

# Circuit Implementation of FitzHugh-Nagumo Neuron Model Using Field Programmable Analog Arrays

Jun Zhao

Department of Electrical and Computer Engineering  
Northeastern University  
Boston, MA 02115  
jzhao@ece.neu.edu

Yong-Bin Kim

Department of Electrical and Computer Engineering  
Northeastern University  
Boston, MA 02115  
ybk@ece.neu.edu

**Abstract**— A simple neuron model, the FitzHugh-Nagumo (FHN) model, is implemented on Field Programmable Analog Arrays (FPAAs). The differential equations of the model is integrated by making arithmetic operations on the reconfigurable voltage model circuits of the FPAAs. The simulation and implementation results demonstrate that FPAA is the viable candidate for the neuron hardware implementation in real time or many orders of magnitude faster.

## I. INTRODUCTION

Neural models are used in both computational neuroscience and in pattern recognition. In both cases, the highly parallel nature of the neural system contrasts with the sequential nature of computer systems, resulting in slow and complex simulation software. More direct hardware implementation holds out the promise of faster emulation both because it is inherently faster than software and the operation is much more parallel. In fact, direct hardware implementation of neural models has a relatively long history. Lewis published several electronic membrane models that are based on the Hodgkin-Huxley equations with discrete transistors and associated components [1]. Roy published an electronic membrane model, analogous to Lewis's, using FET transistors as adjustable conductors [2]. Harmon constructed a simple neuron model, which produces an action pulse in response to the stimuli in a similar manner to an actual neuron [3]. This model has reduced the complexity of neural network simulation. With the development of integrated circuit technology, it is also possible to produce electronic neuron models in large quantities [4,5].

However, the design, analysis, and verification of analog circuits are a very difficult task due, in part, to the inherent complexities of the basic techniques. Also, the aforementioned electronic neuron models have limited applications due to their manufacturing viability and bias voltages tuning problems. The best choice for electronic neuron implementation appears to be programmable hardware approaches like FPGAs and FPAAs. Rossmann [6] implemented a biologically inspired neuron model in FPGAs. Compared to digital circuitry, analog circuitry can perform complex, non-linear operations by a handful of transistors while digital circuitry requires hundreds or even thousands of wires and transistors to perform the same

operations. Analog computation also performs these operations using far less power per computation than digital computation. The dynamical reconfigurability of the circuits also provides the realization of robust and adapting systems, which is indispensable in neural systems.

This paper implements FitzHugh-Nagumo Neuron Model using FPAA to demonstrate the effectiveness of FPAA for analog neuron implementation.

## II. FITZHUGH-NAGUMO NEURON MODEL

The FitzHugh-Nagumo neuron model [7] is a mathematical abstraction for an excitable-oscillatory membrane, which is very adequate for emulation of small biological systems. The simplicity of the FitzHugh-Nagumo model permits the entire solution to be viewed at once. This allows a geometrical explanation of important biological phenomena related to neuronal excitability and spike-generating mechanism. The equations describing the dynamics are shown in (1) and (2):

$$\frac{dx}{dt} = c(x - y + z - x^3 / 3) \quad (1)$$

$$\frac{dy}{dt} = (x - by + a) / c \quad (2)$$

The variable  $x$  represents the membrane potential of the neuron, while the  $y$  is a recovery variable, causing the neuron to return to its resting state. The parameter  $z$  represents the current stimulation to the membrane model. It corresponds to the externally applied current intensity. We set  $a=0.7$ ,  $b=0.8$ ,  $c=3$  throughout the paper.

The dynamics of the FitzHugh-Nagumo neuron model with different values of the parameter  $z$  are illustrated in Fig.1. When  $z$  is close to zero, the system has a unique equilibrium point. The model behaves as an excitable membrane (*Fig.1(a)*). A strong enough excitatory stimulation delivered to the system at the equilibrium point leads to an action potential. If the stimulation is below threshold, the state point returns to the equilibrium point without passing the active region. As the  $z$  value increases, the focal equilibrium becomes less attractive, which leads to a relatively large radius around the equilibrium and finally converges to the equilibrium (*Fig.1(b)*). As  $z$  increase further, a neutral stable limit cycle (inside stable and outside

unstable) appears abruptly through a double cycle bifurcation (Figure 1(c)), leading to bistability. Finally, for still larger  $z$ , the stable equilibrium point loses its stability and the system shows the monostable limit cycle oscillation (Figure 1(d)).

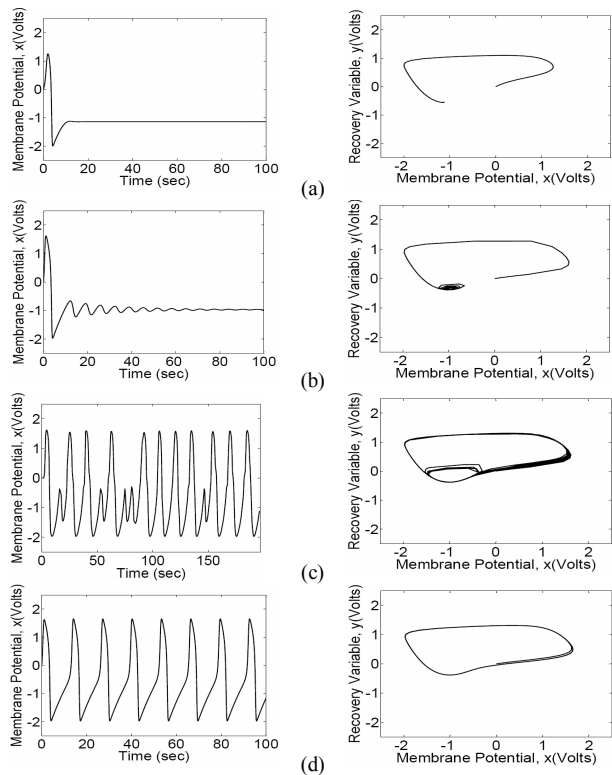


Fig. 1. Wave forms of the model in response to different values of parameter  $z$  and the corresponding  $x$ - $y$  phase portrait: (a)  $z=0.1$ , an excitable membrane with a small spiral; (b)  $z=0.33$ , an excitable membrane with a big spiral; (c)  $z=0.337$ , bistable with a stable equilibrium point and a stable limit cycle; and (d)  $z=0.34$ , an oscillatory membrane.

### III. CIRCUIT IMPLEMENTATION OF THE FITZHUGH-NAGUMO NEURON MODEL

#### A. FPA Technology

The Field-Programmable Analog Array is the analog equivalent of the FPGA. Unlike the FPGA, which contain a large number of modules and interconnections allowing arbitrary configurations of combinatorial and sequential logic, FPA devices typically contain a small number of CABs (Configurable Analog Blocks). They give designers the ability to describe analog functions such as gain stages and filters without reference to underlying function-in other words, without having to think on the level of such components as op amps, capacitors, resistors, transconductors, and current mirrors. Lifted to this higher level of abstraction, the FPA implementation allows the experimental characterization of the model dynamics with a low cost, reconfigurable and fast experimental setup.

The chips we have chosen to implement the neuron model are AN231E04's from Anadigm Inc. The core of the device is a two-by-two matrix of CABs (Configurable Analog Blocks) that can be connected with each other and with external I/O blocks. Each CAB contains two operational amplifiers, a comparator, and a set of capacitors. The FPA chip is programmed by switching the capacitors at a system frequency. Configurable analog modules (CAMs) such as integrators, inverters, sum blocks, and filters can be implemented easily due to the block level design. The modules in the chip samples their inputs at a specific phase and provide the output at the same phase or other phases. Each input branch has a programmable weight that can be selected independently.

The signals processed inside the FPA are fully differential voltage signals, but an external circuit can be used on the evaluation board to realized differential to single/single to differential conversion.

#### B. Implementation of the Model

The block diagram of the analog circuit to present the FitzHugh-Nagumo equations is shown in Fig.2. Two multiplier blocks are used to realize the function of  $x^3$  (Fig.2(a)). The differential equation of  $y$  is implemented by use of a three input SumIntegrator (Fig.2(b)). Similarly, the differential equation of  $x$  is implemented using a three input SumIntegrator (Fig.2(c)). More blocks have been used in this case because of the limited maximum input branches of 3. The parameters  $\alpha$  and  $z$  are realized by Voltage blocks. The coefficients of the two equations, which will be scaled by magnitude and time variables, are set by the internal programmable input weights.

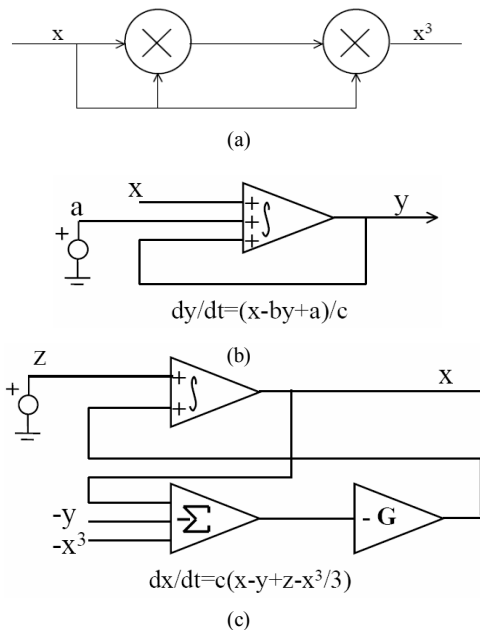


Figure 2. Block diagram for the FitzHugh-Nagumo equations

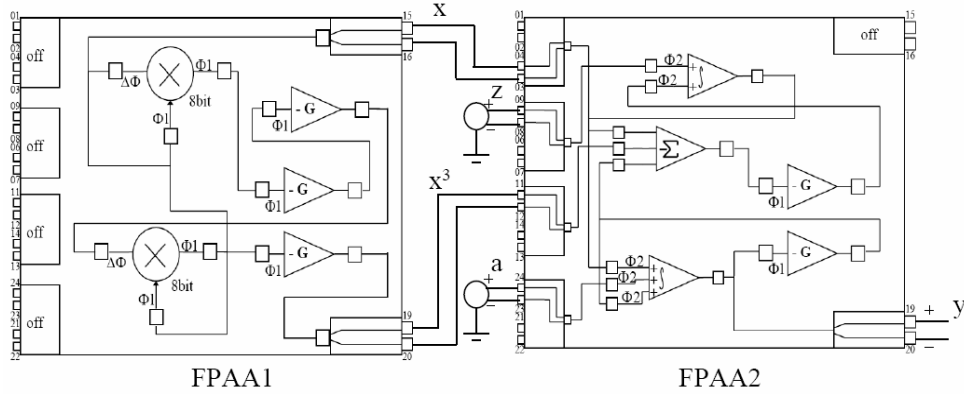


Figure.3. The two-chip FPAA based FitzHugh-Nagumo model circuit schematic.

The dynamic range of the signals in the FPAA device is bounded by physical constraints. The AN231E04's uses a reference voltage of 1.5V with upper and lower limits of 3.3V and  $-3.3V$  with respect to the reference voltage so that the magnitude and the frequency of the neuron output have to be scaled. The magnitude scaled and time scaled variables are defined as

$$x_{new} = \frac{x}{x_{ms}}, \quad t_{new} = T_s \times t \quad (3)$$

After these variables are applied to the original FitzHugh-Nagumo equations, the following new equations are obtained.

$$\frac{dx}{dt} = \frac{c}{T_s} \left( x - \frac{x_{ms}^2 x^3}{3} - \frac{y}{x_{ms}} + \frac{z}{x_{ms}} \right) \quad (4)$$

$$\frac{dy}{dt} = \frac{1}{cT_s} (x_{ms}x - by + a) \quad (5)$$

Since there are only four CABs for each chip, the dynamics that a single board could implement is relatively simple. However, the AN231E04's can be chained together

to realize more complex dynamics. In this paper, two chained boards are used to implement the FitzHugh-Nagumo neuron model (Figure.3). The FPAA1 chip converts the input of  $x$  to the output of  $x^3$ . The two GainHold blocks used here are to avoid the phase mismatch problems. The two separate differential equations in Fig. 2b and Fig. 2c are combined together and implemented as in the FPAA2 chip. It takes the FPAA1 output of  $x^3$  and two voltage variables  $a$  and  $z$ , controlled by the voltage blocks, as the inputs and finally outputs  $x$  and  $y$ . The FitzHugh-Nagumo equations are implemented effectively using two chained FPAAs.

#### IV. RESULTS

The response of FitzHugh-Nagumo neuron model with respect to different values of parameters  $z$  has been recorded with a 12-bit NI 6024E data acquisition card. Two external op-amp circuits have been used to convert differential output to single ended in order to draw the  $x$ - $y$  phase portraits. The amplitude of the membrane voltage  $x$  has been scaled down by 2 and the time variable  $y$  has been scaled down by 100 in order to meet the dynamic range of the FPAA circuitry. However the system is able to output oscillations with a wide range of scaling factors from real time to many orders of magnitude faster.

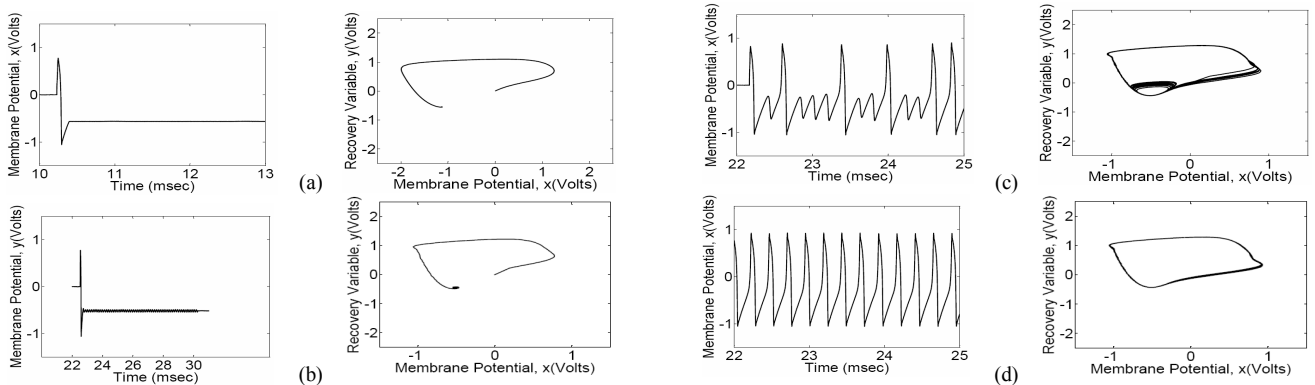


Fig.4. Four stages of the FitzHugh-Nagumo neuron model: (a) Small spiral at  $z=0.1V$ , (b) Big spiral at  $z=0.3V$ , (c) Bistable at  $z=0.32V$ , and (d) Oscillatory at  $z=0.35V$ .

Fig.4 shows the waveforms of the four stages of the FitzHugh-Nagumo neuron model recorded by NI DAQ system. When  $z$  is close to zero, the system has a unique equilibrium point with small spiral. As the  $z$  value increases, it has a large spiral around the equilibrium. As  $z$  increase further, a neutral stable limit cycle appears abruptly. Finally, for still larger  $z$ , the response of the neuron will become oscillatory. The results demonstrate the FPAA is a feasible technology to implement electronic neuron in a fast and low cost way.

## V. CONCLUSION

In this paper, the hardware implementation of the FHN neuron model using low-cost commercially available FPAA chips is presented. We found the proposed FPAA technology is simple and widely applicable. By choosing appropriate scaling factors, it could also realize neuron dynamics in real time or faster. Therefore it provides a low-cost, high performance, and dynamical reconfigurable analog circuit solution. This paper will be a good reference for the future implementations of the electron neurons and synapses using FPAA.

## REFERENCES

- [1] E.R. Lewis, "An electronic model of the neuron based on the dynamics of potassium and sodium ion fluxes," in *Neural Theory and Modeling. Proceedings of the 1962 Ojai Symposium*, RF Reiss, HJ Hamilton, LD Harmon, G Hoyle, D Kennedy, O Schmitt, CAG Wiersma, Eds., Stanford, 1964, p.427, Stanford University Press.
- [2] G. Roy, "A simple electronic analog of the squid axon membrane: The neurofet," *IEEE Trans. Biomed. Eng.*, vol. 19, no. 1, pp. 60-63, 1972.
- [3] L. D. Harmon, "Studies with artificial neurons, I: Properties and functions of an artificial neuron," *Kybernetik*, vol. Heft 3, pp. 89-101, 1961.
- [4] S. Prange, "Emulation of biology-oriented neural networks," in *Proc. Int. Conf. On Parallel Processing in Neural Systems and Computers (ICNC)*, M. Eckmiller, Ed., Dusseldorf, 1985. R. Nicole, "Title of paper with only first word capitalized," *J. Name Stand. Abbrev.*, in press.
- [5] M. Mahowald, and R. J. Douglas, "A silicon neuron," *Nature*, vol. 354, pp. 515-518, 1991.
- [6] M. Rossmann, B. Hesse, K. Doser, A. Buhlmeier, and G. Manteuffel, "Implementation of a biologically inspired neuron model in FPGA," in *Proc. 5th Int. Conf. On Microelectronics for Neural Networks and Fuzzy Systems*, Lausanne, 1996, pp. 322-330.
- [7] R. FitzHugh, "Impulses and physiological States in theoretical models of nerve membrane," *Biophys. J.*, 1961, 1, pp. 44-466.