DYNAMIC PHASORS IN MODELING, ANALYSIS AND CONTROL OF ENERGY PROCESSING SYSTEMS

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Research Program Overview

My research program focuses on the interface of control and energy processing.

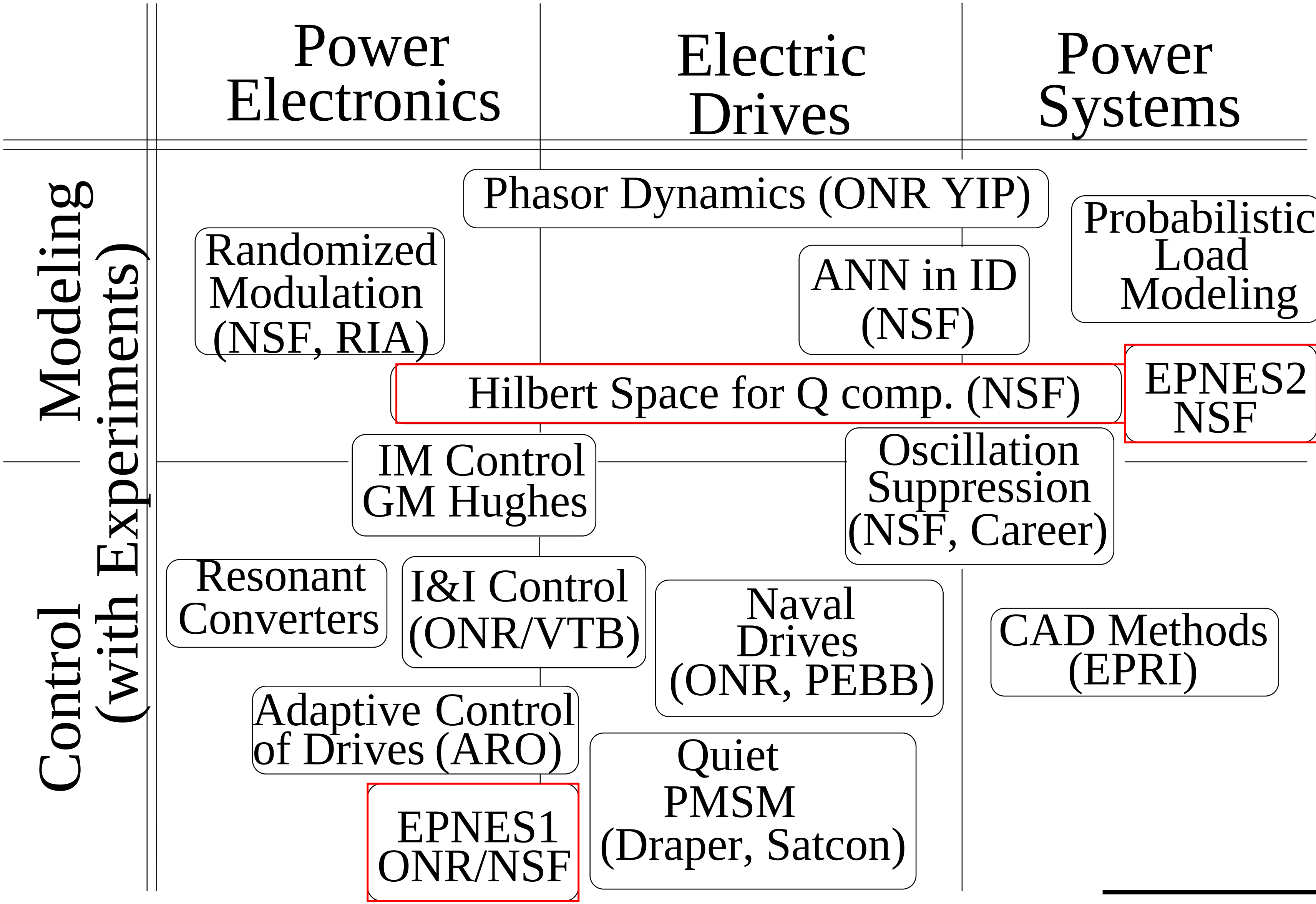
Emerging energy conversion technologies: **1.** efficiency driven and **2.** require closed–loop control - **nonlinearity** and **uncertainty**.

Developments in: power electronics, electric drives and power systems.

NEU Energy Processing Laboratory (1994) is a confluence of research and educational efforts:
1. Areas: power electronics, electric drives and power systems,
2. Graduate students,
3. Sponsors in government and industry,

4. Technical collaborations.

Research Program Overview



Presentation Overview

- **Background and definitions,**
- POWER SYSTEMS - Flexible AC Transmission Systems,
- ELECTRIC DRIVES - AC machine modeling,
- An interlude - extension to polyphase systems,
- ESTIMATION - dynamic phasors and symmetrical components,
- POWER ELECTRONICS - model reduction in switched-mode DC/DC converters.

Background

New challenges in energy processing (power electronics, electric drives, power systems):

- reliance on switching operation for efficiency,
- new dynamic couplings,
- increased performance specifications,
- new problems (e.g., active filtering, pulsed power).

Analytical tools for addressing ("close-to" periodic) system operation:

- "sinusoidal quasi-steady-state" approximation in drives and power systems,
- time-domain simulations (in power electronics and drives),
- systematic exploration of the "middle ground" is largely missing.

Background

Main features of dynamic phasors:

- large signal models,
- nonlinear,
- physical intuition used for simplifications,
- appealing mathematical structure.

Origins and ideas related to our work:

- power electronics,
- power engineering (“space vectors”, “spiral vectors”, polyphasors),
- nonlinear oscillations (classical averaging and recent variants),
- signal processing.

Definitions

A (possibly complex) waveform $x(\cdot)$ can be represented on the interval $(t - T_0, t]$ using (short-time) Fourier series:

$$x(\tau) = \sum_{k=-\infty}^{\infty} X_k(t) e^{jk\omega_0 \tau}$$

where $X_k(t)$ are the complex, slowly time-varying Fourier coefficients, or *dynamic phasors*.

$$X_k(t) = \frac{1}{T_0} \int_{t-T_0}^{t} x(\tau) e^{-jk\omega_0 \tau} d\tau = < x >_k (t)$$

Our dynamical models describe evolution of $X_k(t)$; for real $x(\cdot)$ we have $X_{-k} = X_k^*$

Definitions

Two useful facts:

Derivative of the k-th dynamic phasor:

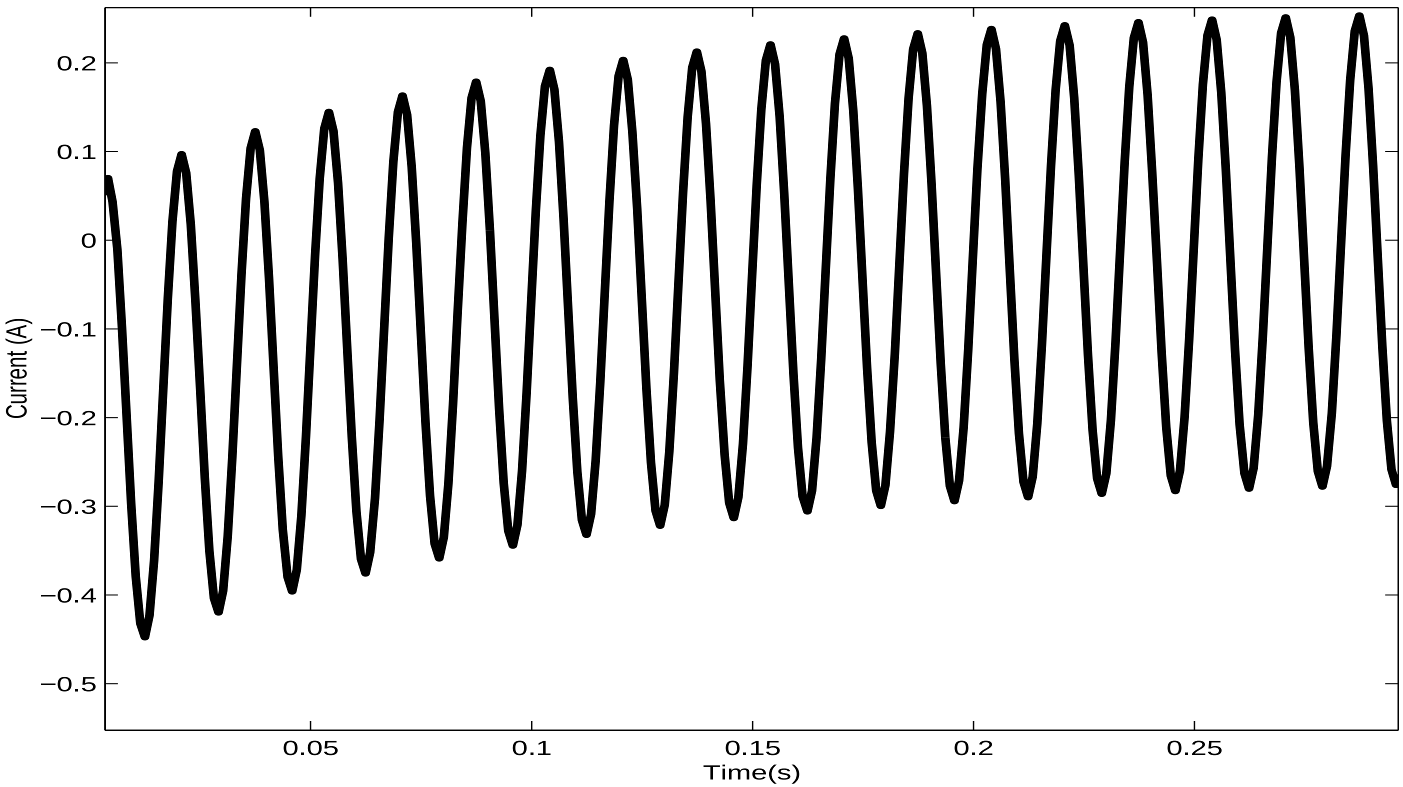
$$\frac{dX_k}{dt} = \left\langle \frac{d}{dt}x \right\rangle_k - j\,k\,\omega_0 X_k$$

Multiplication in time domain:

$$< xy >_k = \sum_\ell < x >_{k-\ell} < y >_\ell$$

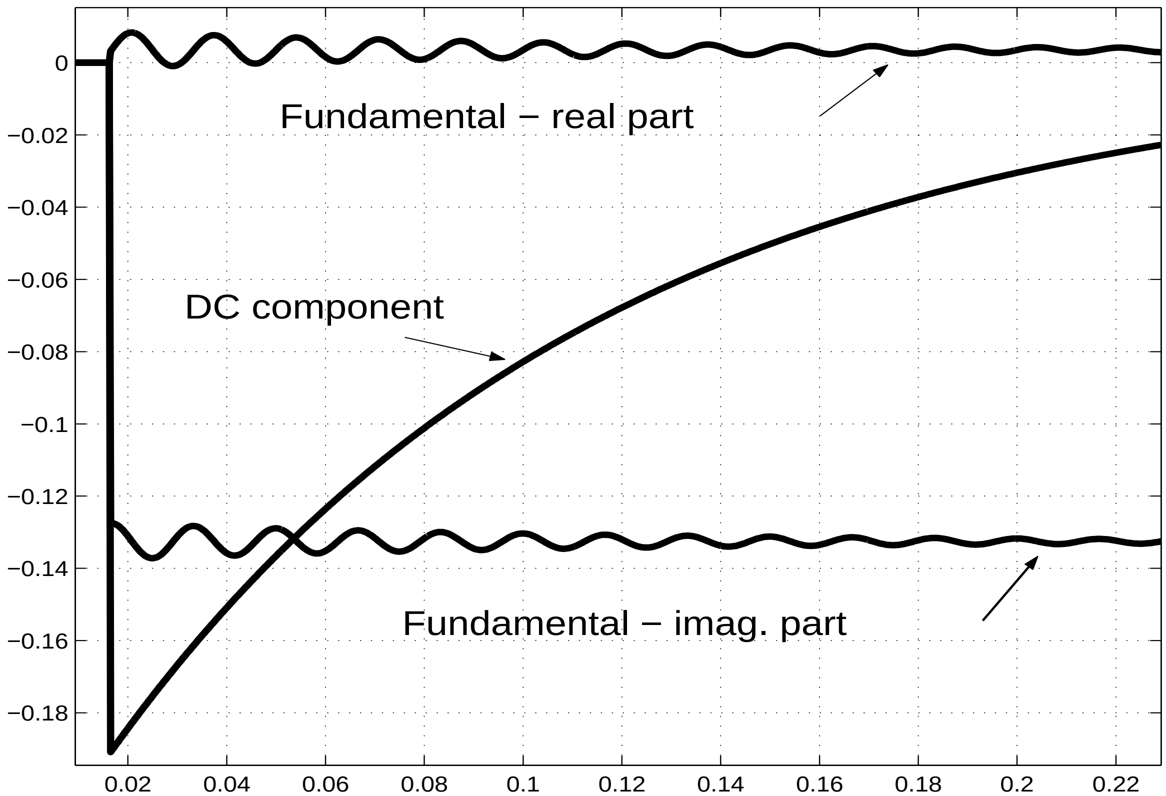
A Simple Example

Consider a simple RL circuit (R=1, L=0.1) with a cos excitation (V=10), at 60Hz



A Simple Example, cont. 1

The dynamic phasors (according to our definition)



Generalized Orthonormal Series

Instead of the Fourier series we can use any other **orthonormal** basis (or a self-dual frame) of the Hilbert space of square integrable waveforms on $[0, T_0]$

$$x(\tau) = \sum_k \chi_k(t)\phi_k(t-\tau), \qquad t - T_0 \ < \tau \ \leq t$$
$$\chi_k(t) = \frac{1}{T_0} \int_{t-T_0}^{t} x(\tau)\phi_k^*(t-\tau)d\tau$$

where because of orthonormality

$$\frac{1}{T_0}\int_0^{T_0} \phi_k(\tau)\phi_\ell^*(\tau)d\tau = \delta_{k,\ell}$$

Note that our Fourier series representation is recovered by setting

$$\phi_k(t) = e^{-jk\omega_0 t} \quad \text{and} \quad X_k(t) = \chi_k(t)e^{-jk\omega_0 t}$$

The Parseval equality still holds, with χ_k replacing X_k.

Generalized Orthonormal Series (2)

A filtering interpretation of the inner product expression – via change of the integration variable

$$\chi_k(t) = \frac{1}{T_0}\int_0^{T_0} x(t-\tau)\phi_k^*(\tau)d\tau$$

which identifies $\chi_k(t)$ as the output of a **linear time-invariant filter** with impulse response

$$h_k(\tau) = \frac{1}{T_0}\phi_k^*(\tau)$$

For Fourier series, $\chi_k(t)$ is centered at the frequency $k\omega$, which motivates the definition of a *baseband equivalent*

$$X_k(t) = \chi_k(t)e^{-jk\omega_0 t}$$

Dynamics of Local Coefficients

The interpretation of $\chi_k(t)$ as the output of a LTI filter makes it possible to translate state-space representations of signals into equivalent representations of their expansion coefficients.

Thus, if

$$\dot{x}(t) = Ax(t) + Bu(t)$$

then

$$\dot{\chi}_k(t) = A\chi_k(t) + B\upsilon_k(t)$$

where $\upsilon_k(t)$ are the expansion coefficients of the input $u(t)$.

This holds for **any** orthonormal representation – in the case of Fourier series coefficients (since $X_k(t) = \chi_k(t)e^{-jk\omega_0 t}$),

$$\dot{X}_k(t) = (A - jk\omega_0 I)X_k(t) + BU_k(t)$$

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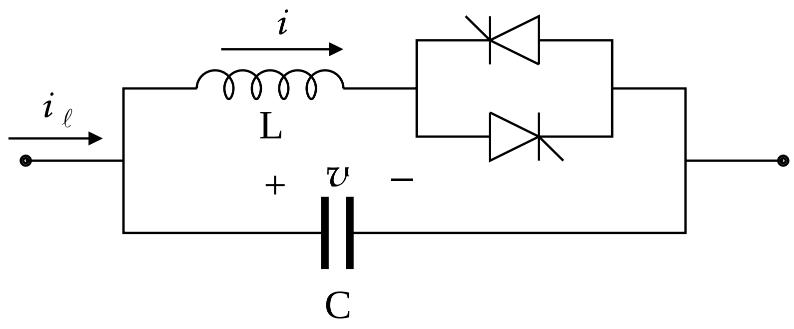
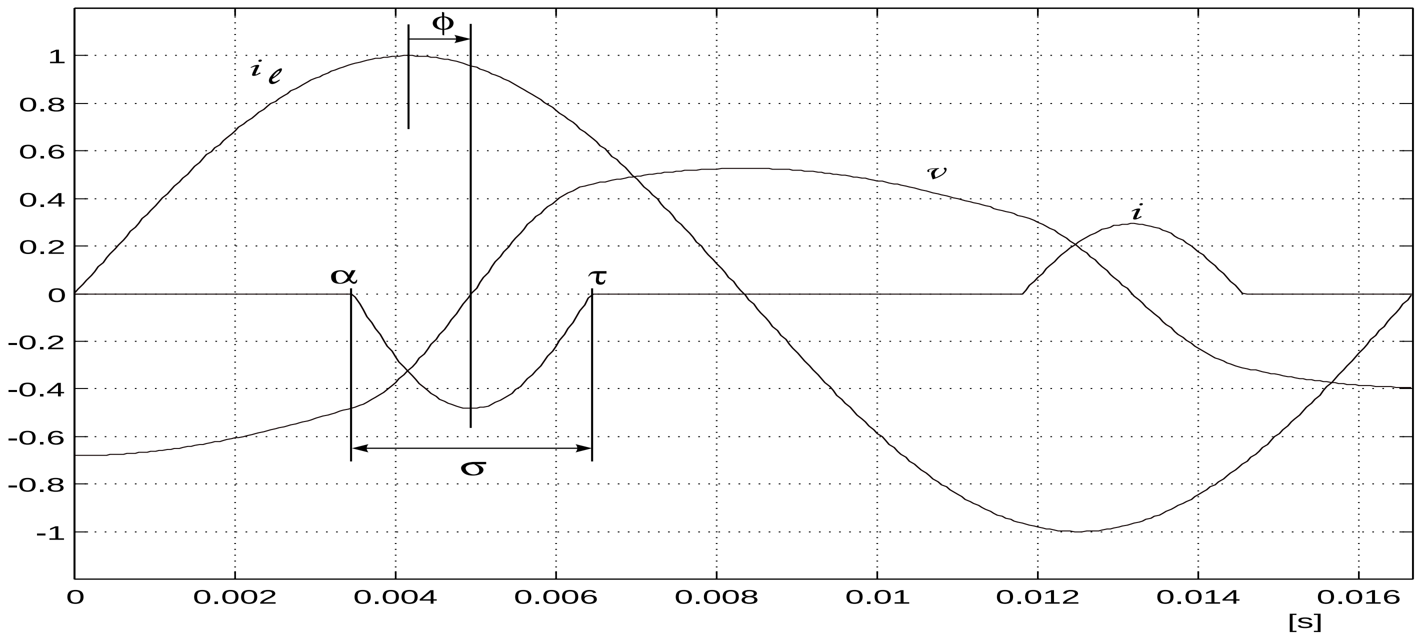
Flexible AC Transmission

Thyristor Controlled Series Capacitors (TCSCs) are finding increasing application in power systems.





Flexible AC Transmission

Fast and accurate models are needed for simulation and control.

Analytical difficulties stem from the nature of TCSC that amalgamates continuous-time dynamics with discrete events (thyristor firings).

Sampled-data models were the first to offer the needed accuracy, but

- model structure has no clear relation to the system configuration,
- hard to interface with the rest of the system which is usually described with phasor-based continuous-time models.

Flexible AC Transmission

A state-space model for basic TCSC configuration:

$$C\frac{dv}{dt} = i_\ell - i$$
$$L\frac{di}{dt} = q\,v$$

where q is a $0-1$ switching function.
Evaluating the 1-phasor on both sides of each equation (and assuming i_ℓ is sinusoidal) we obtain a 2nd-order (complex) phasor model:

$$C\,\frac{dV_1}{dt} = I_\ell - I_1 - j\,\omega_0\,C\,V_1$$
$$L\,\frac{dI_1}{dt} = < q\,v >_1 - j\,\omega_0\,L\,I_1$$

where $< q\,v >_1$ is:

$$< q\,v >_1 = \frac{2}{\pi}\int_\alpha^\tau v\,e^{-j\theta}d\theta.$$

Flexible AC Transmission

I_1 has fast dynamics compared to V_1, so we assume

$$I_1 \approx \frac{V_1}{j\omega_0 L_{eff}(\sigma)}$$

yielding

$$C\frac{dV_1}{dt} = I_\ell - (j\omega_0 C + \frac{1}{j\omega_0 L_{eff}(\sigma)})V_1 = I_\ell - j\omega_0 C_{eff}(\sigma)V_1$$

where σ is the prevailing conduction angle

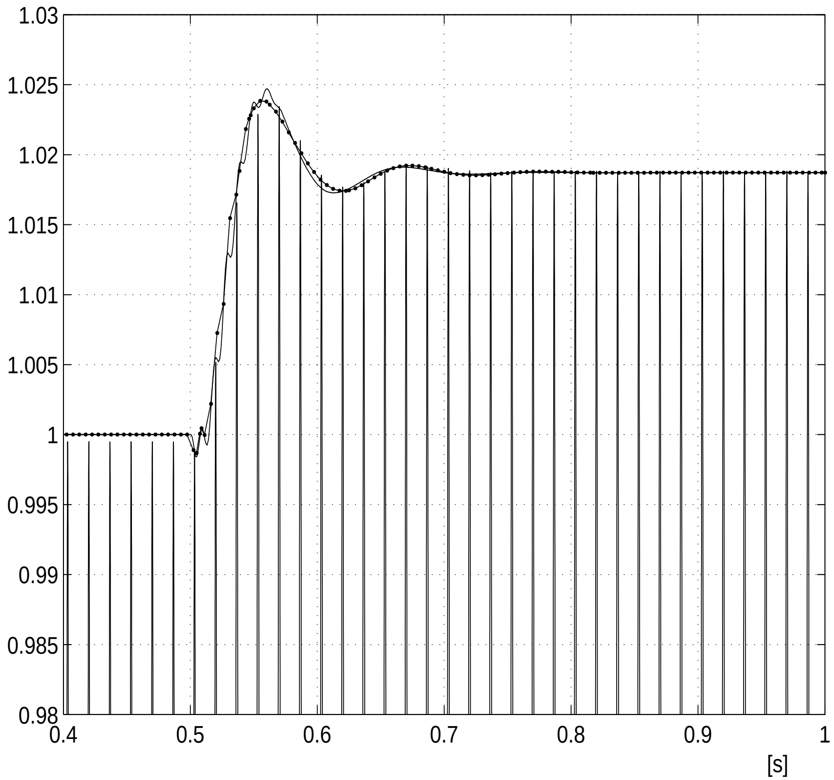
$$\sigma = \sigma^0 + 2\phi \approx \sigma^0 + 2\arg\left[-jI_\ell(V_1)^*\right]$$

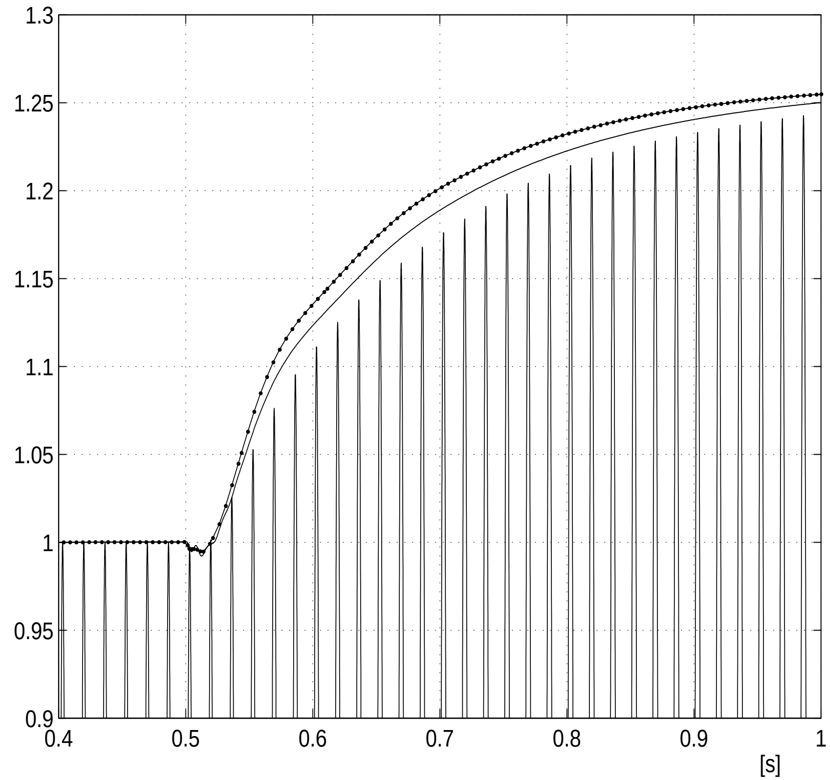
and σ^0 is the reference.

C_{eff} is computed from steady-state assuming sinusoidal line current i_ℓ (in contrast to the conventional approach which assumes v to be sinusoidal).

Flexible AC Transmission

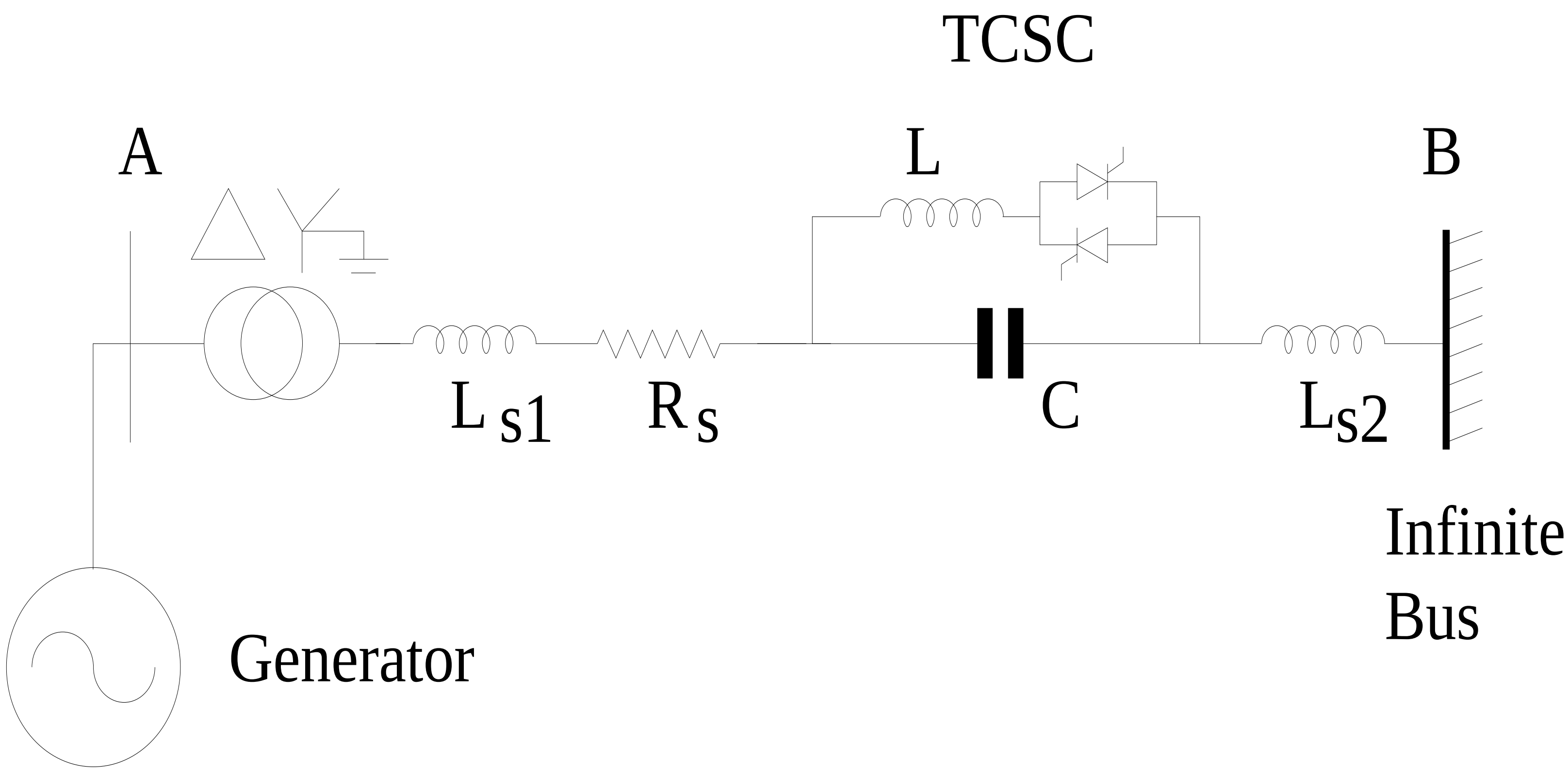
Line current for 2° step changes of the firing angle:





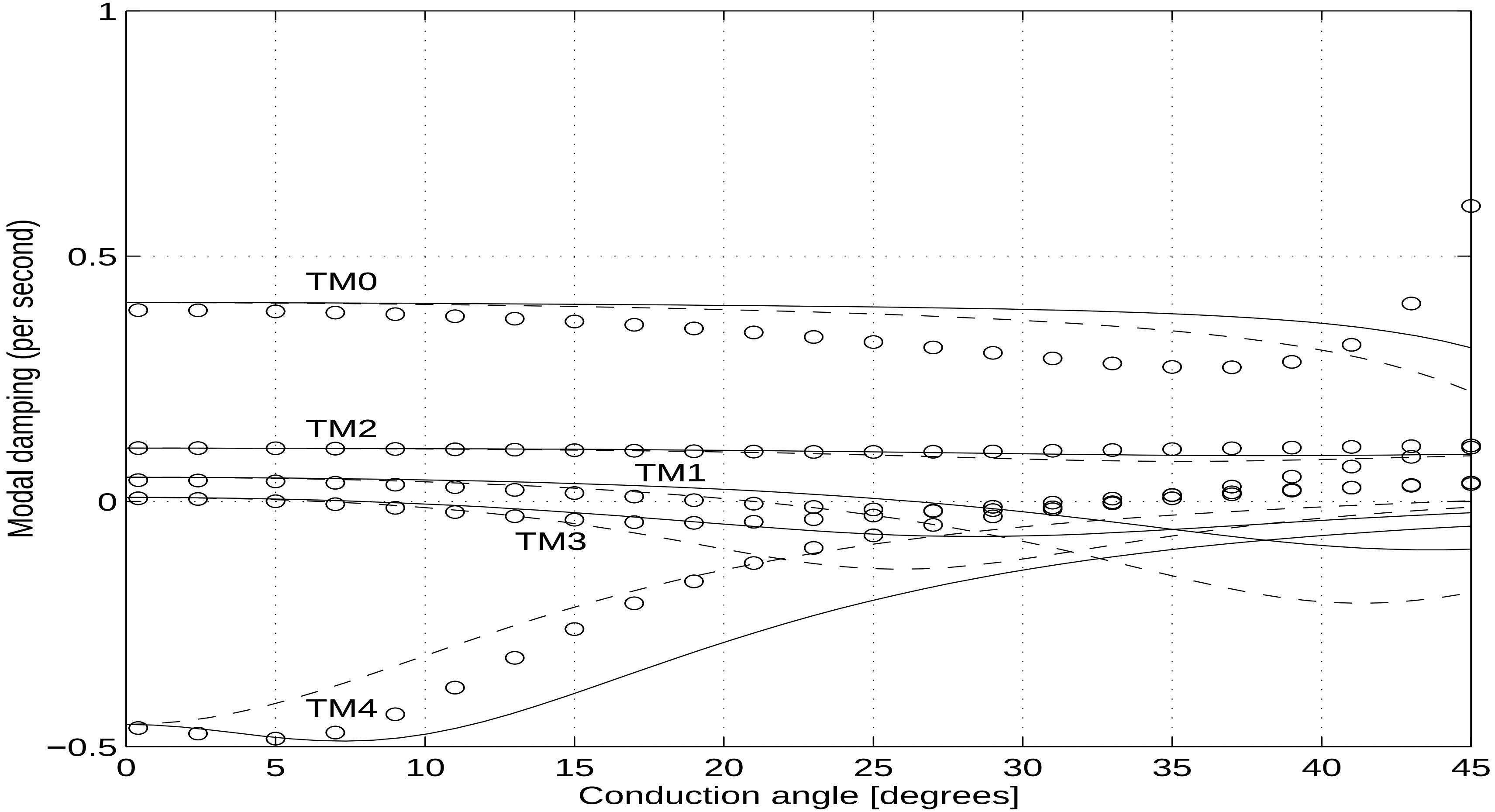
Subsynchronous Resonance

First IEEE benchmark test with TCSC:

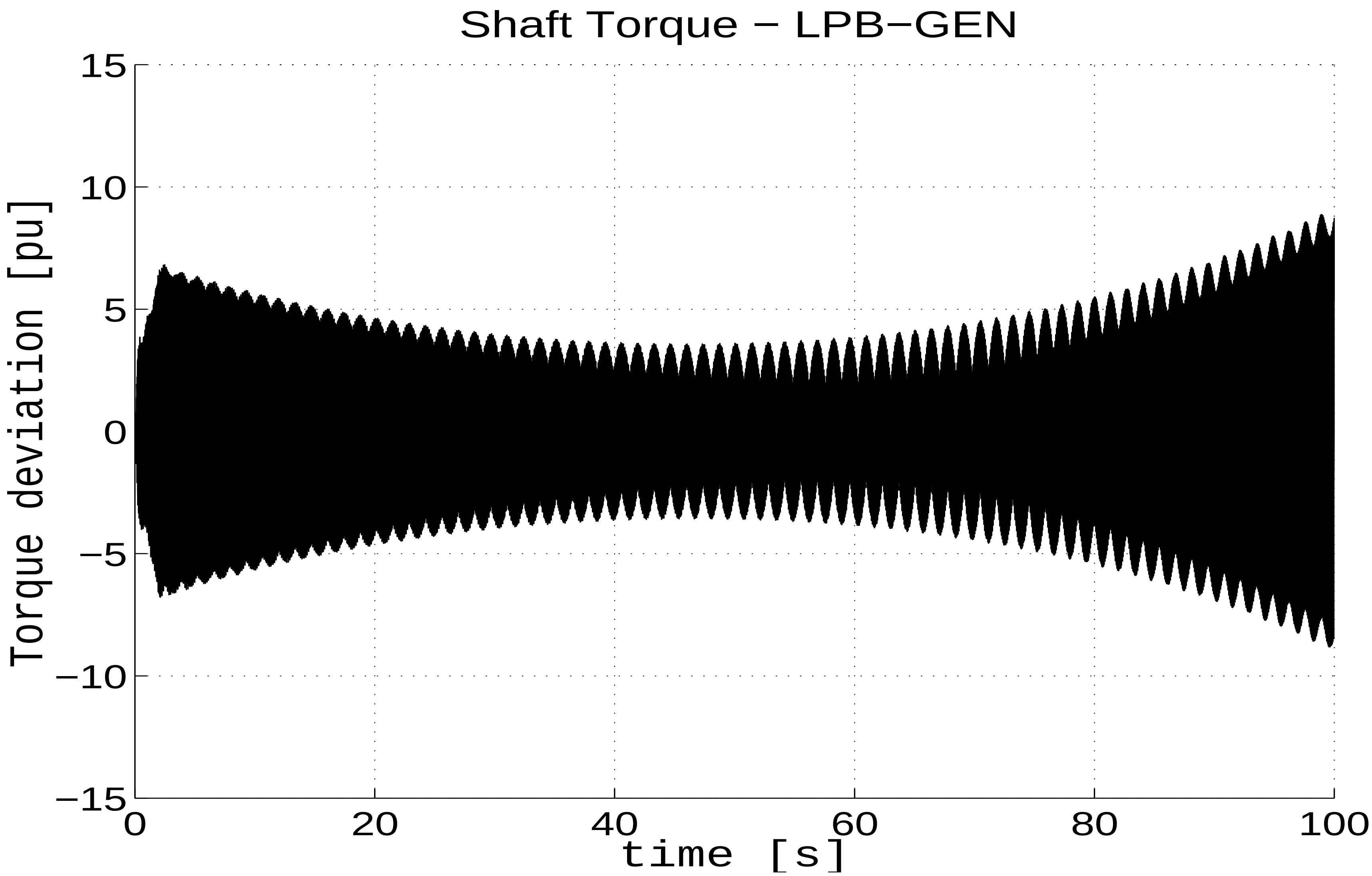


Subsynchronous Resonance

TCSC model without (solid line) and with the phase correction (dashed line); "○" sampled-data for the *equidistant* synchronization:

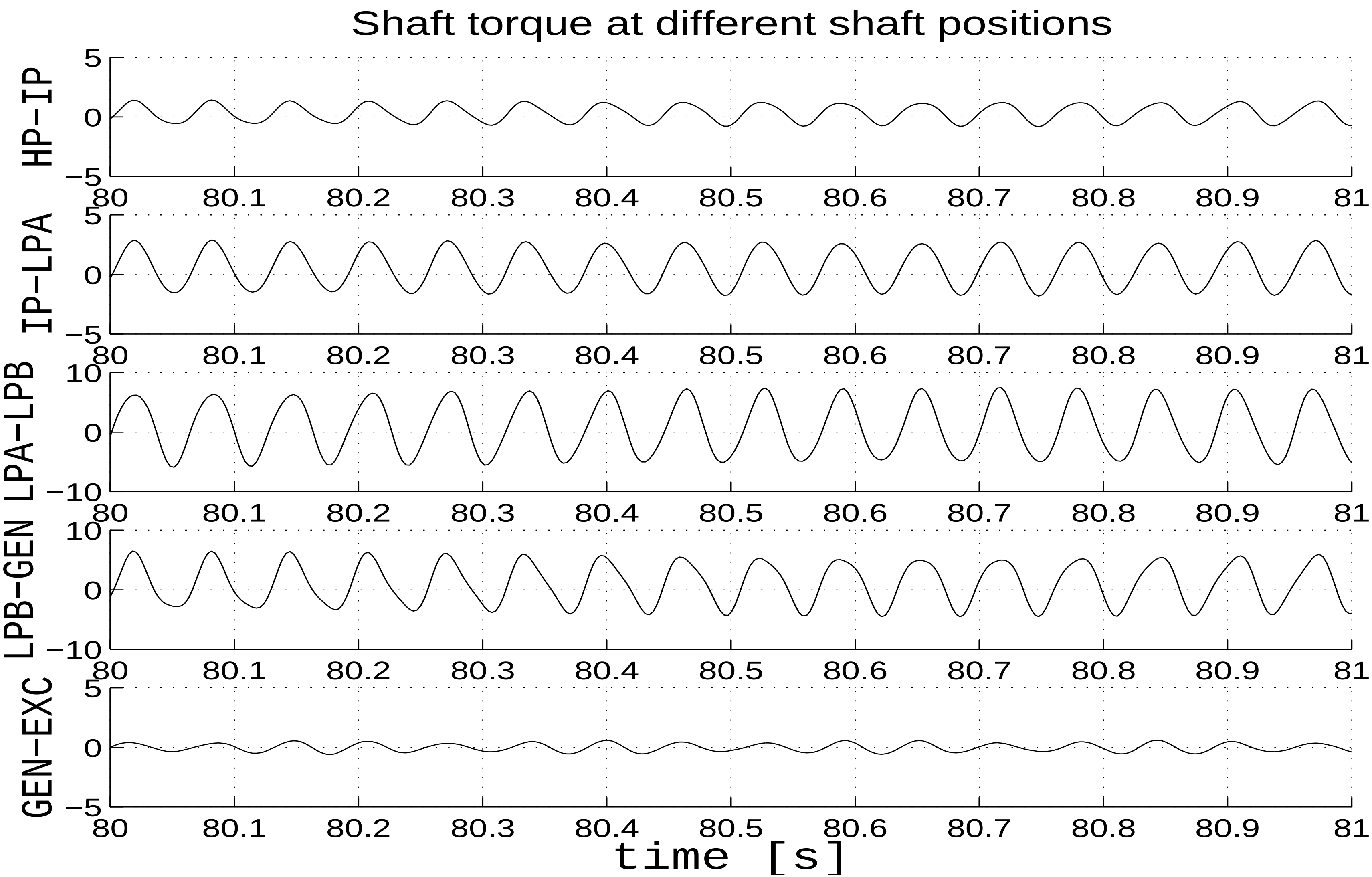


Subsynchronous Resonance



Subsynchronous Resonance

Expanded view (between 80s and 81s) of torque variations



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Dynamic Phasors and Space Vectors

Assuming now that all phase quantities are real; let $\alpha = e^{j2\pi/3}$:

$$\vec{x}(t) = \frac{2}{3}(x_a(t) + \alpha x_b(t) + \alpha^* x_c(t))$$

This complex (scalar) quantity can encode two-dimensional information; for example, $< \vec{x} >_{-k} = < \vec{x}^* >_k^*$.

$$X_{p,1}(t) := < \vec{x} >_1 (t) = < \vec{x}^* >_{-1}^* (t),$$

$$X_{n,1}(t) := < \vec{x} >_{-1}^* (t) = < \vec{x}^* >_1 (t).$$

Electrical Machines

Space vector model of a three-phase induction machine:

$$\begin{aligned}
\vec{v_s} &= (r_s + L_s \frac{d}{dt})\vec{i_s} + L_m \frac{d}{dt}\vec{i_r} \\
0 &= L_m \frac{d}{dt}\vec{i_s} + (r_r + L_r \frac{d}{dt})\vec{i_r} - j\omega_r \frac{P}{2}(L_m \vec{i_s} + L_r \vec{i_r}) \\
J\frac{d}{dt}\omega_r &= \frac{3P}{4} L_m \Im(\vec{i_s}\vec{i_r}^*) - B\omega_r - T_L
\end{aligned}$$

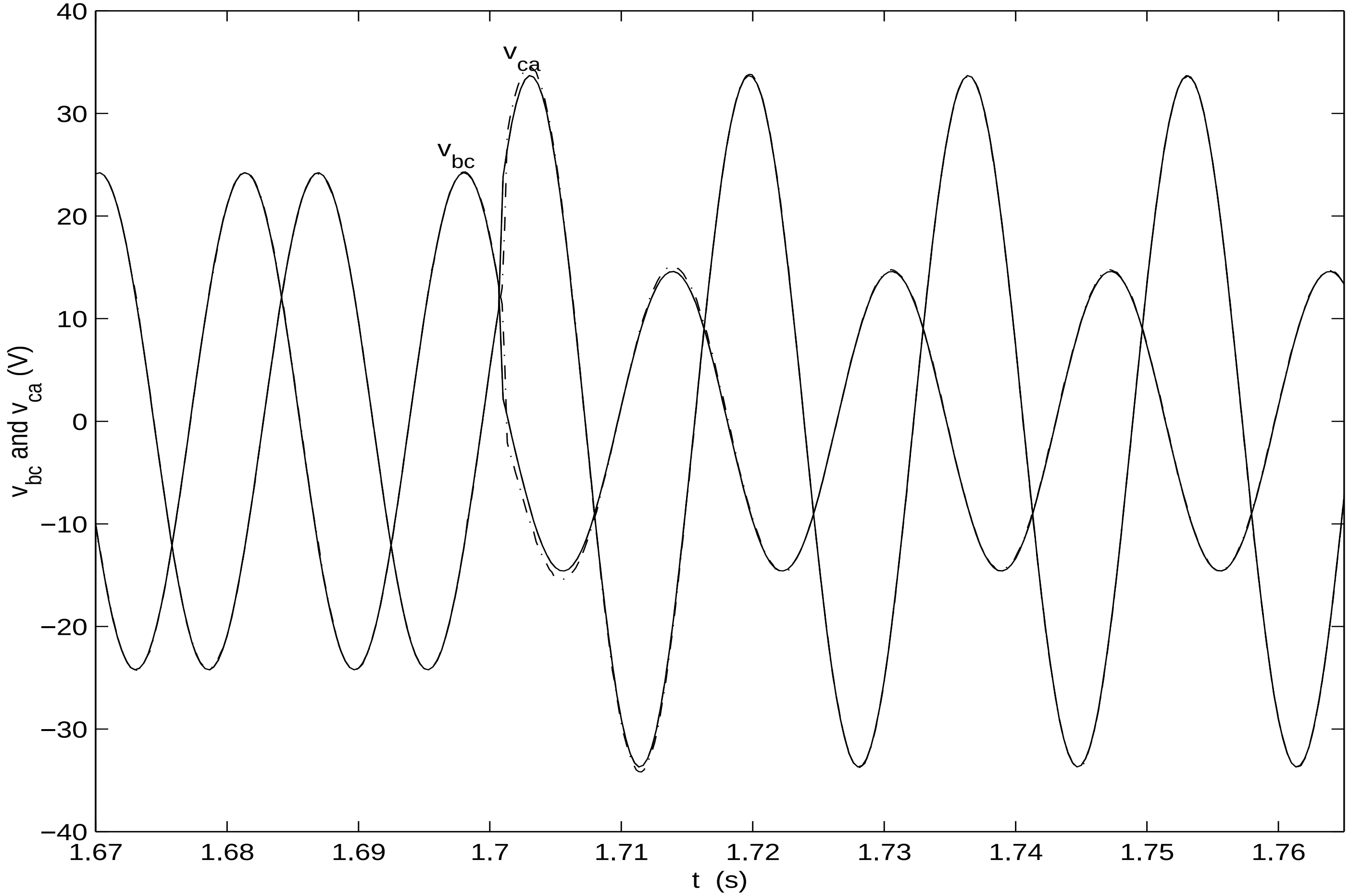
Electrical Machines

Dynamic phasor model of a three-phase induction machine:

$$
\begin{aligned}
V_p &= (r_s + j\omega_0 L_s + L_s \frac{d}{dt}) I_{p,s} + (j\omega_0 L_m + L_m \frac{d}{dt}) I_{p,r} \\
0 &= (j\omega_0 L_m + L_m \frac{d}{dt}) I_{p,s} + [r_r + (j\omega_0 L_r + L_r \frac{d}{dt})] I_{p,r} \\
&\quad - j\Omega_{r,0} \frac{P}{2} (L_m I_{p,s} + L_r I_{p,r}) - j\Omega_{r,2} \frac{P}{2} (L_m I_{n,s}^* + L_r I_{n,r}^*) \\
V_n^* &= (r_s - j\omega_0 L_s + L_s \frac{d}{dt}) I_{n,s}^* + (-j\omega_0 L_m + L_m \frac{d}{dt}) I_{n,r}^* \\
0 &= (-j\omega_0 L_m + L_m \frac{d}{dt}) I_{n,s}^* + [r_r + (-j\omega_0 L_r + L_r \frac{d}{dt})] I_{n,r}^* \\
&\quad - j\Omega_{r,0} \frac{P}{2} (L_m I_{n,s}^* + L_r I_{n,r}^*) - j\Omega_{r,2}^* \frac{P}{2} (L_m I_{p,s} + L_r I_{p,r}) \\
J \frac{d}{dt} \Omega_{r,0} &= \frac{3P}{4} L_m \Im (I_{p,s} I_{p,r}^* + I_{n,s}^* I_{n,r}) - B\Omega_{r,0} - T_L \\
J \frac{d}{dt} \Omega_{r,2} &= \frac{3P}{j8} L_m (I_{p,s} I_{n,r} - I_{n,s} I_{p,r}) - (B + j2J\omega_0) \Omega_{r,2}
\end{aligned}
$$

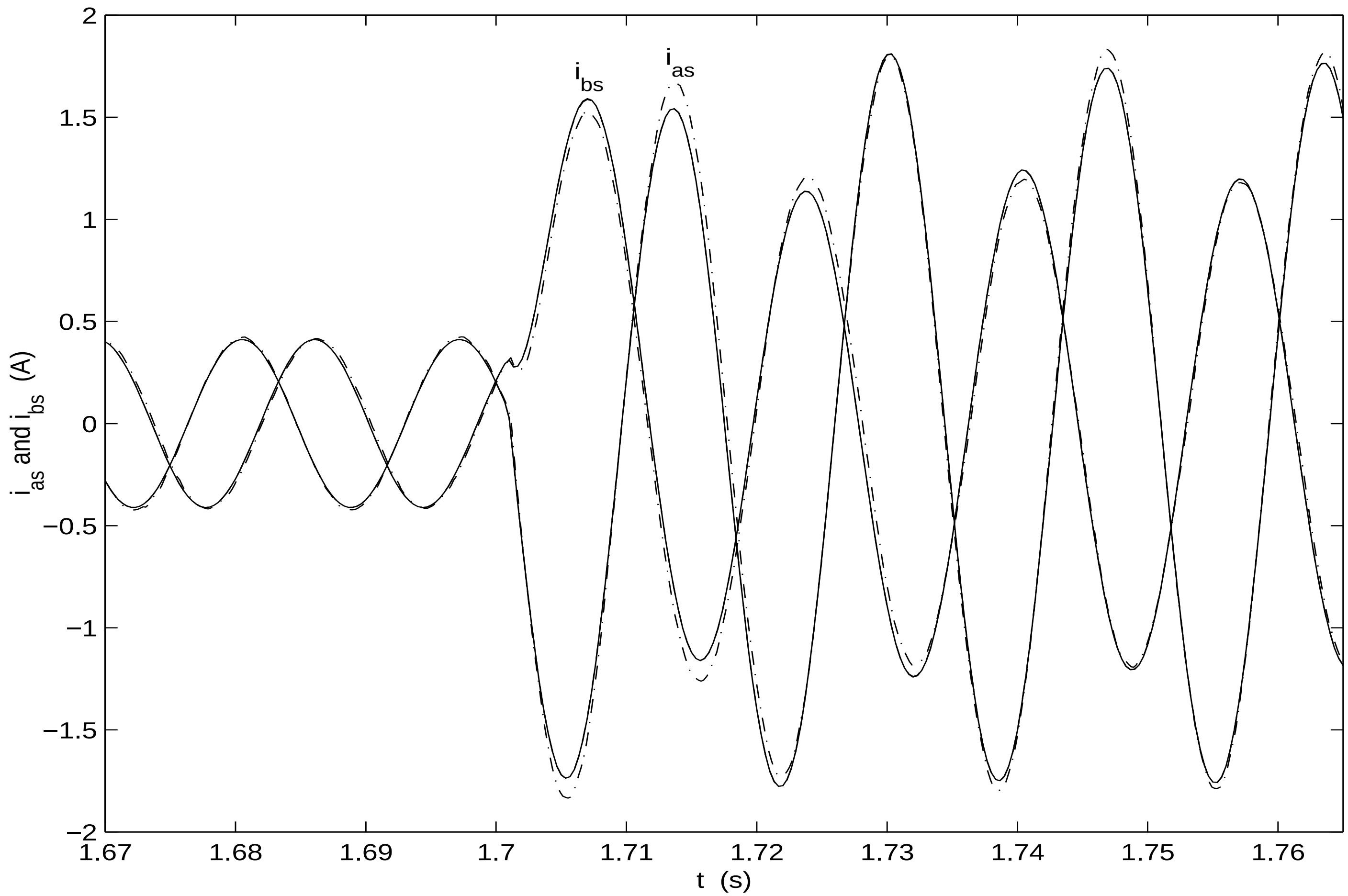
Electrical Machines

Experimental evaluation:



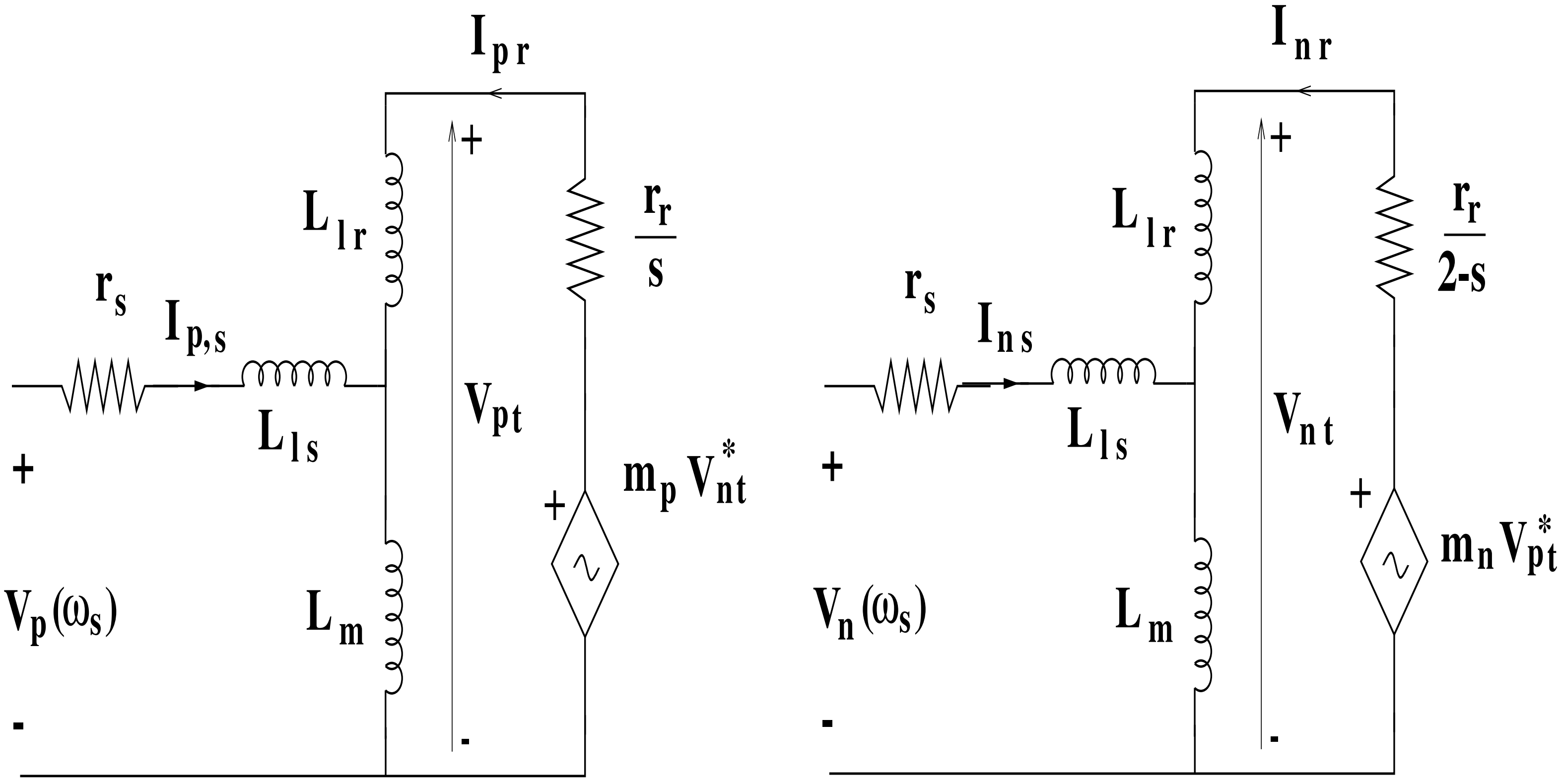
Electrical Machines

Experimental evaluation, cont.



Electrical Machines

Steady-state equivalent circuit ($m_p = \Omega_{r,2}/(\omega_0 s)$ and $m_n = -\Omega_{r,2}/[\omega_0(2-s)]$):



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Extension to Polyphase Systems

Dynamical symmetric components - recall $\alpha = e^{j\frac{2\pi}{3}}$,

$$\begin{bmatrix} x_a \\ x_b \\ x_c \end{bmatrix}(\tau) = \sum_{\ell=-\infty}^{\infty} e^{j\ell\omega_0\tau} \underbrace{\frac{1}{\sqrt{3}} \begin{bmatrix} 1 & 1 & 1 \\ \alpha^* & \alpha & 1 \\ \alpha & \alpha^* & 1 \end{bmatrix}}_{A} \begin{bmatrix} X_{p,\ell} \\ X_{n,\ell} \\ X_{z,\ell} \end{bmatrix}(t)$$

$$\begin{bmatrix} X_{p,\ell} \\ X_{n,\ell} \\ X_{z,\ell} \end{bmatrix}(t) = \frac{1}{T_0} \int_{t-T_0}^{t} e^{-j\ell\omega_0\tau} A^H \begin{bmatrix} x_a \\ x_b \\ x_c \end{bmatrix}(\tau) d\tau = \begin{bmatrix} < x >_{p,\ell} \\ < x >_{n,\ell} \\ < x >_{z,\ell} \end{bmatrix}(t).$$

$$\frac{d}{dt} \begin{bmatrix} X_{p,\ell} \\ X_{n,\ell} \\ X_{z,\ell} \end{bmatrix}(t) = A^H \begin{bmatrix} < {}^{d}\!/_{d\tau} x_a(\tau) >_\ell \\ < {}^{d}\!/_{d\tau} x_b(\tau) >_\ell \\ < {}^{d}\!/_{d\tau} x_c(\tau) >_\ell \end{bmatrix}(t) - j\ell\omega_0 \begin{bmatrix} X_{p,\ell} \\ X_{n,\ell} \\ X_{z,\ell} \end{bmatrix}(t)$$

Properties of Dynamic Symmetric Components

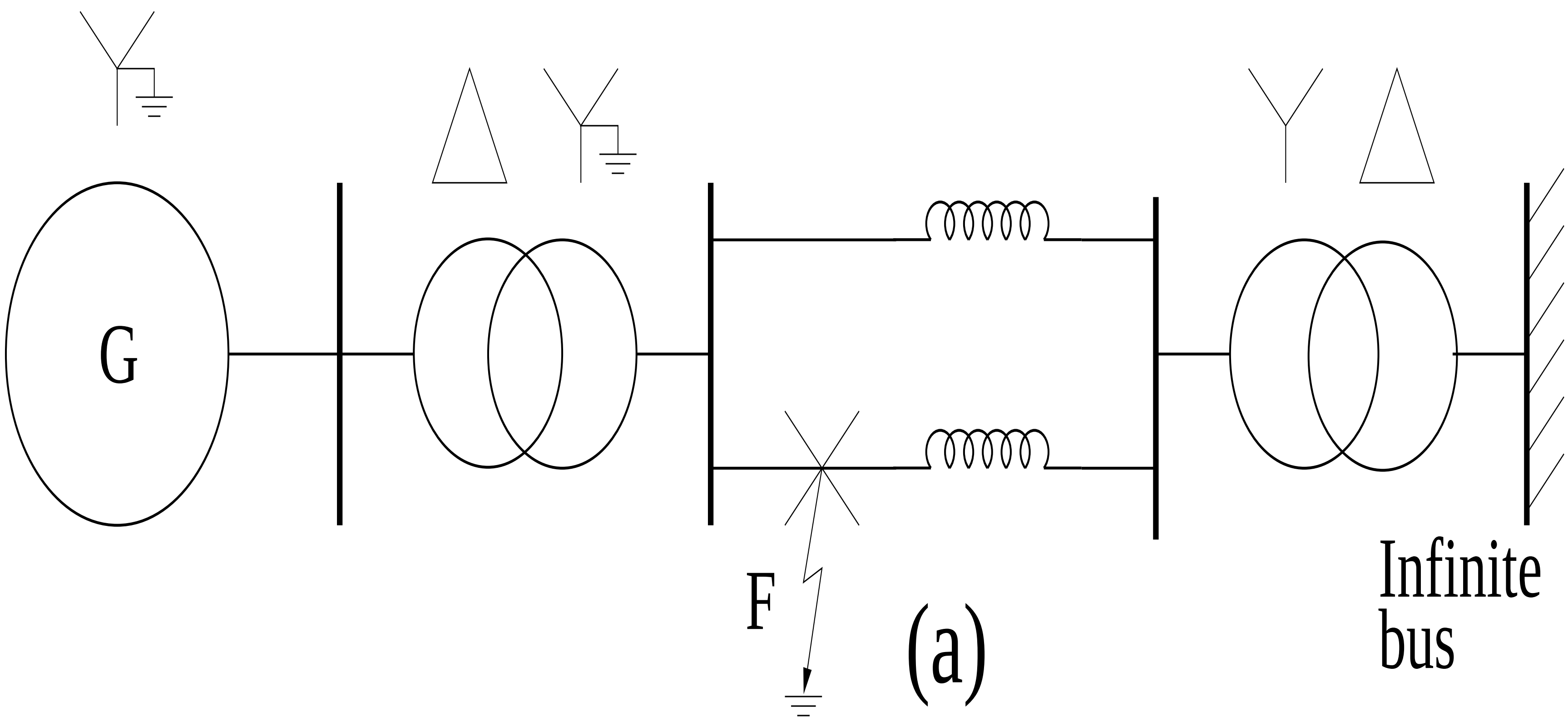
For real waveforms:

$$X_{p,\ell} = X^*_{n,-\ell} \qquad X_{n,\ell} = X^*_{p,-\ell} \qquad X_{z,\ell} = X^*_{z,-\ell}.$$

Connection with space vectors:

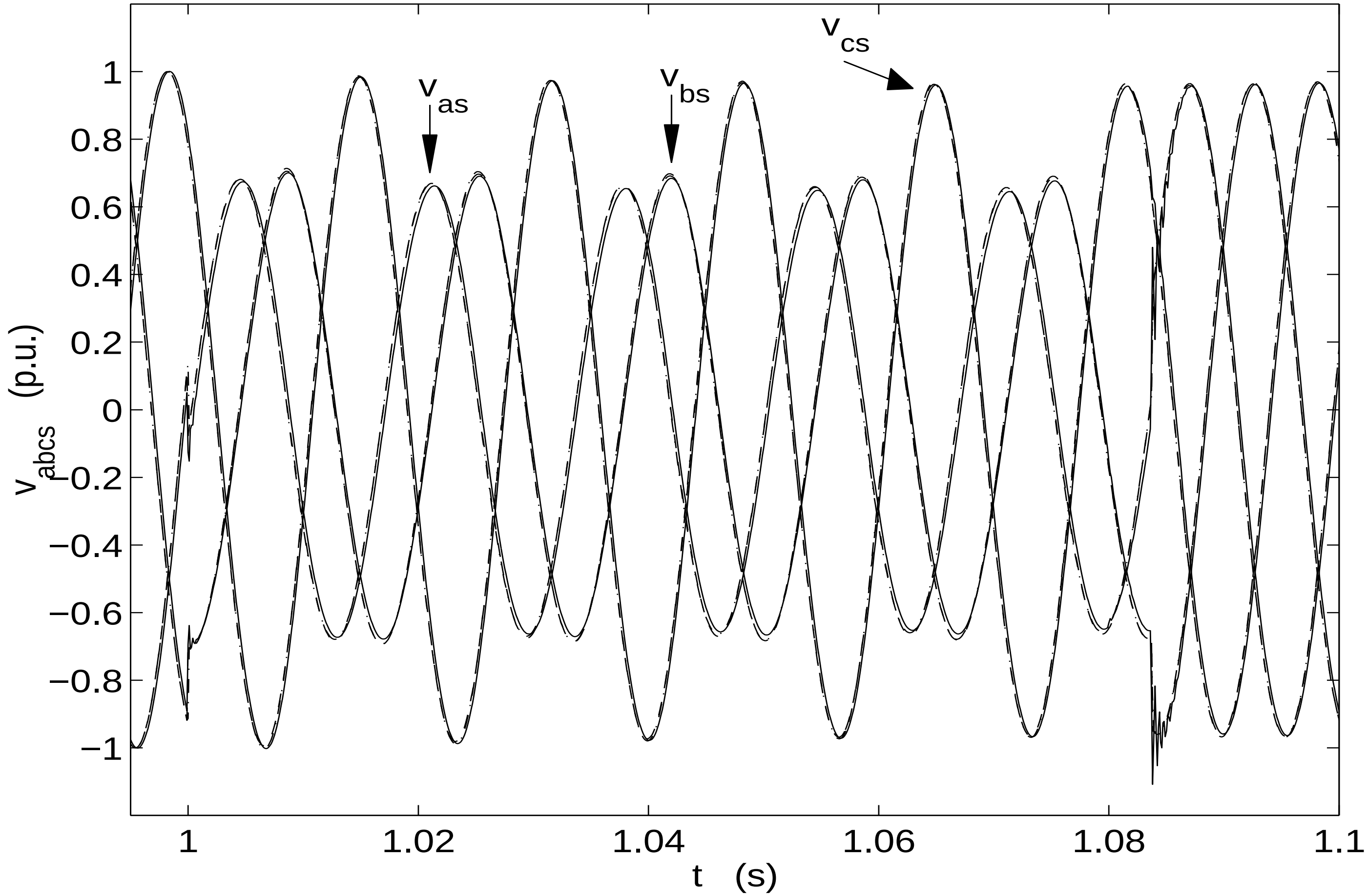
$$\vec{x}(\tau) = \frac{2}{\sqrt{3}} \sum_{\ell=-\infty}^{\infty} e^{j\ell\omega_0\tau} X_{p,\ell}(\tau).$$

Asymmetric Faults in Power Systems



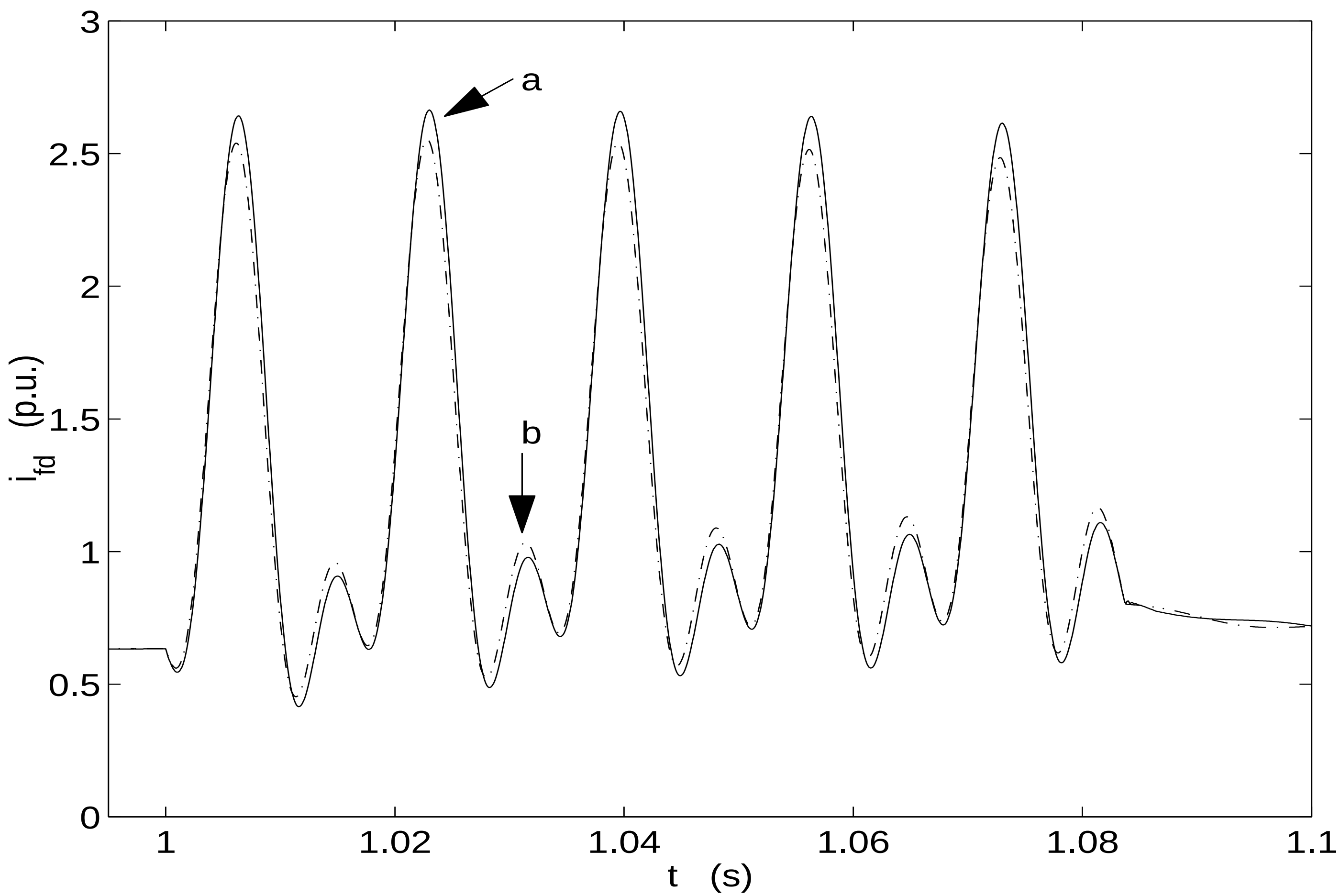
Asymmetric Faults in Power Systems

Line voltages



Asymmetric Faults in Power Systems

Field current



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Phasor Estimation

Starting with a standard (A, B, C) formulation of an LTI model, our state x is n-dimensional, and it contains a finite number of non-negligible harmonics, collected in a (symmetric) set S

$$x(\tau) = \sum_{\ell \in S} X_\ell(t) e^{j\ell\omega_0\tau}, \quad \tau \in (t - T_0,\ t]$$

For each phasor we can write

$$\dot{X}_\ell(t) = (A - j\ell\omega_0) X_\ell(t) + BU_\ell(t)$$

or we can combine all such equations as

$$\dot{\mathbf{X}}(t) = [\mathcal{A} - j\ \omega_0\ diag(S) \otimes I_n]\,\mathbf{X}(t) + \mathcal{B}\mathbf{U}(t)$$

where $\mathbf{X}$ (resp. $\mathbf{U}$) is a column vector consisting of all the phasors X_ℓ (resp. U_ℓ) for $\ell \in S$ and $\mathcal{A} = I_K \otimes A$, $\mathcal{B} = I_K \otimes B$.

Phasor Estimation, cont 1.

The (p-dimensional) output equation is (for all $\tau \in (t - T_0,\ t]$):

$$y(\tau) = Cx(\tau) = C\theta(\tau)\mathbf{X}(t)$$

where $\theta(\tau)$ is a block row matrix – for instance, if $S = \{\ell; \mid \ell \mid \leq L\}$, the

$$\theta(\tau) = \left[e^{-jL\omega_0\tau} I_n \ldots I_n \ldots e^{jL\omega_0\tau} I_n\right]$$

It turns out that in order to ensure observability, we arrange to have M output samples, obtained at $t,\ t-h, \cdots, t-(M-1)h$, all within $(t - T_0,\ t]$, and ordered chronologically in a vector $\mathbf{y}(t)$.

$$\mathbf{y}(t) \quad = \quad \begin{bmatrix} C\ \theta(t-(M-1)h) \\ \vdots \\ C\ \theta(t-h) \\ C\ \theta(t) \end{bmatrix} \mathbf{X}(t) \quad = \quad \mathcal{C}(t)\mathbf{X}(t)$$

The task is now to estimate the phasor state vector $\mathbf{X}(t)$, which satisfies the corresponding differential equation, from possibly noisy measurements of $\mathbf{y}(t)$.

Model Observability

Important for transient performance; the overall model is periodic due to the output map - to simplify, consider the change of coordinates

$$\chi_\ell(t) = X_\ell(t) e^{j\ell\omega_0 t}$$

A direct check shows that the corresponding state equation is

$$\dot{\boldsymbol{\chi}}(t) = \mathcal{A}\boldsymbol{\chi}(t) + \mathcal{B}\boldsymbol{\upsilon}(t)$$

the output equation simplifies as well

$$\mathbf{y}(t) = \mathcal{C}(0)\,\boldsymbol{\chi}(t)$$

The overall model is now LTI so we consider the observability matrix

$$\mathcal{O} = \begin{bmatrix} \mathcal{C}(0) & \mathcal{C}(0)\,\mathcal{A} & \mathcal{C}(0)\,\mathcal{A}^2 & \cdots & \mathcal{C}(0)\,\mathcal{A}^{n-1} \end{bmatrix}^\top$$

For a scalar system with state measurement ($n = 1 = p$,$C = c = 1$) and $K = \mid S \mid$, need at least $M \geq K$ measurements.

Estimation with Prescribed Harmonics

$$x(\tau) = \sum_{\ell \in S} X_\ell(t) e^{j\ell\omega_0\tau} + \sum_{\ell \notin S} X_\ell(t) e^{j\ell\omega_0\tau}$$

the output equation now becomes

$$\boldsymbol{y}(t) = \mathcal{C}(t)\boldsymbol{X}(t) + \boldsymbol{N}(t)$$

where $\boldsymbol{N}(t)$ represents the contribution of the unmodeled phasors. Since $\boldsymbol{N}(t)$ is, in general, correlated with $\boldsymbol{X}(t)$, the Kalman filter is no longer optimal in the sense of minimizing the probabilistic mean-square error $E|X_\ell(t) - \hat{X}_\ell(t)|^2$.
Nevertheless, it is still optimal in the sense of minimizing the *deterministic* cost

$$\min_{\boldsymbol{X}(0)} \left[\boldsymbol{X}(0) - \boldsymbol{X}_0\right]^H \Pi_0^{-1} \left[\boldsymbol{X}(0) - \boldsymbol{X}_0\right] +$$

$$\int_0^t \left[\boldsymbol{y}(s) - \mathcal{C}(s)\boldsymbol{X}(s)\right]^H R_{vv}^{-1} \left[\boldsymbol{y}(s) - \mathcal{C}(s)\boldsymbol{X}(s)\right] ds$$

Single Phase Inductor

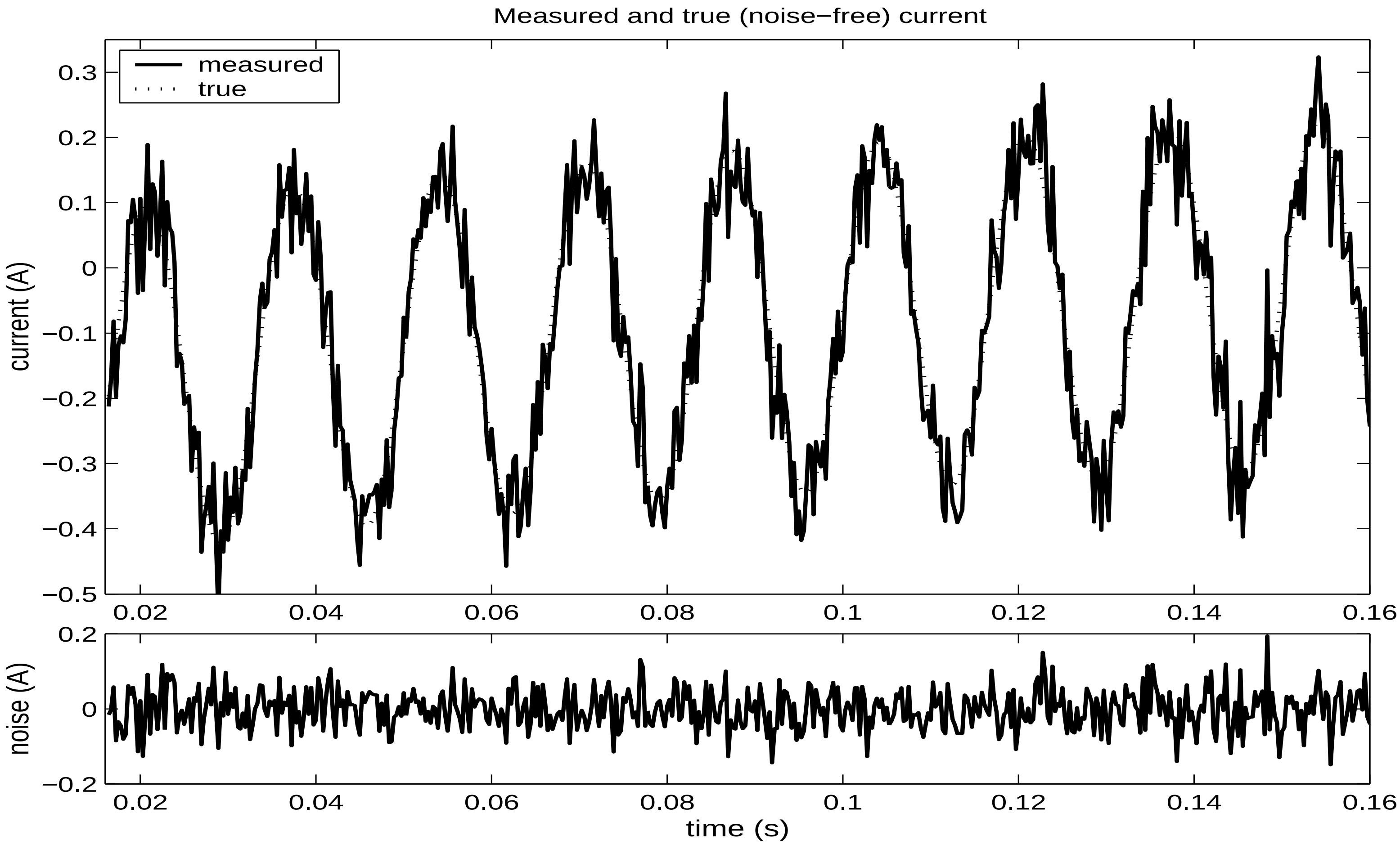
Consider an inductor L to which we connect an ac source $V\cos(\omega_0 t)$ at time $t = 0$; the solution will have a dc component I_0 and a fundamental component - we denote the fundamental phasor with $I_1 = \mu + j\nu$.

$$\frac{d}{dt}\begin{bmatrix} I_0 \\ \mu \\ \nu \end{bmatrix} = \begin{bmatrix} -R/L & 0 & 0 \\ 0 & -R/L & \omega_0 \\ 0 & -\omega_0 & -R/L \end{bmatrix}\begin{bmatrix} I_0 \\ \mu \\ \nu \end{bmatrix} + \begin{bmatrix} 0 \\ 1/2L \\ 0 \end{bmatrix} V$$

We assume that only noisy measurements of the current are available.

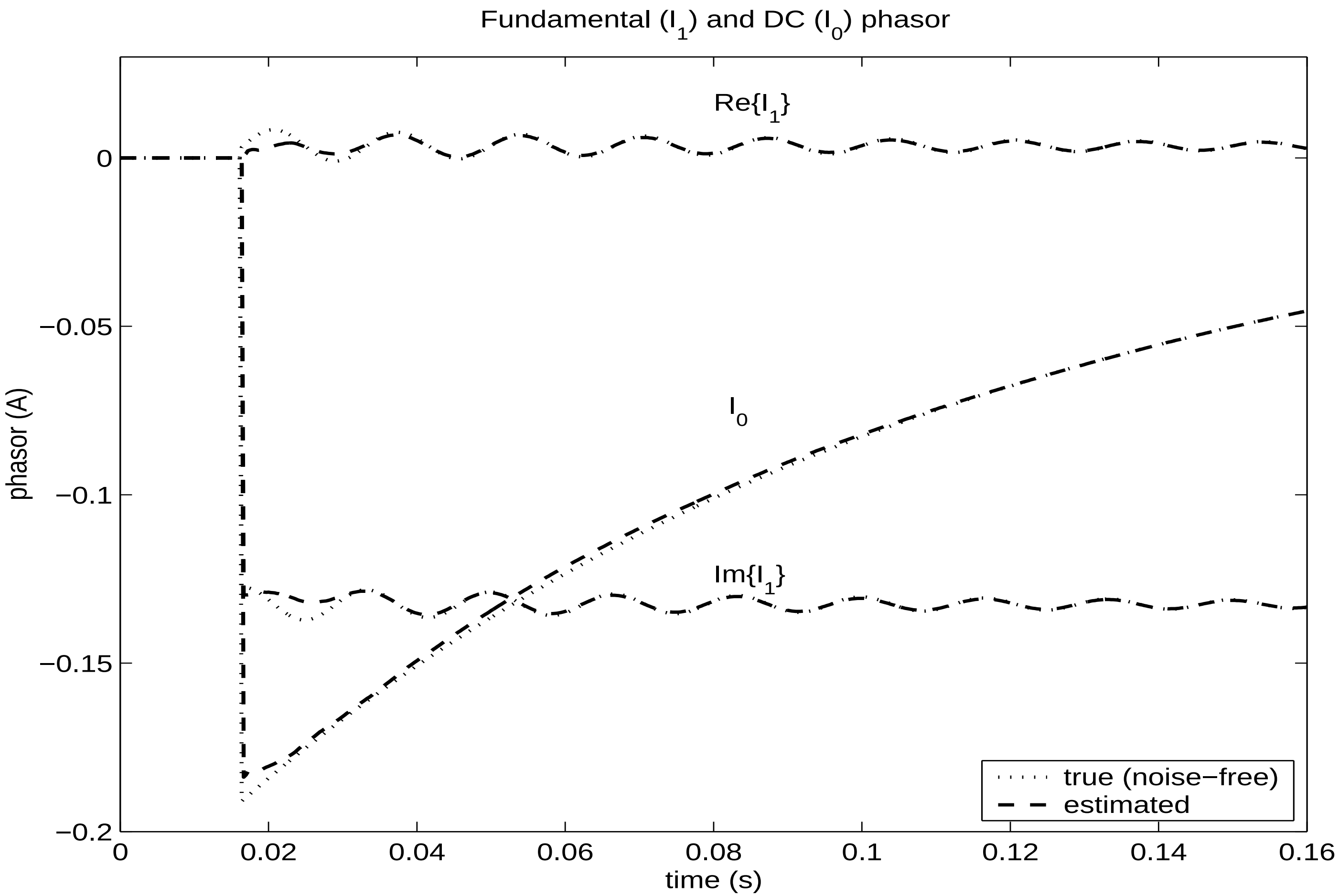
Single Phase Inductor, cont. 1

Noisy “measured” current (solid line) and the true current (dotted line):



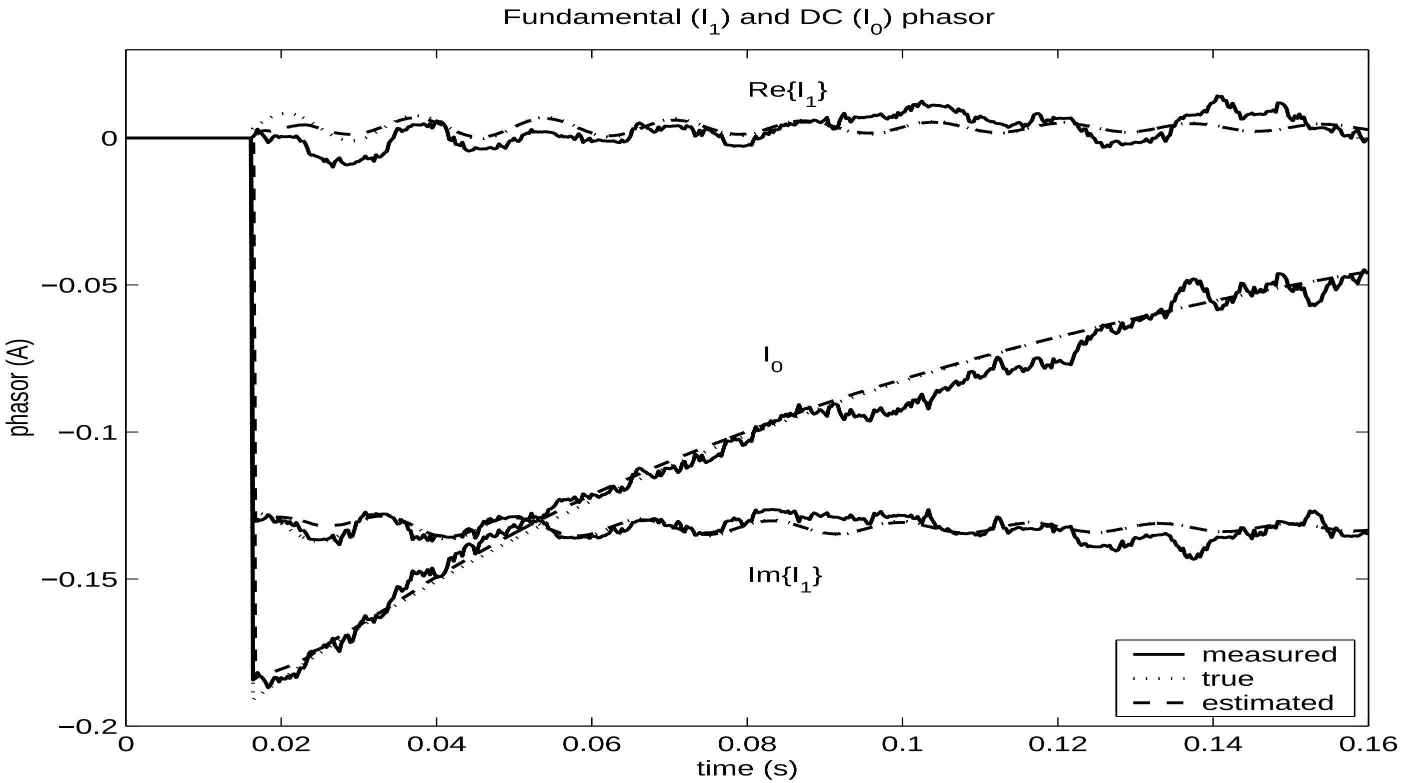
Single Phase Inductor, cont. 2

Time evolution of various phasor components (true - dotted line) and their estimates: (dashed line)



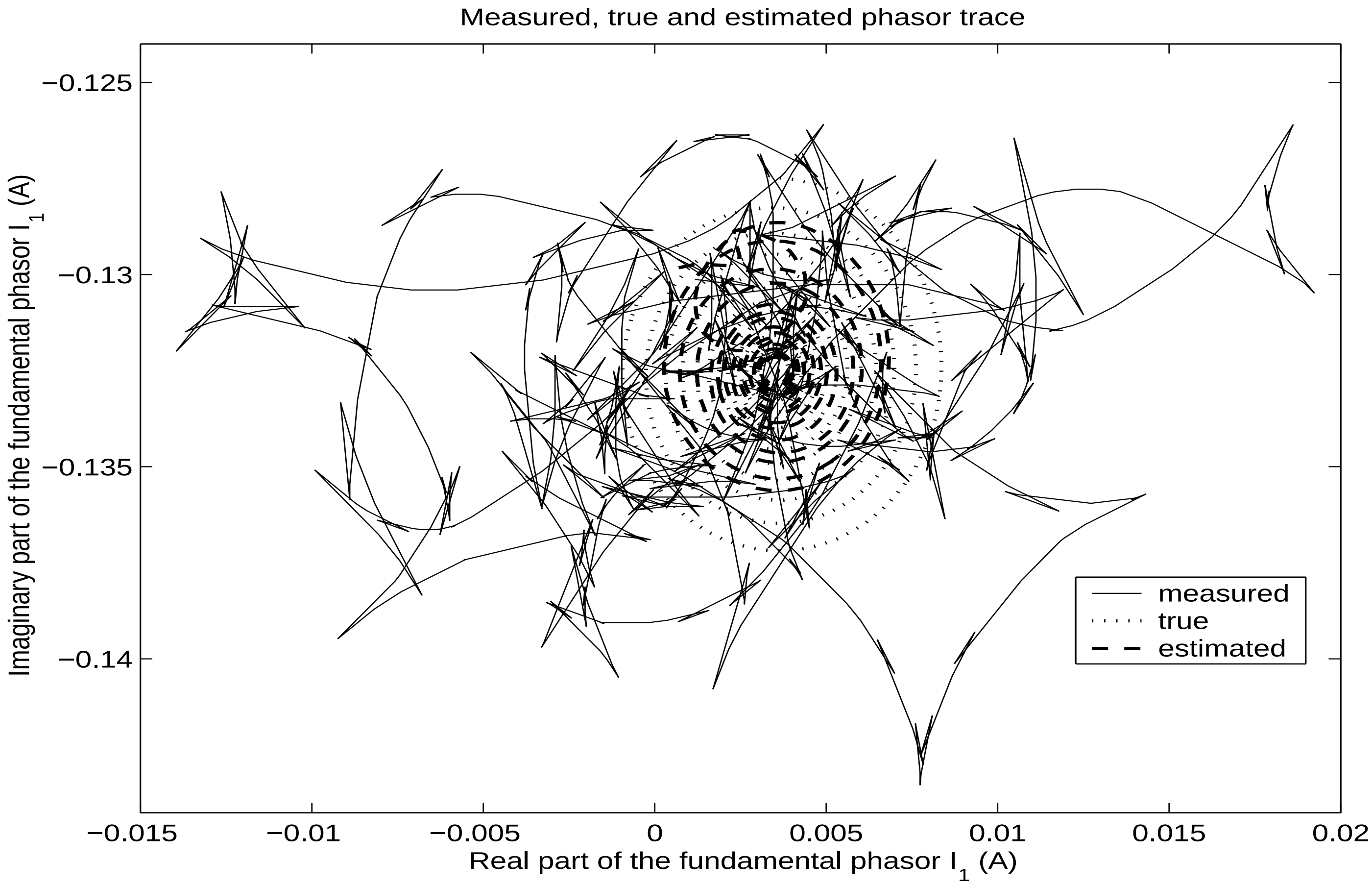
Single Phase Inductor, cont. 3

Tracking performance of the current phasors - “measured” running window DFT (solid line), true values (dotted line) and Kalman-based estimate (dashed line):

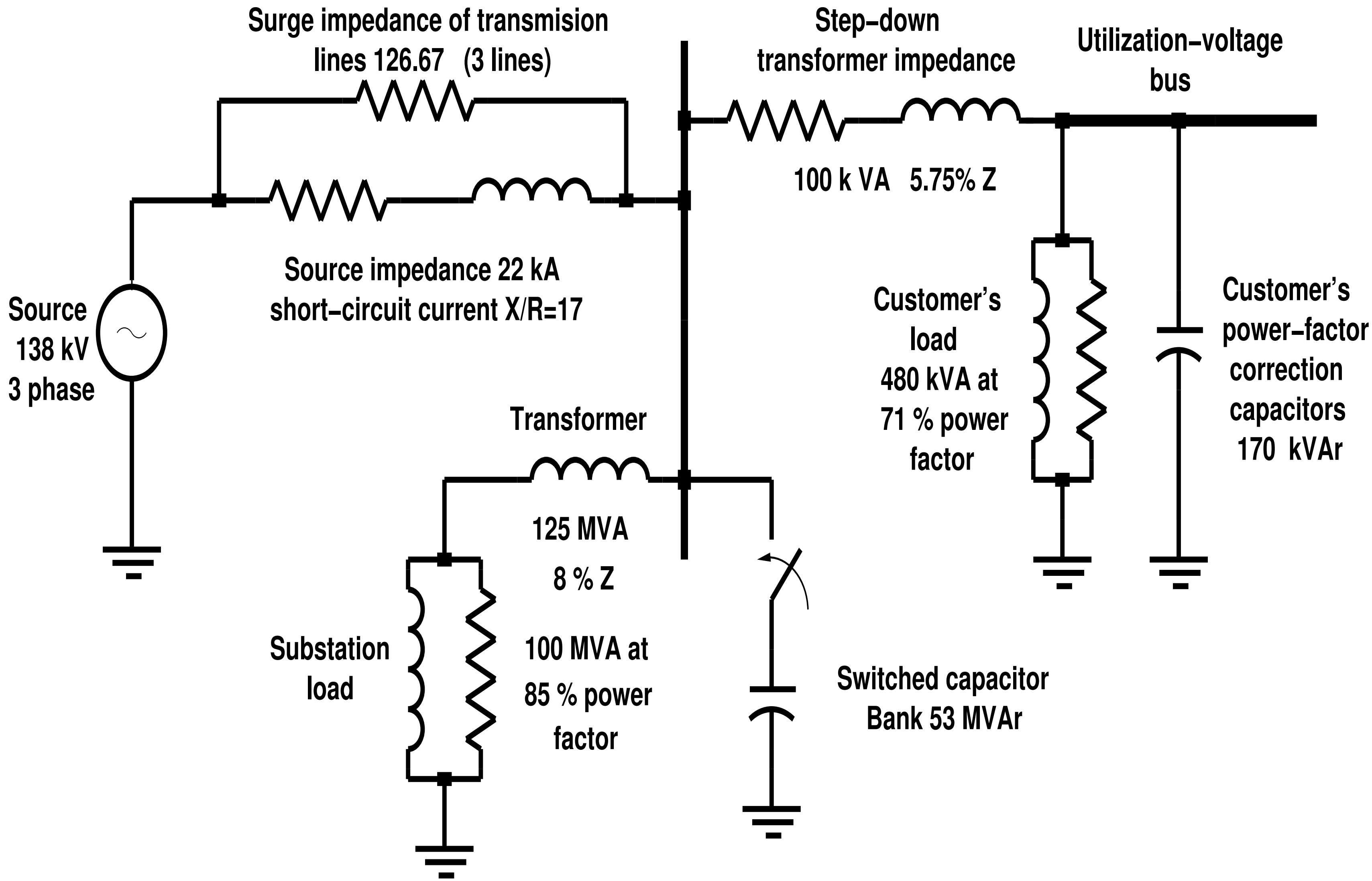


Single Phase Inductor, cont. 4

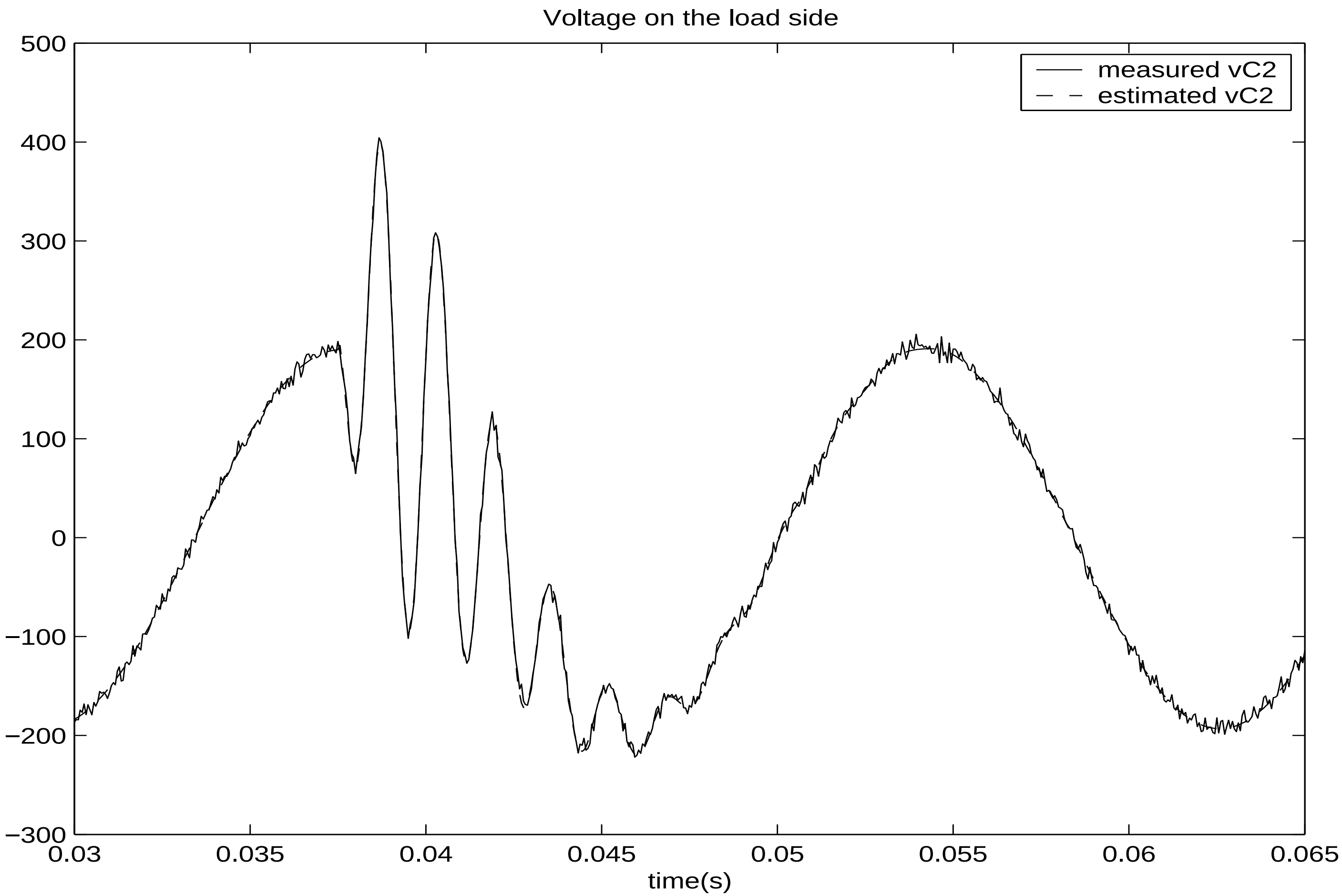
Tracking performance of the **fundamental** current phasor - DFT-based estimate (solid line), true values (dotted line) and Kalman filter-based estimate (dashed line)



Power Systems Example



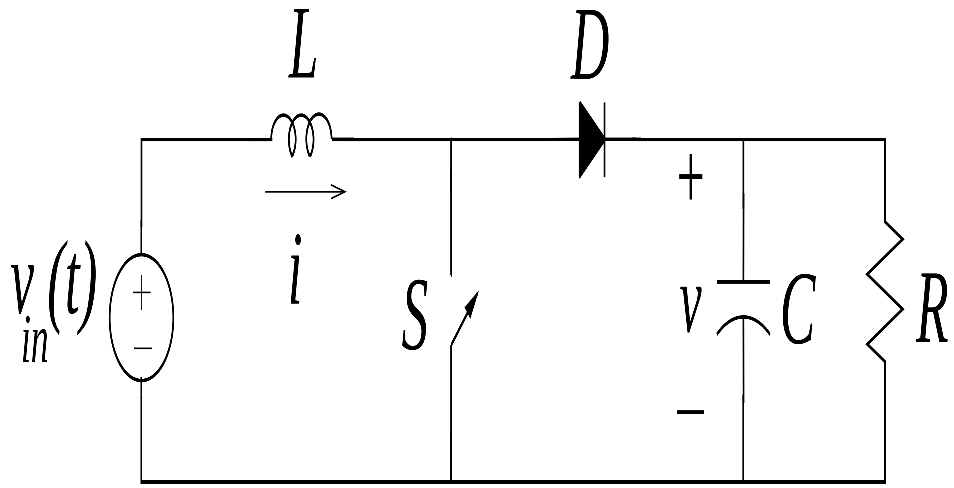
Power Systems Example, cont. 1



Presentation Map

- Background and definitions,
- POWER SYSTEMS - Flexible AC Transmission Systems,
- ELECTRIC DRIVES - AC machine modeling,
- An interlude - extension to polyphase systems,
- ESTIMATION - dynamic phasors and symmetrical components,
- **POWER ELECTRONICS - model reduction in switched-mode DC/DC converters.**

Model Reduction - DC/DC Converters



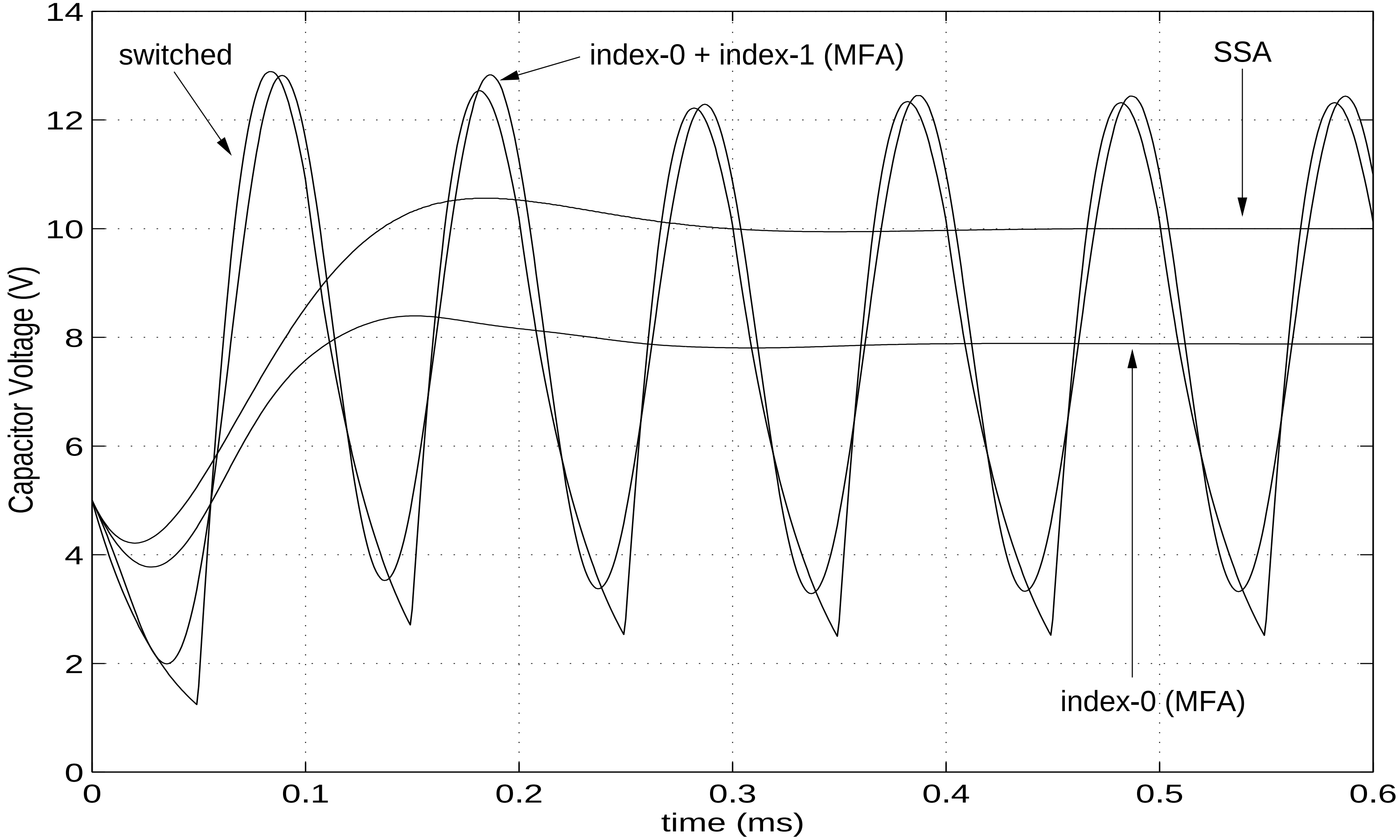
Model in continuous conduction ($q(\cdot) = 1$ when S closed):

$$
\begin{aligned}
L\frac{di}{dt} &= v_{in}(t) - [1 - q(t)]v(t) \\
C\frac{dv}{dt} &= [1 - q(t)]i(t) - \frac{1}{R}v(t)
\end{aligned}
$$

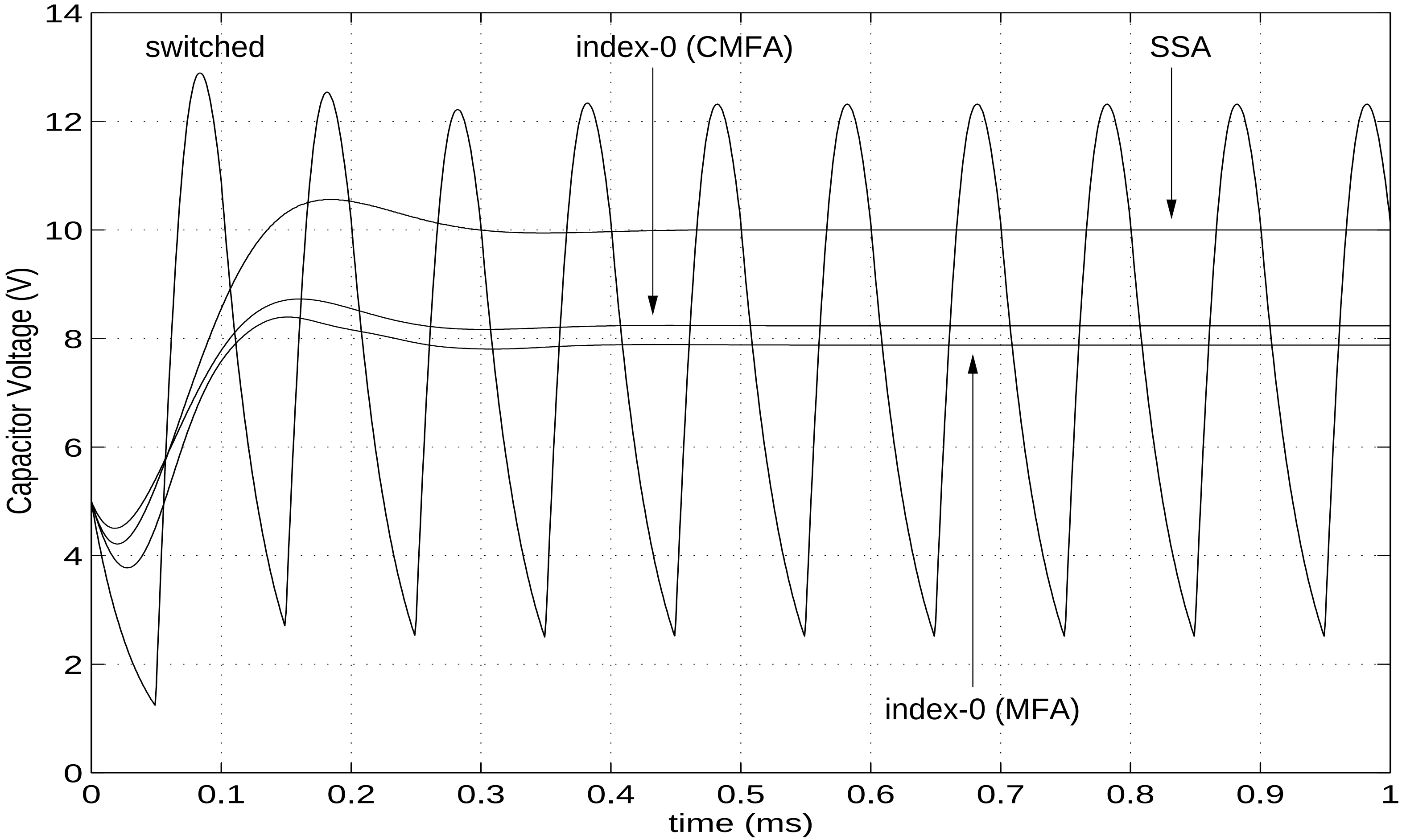
Model Reduction - DC/DC Converters

$$
\begin{aligned}
L\frac{d<i>_0}{dt} &= V_{in} - (1- < q >_0) < v >_0 + 2 < q >_1^R < v >_1^R \\
&\quad + 2 < q >_1^I < v >_1^I \\
C\frac{d<v>_0}{dt} &= (1- < q >_0) < i >_0 - \frac{<v>_0}{R} - 2 < q >_1^R < i >_1^R \\
&\quad - 2 < q >_1^I < i >_1^I \\
L\frac{d<i>_1^R}{dt} &= L\omega_0 < i >_1^I + (1- < q >_0) < v >_1^R + < v >_0 < q >_1^R \\
L\frac{d<i>_1^I}{dt} &= -L\omega_0 < i >_1^R - (1- < q >_0) < v >_1^I + < v >_0 < q >_1^I \\
C\frac{d<v>_1^R}{dt} &= C\omega_0 < v >_1^I + (1- < q >_0) < i >_1^R - < i >_0 < q >_1^R \\
&\quad - \frac{<v>_1^R}{R} \\
C\frac{d<v>_1^I}{dt} &= -C\omega_0 < v >_1^R + (1- < q >_0) < i >_1^I - < i >_0 < q >_1^I \\
&\quad - \frac{<v>_1^I}{R}
\end{aligned}
$$

Model Reduction - DC/DC Converters



Model Reduction - DC/DC Converters



Summary

- Dynamic phasors yield simple, but powerful large-signal dynamic models.
- A unified approach with additional applications in power electronics (resonant converters, active filters), electric drives (torque ripple minimization, position-dependent loads) and power systems (UPFC, protection).
- Both refinements and simplifications are possible for various accuracy requirements.
- Models are modular, and compatible with engineering experience and intuition,
- A timely re-examination of analytical tools in energy processing addresses challenges posed by advances in semiconductor and computer technology.