

$$J_n = \begin{bmatrix} \gamma_1 & 0 & & & \\ 0 & \gamma_2 & 1 & & \\ & 0 & \gamma_2 & 1 & \\ & & \ddots & & \\ & & & \ddots & \\ & & & & \gamma_k \end{bmatrix}, \gamma_j \in C,\ j = 1,2,...k, \tag{4}$$

where $P^{-1}NP = J_n$, the nxn Jordan canonical matrix of N.

The transcendental characteristic equation of system (4) is

$$\Delta(s,r) = \sigma_1(s,r)^{m_1} \cdot \sigma_2(s,r)^{m_2} \cdots \sigma_k(s,r)^{m_k},$$

where $\sigma_j(s,r) = (s - \gamma_j - be^{-rs})$, m_j is the multiplicity of eigenvalue $\gamma_j \in C$, and $\sum_{j=1}^{k} m_j = n$.

In order for the solutions to Eq. (4) to be asymptotically stable for any $r \geq 0$, they must first be stable for $r = 0$. When $r = 0$, $\Delta(s,0) = \prod_{j=1}^{k} \sigma_j(s,0)^{m_j}$, where $\sigma_j(s,0) = s - \gamma_j - b$. Therefore, $\Delta(s,0) = 0$ has all solutions with $Re(s) < 0$ when $Re[\gamma_j] + b < 0$ which is guaranteed by condition ii of the theorem.

Since each $\sigma_j(s,r)$ is continuous in both s and r, for a solution of $\sigma_j(s,r) = 0$ to move into the right half plane, there must exist some fixed $r_o > 0$ such that $\sigma_j(i\varphi,r_o) = 0$, where $\varphi \in \mathbf{R}$, i.e.

$$b\cos(r_o\varphi) + Re(\gamma_j) = 0 \tag{5}$$

$$b\sin(r_o\varphi) - Im(\gamma_j) + \varphi = 0 \tag{6}$$

Condition ii. of the theorem guarantees that Eq. (5) is never true. This proves the theorem. •

Applying the Theorem to the linear variational equation of the CSTR with delayed recycle stream as given in Eq. (2), it is seen that solutions to Eq. (2) are asymptotically stable for any recycle delay if it is stable when there is no recycle delay, $(\alpha = 0)$.

This is a surprising result since it is often thought that delays are destabilizing, but, in fact, local stability of the steady states of this first order reaction are not affected by the size of recycle delay. While these results are only valid for the first order reaction supplied, it does suggest that in some cases it may be possible to ignore the recycle delay when modeling the CSTR.

IV. CONCLUSIONS

The surprising results of this paper suggest that in a first order, irreversible, exothermic reaction $A \rightarrow B$ carried out in a CSTR, the delay in the recycle stream does not affect stability of steady states. Of course, the delay still may cause increased oscillation and perhaps, a slower response time, however, local stability is unchanged. This may allow the engineer the ability to ignore the recycle delay in the model when developing a control scheme. However, further investigation of this statement still needs to be performed.

The results of this paper are only valid for the first order reaction modeled. Higher order reactions may not be so simple since the linearized model may not be in the form of Eq. (2). However, this paper does present analysis techniques for delay equations which should be valid for all chemical reactions.

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