On the Modeling and Analysis of Jitter in ATE Using Matlab

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Abstract

This paper presents a new jitter component analysis method for mixed mode VLSI chip testing in Automatic Test Equipment (ATE). The separate components are analyzed individually and then combined using Matlab. The Matlab simulation shows how jitter components combine and how the total jitter depends on the jitter injection sequence. The relationship among jitter components is presented and the superposition of the jitter components is verified. This new technique gives test engineers an insight into how the jitter components interact.

1. Introduction

As the data rates of VLSI systems reach several gigabits per second, timing jitter have become more significant in ATE (Automatic Test Equipment) systems. Therefore, a correct model and analysis of the timing errors and jitter will provide more accurate characterizations of high-speed VLSI systems.

Timing Jitter (henceforth referred to as jitter) is defined as the deviation of a signal transition from its ideal position in time. The Total Jitter (TJ) consists of two components: the Deterministic Jitter (DJ) and Random Jitter (RJ). Assuming that the each jitter component is independent, the distribution of TJ will be the convolution of the distributions of DJ and RJ. The DJ consists of several subcomponents. These may include Electromagnetic Interference, Cross-Talk, Bandwidth Limitation, and etc. DJ has a bounded peak-to-peak value that does not increase when more samples are taken. The RJ comes from device noise sources such as thermal and flicker noise. It is theoretically unbounded in amplitude, and is characterized by a Gaussian distribution. Multiple random jitter sources add in an rms fashion, but a peak-to-peak value is needed to get total peak-to-peak jitter when RJ is combined with DJ. Although Gaussian statistics imply an "infinite" peak-to-peak amplitude, a useful peak-to-peak value can be calculated from the rms value after a probability of exceeding the peak-to-peak value is established. For example, the peak-to-peak random jitter has less than 10^{-12} probability of exceeding is 14.1 times the rms value [1].

Many works have been reported on jitter measurement and analysis. It is relatively simple to measure each jitter component but it is challenging to measure and analyze them if multiple jitter components are simultaneously injected. However, there has not been a consensus on a standard methodology for separating measured total jitter into components. Only a few methods such as the Tailfit Algorithm, One-Shot Time Interval Methodology, and Spectral Methodology have been proposed on the this critical issue [2][3][4][5].

The objective of this paper is to determine how jitter components can be modeled and combined, and how the total jitter can be changed according to different injection sequence.

In this paper, a novel and standard amenable jitter component analysis and combining methodology are proposed, and Matlab is used to demonstrate the effectiveness of the methodology. The remainder of this paper is organized as follows. Section 2 shows the jitter classification and the definition of each model for the components. Section 3 presents jitter combining and measurement experiments followed by conclusion in Section 4.

2. Jitter Classification

Deterministic Jitter (DJ) consists of Duty-Cycle Distortion (DCD), Inter-Symbol Interference (ISI), Periodic Jitter (PJ), and Bounded Uncorrelated Jitter (BUJ). DCD and ISI are referred to as data correlated jitter, while PJ, RJ and BUJ are referred to as data uncorrelated jitter [6][7]. Figure 1 shows a block diagram of the jitter classification. Accurate jitter models and their analysis are essential for better prediction and characterization of jitter effects in high-speed VLSI systems.

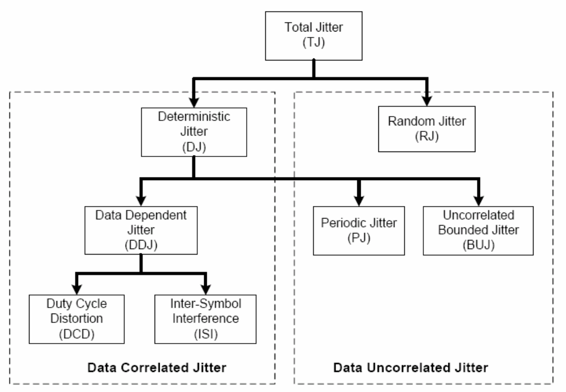


Figure 1. Jitter classification

2.1. Periodic Jitter (PJ) Model

Electromagnetic Interference (EMI) can cause a periodic deviation of a signal transition from its expected location. This type of deviation is referred to as Sinusoidal or Periodic Jitter, which repeats in a cyclic fashion. PJ is typically uncorrelated to any periodically repeating patterns in the data stream. The model of PJ is summation of cosine functions with phase deviation, modulation frequency, and peak amplitude [6]. The model is represented by

$$PJ_{total}(t) = \sum_{i=0}^{N} A_i \cos(\omega_i t + \theta_i) \tag{1}$$

where $PJ_{Total}(t)$ denotes the total periodic jitter, N is the number of cosine components (tones), A_i is the amplitude in units of time, ω_i is the modulation frequency, t is the time, and θ_i is the initial phase.

2.2. Duty Cycle Distortion (DCD) Model

DCD results in high bit cells having a different width from low bit cells. It is caused by a difference in propagation delay between low to high transitions and high to low transitions. The sources of the DCD can be offset errors, turn-on delays and saturation. Figure 2 is the proposed DCD model that generates the duty cycle effect in a signal.

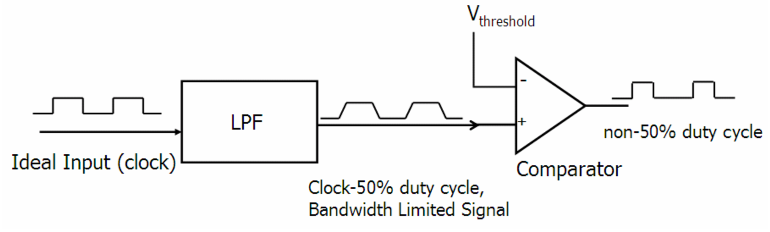


Figure 2. DCD model

2.3. Inter-Symbolic Interference (ISI) Model

ISI is dependent on the data pattern, date rate, data slew rate, and frequency response of the data path. The step response of ISI model might be approximated by the three-pole response. Therefore, our ISI model is a 3-pole LPF system that consists of a 1-pole and 2-poles LPF system. It is given by

$$H_{ISI}(s) = \frac{a}{(s-\omega_1)} \cdot \frac{b}{(s^2 - 2\xi\omega_2 s + \omega_2{}^2)} \tag{2}$$

where a and b are constants. ω_1 is a single pole of the 1-pole system; ω_2 is the natural frequency and ξ is the damping ratio of the 2-poles system.

The ISI model may not reflect all the effect of a lossy line, which is the dominant cause of ISI in the real world, but if the settling time of the 3-pole LPF system is greater than 2-bit Unit Interval (UI), it will be a good estimate to the lossy line [8].

High-frequency losses caused by the skin effect and dielectric loss also affect ISI. These effects are frequency-related. The skin effect is proportional to the square root of the frequency, while the dielectric loss is proportional to the frequency [9]. Therefore, the skin effect dominates data loss at a lower frequency, whereas the dielectric loss dominates at a higher frequency.

2.4. Random Jitter (RJ) Model

RJ is theoretically unbounded in amplitude, and is characterized by both Gaussian and non-Gaussian Distributions. In this paper, it is assumed that RJ has simple Random Gaussian distribution (like thermal and flicker noise) and it is given by

$$J_{RJ}(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{-(\frac{x^2}{2\sigma^2})} \tag{3}$$

where σ is the standard deviation of the jitter distribution or the rms value, and J_{RJ} is the probability that the edge will occur at time x, where x is the deviation from the mean value of the transition time.

The observed effect of RJ depends on the number of samples. The relationship between rms and peak-to-peak jitter conversion as shown in Equation (4) is used to compute the peak-to-peak jitter [1]. This relationship is given by

$$Jitter_{p-p} = \alpha \times Jitter_{RMS} \tag{4}$$

where α is determined by

$$0.5 \cdot erfc\left(\frac{\alpha}{2 \cdot \sqrt{2}}\right) = BER \text{ (Bit Error Rate)} \tag{5}$$

3. Jitter Combining Experiments

This section presents different jitter combining methods to illustrate the injection sequence dependence of the jitter components on the injection sequence.

3.1. Jitter Modeling Specification

The jitter combining scheme is simulated using Matlab. Input data is an ideal square wave sequence of 2,540 bits in length such that Pseudo-Random Binary Sequence (PRBS)-7 pattern is 40. The bit rate of the data pattern generator is 5G bits/sec, and the sampling rate of the simulator is 1,000 samples/bit cell.

Figure 3 illustrates the combining method. The sequence of each jitter model can be interchanged to study the impact of different injection sequences. The PRBS-7 pattern as the data source 1 is generated and applied to a low-pass filter block. The LPF is selected such that the slope of the waveform going into the driver input introduced the appropriate jitter at the driver input. The PJ model is a single-tone sinusoidal jitter with rms value β injected at a comparator input, and the RJ model is a Gaussian noise signal with rms value α. In addition, the DCD model changes the duty cycle of a signal, and the ISI model is the three-pole Low-Pass Filter (LPF) discussed in Section 2.3. The pole locations is (-10±17.3i)GHz and 17GHz. In addition, the data source 2 related to crosstalk of transmission line is not considered active in this case.

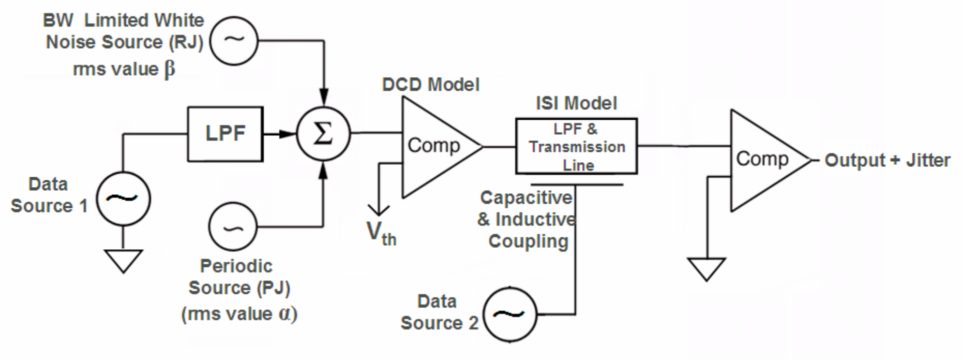


Figure 3. Basic combining method for jitter

Assuming that each jitter component is already known, the total jitter Probability Density Function (PDF) is given by the convolution of the PDF's of each component [6][7] as shown in Equation (6).

$$TJ(x) = RJ(x) * PJ(x) * ISI(x) * DCD(x) \tag{6}$$

where each function is the Probability Density Function (PDF) of TJ and each subcomponents, and "*" is a convolution operation.

In this experiment, the above two approaches is used to combine and measure jitter.

3.2. Jitter Measurement

The jitter is measured using the eye-diagram, histogram, and the frequency spectrum. The jitter rms and peak-to-peak jitter value define the amount of the jitter. The Tailfit algorithm [2] is used to separate DJ and RJ and measure peak-to-peak TJ as shown in Figure 4. For a BER of 10^{-12}, the peak-to-peak value of TJ is given by

$$\begin{aligned} Jitter_{total\_p-p} &= Jitter_{DJ} + 14.069 \times Jitter_{RJ\_RMS} \\ Jitter_{DJ} &= a - b \\ Jitter_{RJ\_RMS} &= \frac{\sigma_a + \sigma_b}{2} \end{aligned} \tag{7}$$

where $Jitter_{DJ}$ is the deterministic jitter, and $Jitter_{RJ\_RMS}$ is the rms value of the RJ. The RMS value of the jitter is the standard deviation of the jitter. The standard deviation of the two side lobes are extracted by the algorithm and evaluated separately as σ_a and σ_b.

In case of jitter rms, total rms (σ_{total}) is calculated from the rms values of each components as shown in Equation (8); direct measure from TJ distribution was used to represent TJ rms.

$$\sigma_{total} = \sqrt{\sigma_{RJ}^2 + \sigma_{PJ}^2 + \sigma_{ISI}^2 + \sigma_{DCD}^2} \tag{8}$$

In addition, the frequency spectrum is performed by FFT on the jitter to show the spectral content of the jitter.

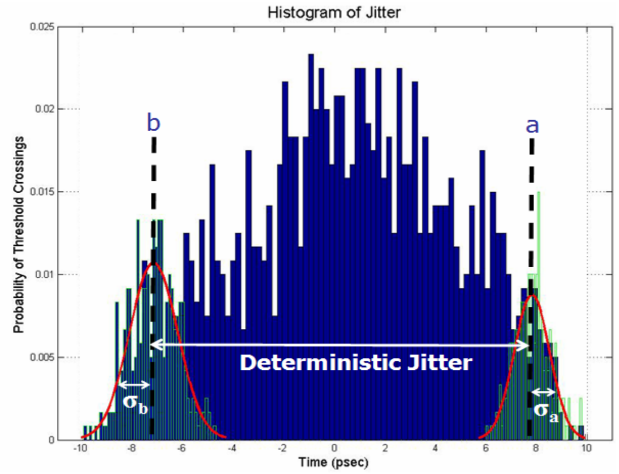


Figure 4. Jitter measurement using Tailfit algorithm

3.3. The First Experiment

In the first experiment, PJ model is a single-tone sinusoidal jitter with an rms value of 3.57ps. The RJ model is a Gaussian noise signal with a rms value of 0.7ps. The ISI model is simulated using the three pole low-pass filter described above and the DCD model changes the duty cycle of data signal. All values are typical of those observed in a serial data system. Based on the proposed method, the sequence of each jitter model is changed, and Table 1 shows five different sequences and a convolution of the jitter subcomponents. The Matlab system permits random numbers to be changed during every simulation. In order to obtain repeatable results, a PRBS-7 signal and a Gaussian noise are generated once, and used in all sequences.

The individually injected jitter components are shown in Figure 5(a). The jitter peak-to-peak of RJ is not the peak-to-peak jitter with a value 14.1 times the rms jitter but the measured peak-to-peak jitter within the 2,540 data sequence in order to illustrate actual effect of RJ on the other jitter components. The PJ has a peak-to-peak value that is $2\sqrt{2}$ times the rms value. The DCD has a peak-to-peak value that is twice the rms value.

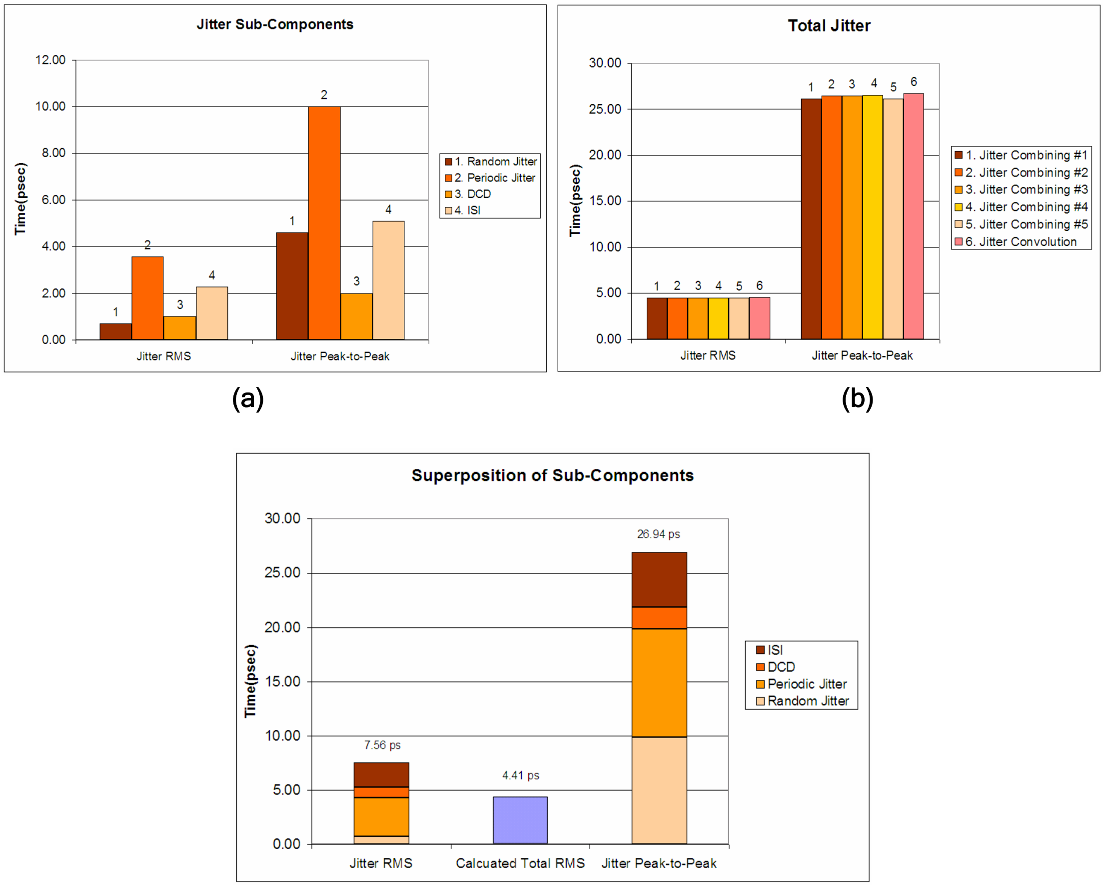
Table 1: Jitter combining methods

# Method	Subcomponent Sequence of Combining Jitter
Combining #1	Data Generator → (PJ+RJ) → DCD → ISI
Combining #2	Data Generator → PJ → DCD → ISI → RJ
Combining #3	Data Generator → DCD → (PJ+RJ) → ISI
Combining #4	Data Generator → DCD → ISI → (PJ+RJ)
Combining #5	Data Generator → ISI → DCD → (PJ+RJ)
Combining #6	Convolution of (PJ & RJ & ISI & DCD)

In this experiment, the jitter components shown in Figure 5(a) are applied in the different sequences that are shown in Table 1. TJ is measured using the rms and peak-to-peak jitter as mentioned in Section 3.2. Assuming that each jitter component is independent, TJ will be convolution of each jitter subcomponents, and the total jitter is obtained as shown in Figure 6. Since each jitter component is independent, in this experiment the conventional convolution-based jitter combining method is regarded as the golden model and compared with the other combining method.

This experiment shows that the combining method changes the TJ, but the difference is not significant as shown in Figure 5(b). Apparently, the sequence of injection did not significantly affect the results. All combining methods results in almost the same TJ as the convolution-based method. The peak-to-peak results appear to be about 6 times the rms values.

Figure 5(c) represents the superposition of jitter component, where RJ peak-to-peak is 14.1 times RJ rms. It shows that superposition applies when jitter components are combined, and that the calculated jitter rms can be obtained by Equation (8). When Figure 5(c) compares with Figure 5(b), peak-to-peak jitter is almost same within 3% difference which can come from sampling error, or measurement error. However, the superposition of jitter rms is not the same as the total jitter rms, while the calculated jitter rms is almost same within 3% difference, which means Equation (8) is valid. As a result, superposition can apply to jitter combining only in case of peak-to-peak jitter.



(a)

(b)

(c)

Figure 5. Jitter measurement, (a) Injected jitter subcomponents, (b) Total jitter depending on combining sequence, (c) Superposition of jitter subcomponents

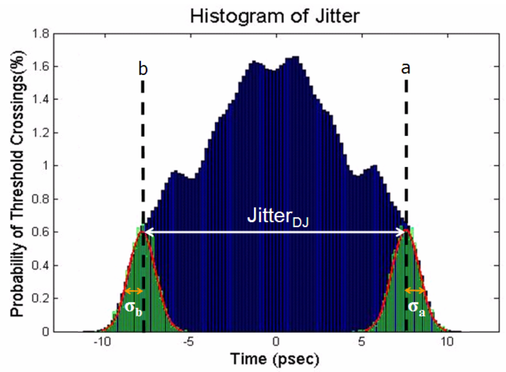


Figure 6. Total jitter using the convolution method

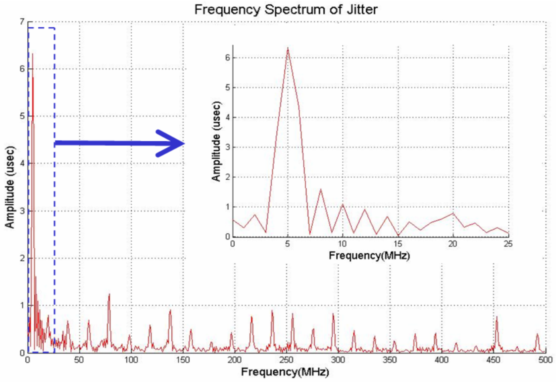


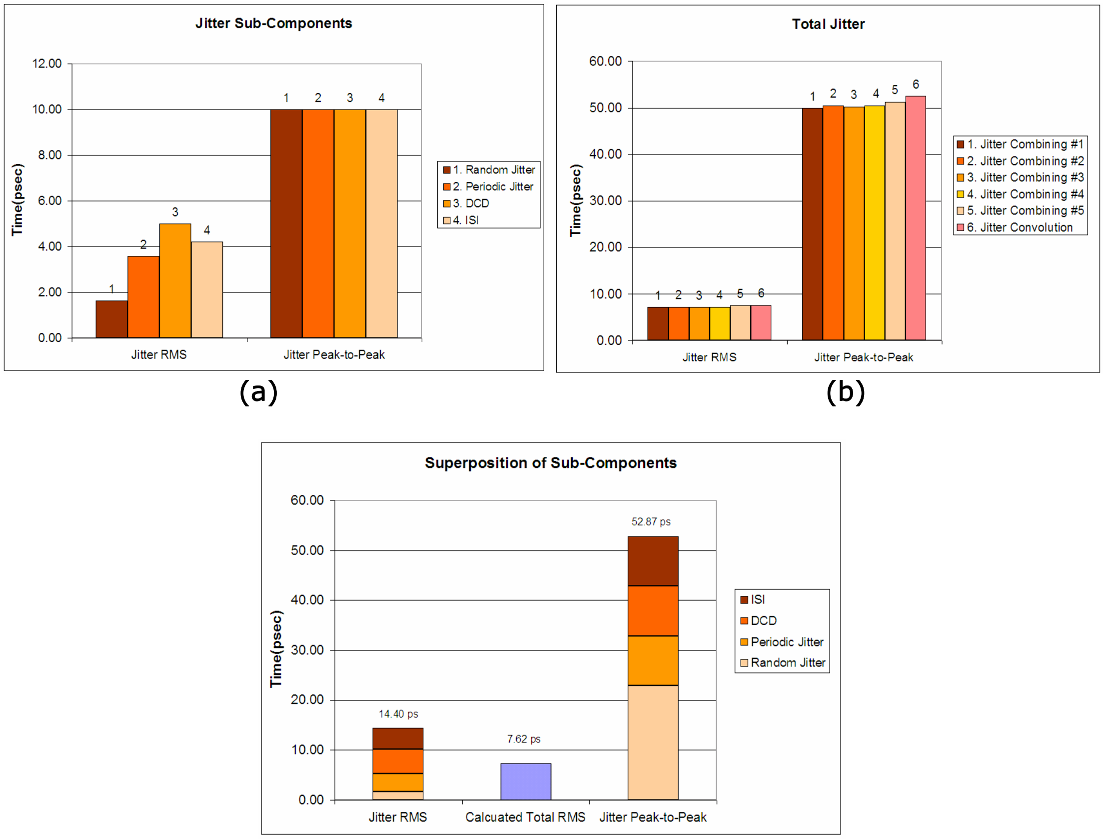
Figure 7. Frequency spectrum of TJ

Figure 7 shows the frequency spectrum of jitter combining #1 which represents the 1-tone PJ (5MHz), ISI, DCD, and RJ. The other combining methods also generate the similar frequency spectrum with only slight amplitude difference, i.e. the combining methods have little effect on the frequency spectrum of TJ.

3.4. The Second Experiment

In the second experiments, all the conditions are the same except that the amplitude of the injected subcomponents is changed. Figure 8(a) shows the magnitude of each of the injected jitter subcomponents. Since the peak-to-peak jitter shown in Figure 8(b) are almost the same, it appears that the assumption that superposition holds is correct.

Figure 8(c) shows the superposition of jitter subcomponent, where RJ peak-to-peak is set to be 14.1 times RJ rms. It again shows that superposition holds just in case of peak-to-peak jitter. If Figure 8(b) and (c) demonstrate that the peak-to-peak jitter by both methods of calculation are within 5% difference. As a result, superposition of jitter is demonstrated in both experiments. In addition, the calculated jitter rms by Equation (8) is again valid.



(a)

(b)

(c)

Figure 8. Jitter measurement of the second experiment, (a) Injected jitter subcomponents, (b) Total jitter depending on combining sequence, (c) Superposition of jitter subcomponents

4. Conclusion

This paper has developed models of jitter components and jitter combining methods using Matlab as a simulator of an ATE system. In the jitter modeling, PJ model was a single-tone sinusoidal jitter, RJ was a Gaussian noise signal, ISI model was a 3-pole LPF, and DCD model generated the duty cycle effect of a signal. The components models have been developed and characterized in order to predict overall system jitter.

It has been demonstrated how jitters combine, and how the jitter varies with jitter injection sequence. Matlab was used to inject each subcomponent in five injection sequences. The jitter rms values and peak-to-peak values were compared with one another. The convolution of each jitter components was presented, and compared with the TJ of each jitter combining. As a result, it has been shown that TJ does not depend on the jitter injection sequence, and that superposition applied. The jitter modeling and combining method using Matlab should contribute to standardization of a total jitter simulation. More detailed models are currently under development.

5. References

[1] Maxim, "Converting between RMS and Peak-to-Peak Jitter at a Specified BER." Application Note, HFAN-4.0.2, Rev1; 2/-3; http://pdfserv.maxim-ic.com/arpdf/AppNotes/3hfan402.pdf

[2] Mike P.Li, Jan Wilstrup, et. al, "A New Method for Jitter Decomposition Through Its Distribution Tail Fitting." IEEE International Test Conference, pp788-794, 1999

[3] Jan Wilstrup, Corporate Consultant, " A Method of Serial Data Jitter Analysis Using One-Shot Time Interval Measurements.", IEEE International Test Conference, page 819-823, Oct. 1998.

[4] John Patrin, Mike Li, "Comparison and Correlation of Signal Integrity Measurement Techniques," DesignCon 2002.

[5] Agilent Technologies Inc. "Jitter Analysis Techniques for High Data Rates", Application Note 1432, February 2003;http://cp.literature.agilent.com/litweb/pdf/5988-8425EN.pdf.

[6] Wavecrest Corp. "Understanding Jitter," application note, 2001;

http://www.wavecrest.com/technical/VISI_6_Getting_Started_Guides/6understanding.PDF

[7] Nelson Ou, Touraj Farahmand, et al., "Jitter Models for the Design and Test of Gbps-Speed Serial Interconnects," IEEE Design and Test of Computers, vol. 21, Jul-Aug 2004.

[8] Jie Sun, and Mike Li, "A Generic Test Path and DUT Model for DataCom ATE", IEEE International Test Conference(ITC), Vol. 1, page 528-536, 2003

[9] Howard W. Jonson, Martin Graham, "High-Speed Digital Design-A Handbook of Black Magic." PTR Prentice-Hall, 1993.