DYNAMIC DECOMPOSITION OF APPARENT POWER IN POLYPHASE UNBALANCED NETWORKS WITH APPLICATION TO TRANSIENTS IN AN INDUSTRIAL LOAD

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PREVIEW AND MOTIVATION

- Our (earlier) static 7-component decomposition of apparent power refines several known results, including Czarnecki's 5-component decomposition (CPC).

- We rely on the concept of dynamic phasors to make our decomposition **dynamic**, allowing it to capture transient behavior.

- Two of our 7 components – the real power P and Budeanu's reactive power Q_B – are network conservative. We split these two into their symmetric sequence components (all conservative as well), which results in a new two-level, 7/11-component decomposition of apparent power.

OVERVIEW

- Hilbert Space of Periodic Polyphase Waveforms
- Orthogonal Current Decomposition
- 7-Component Power Decomposition
- Network-Conservative Power Components
- 7/11-Component Power Decomposition
- Local Hilbert Space and Dynamic Phasors
- Industrial Example
- Concluding Remarks

HILBERT SPACE FRAMEWORK

Space elements: real-valued T-periodic square-integrable polyphase waveforms

$$x(t) = \left[\, x_1(t) \quad x_2(t) \quad \dots \quad x_n(t) \,\right] \qquad \text{(row vector)}$$

Inner product and norm:

$$\langle\, x \,,\, y \,\rangle \overset{\text{def}}{=} \frac{1}{T}\int_T x(t)\, y(t)^\top\, dt \qquad \|\, x \,\| \overset{\text{def}}{=} \sqrt{\langle\, x \,,\, x \,\rangle}$$

Fourier coefficients: $X_\ell \overset{\text{def}}{=} \frac{1}{T}\int_T x(t)\, e^{-j\ell\omega t}\, dt$

$X_{k,\ell}$ is the k-th element of the phasor vector X_ℓ .

Parseval identity: $\langle\, x \,,\, y \,\rangle = \sum_{\ell=-\infty}^{\infty} X_\ell\, Y_\ell^H = \sum_{k=1}^{n} \sum_{\ell=-\infty}^{\infty} X_{k,\ell}\, Y_{k,\ell}^*$

STATIC ORTHOGONAL CURRENT DECOMPOSITION

$$i(t) \;=\; \underbrace{\sum_{k,\ell} g_{k,\ell}\, v_{k,\ell}(t)}_{i_v(t)} \;\oplus\; \underbrace{\sum_{k,\ell} b_{k,\ell}\, \mathcal{H}v_{k,\ell}(t)}_{i_w(t)} \;\oplus\; i_\perp(t)$$

where

$$g_{k,\ell} \overset{\text{def}}{=} \frac{\langle\, i\,,\, v_{k,\ell}\,\rangle}{\|v_{k,\ell}\|^2}, \qquad b_{k,\ell} \overset{\text{def}}{=} \frac{\langle\, i\,,\, \mathcal{H}v_{k,\ell}\,\rangle}{\|v_{k,\ell}\|^2}$$

Equivalent linear load:

$$i_{k,\ell}(t) \;=\; g_{k,\ell}\, v_{k,\ell}(t) + b_{k,\ell}\, \mathcal{H}v_{k,\ell}(t) \;=\; 2\,\Re\{\, \underbrace{(g_{k,\ell} - j b_{k,\ell}) V_{k,\ell}}_{I_{k,\ell}}\; e^{j\ell\omega t} \,\}\;\; [\underbrace{0 \ldots 0\; 1}_{k}\;\; 0 \ldots 0]$$

$$\Downarrow$$

$$(k,\ell)\text{-admittance} \;\overset{\text{def}}{=}\; \frac{I_{k,\ell}}{V_{k,\ell}} \;=\; g_{k,\ell} - j\, b_{k,\ell}$$

The Hilbert transform:

$$\mathcal{H}\,\{e^{j\ell\omega t}\} \;\overset{\text{def}}{=}\; -\,j\;\mathsf{sgn}(\ell)\; e^{j\ell\omega t}$$

5-COMPONENT DECOMPOSITION

Conductance spread (average, variance): $i_v(t) = i_F(t) \oplus i_g(t)$

$i_F(t) \iff \mu_g$, weighted mean of $g_{k,\ell}$

$i_g(t) \iff$ spread of $g_{k,\ell}$ around μ_g

Susceptance spread (average, variance): $i_w(t) = i_B(t) \oplus i_b(t)$

$i_B(t) \iff \mu_b$, weighted mean of $b_{k,\ell}$

$i_b(t) \iff$ spread of $b_{k,\ell}$ around μ_b

5-COMPONENT DECOMPOSITION (continued)

5-component decomposition:

$$i(t) = \underbrace{i_F(t) \;\oplus\; i_g(t)}_{i_v(t)} \;\oplus\; \underbrace{i_B(t) \;\oplus\; i_b(t)}_{i_w(t)} \;\oplus\; i_\perp(t)$$

$$S^2 \;=\; (P^2 + N_g^2) \;+\; (\,Q_B^2 + Q_b^2\,) \;+\; S_\perp^2$$

$$S \overset{\text{def}}{=} \|i\|\,\|v\| \;, \qquad N_g \overset{\text{def}}{=} \|i_g\|\,\|v\| \;, \qquad Q_b \overset{\text{def}}{=} \|i_b\|\,\|v\| \;, \qquad S_\perp \overset{\text{def}}{=} \|i_\perp\|\,\|v\|$$

Shepherd & Zakikhani	S_R	S_X
Ours	$i_v,\ S_v$	$i_w,\ Q_w$

Sharon	P	S_c	S_Q
Ours	i_F, P	$i_g,\ N_g$	$i_w,\ Q_w$

$$S_v \overset{\text{def}}{=} \|i_v\|\,\|v\| = \sqrt{P^2 + N_g^2} \;, \qquad Q_w \overset{\text{def}}{=} \|i_w\|\,\|v\| = \sqrt{Q_B^2 + Q_b^2}$$

EFFECT OF UNBALANCED LOAD

Splitting of i_g: $$i_v(t) = i_F(t) \oplus \underbrace{i_{gs}(t) \oplus i_{gu}(t)}_{i_g(t)}$$

$i_{gs}(t) \iff \mu_g(\ell)$, weighted conditional mean of $g_{k,\ell}$

$i_{gu}(t) \iff$ spread of $g_{k,\ell}$ around $\mu_g(\ell)$ (g-balanced load $\Leftrightarrow$ $i_{gu} = 0$)

Splitting of i_b: $$i_w(t) = i_B(t) \oplus \underbrace{i_{bs}(t) \oplus i_{bu}(t)}_{i_b(t)}$$

$i_{bs}(t) \iff \mu_b(\ell)$, weighted conditional mean of $b_{k,\ell}$

$i_{bu}(t) \iff$ spread of $b_{k,\ell}$ around $\mu_b(\ell)$ (b-balanced load $\Leftrightarrow$ $i_{bu} = 0$)

7-COMPONENT DECOMPOSITION

$$i(t) = \underbrace{i_F(t) \ \oplus \ i_{gs}(t) \ \oplus \ i_{gu}(t)}_{i_v(t)} \ \oplus \ \underbrace{i_B(t) \ \oplus \ i_{bs}(t) \ \oplus \ i_{bu}(t)}_{i_w(t)} \ \oplus \ i_\perp(t)$$

$$S^2 = P^2 \ + \ \underbrace{N_s^2 \ + \ N_u^2}_{N_g^2} \ + \ Q_B^2 \ + \ \underbrace{Q_s^2 \ + \ Q_u^2}_{Q_b^2} \ + \ S_\perp^2$$

$$N_s \ \overset{\text{def}}{=} \ \|i_{gs}\| \, \|v\| \ , \quad N_u \ \overset{\text{def}}{=} \ \|i_{gu}\| \, \|v\| \ , \quad Q_s \ \overset{\text{def}}{=} \ \|i_{bs}\| \, \|v\| \ , \quad Q_u \ \overset{\text{def}}{=} \ \|i_{bu}\| \, \|v\|$$

Czarnecki	i_a	i_s	i_r	i_u	i_g
Ours	i_F	i_{gs}	$i_B \oplus i_{bs}$	$i_{gu} \oplus i_{bu}$	$i_\perp$

Notice: Budeanu's reactive power Q_B is a subcomponent of Czarnecki's Q_r .

NETWORK-CONSERVATIVE POWER COMPONENTS

Telegen's theorem: implies that the quantity $\langle\, \mathcal{A}\, i \,,\, \mathcal{B}\, v \,\rangle$ is *network-conservative* for any two linear operators $\mathcal{A}$ and $\mathcal{B}$ that are invariant across the network.

This means that

$$\sum_m \langle\, \mathcal{A}\, i_m \,,\, \mathcal{B}\, v_m \,\rangle = 0$$

in any closed system of interconnected (lumped) elements, where $v_m(t)$ denotes the voltage across the m-th element, and $i_m(t)$ is the current flowing into the same element.

Example: our two signed power components

$$P \;=\; \langle\, i \,,\, v \,\rangle \;=\; \mu_g \,\|v\|^2 = 2 \sum_{k,\ell} |I_{k,\ell}||V_{k,\ell}| \cos(\arg(V_{k,\ell} I^*_{k,\ell}))$$

$$Q_B \;=\; \langle\, i \,,\, \mathcal{H} v \,\rangle \;=\; \mu_b \,\|v\|^2 = 2 \sum_{k,\ell} |I_{k,\ell}||V_{k,\ell}| \sin(\arg(V_{k,\ell} I^*_{k,\ell}))$$

7/11-COMPONENT POWER DECOMPOSITION

Decompose signed power components:

$$P = \underbrace{\langle\, i_+\,,\, v \,\rangle}_{P_+} + \underbrace{\langle\, i_-\,,\, v \,\rangle}_{P_-} + \underbrace{\langle\, i_0\,,\, v \,\rangle}_{P_0}$$

$$Q_B = \underbrace{\langle\, i_+\,,\, \mathcal{H}v \,\rangle}_{Q_{B,+}} + \underbrace{\langle\, i_-\,,\, \mathcal{H}v \,\rangle}_{Q_{B,-}} + \underbrace{\langle\, i_0\,,\, \mathcal{H}v \,\rangle}_{Q_{B,0}}$$

$i_+(t)$, $i_-(t)$, $i_0(t)$ are the positive, negative, and zero-sequence components of $i(t)$.

All six subcomponents – $P_+\,;P_-\,;P_0\,;Q_{B,+}\,;Q_{B,-}\,;Q_{B,0}$ – are network-conservative.

Two-level power decomposition:

$$S^2 = (P_+ + P_- + P_0)^2 + (Q_{B,+} + Q_{B,-} + Q_{B,0})^2 + N_s^2 + N_u^2 + Q_s^2 + Q_u^2 + S_\perp^2$$

LOCAL HILBERT SPACE AND DYNAMIC PHASORS

Space elements: real-valued polyphase waveforms that are square-integrable on the interval $(t-T \qquad t]$ for every t.

$$x(t) = \left[\; x_1(t) \quad x_2(t) \quad \ldots \quad x_n(t) \;\right] \qquad \text{(row vector)}$$

Inner product and norm:

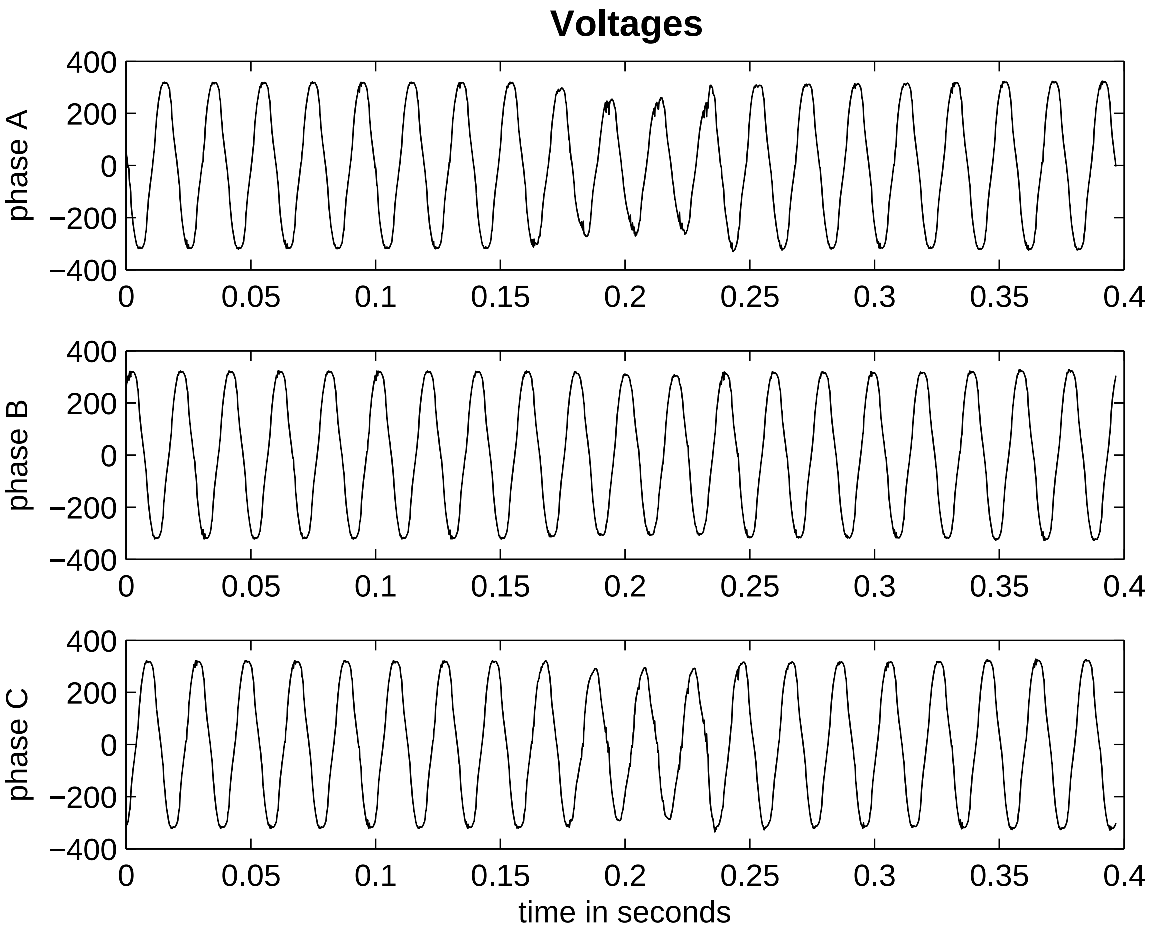
$$\langle\, x \,,\, y \,\rangle(t) \;\overset{\text{def}}{=}\; \tfrac{1}{T}\int_{t-T}^{T} x(s)\, y(s)^{\top}\, ds \qquad\qquad \|\, x \,\|(t) \;\overset{\text{def}}{=}\; \sqrt{\langle\, x \,,\, x \,\rangle}(t)$$

Dynamic phasors: $$X_{\ell}(t) \;\overset{\text{def}}{=}\; \tfrac{1}{T}\int_{t-T}^{T} x(s)\, e^{-j\ell\omega s}\, ds$$

$X_{k,\ell}(t)$ is the k-th element of the phasor vector $X_{\ell}(t)$.

Parseval: $$\langle\, x \,,\, y \,\rangle(t) \;=\; \sum_{\ell=-\infty}^{\infty} X_{\ell}(t)\, Y_{\ell}^{H}(t) \;=\; \sum_{k=1}^{n}\sum_{\ell=-\infty}^{\infty} X_{k,\ell}(t)\, Y_{k,\ell}^{*}(t)$$

INDUSTRIAL EXAMPLE – paper plant during an outage [14]



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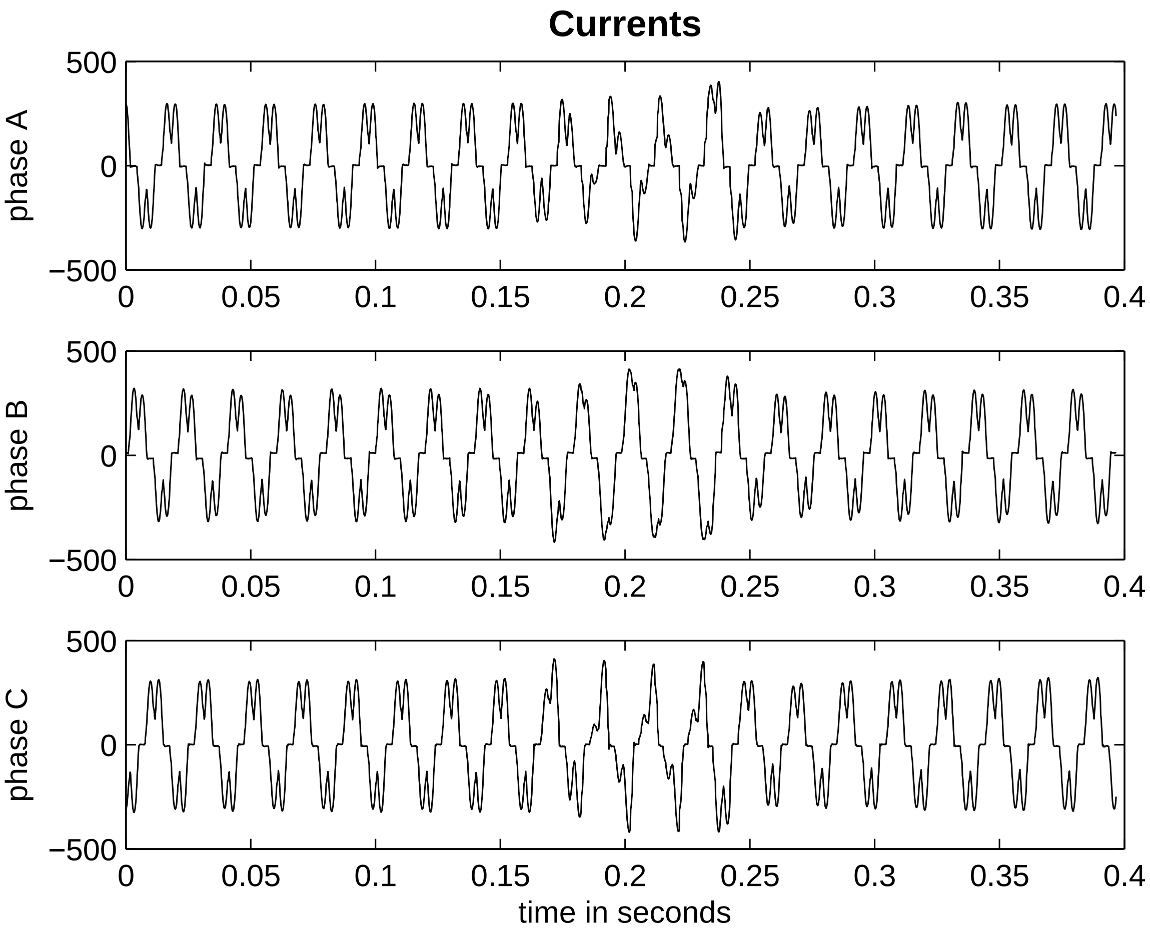
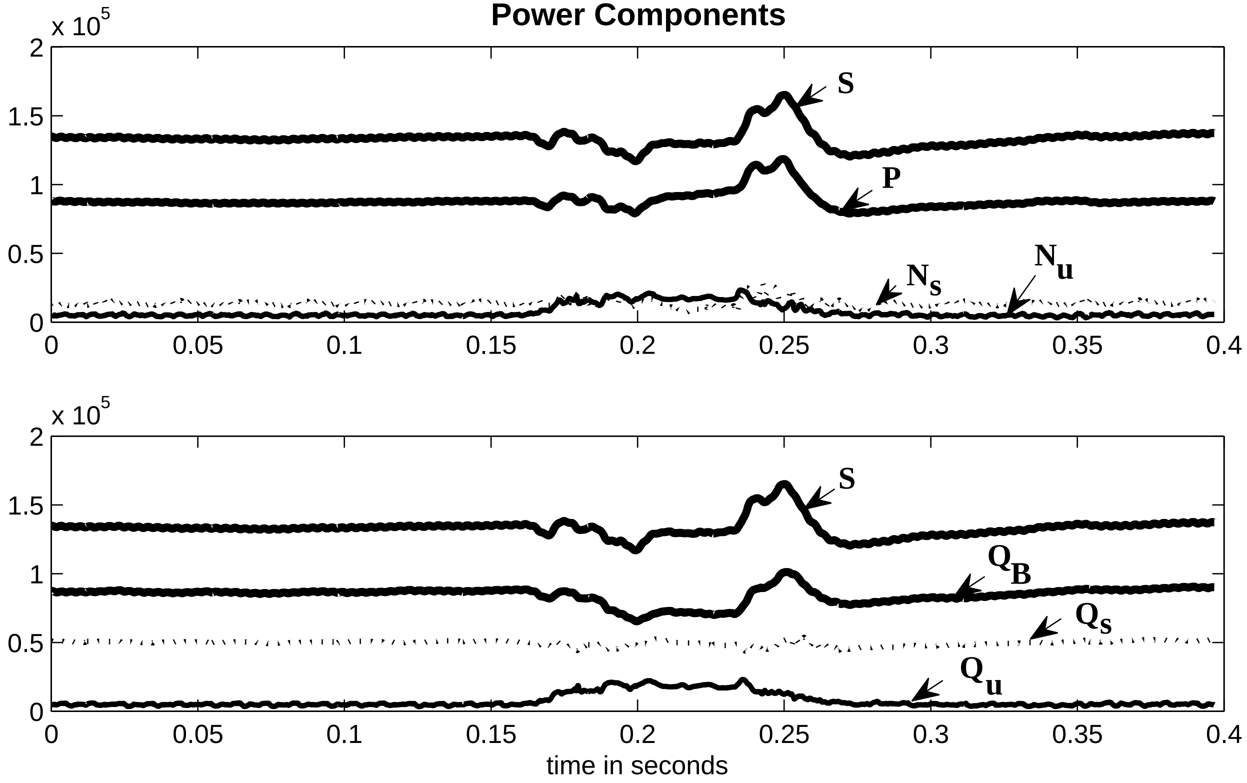
INDUSTRIAL EXAMPLE – continued

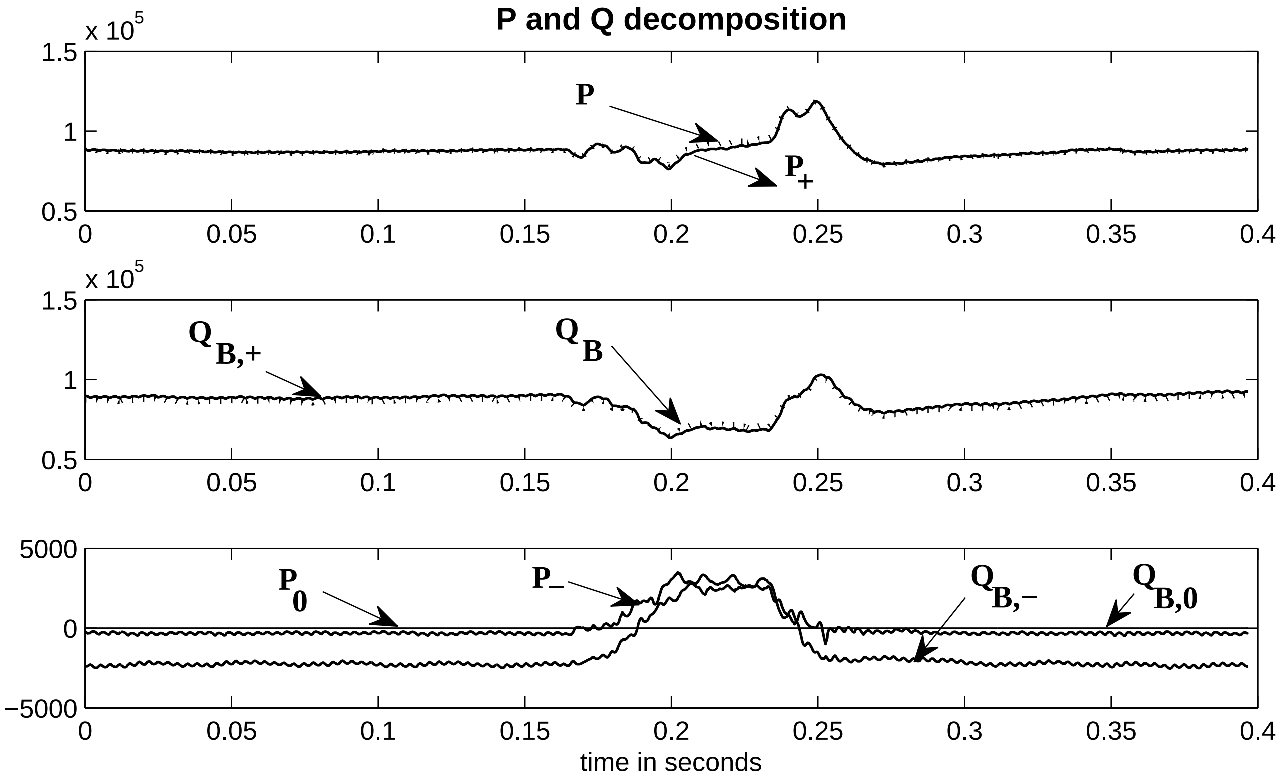


INDUSTRIAL EXAMPLE – continued



Notice: $N_u(t)$ and $Q_u(t)$ have significant values only during the fault.

INDUSTRIAL EXAMPLE – continued



Notice: $P_{-}(t)$ (and $Q_{B,-}(t)$) exhibit significant transients during the fault.

CONCLUDING REMARKS

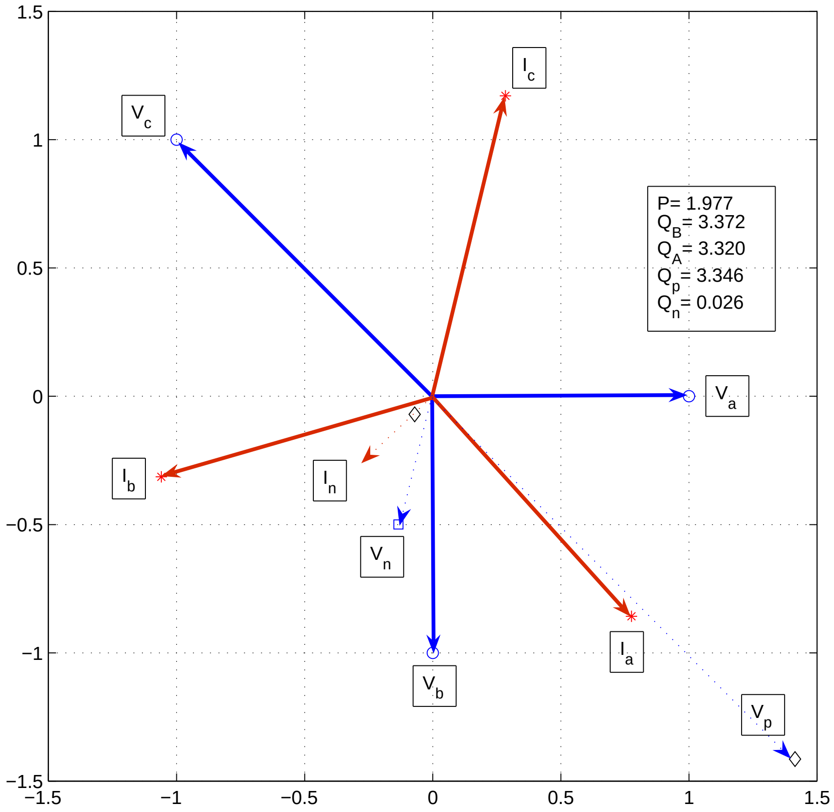
- Our 7-component decomposition is a refinement of Czarnecki, Sharon, Shepherd & Zakikhani, as well as Budeanu and Fryze. It serves to illuminate the relations between these classical decompositions.

- The refined, two-level, 7/11-component decomposition provides further detail – the sequence components of P and Q_B (all network-conservative).

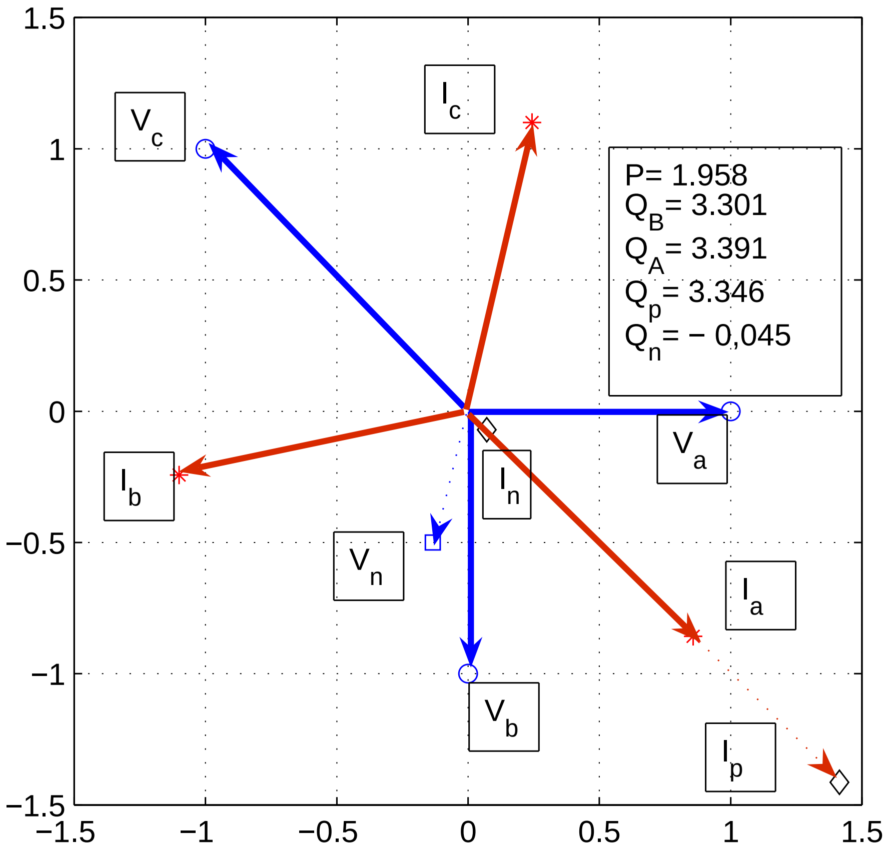
- Relying on dynamic phasors makes our decompositions time-variant, allowing to capture transient behavior.

- The example demonstrates the intuitive appeal of our dynamic current/power decompositions.

INDUSTRIAL EXAMPLE – continued



INDUSTRIAL EXAMPLE – continued



5-COMPONENT DECOMPOSITION

Conductance spread (average, variance):

$$i_F(t) = \mu_g \, v(t) \quad , \qquad \mu_g \overset{\text{def}}{=} \frac{P}{\|v\|^2} = \sum_{k,\ell} p_{k,\ell} \; g_{k,\ell} \quad , \qquad p_{k,\ell} \overset{\text{def}}{=} \frac{\|v_{k,\ell}\|^2}{\|v\|^2}$$

$$i_v(t) = i_F(t) \; \oplus \; i_g(t) \quad , \qquad i_g(t) = \sum_{k,\ell} (\, g_{k,\ell} - \mu_g \,) \, v_{k,\ell}(t)$$

$i_F(t) \quad \Longleftrightarrow \quad$ weighted mean of $g_{k,\ell}$

$i_g(t) \quad \Longleftrightarrow \quad$ spread of $g_{k,\ell}$ around μ_g

$$\|i_g\| = \sigma_g \, \|v\| \quad , \qquad \sigma_g \overset{\text{def}}{=} \sqrt{\sum_{k,\ell} p_{k,\ell} \, (g_{k,\ell} - \mu_g)^2} \qquad (\, g_{k,\ell} = \text{const.} \;\; \Leftrightarrow \;\; i_g = 0 \,)$$

5-COMPONENT DECOMPOSITION (continued)

Susceptance spread (average, variance):

$$i_B(t) = \mu_b \, \mathcal{H} v(t) \quad , \qquad \mu_b \overset{\text{def}}{=} \sum_{k,\ell} p_{k,\ell} \, b_{k,\ell}$$

$$i_w(t) = i_B(t) \; \oplus \; i_b(t) \quad , \qquad i_b(t) = \sum_{k,\ell} (\, b_{k,\ell} - \mu_b \,) \, \mathcal{H} v_{k,\ell}(t)$$

$i_B(t) \quad \Longleftrightarrow \quad$ weighted mean of $b_{k,\ell}$

$i_b(t) \quad \Longleftrightarrow \quad$ spread of $b_{k,\ell}$ around μ_b

$$\|i_b\| = \sigma_b \, \|v\| \quad , \qquad \sigma_b \overset{\text{def}}{=} \sqrt{\sum_{k,\ell} p_{k,\ell} \, (b_{k,\ell} - \mu_b)^2} \qquad (\, b_{k,\ell} = \text{const.} \quad \Leftrightarrow \quad i_b = 0 \,)$$

EFFECT OF UNBALANCED LOAD – conductances

Splitting of i_g :

$$i_v(t) \;=\; i_F(t) \oplus \underbrace{i_{gs}(t) \oplus i_{gu}(t)}_{i_g(t)} \quad , \qquad \mu_g(\ell) \overset{\text{def}}{=} \frac{\Sigma_k \; g_{k,\ell}\, p_{k,\ell}}{\Sigma_k \; p_{k,\ell}}$$

$$i_{gs}(t) \;=\; \sum_{k,\ell} [\mu_g(\ell) - \mu_g]\, v_{k,\ell}(t) \quad , \qquad i_{gu}(t) \;=\; \sum_{k,\ell} [g_{k,\ell} - \mu_g(\ell)]\, v_{k,\ell}(t)$$

Spread of $g_{k,\ell}$:

$$\|i_{gu}\| = \sigma_{gu}\|v\| \quad , \quad \sigma_{gu} \overset{\text{def}}{=} \sqrt{\sum_{k,\ell} p_{k,\ell}\, |g_{k,\ell} - \mu_g(\ell)|^2} \qquad (\,\text{g-balanced load} \quad \Leftrightarrow \quad i_{gu} = 0\,)$$

$$\|i_{gs}\| = \sigma_{gs}\|v\| \quad , \quad \sigma_{gs} \overset{\text{def}}{=} \sqrt{\sum_{k,\ell} p_{k,\ell}\, |\mu_g(\ell) - \mu_g|^2} \qquad (\,\sigma_g^2 = \sigma_{gs}^2 + \sigma_{gu}^2\,)$$

EFFECT OF UNBALANCED LOAD - susceptances

Splitting of i_b:

$$i_w(t) \;=\; i_B(t) \oplus \underbrace{i_{bs}(t) \oplus i_{bu}(t)}_{i_b(t)} \quad , \qquad \mu_b(\ell) \overset{\text{def}}{=} \frac{\sum_k \; b_{k,\ell}\, p_{k,\ell}}{\sum_k \; p_{k,\ell}}$$

$$i_{bs}(t) \;=\; \sum_{k,\ell} \left[\mu_b(\ell) - \mu_b\right] \mathcal{H} v_{k,\ell}(t) \quad , \qquad i_{bu}(t) \;=\; \sum_{k,\ell} \left[b_{k,\ell} - \mu_b(\ell)\right] \mathcal{H} v_{k,\ell}(t)$$

Spread of $b_{k,\ell}$:

$$\|i_{bu}\| = \sigma_{bu} \|v\| \quad , \quad \sigma_{bu} \overset{\text{def}}{=} \sqrt{\sum_{k,\ell} p_{k,\ell}\, |b_{k,\ell} - \mu_b(\ell)|^2} \qquad (\text{b-balanced load} \quad \Leftrightarrow \quad i_{bu} = 0)$$

$$|i_{bs}\| = \sigma_{bs} \|v\| \quad , \quad \sigma_{bs} \overset{\text{def}}{=} \sqrt{\sum_{k,\ell} p_{k,\ell}\, |\mu_b(\ell) - \mu_b|^2} \qquad (\sigma_b^2 = \sigma_{bs}^2 + \sigma_{bu}^2)$$