A MULTI-RESOLUTION ADMISSIBLE SOLUTION APPROACH FOR DIFFUSE OPTICAL TOMOGRAPHY

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ABSTRACT

We address the use of diffuse light to characterize the space-varying absorption coefficient in tissue, posed as an inverse problem, ill-posed due to the physics and limitations on source-detector location. Accurate reliable solutions require a priori constraints. We extend our previously utilized admissible solution approach, with convex constraint functions defining admissibility conditions, by using the deep-cut ellipsoid algorithm, iteratively choosing the most important constraint value, and introducing a multi-resolution grid method to decrease the computational burden. Simulations in a representative 2-D scenarios indicate that we successfully reconstruct relatively deep anomalies while reducing the computational time more than 95%.

1. INTRODUCTION

Three-dimensional mapping of the blood volume and oxygenation of biological tissues to depths of up to several cm would be of significant benefit for a number of medical applications [1]. Potential applications include tumor detection and mapping of brain function. Using near infrared light to probe tissue offers the possibility of estimating just such a map by solving a diffuse optical tomography (DOT) problem, and there has been a significant increase in attention to DOT by a variety of investigative groups over the last several years [1, 2].

In this work we assume the scattering coefficient is constant and known and reconstruct only the optical absorption coefficient. We assume that there are a relatively small number of absorption inhomogeneities in an otherwise reasonably uniform medium and that both background properties of the medium and coupling coefficients of the sources to the medium can be accurately estimated. Thus we concentrate here on the estimation of location and amplitude of absorption coefficient perturbation using a linear forward model.

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cally determines the constraint bound to be used for the residual error, based on an estimate of the background noise variance, and we introduce a multi-resolution grid approach to allow control over the computational burden of the algorithm.

2. PROBLEM FORMULATION

Due to space restrictions, we refer the reader to the references for further description of our forward model and admissible solution approach and the Ellipsoid Algorithm. First we describe the specific constraints we employ. Then we describe our method for determining the appropriate residual constraint bound, followed by a description of our new multi-resolution method. Details on subgradients of the constraints can be found in the references.

2.1. Residual 2-norm subgradient

To enforce reasonable consistency with the forward model and the measured data, we require that the residual between the scattered fluence on the surface predicted by any solution and that measured by the sensors be less in 2-norm than a known bound. Thus we have

\[ \| Gx - b \|_2^2 < \epsilon \]  

(1)

where G is forward model approximation matrix, x is a vector of candidate solutions for the absorption perturbation, and b represents the surface fluence measurements.

2.2. Total variation subgradient

Generally, many non-physical small regions of absorption inhomogeneity and large amplitude artifacts near the source/detector surface(s) appear in reconstructions. Commonly-employed smoothing regularizers suppress them, but at the cost of smoothing out deeper true inhomogeneities. Here we use an approximation of total variation for a discretized absorption coefficient vector x

\[ tv(x) = \| Lx \|_1. \]  

(2)

where L contains appropriately-ordered discrete approximations to a gradient operator.

2.3. Min/max subgradient

To further suppress the large amplitude artifacts we also used constraints on the maximum and minimum values of the perturbation.

2.4. Residual constraint determination

Performance of the algorithm is strongly sensitive to the bound used for the residual constraint. In our previous work, where we were investigating feasibility of the approach, we tested a range of residual constraint bounds and looked at best-case performance and sensitivity to the choice of bound. We have discovered experimentally, using Monte-Carlo simulations, that an iterative approach to choosing the constraint bound works well: it is both robust to noise realization and chooses a high quality reconstruction. The approach is to set the constraint to a dimensionless constant \(\epsilon\) multiplying the estimated background noise standard deviation and a factor that reflects the number of measurements being considered. We start with a large value of \(\epsilon\), such that we are sure that we encompass all “good” solutions and that we converge rapidly. Once we have converged, we decrease the value of \(\epsilon\) in a systematic fashion until we reach a value slightly smaller than 1 (0.8 or 0.7). Each time we change the value of \(\epsilon\) we restart the algorithm from the solution to which we previously converged. Our experience is that

1. The algorithm is both much faster and more accurate with this restart technique than if we start from an unbiased initial guess (normally, setting the initial value to all zeros).

2. On occasion we no longer have a feasible solution as \(\epsilon\) becomes less than 1. However in this case we can detect the non-feasibility and simply keep the previous solution.

3. Frequently, the result improves for values of \(\epsilon\) slightly less than 1. Although \(\epsilon = 1\) would be the nominally “optimal” value in a mean-square averaged sense, it appears that better solutions may not necessarily follow this average.

2.5. Multi-resolution algorithm

In our current work we are using a variant of the ellipsoid algorithm, the deep-cut ellipsoid algorithm, which shrinks the feasibility region faster at each iteration than the basic algorithm does. However, the computational complexity of the algorithm is strongly tied to the dimension of the problem space, the number of voxels we reconstruct, and therefore the resolution, in two ways:

1. the number of iterations increases almost linearly with the problem dimension, and

2. the computational cost of each iteration increases with the square of the problem dimension.

Thus there is strong motivation to look for a way to control the problem dimension while maximizing resolution in
areas where there may be inhomogeneities. Based on our assumption that there are only a relatively small number of absorption anomalies in an otherwise reasonably uniform medium, we observed that during the iteration process, there are many voxels that change their value very slightly if at all — since the anomalies only occupy a few percent of all voxels. So if we focus on relatively small domains containing the possible anomalies, we can hope to achieve good resolution in these regions and low resolution in all the other regions, holding the problem dimensionality to a reasonable level. Moreover, the solution obtained at a given resolution is likely to be a good initial starting point for the next higher resolution, which should in itself decrease the number of iterations required to converge.

Motivated by this reasoning, we developed a multi-resolution mesh method and an associated algorithm. The basic idea is as follows:

1. Use a coarse mesh to get an initial result.

2. Use this result to selectively refine the mesh. We identify regions which are likely to contain anomalies, refining the mesh in these regions only. Regions that are identified as not containing anomalies are consolidated into a single region. Thus we compensate for the increased dimensionality in the “potentially anomalous” regions by reducing resolution in the regions identified as background.

3. Use the initial result, interpolated to the new grid, to re-estimate the result. In doing so, we use the same range of c parameters as described above for the residual constraint. The minimum and maximum value constraints are held constant across all iterations, while the total variation constraint is changed by a factor of 2 on the anomalous region to reflect the larger number of voxels in that region.

4. Iterate the decision and refinement/consolidation process until we reach the desired resolution.

3. RESULTS

First we describe our simulation scenario and then present some initial results.

3.1. Simulation Scenario

To perform an initial study of the performance of the admissible solution approach with limited computation cost we used the same geometry as in [1, 3]. We have a single vertical 2-D slab. The slab was positioned vertically to extend into the medium so that we could study performance as a function of depth—generally the most difficult aspect of DOT reconstructions. A rectangular absorption anomaly was simulated centered at 2.25 cm deep and 0.75 cm to the right of the source-detector array center line. The width of the anomaly was the same as the width of the reconstruction slab, 0.25 cm. An image of the anomaly is shown in Fig.2(a).

We simulated four sources and four detectors, all collinear along the air-tissue boundary of the slab. The forward and inverse models were identical (Born-1) including the discretization of the slab. The discretization was on a 0.25 cm grid with 16 x 16 (256) voxels. The simulated modulation frequency was 200 MHz, resulting in 32 measurements (in-phase and quadrature) and 256 unknowns. This maintained a similar measurements-to-unknowns-ratio as one might meet in a 3D reflection measurement scenario [4]. In addition we ensured that the singular spectrum of the resulting forward operator had a similar decay to a corresponding 3D model. The background absorption coefficient was 0.041 cm$^{-1}$, while the anomaly was 0.139 cm$^{-1}$ above the background.

We examined several aspects of the performance of the admissible solution approach applied to DOT. Fig.1 shows a qualitative comparison between the TSVD linear algebraic approach and admissible solution approach using the three constraints described above when 30 dB SNR noise has been added to the forward-simulated detected light before inversion. The latter method correctly identifies the boundaries of the anomaly and very accurately reconstructs its amplitude as well. The only significant distortion is the “shadowing” under the anomaly. But the computational burden is quite high: 203,479 iterations were required.

![Reconstruction using (a) TSVD and (b) Admissible Solution](image)

Fig.1. Reconstruction using (a) TSVD and (b) Admissible Solution

Fig.2(b)-(d) show results from multi-resolution algorithm anomaly, while Fig.2(f)-(h) show the result with two anomalies. All these results are reconstructed under 30 dB SNR noise and with 3 different mesh size of 4X4, 8X8, 16X16 (the number in parentheses below each plot is the number of iterations needed for each resolution). For both cases, at resolution 4X4, we set the initial value to zero, and as shown we obtained a very coarse image. Then we refined the whole domain to resolution 8X8. After the reconstruction, we can distinguish some anomalous domains (indicated by the dashed lines), and we refined just these domains to 16X16 mesh size while combining all other voxels
4. CONCLUSIONS AND FUTURE WORK

Using a combination of residual two-norm, total variation, and min/max constraint sets in an admissible solution approach implemented with the Deep-Cut Ellipsoid Algorithm, we were able to achieve significantly improved reconstruction of the absorption parameter in a two-dimensional model of the full three-dimensional problem. Additionally, the multi-resolution method dramatically decreased the computational time while obtaining an almost equivalent result.

Methods to adequately estimate the bounds of the anomalous domain need to be investigated. In the situation with more than one anomaly, an algorithm that can automatically distinguish each anomalous domain should be developed. In addition, systematic multi-grid methods should be added to flexibly supply an appropriate resolution mesh. Moreover, we intend to combine this approach with non-linear forward models and parameterized solution spaces currently being investigated concurrently by our group, with the intention of applying our methods to clinically-obtained mammography and brain function data in the near future.

5. REFERENCES


