HILBERT SPACE APPROACH TO MODELING AND COMPENSATION OF REACTIVE POWER

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A.M. Stanković and **H. Lev-Ari**
Northeastern University, Boston, USA
`astankov, levari@ece.neu.edu`

ASU Dept. of Electrical Engineering

Goals

Main goal - a comprehensive framework for modeling and compensation of reactive (inactive) power in energy processing systems - multiple criteria in steady-state (with harmonics) and in transients.

Applications - power systems, industrial electric drives, and power electronic systems; also emerging electromechanical systems with complex power flow patterns (e.g. piezoelectric transducers), and naval and aerospace applications where energy efficiency is *critical*.

The key concept is that of a *projection* (often orthogonal) on a chain of nested subspaces in a Hilbert space.

Technical relevance - our development is in frequency domain, where standards for harmonic distortion are specified; technical means for on-line compensation of polyphase systems are just becoming available – power converters and DSP.

Motivating Examples

Unbalanced R - two phase system (say $V_1 \cos(\omega t), \quad V_2 \sin(\omega t)$) with (very) unbalanced voltages (say $V_1 \; = \; 10\ V_2$) and resistances ($I_2 \; = \; 10\ I_1$).

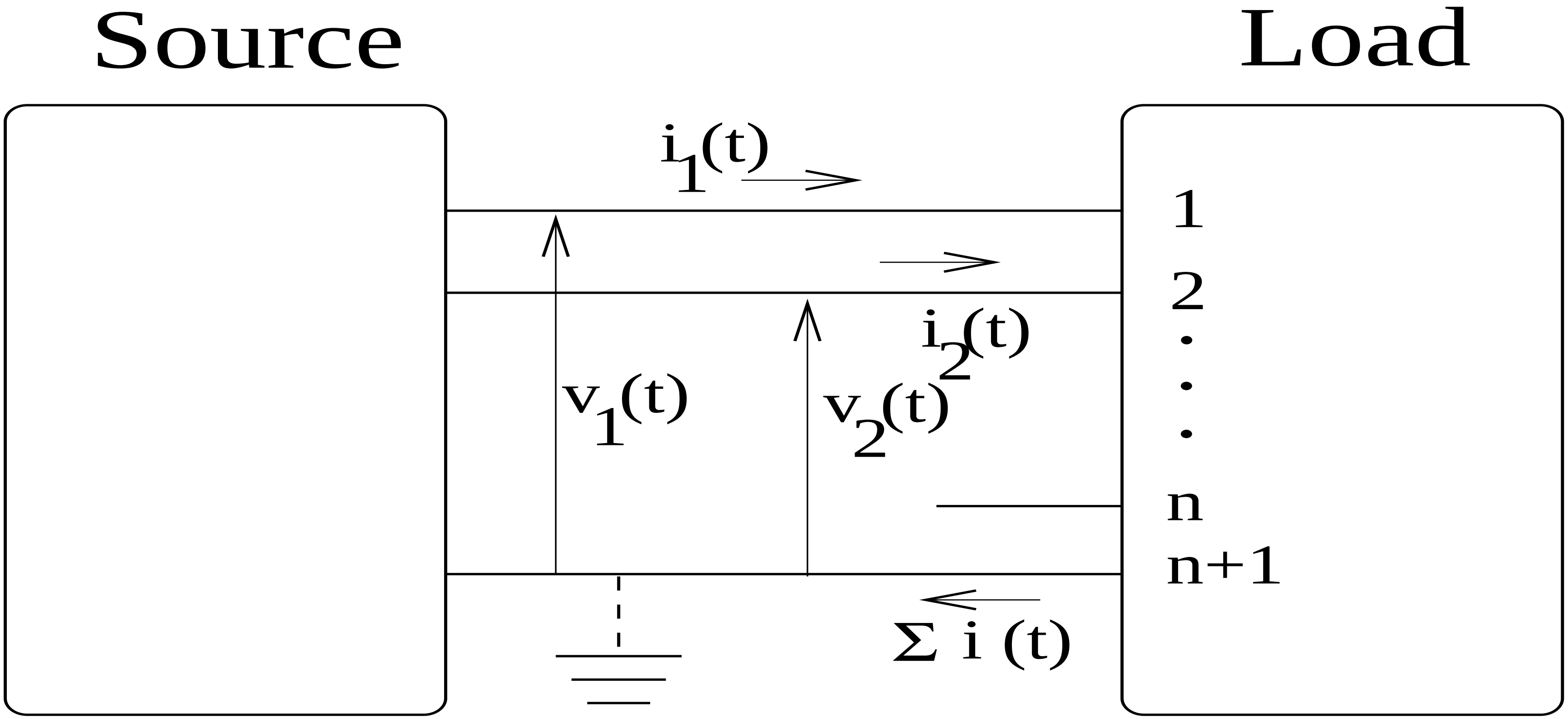
Three phase RLC - assume balanced single-frequency voltages, purely resistive phase *a*, purely inductive phase *b*, and purely capacitive phase *c*, with $L\omega = {}^1\!/_{C\omega}$,

Linear L with harmonics - single phase, with two (or more) harmonics.

Transients

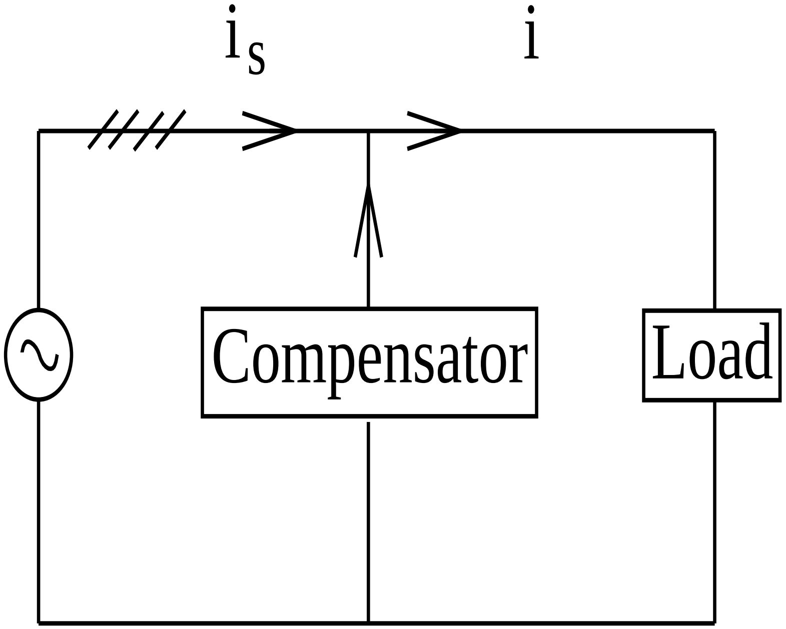
Problem Formulation

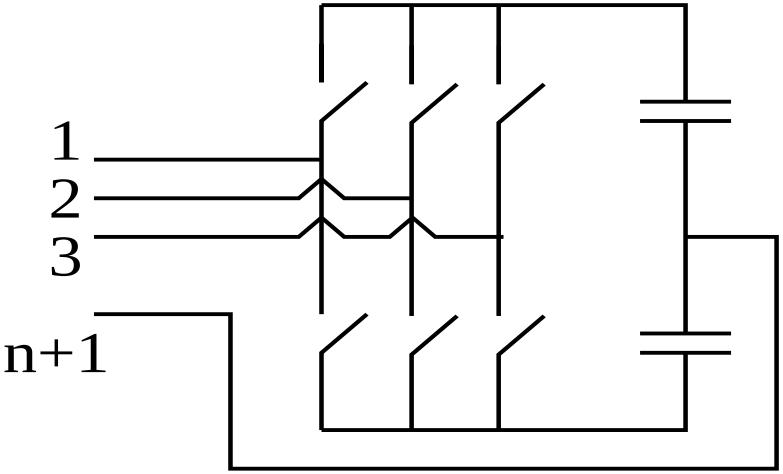
Consider a possibly unbalanced system with n phases, $n + 1$ conductors (with possibly different resistances), and arbitrary many harmonics:



Problem Formulation (2)

Compensation framework





Compensator

For now, all time domain vector waveforms $x(t)$ are periodic, with period T_0; let $\omega_0 = {}^{2\pi}/_{T_0}$ so the Fourier coefficients are

$$X_\ell = \frac{1}{T_0}\int_{T_0} x(t)e^{-j\ell\omega_0 t}dt \overset{\text{def}}{=} < x(t) >_\ell \qquad (1)$$

Problem Formulation (3)

Some time-domain quantities - all vectors are *row* vectors:
The instantaneous power

$$p(t) = v(t)i(t)^\top . \tag{2}$$

The real or average power is the DC component $P_0 =< p(t) >_0$

The rms quantities $\|v\| \overset{\text{def}}{=} \sqrt{< v(t)v(t)^\top >_0}$, $\|i\| \overset{\text{def}}{=} \sqrt{< i(t)i(t)^\top >_0}$
The instantaneous active current $i_a(t)$:

$$i_a(t) = \frac{p(t)}{v(t)v(t)^\top} v(t) = \frac{i(t)v(t)^\top}{v(t)v(t)^\top} v(t) \tag{3}$$

It is the smallest current (by rms) that supplies the same $p(t)$ at $i(t)$.
The Fryze current:

$$i_F(t) = \frac{< p(t) >_0}{\|v\|^2} v(t) = \frac{< p(t) >_0}{< v(t)v(t)^\top >_0} v(t) \tag{4}$$

is the smallest current (by rms) with the same DC component $< p(t) >_0$ as $i(t)$.

Example 1 (1)

Consider a *resistive* two phase circuit with

$$v(t) = [V_1 \cos(\omega_0 t) \quad V_2 \sin(\omega_0 t)]\,, \qquad i(t) = \left[{}^{P_0}\!/_{V_1} \cos(\omega_0 t) \quad {}^{P_0}\!/_{V_2} \sin(\omega_0 t)\right]$$

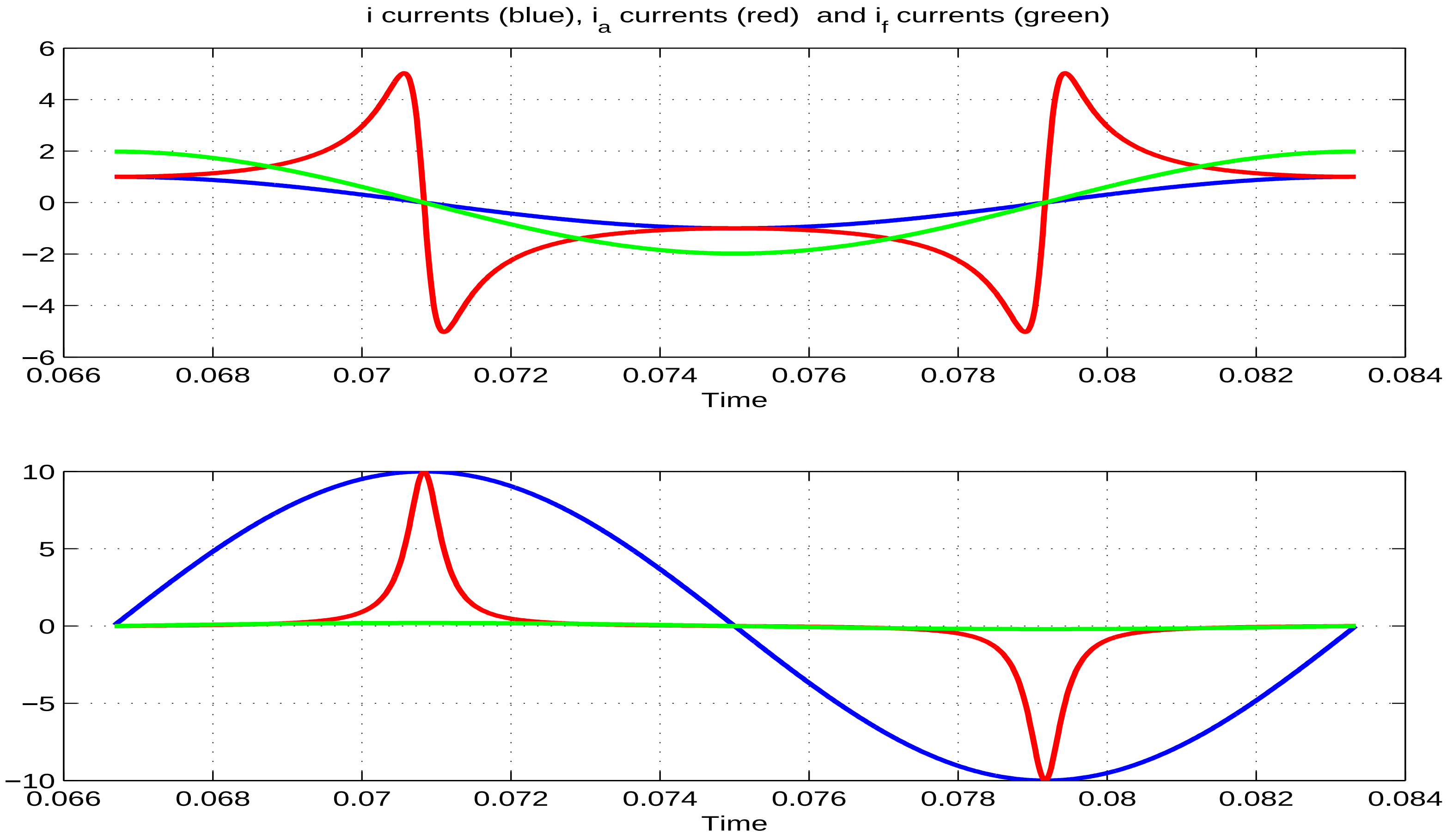
- The resistance in each phase is proportional to the square of the voltage magnitude in that phase. Then $v(t)v(t)^\top = V_1^2 \cos^2(\omega_0 t) + V_2^2 \sin^2(\omega_0 t)$, and $p(t) = v(t)i(t)^\top = P_0$.
- $p(t)$ has only a DC component, but $v(t)v(t)^\top$, which *divides* $p(t)$ in the definition of i_a, has a DC and a second harmonic (for $V_1 \neq V_2$).

The resulting $i_a(t)$ in this example has infinitely many harmonics

$$i_a(t) = \frac{2P_0}{V_1 + V_2} \sum_{\ell=0}^{\infty} \left(-\frac{V_1 - V_2}{V_1 + V_2}\right)^\ell \left[\cos(2\ell + 1)\omega_0 t \quad \sin(2\ell + 1)\omega_0 t\right]$$

Example 1 (2)

With $V_1 = 1,\ V_2 = 0.1,\ P_0 = 1,$



Currents in phase 1 (top panel) and phase 2 (bottom panel): i (blue), i_a (red), and i_f (green).

A Recapitulation

- The Fryze and the instantaneous compensator are quite different.
- The Fryze compensator minimizes the line current (good for the energy source!), but may require elaborate compensation, including significant energy storage.
- The instantaneous compensator has no energy storage requirements, but may alter the frequency content of the line current in undesirable ways.
- Unclear how to include practical constraints - limitations of the compensator current bandwidth and unequal line resistances.
- The two compensators are members of a much larger family - power harmonic matching compensators.
- A larger framework is needed for effective study - we propose a Hilbert space.

Hilbert Space Interpretation

We define the inner product of two polyphase signals $x(t)$ and $y(t)$ as

$$\langle \, x \, , \, y \, \rangle \overset{\text{def}}{=} \frac{1}{T_0} \int_{T_0} x(t) y(t)^\top dt \tag{5}$$

Using Parseval's identity

$$\langle \, x \, , \, y \, \rangle = \sum_{\ell} X_\ell Y_\ell^H \overset{\text{def}}{=} \langle \, X \, , \, Y \, \rangle \tag{6}$$

where $\{X_\ell\}$ is the sequence of Fourier coefficients of $x(t)$.
An orthogonal projection of Y on the subspace spanned by signal X

Projection of Y on $X = \dfrac{\langle \, Y \, , \, X \, \rangle}{\langle \, X \, , \, X \, \rangle} X \overset{\text{def}}{=} \Pi_X\{Y\}$

Projection Interpretation of I_f and I_a

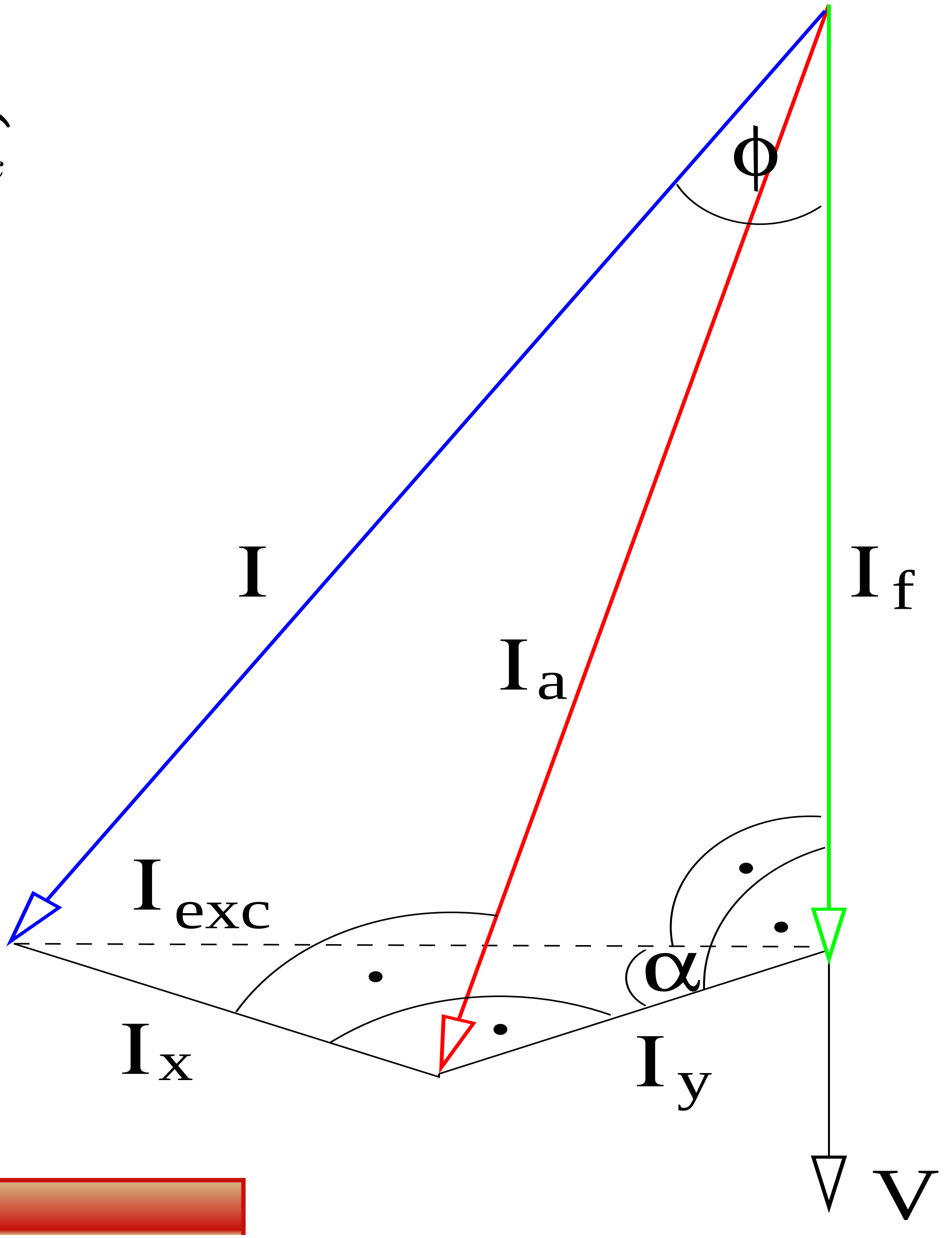
It follows, for example, that $I_f = \Pi_V\{I\}$, the projection of the current I onto the *one-dimensional* subspace spanned by the voltage V.

The mapping $i(t) \mapsto i_a(t)$ (equivalently $I \;\mapsto\; I_a$) is an orthogonal projection onto an (infinite dimensional) subspace spanned by the set of all signals of the type $\alpha(t)v(t)$, where $\alpha(t)$ is an arbitrary T_0-periodic real **scalar** signal. One possible (non-orthogonal) basis for this subspace is the following collection of polyphase signals

$$\left\{ v(t)\,\cos\ell\omega_0 t\;,\;\; v(t)\,\sin\ell\omega_0 t\;;\;\; \ell \geq 0 \right\}.$$

Projections in the Hilbert Space

$$I = I_F \oplus \overbrace{I_y \oplus I_x}^{I_{exc}}$$



Orthogonal Basis for Bandlimited Periodic Waveforms

Harmonic/phase decomposition:

$$v(t) = \sum_{k=1}^{n} \sum_{\ell=1}^{L} v_{k,\ell}(t) \ , \ v_{k,\ell}(t) \triangleq 2\,\Re\{V_{k,\ell}e^{j\ell\omega t}\}\ e_k$$

$$e_k \triangleq [\underbrace{0\ldots 0\ 1}_{k}\ \ 0\ldots 0] \ , \ V_{k,\ell} \triangleq \frac{1}{T}\int_T \left[v(t)e_k^T\right] e^{-j\ell\omega t}dt$$

So

$v_{k,\ell}(\cdot) \in \mathcal{S}_{k,\ell} \ \triangleq$ Set of all (T-periodic) k-th phase, ℓ-th harmonic waveforms

Orthogonal basis for $\mathcal{S}_{k,\ell}$: $\{v_{k,\ell}(\cdot), \mathcal{H}v_{k,\ell}(\cdot)\}$, $\mathcal{H}$ is the Hilbert transform.

Recall: $\mathcal{H}\{\cos \ell\omega t\} = \sin \ell\omega t, \quad \mathcal{H}\{\sin \ell\omega t\} = -\cos \ell\omega t$

Orthogonal basis for the bandlimited subspace $\bigoplus_{k=1}^{n}\bigoplus_{\ell=1}^{L}\mathcal{S}_{k,\ell}$

$$\left\{ \ [v_{k,\ell}(\cdot)\ ,\ \mathcal{H}v_{k,\ell}(\cdot)] \ ; \ 1 \le k \le n \ \ , \ 1 \le \ell \le L \ \right\}$$

The Fundamental Orthogonal Current Decomposition

$$i(t) \; = \underbrace{\sum_{k,\ell} g_{k,\ell}\, v_{k,\ell}(t)}_{i_v(t)} \oplus \underbrace{\sum_{k,\ell} b_{k,\ell}\, \mathcal{H}v_{k,\ell}(t)}_{i_w(t)} \oplus i_\perp(t)$$

where

$$g_{k,\ell} \triangleq \frac{\langle\, i \;,\; v_{k,\ell}\,\rangle}{\|v_{k,\ell}\|^2}, \qquad b_{k,\ell} \triangleq \frac{\langle\, i \;,\; \mathcal{H}v_{k,\ell}\,\rangle}{\|v_{k,\ell}\|^2}$$

Equivalent linear load: focus on $\mathcal{S}_{k,\ell}$ for fixed k, ℓ

$$i_{k,\ell}(t) \;=\; g_{k,\ell}\, v_{k,\ell}(t) + b_{k,\ell}\, \mathcal{H}v_{k,\ell}(t) \;=\; 2\,\Re\{I_{k,\ell}\; e^{j\ell\omega t}\}\; e_k$$

$$\Downarrow$$

$$(k,\ell)\text{-admittance} \;\triangleq\; \frac{I_{k,\ell}}{V_{k,\ell}} \;=\; g_{k,\ell} - j\, b_{k,\ell}$$

The (2+1)-component decomposition:

$$i(t) = i_v(t) \oplus i_w(t) \oplus i_\perp(t) \qquad\qquad (\; b_{k,\ell} = 0 \;\Leftrightarrow\; i_w = 0 \;)$$

Reconciling Fryze and Budeanu

Geometric interpretation:

$$i_v(\cdot) \;\equiv\; \text{projection of } i(\cdot) \text{ on } \mathcal{V}\ ,\ i_w(\cdot) \;\equiv\; \text{projection of } i(\cdot) \text{ on } \mathcal{W}$$

$$\mathcal{V} \;\triangleq\; \textit{span}\left\{v_{k,\ell}(\cdot)\right\}\ ,\ \mathcal{W} \;\triangleq\; \textit{span}\left\{\mathcal{H}v_{k,\ell}(\cdot)\right\} \qquad (\,1 \le k \le n,\, 1 \le \ell \le L\,)$$

Also (Fryze current) $i_F(\cdot) \;\sim\; v(\cdot) \;\in\; \mathcal{V}$. So

$$i_v(t) = i_F(t) \;\oplus\; i_g(t)\ ,\ i_g(t) = \sum_{k,\ell} (\,g_{k,\ell} - \mu_g\,)\, v_{k,\ell}(t)$$

Statistical interpretation:

$$i_F(t) = \mu_g\ v(t)\ ,\ \mu_g \triangleq \sum_{k,\ell} p_{k,\ell}\ g_{k,\ell}\ ,\ p_{k,\ell} \triangleq \frac{\|v_{k,\ell}\|^2}{\|v\|^2}$$

$$\|i_g\| = \sigma_g\ \|v\|\ ,\ \sigma_g \triangleq \sqrt{\sum_{k,\ell} p_{k,\ell}\ (g_{k,\ell} - \mu_g)^2} \qquad (\ g_{k,\ell} = \text{const.} \;\Leftrightarrow\; i_g = 0\ )$$

Reconciling Fryze and Budeanu (continued)

Dual decomposition:

$$i_w(t) = i_B(t) \oplus i_b(t) \; , \; i_b(t) = \sum_{k,\ell} (b_{k,\ell} - \mu_b) \, \mathcal{H}v_{k,\ell}(t)$$

Budeanu's reactive power: extended to polyphase waveforms

$$i_B(t) \triangleq \frac{\langle \, i \, , \, \mathcal{H}v \, \rangle}{\|v\|^2} \, \mathcal{H}v(t) \equiv \text{ projection of } i(\cdot) \text{ on } \mathcal{H}v(\cdot)$$

$$Q_B \triangleq \langle \, i \, , \, \mathcal{H}v \, \rangle = \sum_{k,\ell} 2 \, |I_{k,\ell}| \, |V_{k,\ell}| \, \sin\left(\arg(V_{k,\ell} \, I^*_{k,\ell}) \right)$$

Statistical interpretation:

$$i_B(t) = \mu_b \, \mathcal{H}v(t) \; , \; \mu_b \triangleq \sum_{k,\ell} p_{k,\ell} \, b_{k,\ell}$$

$$\|i_b\| = \sigma_b \, \|v\| \; , \; \sigma_b \triangleq \sqrt{\sum_{k,\ell} p_{k,\ell} \, (b_{k,\ell} - \mu_b)^2} \qquad (\, b_{k,\ell} = \text{const.} \; \Leftrightarrow \; i_b = 0 \,)$$

Reconciling with Sharon and with Shepherd&Zakikhani

(4+1)-component decomposition:

$$i(t) = \underbrace{i_F(t) \ \oplus \ i_g(t)}_{i_v(t)} \oplus \underbrace{i_B(t) \ \oplus \ i_b(t)}_{i_w(t)} \oplus \ i_\perp(t)$$

$$S^2 = P^2 + N_g^2 + Q_B^2 + Q_b^2 + S_\perp^2$$

$$S \ \triangleq \ \|i\| \, \|v\| \ , \ N_g \ \triangleq \ \|i_g\| \, \|v\| \ , \ Q_b \ \triangleq \ \|i_b\| \, \|v\| \ , \ S_\perp \ \triangleq \ \|i_\perp\| \, \|v\|$$

Sharon	P	S_c	S_Q
Ours	i_F, P	$i_g, \ N_g$	$i_w, \ Q_w$

Shepherd & Zakikhani	S_R	S_X
Ours	$i_v, \ S_v$	$i_w, \ Q_w$

$$Q_w \ \triangleq \ \|i_w\| \, \|v\| = \sqrt{Q_B^2 + Q_b^2}$$

Effect of Unbalanced Load

Splitting of i_g :

$$i_v(t) \;=\; i_F(t) \oplus \underbrace{i_{gs}(t) \oplus i_{gu}(t)}_{i_g(t)} \,,\; \mu_g(\ell) \triangleq \frac{\sum_k g_{k,\ell}\,\|v_{k,\ell}\|^2}{\sum_k \|v_{k,\ell}\|^2}$$

$$i_{gs}(t) \;=\; \sum_{k,\ell} \Big[\mu_g(\ell) - \mu_g\Big] v_{k,\ell}(t) \,,\; i_{gu}(t) \;=\; \sum_{k,\ell} \Big[g_{k,\ell} - \mu_g(\ell)\Big]\, v_{k,\ell}(t)$$

Splitting of i_b :

$$i_w(t) \;=\; i_B(t) \oplus \underbrace{i_{bs}(t) \oplus i_{bu}(t)}_{i_b(t)} \,,\; \mu_b(\ell) \triangleq \frac{\sum_k b_{k,\ell}\,\|v_{k,\ell}\|^2}{\sum_k \|v_{k,\ell}\|^2}$$

$$i_{bs}(t) \;=\; \sum_{k,\ell} \Big[\mu_b(\ell) - \mu_b\Big] v_{k,\ell}(t) \,,\; i_{bu}(t) \;=\; \sum_{k,\ell} \Big[b_{k,\ell} - \mu_b(\ell)\Big]\, v_{k,\ell}(t)$$

Effect of Unbalanced Load - Statistical Interpretation

Conductances:

$$\|i_{gu}\| = \sigma_{gu}\|v\| \; , \; \sigma_{gu} \triangleq \sqrt{\sum_{k,\ell} p_{k,\ell}\, |g_{k,\ell} - \mu_g(\ell)|^2} \qquad (\,\text{G-balanced load} \;\Leftrightarrow\; i_{gu} = 0\,)$$

$$\|i_{gs}\| = \sigma_{gs}\|v\| \; , \; \sigma_{gs} \triangleq \sqrt{\sum_{k,\ell} p_{k,\ell}\, |\mu_g(\ell) - \mu_g|^2} \qquad (\,\sigma_g^2 = \sigma_{gs}^2 + \sigma_{gu}^2\,)$$

Susceptances:

$$\|i_{bu}\| = \sigma_{bu}\|v\| \; , \; \sigma_{bu} \triangleq \sqrt{\sum_{k,\ell} p_{k,\ell}\, |b_{k,\ell} - \mu_b(\ell)|^2} \qquad (\,\text{B-balanced load} \;\Leftrightarrow\; i_{bu} = 0\,)$$

$$|i_{bs}\| = \sigma_{bs}\|v\| \; , \; \sigma_{bs} \triangleq \sqrt{\sum_{k,\ell} p_{k,\ell}\, |\mu_b(\ell) - \mu_b|^2} \qquad (\,\sigma_b^2 = \sigma_{bs}^2 + \sigma_{bu}^2\,)$$

Reconciling with Czarnecki …

(6+1)-component decomposition:

$$i(t) = \underbrace{i_F(t) \;\oplus\; i_{gs}(t) \;\oplus\; i_{gu}(t)}_{i_v(t)} \oplus \underbrace{i_B(t) \;\oplus\; i_{bs}(t) \;\oplus\; i_{bu}(t)}_{i_w(t)} \oplus i_\perp(t)$$

$$S^2 = P^2 \;+\; \underbrace{N_s^2 \;+\; N_u^2}_{N_g^2} + Q_B^2 \;+\; \underbrace{Q_s^2 \;+\; Q_u^2}_{Q_b^2} + S_\perp^2$$

$$N_s \;\triangleq\; \|i_{gs}\|\,\|v\| \;,\; N_u \;\triangleq\; \|i_{gu}\|\,\|v\| \;,\; Q_s \;\triangleq\; \|i_{bs}\|\,\|v\| \;,\; Q_u \;\triangleq\; \|i_{bu}\|\,\|v\|$$

Czarnecki	i_a	i_s	i_r	i_u	i_g
Ours	i_F	i_{gs}	$i_B \oplus i_{bs}$	$i_{gu} \oplus i_{bu}$	$i_\perp$

Notice: Budeanu’s reactive power Q_B is a subcomponent of Czarnecki’s Q_r .

Basic Examples

Single-frequency voltage, unbalanced (R,L,C) load: Balanced unit magnitude voltages across a resistor (phase a), inductor (phase b), capacitor (phase c), each drawing a unit magnitude current.

S	P	N_s	N_u	Q_B	Q_s	Q_u	$S_\perp$
3	1	0	$\sqrt{2}$	0	0	$\sqrt{6}$	0

$$\mu_g = 1/3 \; , \; \sigma_g^2 = 2/9 \; , \; \mu_b = 0 \; , \; \sigma_b^2 = 2/3$$

Single-phase inductive load: $\|V_1\| = 1 \; , \; \|V_5\| = 0.1 \; , \; b_1 = 1 \; , \; b_5 = 0.2$

S	P	N_s	N_u	Q_B	Q_s	Q_u	$S_\perp$
1.0052	0	0	0	1.0020	0.0800	0	0

$$\mu_g = 0 \; , \; \sigma_g = 0 \; , \; \mu_b = 0.9921 \; , \; \sigma_b = 0.0792$$

Basic Examples (continued)

Single frequency, two-phase resistive load (our Example 1):

$$v(t) = \left[V_1 \cos(\omega t) \; V_2 \sin(\omega t) \right] \, , \; i(t) = \left[{}^{P_0}\!/_{V_1} \cos(\omega t) \; {}^{P_0}\!/_{V_2} \sin(\omega t) \right]$$

$$\mu_g = \frac{2P_0}{V_1^2 + V_2^2} \, , \; \sigma_g = \frac{P_0}{2\|v\|^2} \left| \frac{V_1}{V_2} - \frac{V_2}{V_1} \right|$$

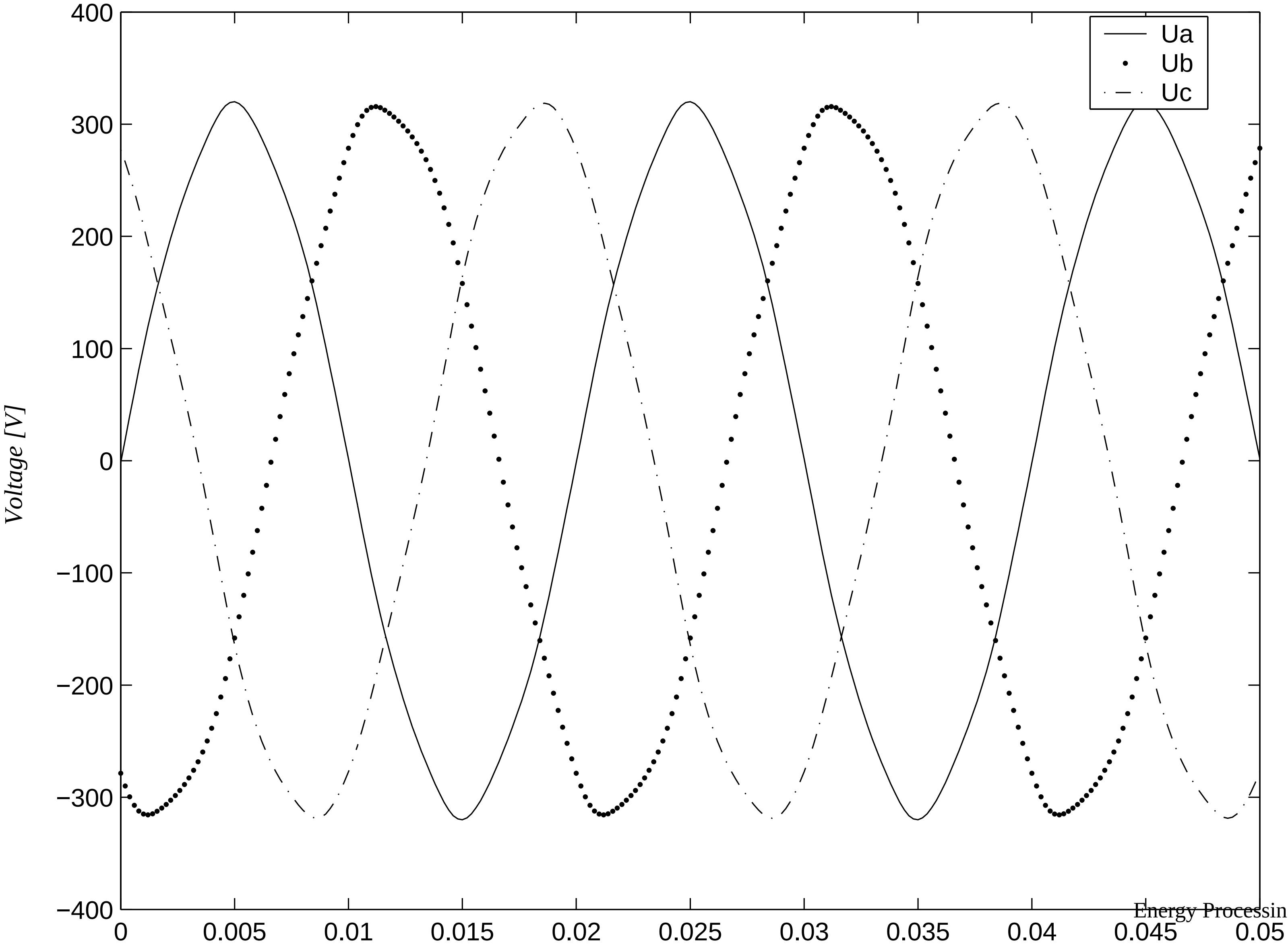
S	P	N_s	N_u	Q_B	Q_s	Q_u	$S_\perp$
5.05	1	0	4.95	0	0	0	0

$V_1 = 1 \, , \; V_2 = 0.1 \, , \; P_0 = 1$

In this *resistive* example, $N_u \equiv N_g = \sigma_g \, \|v\|^2$. In contrast, consider the instantaneous *reactive* power

$$q(t) = N_g \; \sin(2\omega t)$$

Industrial Example – ASD + Unbalanced Load



Industrial Example – Voltage and Current harmonics

	Phase 1		*Phase 2*		*Phase 3*	
Order	*Mag.* $[V]$	*Phase* $[^\circ]$	*Mag.* $[V]$	*Phase* $[^\circ]$	*Mag.* $[V]$	*Phase* $[^\circ]$
1	221.4	-0.18	222.9	-119.6	224.2	119.8
3	0.56	109.4	0.56	-130.6	0.56	-10.56
5	3.74	-142.6	4.25	-27.68	4.32	100.55
7	1.243	26.63	1.69	-127.8	0.784	95.36
9	0.225	-126.6	0.225	-6.62	0.225	113.4
11	0.404	54.43	0.404	174.4	0.404	-65.57

Order	*Mag.* $[A]$	*Phase* $[^\circ]$
1	342.8	-13
3	3.48	33
5	13.37	-16
7	7.20	77
9	0.34	49
11	4.46	145

Industrial Example – Power Decompositions

Unbalanced voltage:

S	P	N_s	N_u	Q_B	Q_s	Q_u	$S_\perp$
282 770	272 540	4 616	24 065	67 720	4 679	20 868	6 078

$$\mu_g = 1.8283[1/\Omega]\ ,\ \sigma_{gs} = \frac{\|i_{gs}\|}{\|v\|} = 0.0310[1/\Omega]\ ,\ \sigma_{gu} = \frac{\|i_{gu}\|}{\|v\|} = 0.1614[1/\Omega]$$

$$\mu_b = 0.4543[1/\Omega]\ ,\ \sigma_{bs} = \frac{\|i_{bs}\|}{\|v\|} = 0.0313[1/\Omega]\ ,\ \sigma_{bu} = \frac{\|i_{bu}\|}{\|v\|} = 0.1399[1/\Omega]$$

Balanced voltage:

S	P	N_s	N_u	Q_B	Q_s	Q_u	$S_\perp$
229 654	223 177	12 480	0	51 302	9 976	0	6 776

Compensation Cost (1)

We characterize the *efficiency* of a compensator by defining the “wasted source power” coefficient

$$\mu = \frac{\|i_s\|^2 - \|i_f\|^2}{\|i\|^2 - \|i_f\|^2} \tag{7}$$

where $i_s(t)$ is the compensated source current.

Storage cost of a compensator is well correlated with the maximal instantaneous energy stored during a period

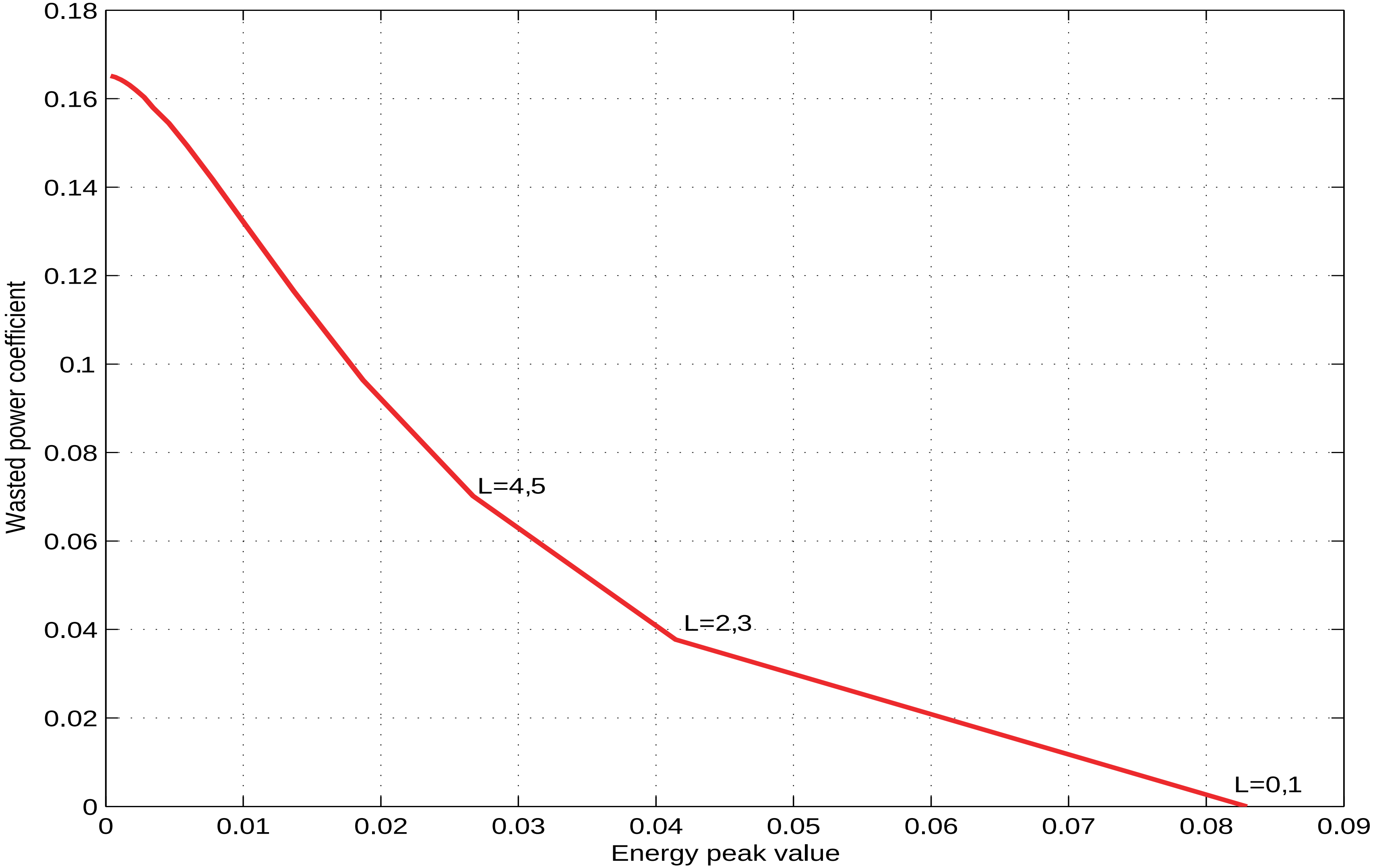
$$E_{max} = \max_{0 \le t \le T_0} \left| \int_0^t (i - i_s)(\tau) v^\top(\tau) d\tau \right|$$

We use the “normalized” peak power E_{max}/T_0 as our measure of hardware (i.e., capacitor) cost.

Compensation Cost (2)

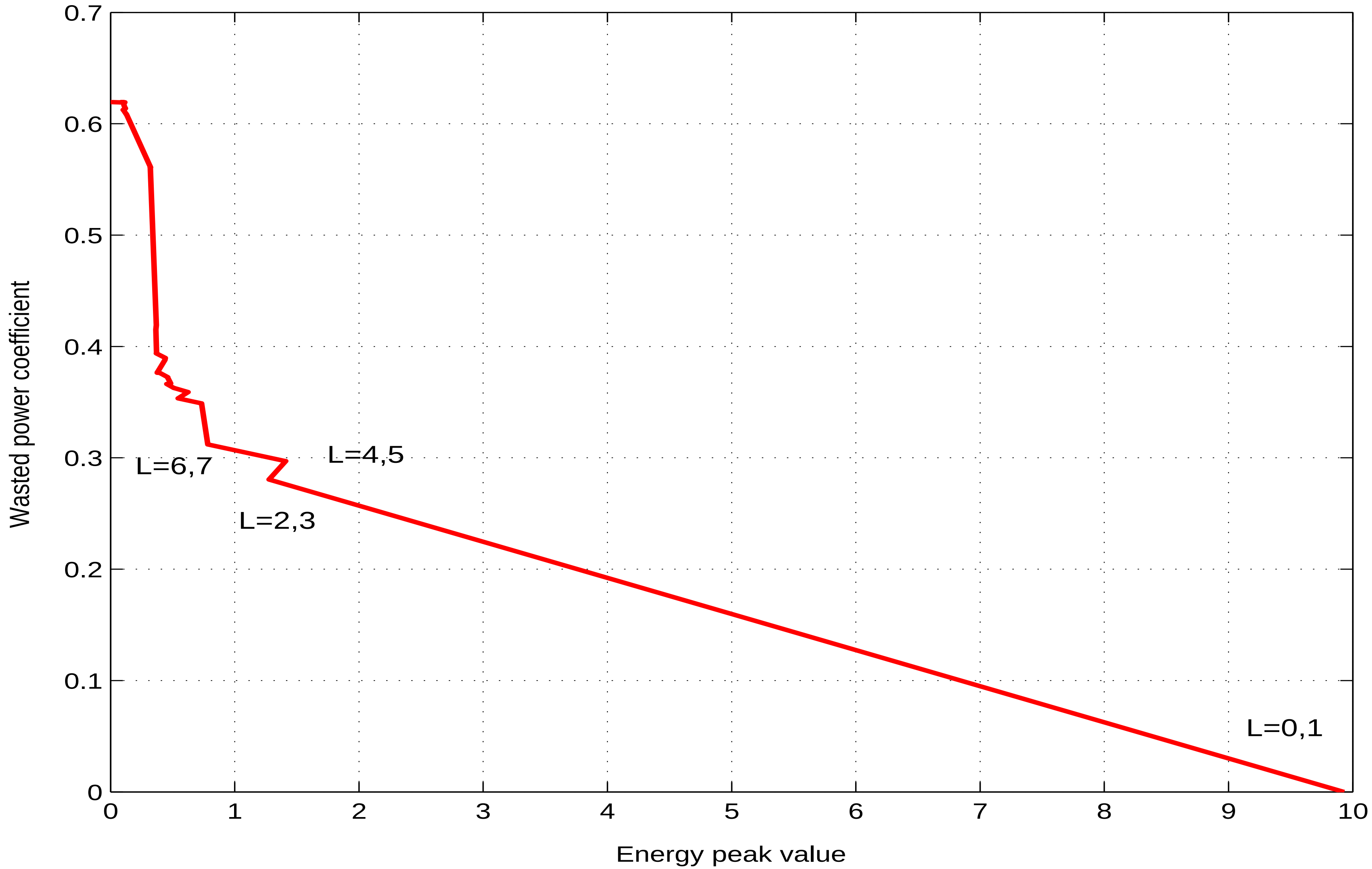
Consider the family of power harmonic matching (PHM) compensators - the “L-compensator” matches on the line power harmonics up to the L-th (so $L = 0$ corresponds to Fryze and $L = \infty$ is the instantaneous compensator).

Compensation Cost - Example 1



Compensation Cost - Nonlinear Load

Consider a three-phase diode bridge rectifier with an RLC load.



Effects of Line Resistances

Let *normalized* line resistances in various phase conductors be ρ_k, and in the neutral conductor ρ_0. The Ohmic losses are now $\sum_0^n \rho_k \|i_k\|^2$, which we can express as the weighted norm $\|i\|_W^2$. Thus consider (pos. definite) weighting matrix

$$W = \mathtt{diag}(\rho_1, \rho_2, \cdots, \rho_n) + \rho_0 \mathbf{1}^T \mathbf{1}$$

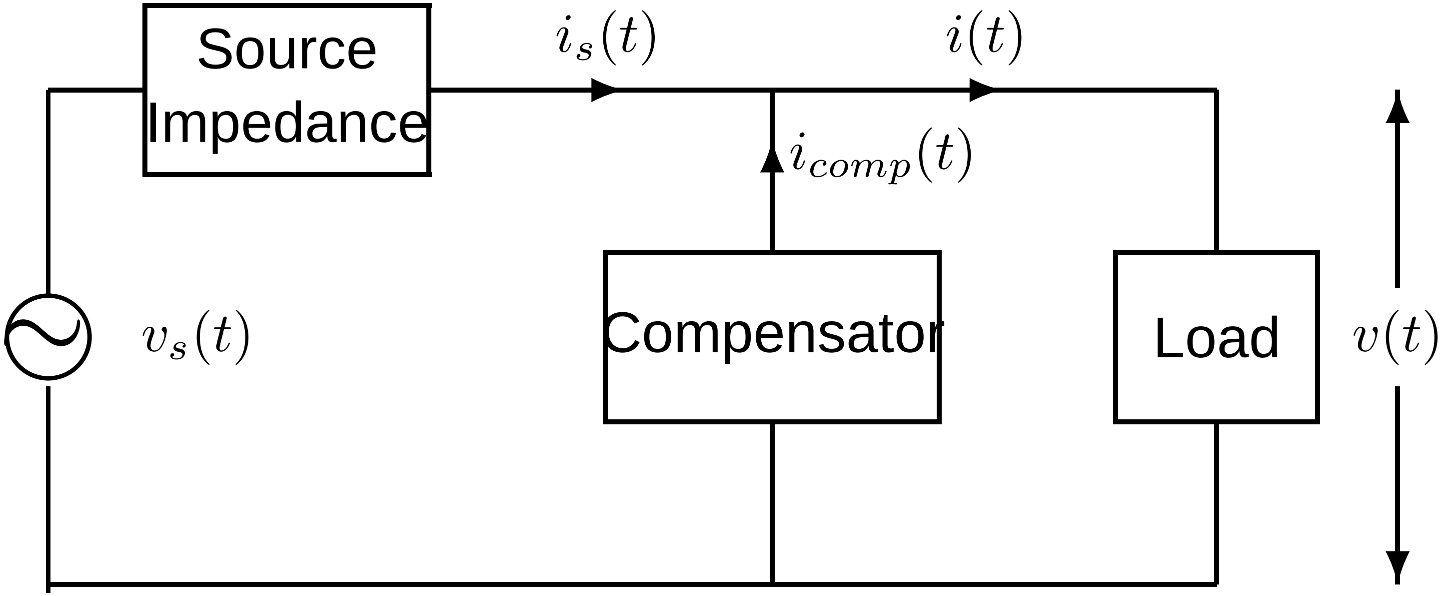
where $\mathbf{1}$ is the n-dimensional row vector of 1's. It turns out that we can just perform symbol replacement in our Hilbert space development

$$i \mapsto i, \quad v \mapsto vW^{-1}, \quad f(t)g(t)^T \mapsto f(t)Wg(t)^T$$

and use the weighted inner product

$$\langle\, x\,,\, y\,\rangle_W = \frac{1}{T_0}\int_{T_0} x(t)Wy(t)^\top dt = \sum_{\ell=-\infty}^{\infty} X_\ell\, W\, Y_\ell^H$$

Source Impedance and Apparent Power



Source Impedance and Apparent Power (2

We need some notation:

- Line losses P_{line},
- Power (real) supplied by the compensator P_{comp},
- Power delivered to the compensator – load combination P_{cload} (supplied by the source),
- Load power before $P_{load}^{(u)}$ and after compensation $P_{load}^{(c)}$

$$P_{cload} = P_{load}^{(c)} - P_{comp}$$

Source Impedance and Apparent Power (3

Two problems with the same solution (by Cauchy-Schwarz) :

Problem 1: Given a polyphase source voltage $v_s(t)$ and a linear time-invariant (polyphase) source impedance $\mathcal{Z}_s$, determine the largest possible P_{load} for a prescribed value of P_{line}.

Problem 2: Given the polyphase source voltage $v_s(t)$ and a linear time-invariant (polyphase) source impedance $\mathcal{Z}_s$, determine the smallest possible P_{line} for a prescribed value of P_{load}.

$$P_{load} \leq \| \mathcal{V}_s \mathcal{R}_s^{-H/2} \|_w \sqrt{P_{line}} - P_{line}$$

allowing us to define an ideal load (a.k.a. the apparent power):

$$S = \| \mathcal{V}_s \mathcal{R}_s^{-H/2} \|_w \| \mathcal{I}_s \mathcal{R}_s^{1/2} \|_w - \| \mathcal{I}_s \mathcal{R}_s^{1/2} \|_w^2$$

Source Impedance and Apparent Power (4)

We can also find the optimized source (generalized Fryze) current:

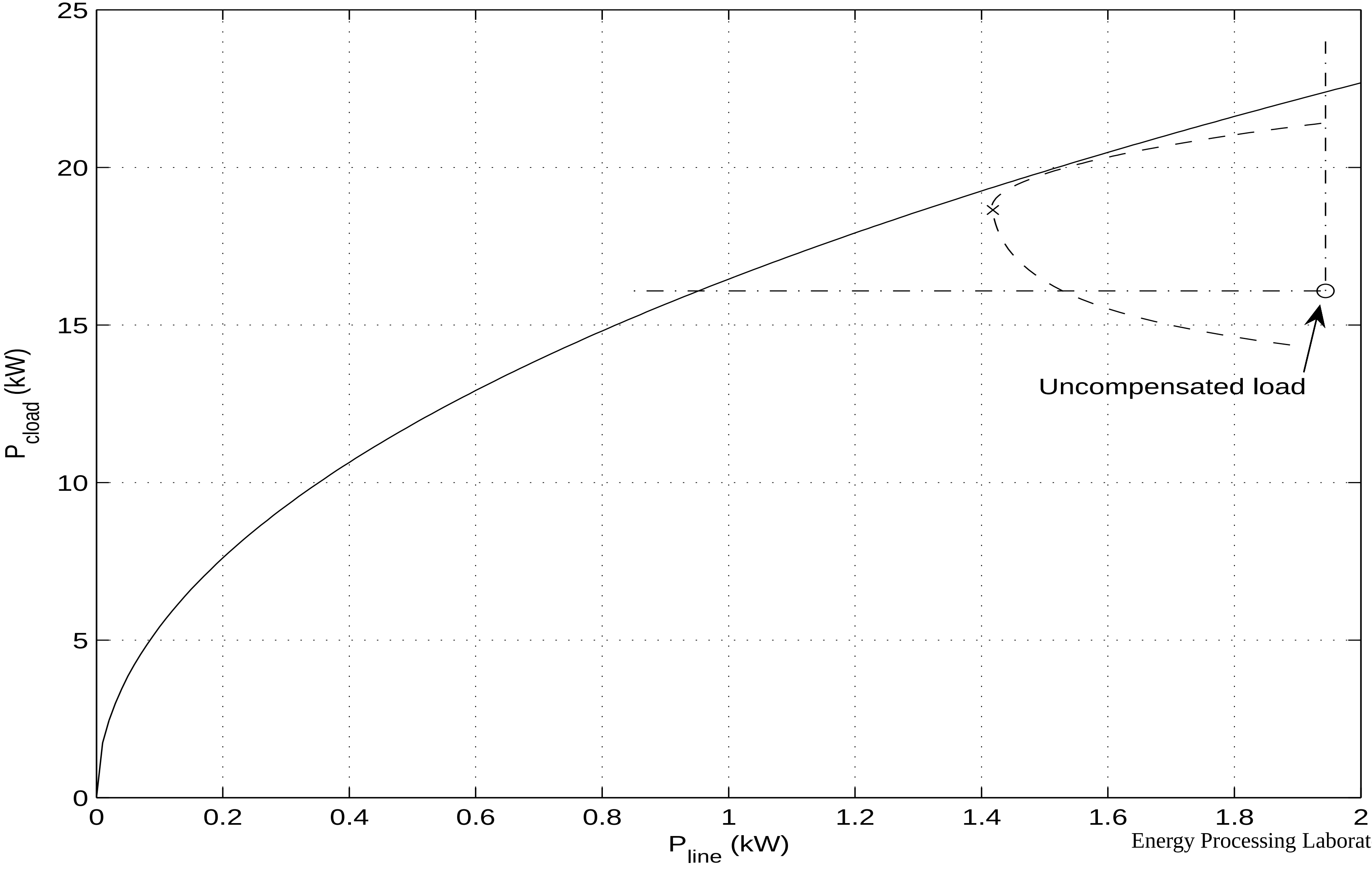
$$\mathcal{I}_{s,opt} = \frac{2\left(\frac{P_{cload}}{\mathcal{V}_s \mathcal{R}_s^{-1} \mathcal{V}_s^H}\right)}{1 + \sqrt{1 - 4\left(\frac{P_{cload}}{\mathcal{V}_s \mathcal{R}_s^{-1} \mathcal{V}_s^H}\right)}} \mathcal{V}_s \mathcal{R}_s^{-1}$$

which in the case of a negligible line voltage drop $P_{load}/(\mathcal{V}_s \mathcal{R}_s^{-1} \mathcal{V}_s^H) \ll 1$ reduces to

$$\mathcal{I}_{s,opt} \approx \frac{P_{load}}{\mathcal{V}_s \mathcal{R}_s^{-1} \mathcal{V}_s^H} \mathcal{V}_s \mathcal{R}_s^{-1}$$

An Example - Unbalanced Induction Mach

This example is taken from the literature - a machine rated at approximately 25 kW, unbalanced line:



Time-Variant Fourier Series

We introduce a time-variant Hilbert space framework for real-valued polyphase signals with finite local power, viz.

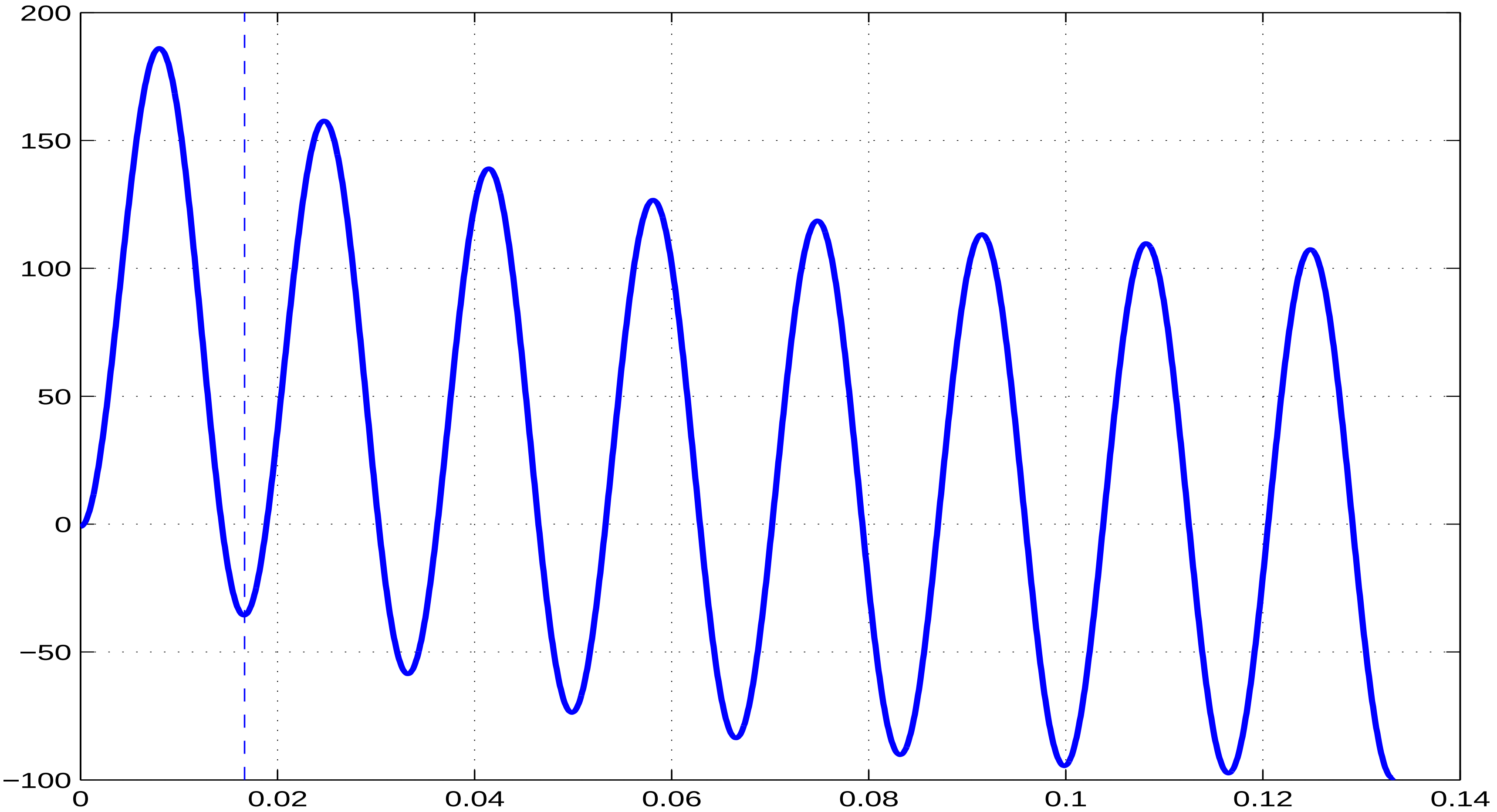
$$\frac{1}{T_0}\int_{t-T_0}^{t} \|x(s)\|^2 ds < \infty, \quad \text{for all } t$$

For a fixed "t" our space consists of finite signal segments, and we define the (time-variant) inner product as

$$\langle\, x \;,\; y \,\rangle(t) \triangleq \frac{1}{T_0}\int_{t-T_0}^{t} x(s)y(s)^\top ds$$

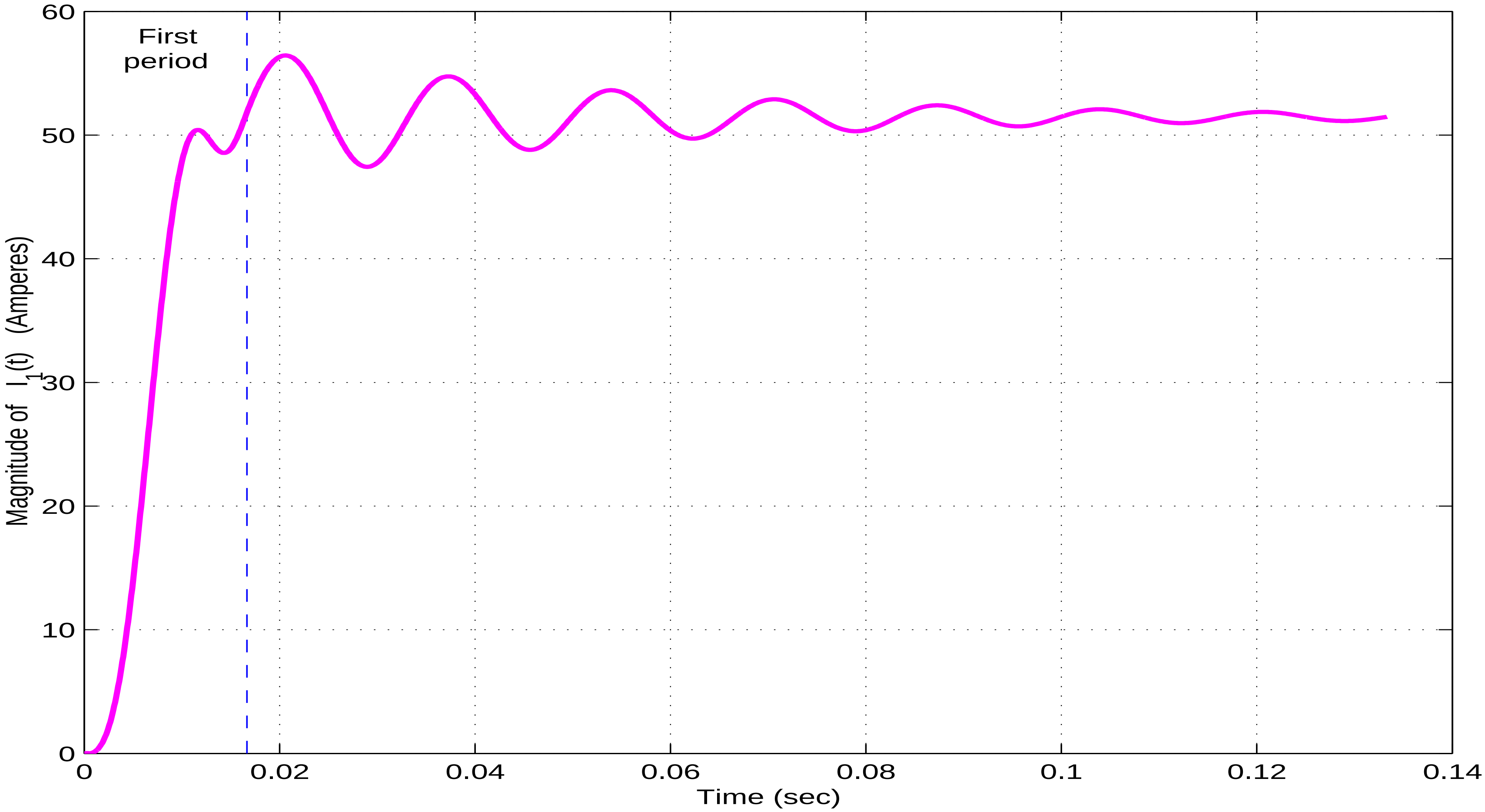
Transient Example (1)

Consider a single-phase RL circuit, driven by the voltage $v(t) = V\sin(\omega_0 t), \;\; t \geq 0$ and with zero initial conditions (with $R = 0.1\Omega, \; L = 1mH, \; f = 60Hz$)– the current is



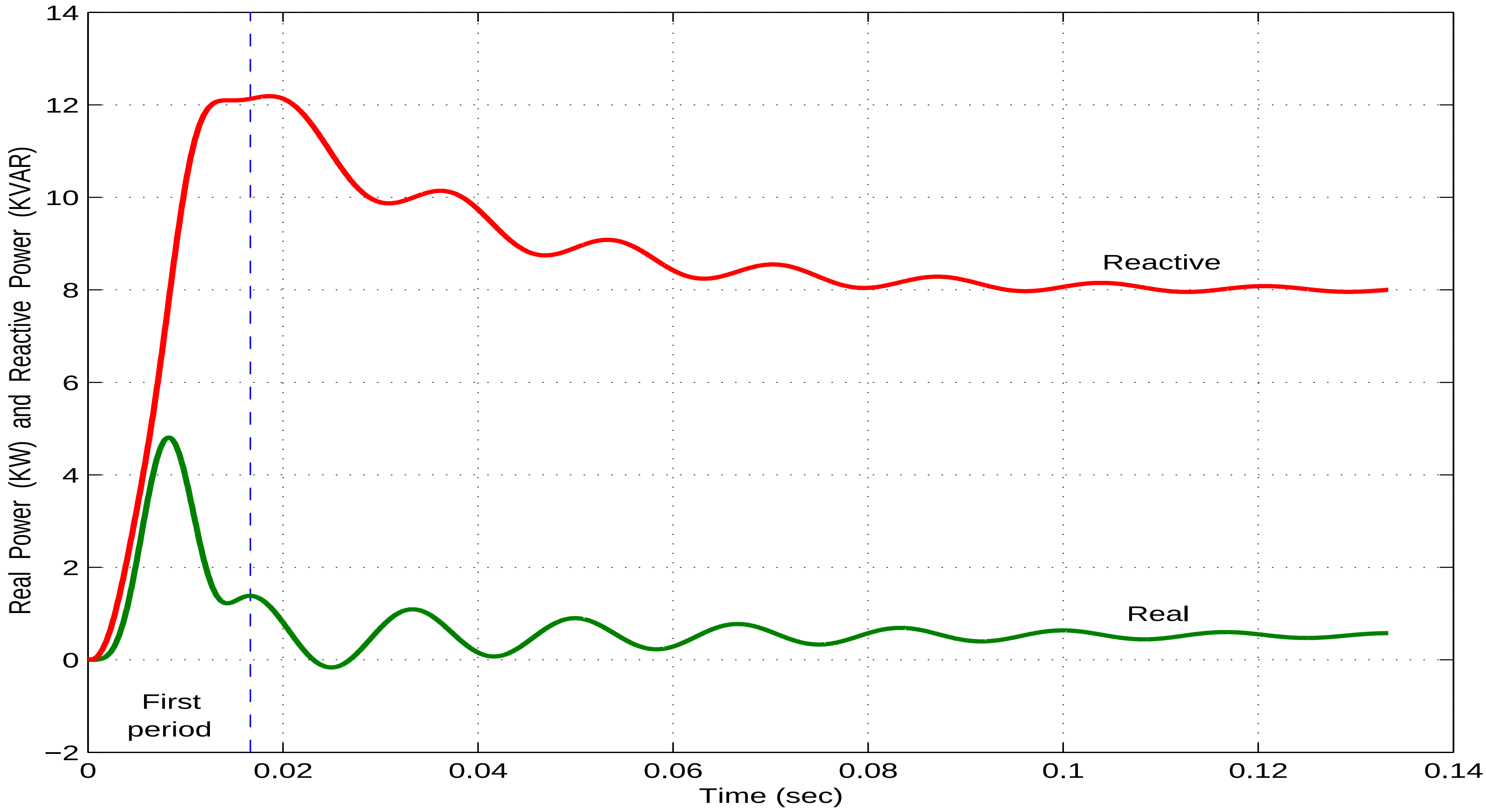
Transient Example (2)

Initialization artifact - if we choose to define $i(t) = 0$ for $t < 0$, then the magnitude of the fundamental $I_1(t)$



Transient Example (3)

$P(t)$ and $Q(t)$ for our example



Concluding remarks

- Our concept of reactive power is intended for effective compensation.
- We provide a physics-based decomposition of apparent power that applies to general systems – any number of phases and harmonics.
- We can accommodate limited bandwidth of compensator current.
- It is extended to transients using the notion of dynamic phasors (Fourier basis) or other orthonormal series representations, or even a self-dual frame.