Java Program that Calculates the Decomposition of Load Current (and consequently the apparent power) into Seven Orthogonal Components

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Project Report

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Abstract - Modern power systems have the complications of nonlinear loads and distributed generations which results in both complicated power flow patterns and the need to precisely characterize them.
Prof. Hanoch Lev-Ari and Prof. Alexander M. Stankovic have found a way to decompose the load current (and consequently the apparent power) into seven mutually orthogonal components [1]. With such a detailed characterization of the load current a more detailed and more effective compensator can be designed.
The purpose of this project is to create a user friendly program that uses these calculations to decompose the load current into these seven components. This program is also available online as an open source.

Introduction

Starting with the work of Budeanu [9], many authors have aimed to characterize the concept of reactive power in the most general case, and to decompose the load current into physically meaningful mutually orthogonal components. The most detailed work to date appears to be that of Czarnecki [10], who introduced a decomposition consisting of five mutually orthogonal components. Used here is the new seven component orthogonal decomposition by Lev-Ari and Stankovic that generalizes and refines the one proposed by Czarnecki, as well as those introduced by Sharon [11], Shepherd and Zakikhani [12].
This seven component decomposition uses only two fundamental mathematical concepts: 1) the Hilbert transform and 2) the Orthogonal projection on a chain of nested subspaces, and treats the conductance and susceptance parts of the load admittance in a symmetric fashion. In table 1 these seven decomposed components are listed with description.

S	Apparent power
P	Real (active) power
N_S	Co-active power, Spread (over frequencies)
N_U	Co-active power, Unbalanced (over phases)
Q_B	Budeanu reactive power
Q_S	Co-reactive power, Spread (over frequencies)
Q_U	Co-reactive power, Unbalanced (over phases)
$S_\perp$	Out of band apparent power

table 1: Description of the seven component decomposition

All of the above mentioned components can be combined into the following equation: $S^2 = P^2 + N_S^2 + N_U^2 + Q_B^2 + Q_S^2 + Q_U^2 + S_\perp^2$

The CPC (Current Physical Component) by Czarnecki that can be read in detail in [5] is here compared to the seven component decomposition by Lev-Ari and Stankovic. As can be seen in [1] they relate in the following way:

Czarnecki		i_a, P	i_S	i_r, Q	i_u, D	i_g
Ours		i_F	i_{gs}	$i_B \oplus i_{bs}$	$i_{gu} \oplus i_{bu}$	$i_\perp$

Graphical User Interface

The program is created in Java where the user inputs the magnitude and phase angle of the load voltage and load current. No limitations are put on the number of harmonics.
The program calculates for one, two or three phase power systems and outputs the apparent power S and the seven component decomposition of S. Each seven component decomposition output with description can be seen in table 1. In addition to the seven component decomposition the program also calculates the values shown in table 2.

$\|V\|$	rms voltage
$\|I\|$	rms current
ρ_v	Normalized zero-sequence rms voltage
ν	Voltage balancing index
ζ	Power balancing index

table 2: Description of values that the program calculates in addition to the seven component decomposition

The program can be accessed through the webpage:
http://www.ece.neu.edu/faculty/stankovic/ReactivePowerCalculator/ it requires Mozilla Firefox browser to function correctly.

Here are brief instructions on how to enter the inputs and using the program:

1. Insert Load voltages and Load Currents harmonics (both even and odd harmonics)
2. If only calculating for 1 or 2 phase, put a zero in first harmonic of each unused phase (as seen in dashed circle below)
3. For each "used" phase the magnitude and phase column need to equal in length (as seen in solid circle on figure below)
4. When done inputting values press the "Calculate" button.

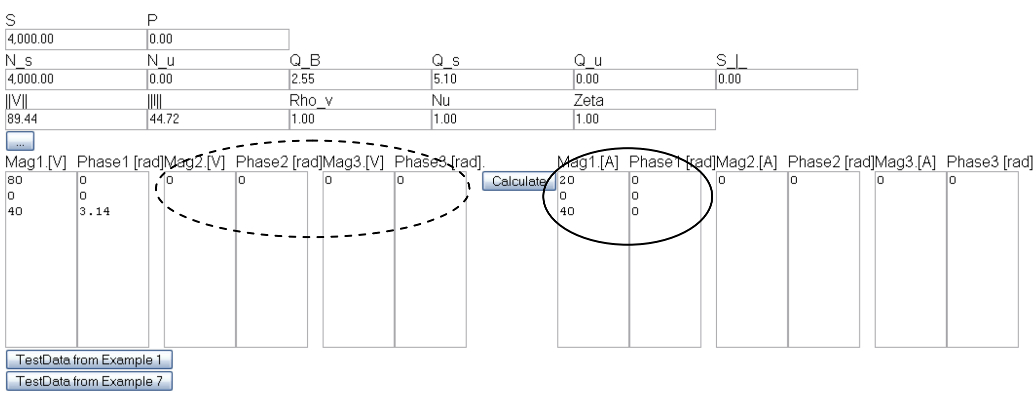


figure 1: Program output, borrowed from example 7

Example 1
Here is an example that further explains how to enter inputs and read outputs from the program. This example deals with an unbalanced load which consists of a resistor capacitor and an inductor, the load is connected as shown on figure5

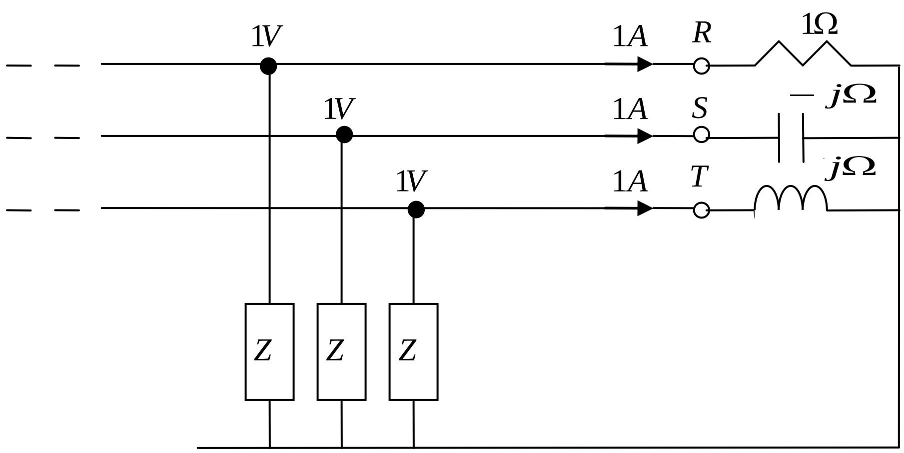
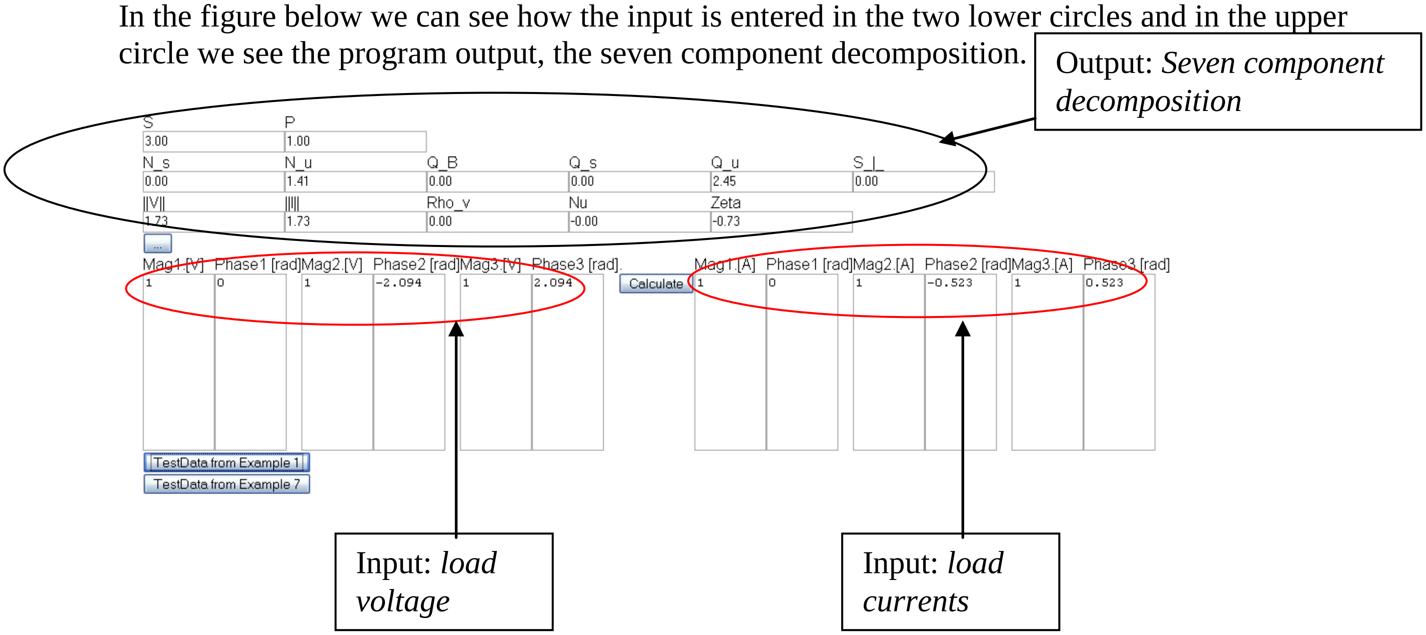


figure 2: Circuit with balanced supply voltage and unbalanced Y connected load.

The load voltages are: $u_R = 1V\angle 0°, u_S = 1V\angle -120° and, u_T = 1V\angle 120°$
The load currents are: $i_R = 1A\angle 0°, i_S = 1A\angle -30° and, i_T = 1A\angle 30°$

In the figure below we can see how the input is entered in the two lower circles and in the upper circle we see the program output, the seven component decomposition.



Industrial Examples

Here are eight examples that should help the reader to understand the program function and theory.
The first five examples, example 2-6 are created by Leszek S. Czarnecki [5]. His five component decomposition is compared to the seven component decomposition used in our program.

Example 2
This example Czarnecki created to find the right definition for apparent power when dealing with an unbalanced load.

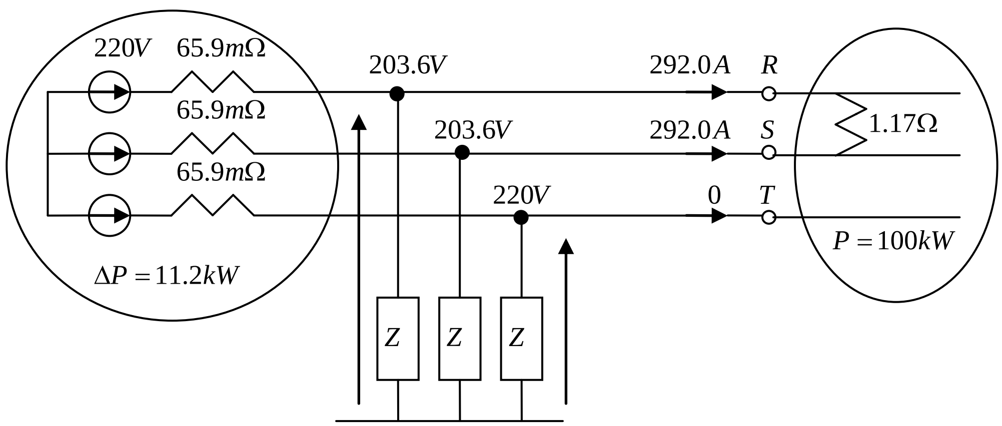


figure 3: A Circuit with an unbalanced resistive load.

In this example we have a three phase circuit with unbalanced resistive load, see figure 3. To calculate the apparent power we have three definitions of apparent power that all give different values:

$$S_A = U_R I_R + U_S I_S + U_T I_T = 119kVA$$
$$S_G = \sqrt{P^2 + Q^2} = 100kVA$$
$$S_B = \sqrt{U_R^2 + U_S^2 + U_T^2} \cdot \sqrt{I_R^2 + I_S^2 + I_T^2} = 149kVA$$

The corresponding power factors are:

$$\lambda_A = 0.84$$
$$\lambda_G = 1$$
$$\lambda_B = 0.67$$

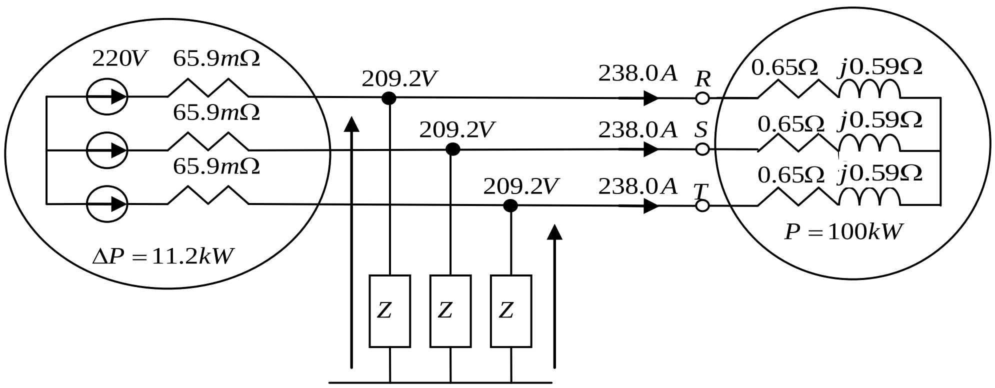


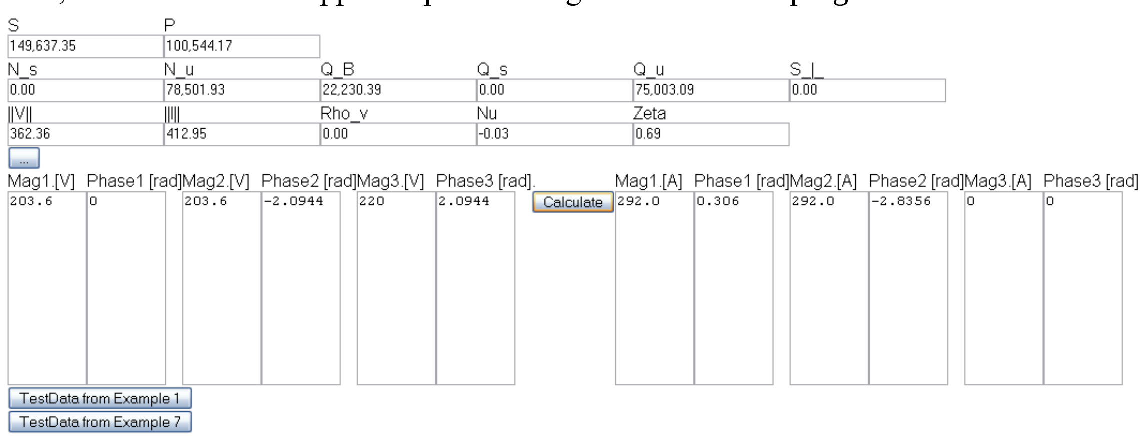
figure 4: A Circuit with a balanced RL load equivalent with respect to load active power and power loss in the supply to the circuit with unbalanced load shown in figure 3.

To prove which of the above definition is right for the unbalanced circuit, Czarnecki creates an equivalent circuit with a balanced RL load see figure 2 calculates the apparent power again with the three definitions. Since the circuit is balanced the same values are obtained for each definition:

$S_A = S_G = S_B = 149kVA$ and the power factor $\lambda_A = \lambda_G = \lambda_B = 0.67$

Now it is clear that S_B is the only correct definition for the apparent power when we are dealing with an unbalanced load.

Now, let's calculate the apparent power using the online Java program:



The result from the program is $S = 149{,}6kVA$ and the power factor $\lambda = \frac{P}{S} = 0.67$. Those are the same values as the ones that Czarnecki has proved in this example to be the only possible correct ones.

Example 3

Here we have an unbalanced resistive load which is connected as shown in figure 5

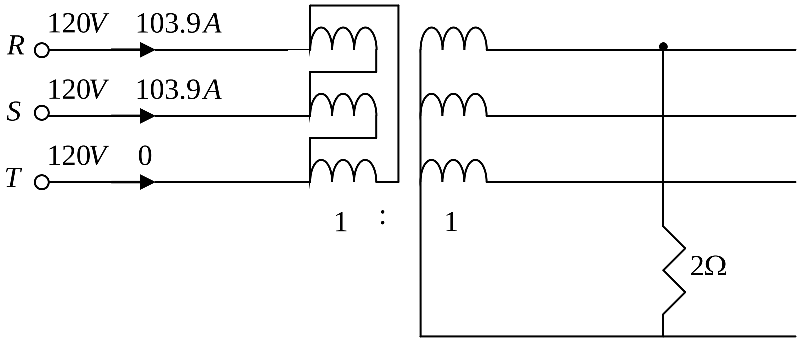


figure 5: Example of a circuit with resistive unbalanced load

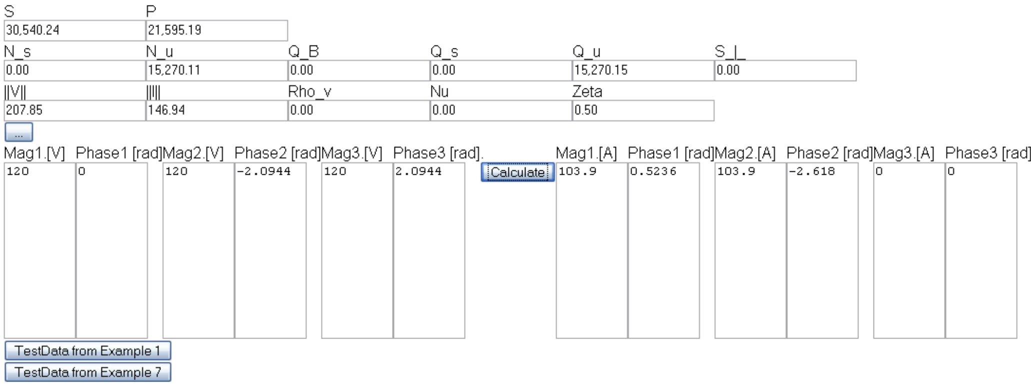
The supply is a symmetrical source of a sinusoidal, positive sequence voltage, with
$u_R = \sqrt{2}U\cos\omega_1 t$, $U = 120V$

And line currents equal to:

$i_R = \sqrt{2}I\cos(\omega_1 t + 30°) = -i_S$, $I = 103.9A$, and $i_T = 0$.

Czarnecki calculates the instantaneous reactive current in the load and shows that it is not equal to zero.

When we put the values into our online Java program to decompose the apparent power we see that the unbalanced reactive power Q_U is not zero along with the unbalanced coactive power N_U . That gives us the same result as Czarnecki, namely that the load has reactive current even though the load is purely resistive. Then, from our decomposition we can calculate the circuit power factor $\lambda = \frac{P}{S} = \frac{21{,}595}{30{,}540} = 0.7$



Example 4
Here is a three phase circuit with unbalanced load.

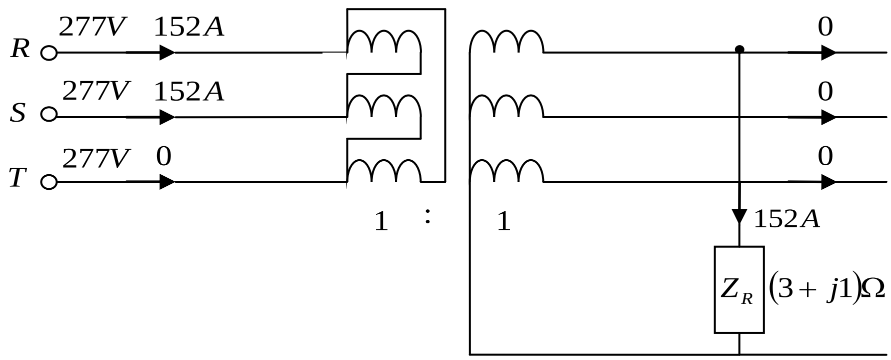
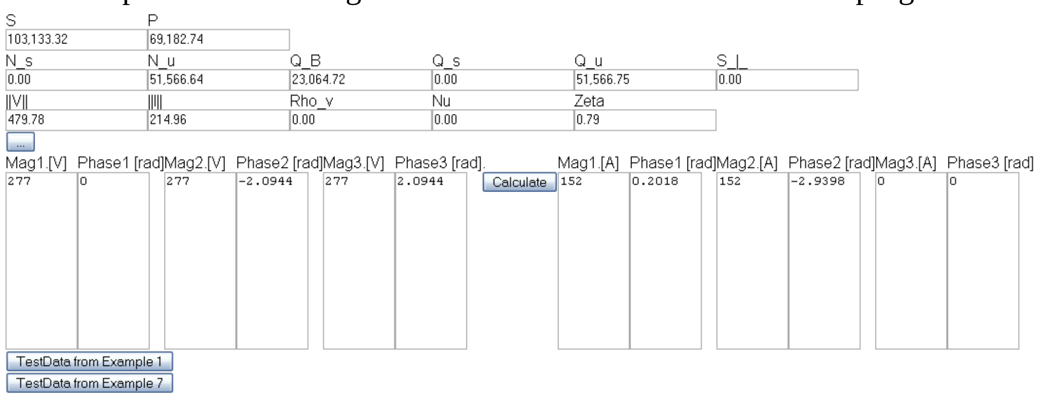


figure 6: Example of three-phase system with unbalanced load

The circuit in figure 6 has the following load voltage and current data:
$u_R = \sqrt{2}U\cos\omega_1 t$, $U = 277V$, $i_R = \sqrt{2}I\cos(\omega_1 t + 11.6°) = -i_S$, $I = 152A$, and $i_T = 0$.
The load impedance is $Z_R = (3 + j1)\Omega$ and the load powers using the CPC theory are:

$$P = 69kW$$
$$Q = 23kVA$$
$$D = 73kVA$$
$$S = 103kVA$$

Now let's put the load voltage and current data into the online Java program to decompose it:



The load powers using the seven component decomposition are:

$P = 69kW$

$Q_B = 23kVA$

$N_U = 51.6kVA$

$Q_U = 51.6kVA$

$S = 103kVA$

The values above are the same as using the CPC theory, the only difference is that the unbalanced power D from the CPC theory has, in our theory, been split into two components:

$N_U^2 + Q_U^2 = D^2 \Rightarrow \sqrt{(51.6kVA)^2 + (51.6kVA)^2} = 73kVA = D$

Example 5

Here we have a single phase RL load supplied through an ideal delta/Y transformer , with the turn ratio 1:1, from a source of a positive sequence voltage, but with zero line-to-ground voltage at terminal S, assuming that the line-to-ground voltage at terminal R and T are $E_R = 100V$ and $E_T = 100e^{j120}V$

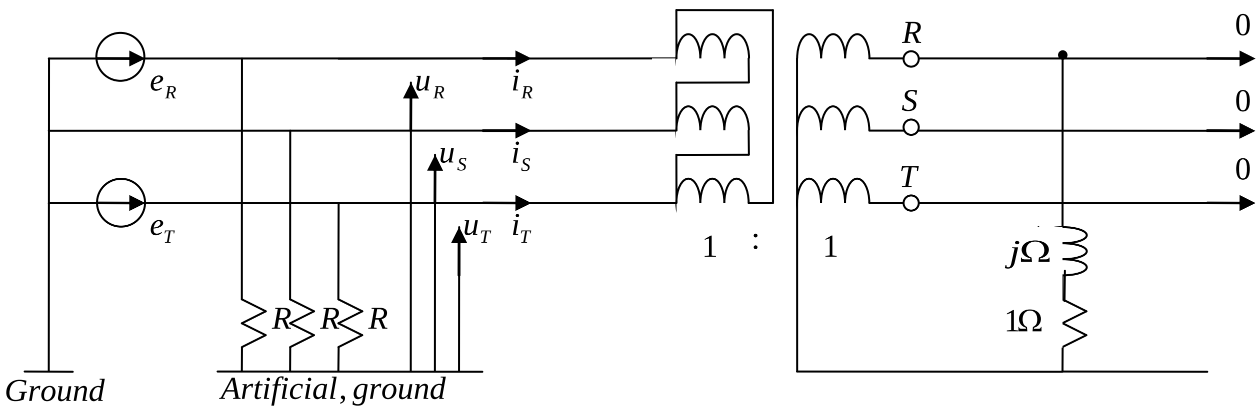


figure 7: Circuit with unbalanced supply voltage and an unbalanced RL load

This example has the following load voltage: and current data:

$U_R = 88.192\angle -19.107°$ V

$U_R = 33.333\angle -120° V$

$U_R = 88.192\angle 139.107° V$

And the load current is:

$I_R = 70.71\angle -45° A$

$I_S = 70.71\angle 135° A$

$I_T = 0A$

The resulting load power Czarnecki gets when using the CPC theory are:

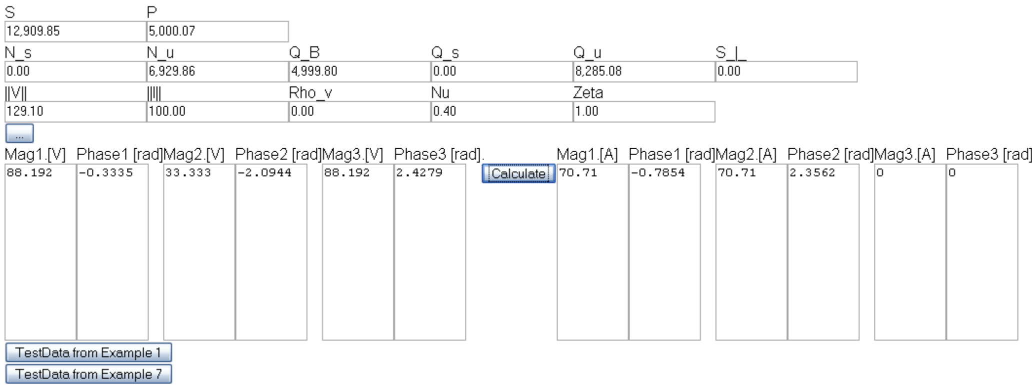
$P = 5.0kW$

$Q = 5.0kVA$

$D = 10.8kVA$

$S = 12.9kVA$

Now we put the load voltage and current data into our program to decompose it:



The load powers using the seven component decomposition are:

$$P = 5.0kW$$

$$Q_B = 5.0kVA$$

$$N_U = 6.9kVA$$

$$Q_U = 8.3kVA$$

$$S = 12.9kVA$$

The values above are the same as using the CPC theory, the only difference is that the unbalanced power D from the CPC theory has in our theory, been split into two components:

$$N_U^2 + Q_U^2 = D^2 \Rightarrow \sqrt{(6.9kVA)^2 + (8.3kVA)^2} = 10.8kVA = D$$

Example 6
In this example we look at circuit with harmonic generating load.

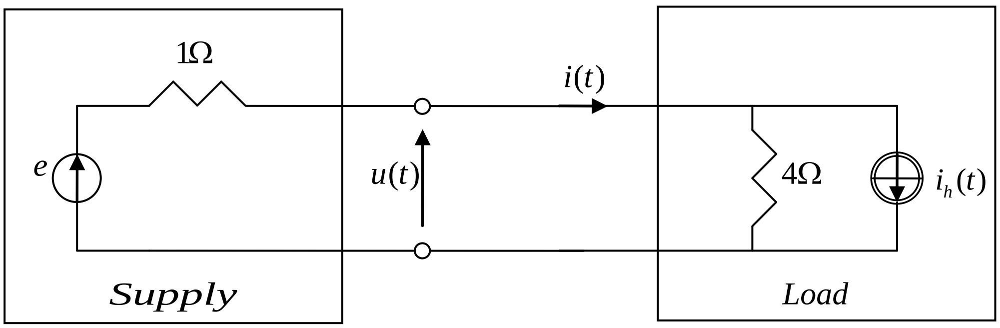


figure 8: Circuit with harmonic generating load

Here we assume that a resistive load in the circuit shown is figure , supplied with a sinusoidal voltage:

$$e = 100\sqrt{2}\sin\left(\omega_1 t\right)\ V$$

Generates a current harmonic of the third order:

$$i_h(t) = 50\sqrt{2}\sin(3\omega_1 t)\ \ A$$

The voltage and current at the load terminals in this circuit are equal to:

$$u = 80\sqrt{2}\sin(\omega_1 t) - 40\sqrt{2}\sin(3\omega_1 t)\ V,$$
$$i = 20\sqrt{2}\sin(\omega_1 t) + 40\sqrt{2}\sin(3\omega_1 t)\ \ A.$$

Thus the load power is:

$$P = \frac{1}{T}\int_0^T ui\ dt = \sum_{n=1,3} U_n I_n \cos(\varphi_n) = 1600 - 1600$$

And the active current is equal to zero, since:

$$i_a = \frac{P}{\|u\|^2} u \equiv 0$$

It means that according to the Fryze power theory, there is no current in this circuit other than the reactive current:

$$i = 20\sqrt{2}\sin(\omega_1 t) + 40\sqrt{2}\sin(3\omega_1 t)\ \ A.$$

Unfortunately this can not substitute as the compensator current, a suggested compensator current can be the following:

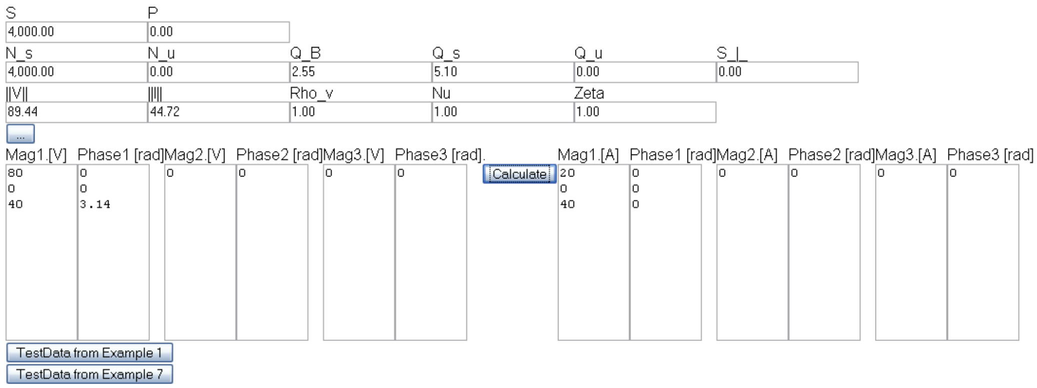
$$i_h(t) = 50\sqrt{2}\sin(3\omega_1 t)\ \ A$$

Now we insert the load voltage and load current in the program.

$u = 80\sqrt{2}\sin(\omega_1 t) - 40\sqrt{2}\sin(3\omega_1 t)\ \ V,$

$i = 20\sqrt{2}\sin(\omega_1 t) + 40\sqrt{2}\sin(3\omega_1 t)\ \ A.$

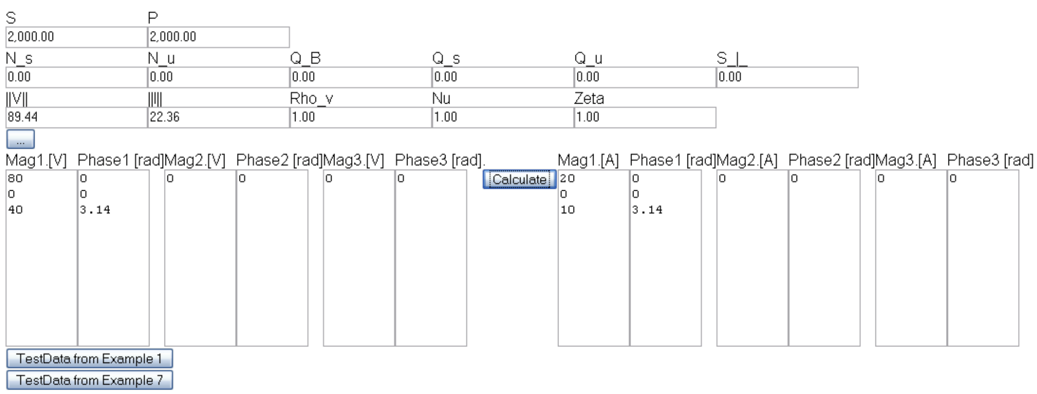
The following figure shows us the values our online Java program generates:



Next we insert the load voltage and the load current when we have added the compensator current in the circuit.

$u = 80\sqrt{2}\sin(\omega_1 t) - 40\sqrt{2}\sin(3\omega_1 t)\ \ V,$

$i = 20\sqrt{2}\sin(\omega_1 t) + 40\sqrt{2}\sin(3\omega_1 t) - 50\sqrt{2}\sin(3\omega_1 t)\ \ A = 20\sqrt{2}\sin(\omega_1 t) - 10\sqrt{2}\sin(3\omega_1 t)\ \ A.$



Example 7

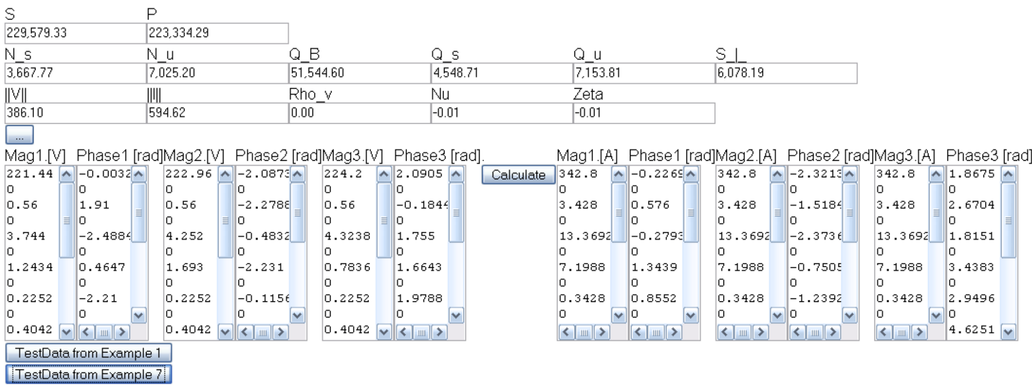
This is an example from Sergio J. Ceballos [2].

Here the load current is nonlinear and balanced. The load voltage has six odd harmonics to the eleventh and the current has ten odd harmonics to the 19th. The load voltage harmonics can be seen in appendix A table 4 and the load current harmonics can be seen in table 5. The seven component decomposition of the apparent power is shown in table 3.

S	P	N_S	N_U	Q_B	Q_S	Q_U	$S_\perp$
229,579	223,334	3,668	7,025	51,545	4,549	7,154	6,078

table 3: Powers of the nonlinear balanced load currents

Below we can see a snapshot of the program after values have been calculated.



Example 8

In this example the load is an aluminum smelter located in Iceland. The power is supplied to the smelter through the two three phase transmission lines NA1 and NA2.

In figure we see the network diagram for NA1. In appendix table 6 and table 7 shows the voltage and current harmonics. NA1 supplies the smelter with a total power of 173.62MW.

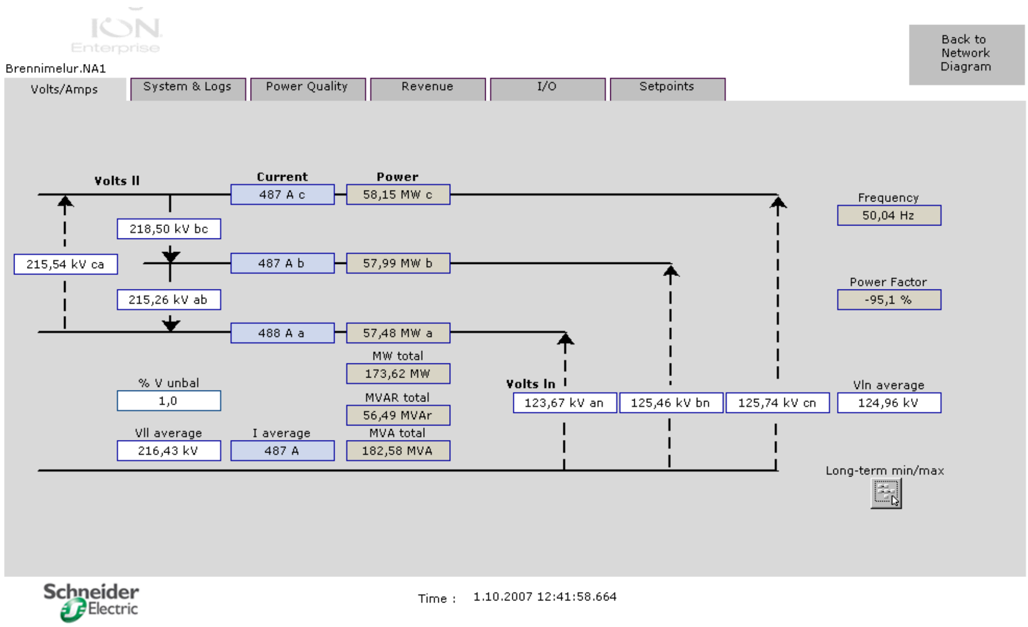
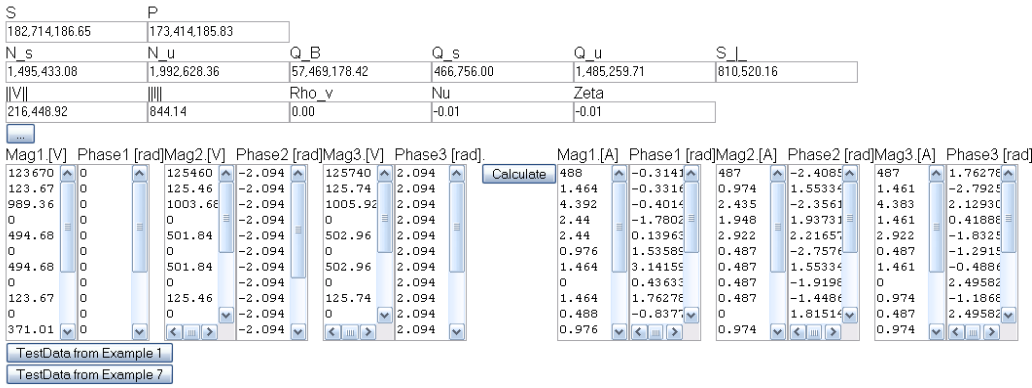


figure 9: Network diagram of the three phase line, NA1 in Iceland

Below we can see the decomposed apparent power:



Example 9
Here we have a 13-bus unbalanced utility distribution system. This system is based on the IEEE 13 bus radial distribution test feeder [8]. This system serves as benchmark system for unbalanced harmonic propagation studies.

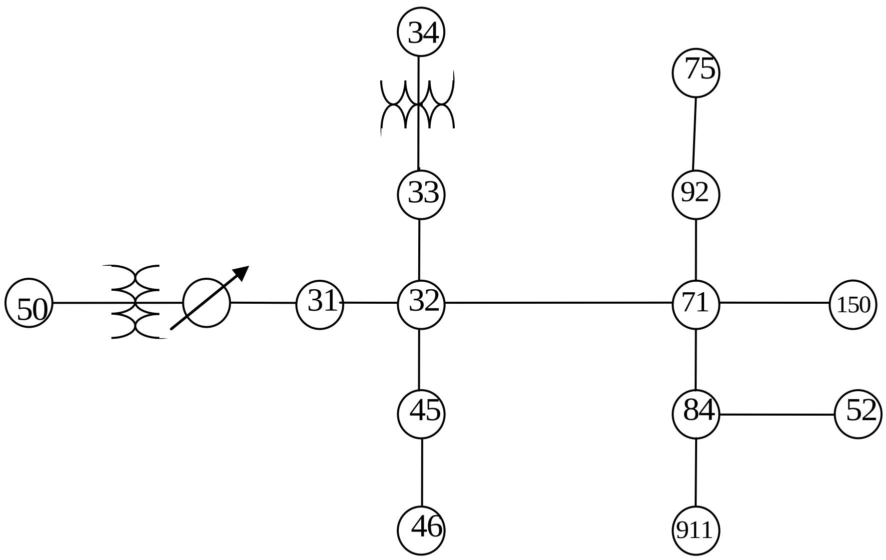


figure 10: Unbalanced distribution system

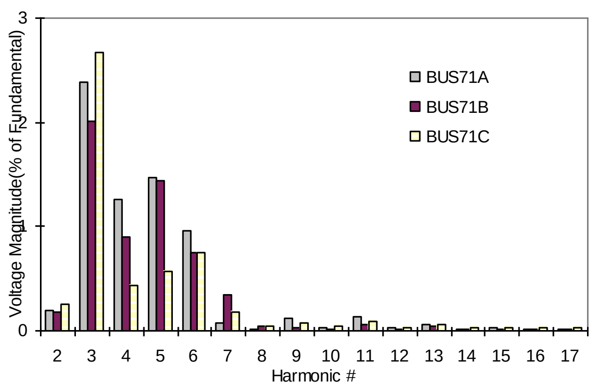
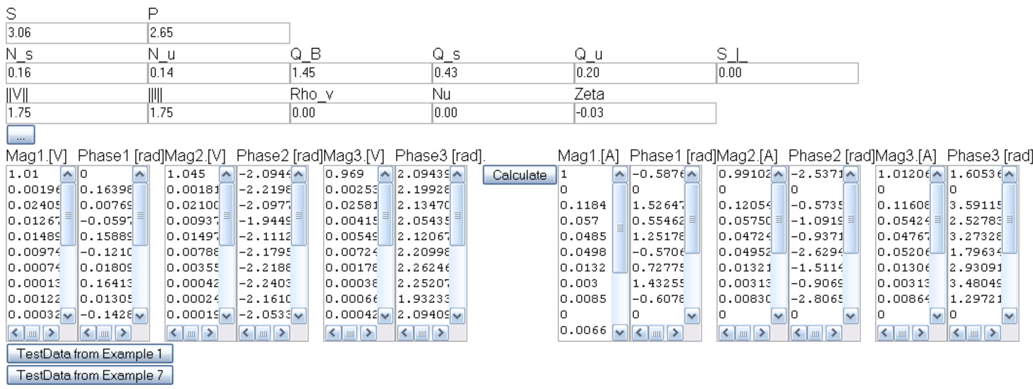


figure 11: Harmonic Voltage Distortion Spectrum at Node 71

The data from node 71 is extracted and used as an input to the decomposition program.

Below we can see the decomposed apparent power:



References

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Appendix

Appendix A – Fundamental Equations $\|V\|, \|I\|, \mu \ and \ \sigma$

Let's look in detail at the calculations behind the decomposition.
First, we need to calculate the polyphase voltage and current rms $\|V\|, \|I\|$ using the following equations:

$\|V\| = \sqrt{\sum_{k.l} |V_{k,l}|^2}$ and $\|I\| = \sqrt{\sum_{k,l} |I_{k,l}|^2}$

The subscript k represents the phase number and the subscript l is the harmonic number.

Next, we calculate the admittance for each phase and harmonic: $g_{k.l} - jb_{k.l} = \frac{I_{k,l}}{V_{k,l}}$.

Next, we calculate μ_g, which is the weighted mean of the load conductance $\{g_{k.l}\}$, and μ_b, the weighted mean of the load susceptance $\{b_{k.l}\}$.

$$\mu_g = \frac{\sum_{k.l} g_{k.l} |V_{k,l}|^2}{\sum_{k.l} |V_{k,l}|^2} \text{ and } \mu_b = \frac{\sum_{k.l} b_{k.l} |V_{k.l}|^2}{\sum_{k.l} |V_{k.l}|^2}$$

Then, we calculate the frequency-dependent weighted means $\mu_g(l)$ and $\mu_b(l)$ in the following way:

$$\mu_g(l) = \frac{\sum_{k} g_{k.l} |V_{k.l}|^2}{\sum_{k} |V_{k.l}|^2} \text{ and } \mu_b(l) = \frac{\sum_{k} b_{k.l} |V_{k.l}|^2}{\sum_{k} |V_{k.l}|^2}$$

Next, we calculate the standard deviation of $\{g_{k.l}\}$ and $\{b_{k.l}\}$, namely σ_g and σ_b.

$$\sigma_g = \sqrt{\frac{\sum_{k.l} (g_{k.l} - \mu_g)^2 |V_{k.l}|^2}{\sum_{k.l} |V_{k.l}|^2}} \text{ and } \sigma_b = \sqrt{\frac{\sum_{k.l} (b_{k.l} - \mu_b)^2 |V_{k.l}|^2}{\sum_{k.l} |V_{k.l}|^2}}$$

Now we need to calculate the standard deviations with respect to the phase, averaged over all frequencies namely σ_{gu} and σ_{bu}.

$$\sigma_{gu} = \sqrt{\frac{\sum_{k.l} |g_{k.l} - \mu_g(l)|^2 |V_{k.l}|^2}{\sum_{k.l} |V_{k.l}|^2}} \text{ and } \sigma_{bu} = \sqrt{\frac{\sum_{k.l} |b_{k.l} - \mu_b(l)|^2 |V_{k.l}|^2}{\sum_{k.l} |V_{k.l}|^2}}$$

Now we can calculate σ_{gs} and σ_{bs}.

$\sigma_{gs} = \sqrt{\sigma_g^2 - \sigma_{gu}^2}$ and $\sigma_{bs} = \sqrt{\sigma_b^2 - \sigma_{bu}^2}$.

Appendix B – Output Equations $S, P, Q_B, N_S, N_U, Q_S, Q_U \text{ and } S_\perp$

$$\|i_{gs}\| = \sigma_{gs}\|V\|$$
$$\|i_{gu}\| = \sigma_{gu}\|V\|$$
$$\|i_{bs}\| = \sigma_{bs}\|V\|$$
$$\|i_{bu}\| = \sigma_{bu}\|V\|$$

Now, when we have completed all the calculations above we can obtain the final result:

$$S = \|V\|\|I\|$$
$$P = \mu_g\|V\|^2$$
$$Q_B = \mu_b\|V\|^2$$
$$N_S = \|i_{gs}\|\|V\|$$
$$N_U = \|i_{gu}\|\|V\|$$
$$Q_S = \|i_{bs}\|\|V\|$$
$$Q_U = \|i_{bu}\|\|V\|$$
$$S_\perp = \|i_\perp\|\|V\| \quad Where \ \|i_\perp\| = \sqrt{\sum_{k,l\in B}\left|I_{k,l}\right|^2} \quad for \ B = \{(k,l); V_{k,l} = 0\}$$
$$S = \sqrt{P^2 + Q_B^2 + N_S^2 + N_U^2 + Q_S^2 + Q_u^2 + S_\perp{}^2}$$

Appendix C – Additional Output Equations ρ_v, v and ζ

The components, ρ_v, v and ζ are calculated using the following formulas:

$$\rho_v = \frac{\sum_{l=1}^{\infty} \|V_l \eta\|^2}{\|V\|^2}$$, where $\eta = \frac{1}{\sqrt{n}}\begin{bmatrix}1\\1\\1\end{bmatrix}$ and l = harmonic number and n = number of phases

$$v = \frac{\Re\left\{\sum_{k,l}^{\infty} V_{k,l} V_{k,l}^T\right\}}{\|V_{k,l}\|^2}$$

$$\zeta = \frac{\Re\left\{\sum_{l=1}^{\infty} I_{k,l} V_{k,l}^T\right\}}{P_0}$$

Appendix D – Data from Example7 and Example8 and Example9

The data from example7 can be seen in table 4 and table 5

	Phase 1		Phase 2		Phase 3	
Order	Mag. [V]	Phase [rad]	Mag. [V]	Phase [rad]	Mag. [V]	Phase [rad]
1	221.44	-0.0032	222.96	-2.0873	224.2	2.0905
2	0	0	0	0	0	0
3	0.56	1.91	0.56	-2.2788	0.56	-0.1844
4	0	0	0	0	0	0
5	3.744	-2.4884	4.252	-0.4832	4.3238	1.755
6	0	0	0	0	0	0
7	1.2434	0.4647	1.693	-2.231	0.7836	1.6643
8	0	0	0	0	0	0
9	0.2252	-2.21	0.2252	-0.1156	0.2252	1.9788
10	0	0	0	0	0	0
11	0.4042	0.95	0.4042	3.0444	0.4042	-1.1444

table 4: Load voltage harmonics in Example 7

	Phase 1		Phase 2		Phase 3	
Order	Mag. [V]	Phase [rad]	Mag. [V]	Phase [rad]	Mag. [V]	Phase [rad]
1	342.8	-0.2269	342.8	-2.3213	342.8	1.8675
2	0	0	0	0	0	0
3	3.428	0.576	3.428	-1.5184	3.428	2.6704
4	0	0	0	0	0	0
5	13.3692	-0.2793	13.3692	-2.3736	13.3692	1.8151
6	0	0	0	0	0	0
7	7.1988	1.3439	7.1988	-0.7505	7.1988	3.4383
8	0	0	0	0	0	0
9	0.3428	0.8552	0.3428	-1.2392	0.3428	2.9496
10	0	0	0	0	0	0
11	4.4564	2.5307	4.4564	0.4363	4.4564	4.6251
12	0	0	0	0	0	0
13	2.0568	-2.8623	2.0568	-4.9567	2.0568	-0.7679
14	0	0	0	0	0	0
15	0.3428	-0.7854	0.3428	-2.8798	0.3428	1.309
16	0	0	0	0	0	0
17	7.1988	-0.7854	7.1988	-2.8798	7.1988	1.309
18	0	0	0	0	0	0
19	5.142	-0.8901	5.142	-2.9845	5.142	1.2043

table 5: Load current harmonics in Example 7

The data from example8 can be seen in table 6 and table 7

	Phase 1		Phase 2		Phase 3	
Order	Mag. [V]	Phase [rad]	Mag. [V]	Phase [rad]	Mag. [V]	Phase [rad]
1	123670	0	125460	-2.094	125740	2.094
2	123.67	0	125.46	-2.094	125.74	2.094
3	989.36	0	1003.68	-2.094	1005.92	2.094
4	0	0	0	-2.094	0	2.094
5	494.68	0	501.84	-2.094	502.96	2.094
6	0	0	0	-2.094	0	2.094
7	494.68	0	501.84	-2.094	502.96	2.094
8	0	0	0	-2.094	0	2.094
9	123.67	0	125.46	-2.094	125.74	2.094
10	0	0	0	-2.094	0	2.094
11	371.01	0	376.38	-2.094	377.22	2.094
12	0	0	0	-2.094	0	2.094
13	123.67	0	125.46	-2.094	125.74	2.094
14	0	0	0	-2.094	0	2.094
15	123.67	0	125.46	-2.094	125.74	2.094

table 6: Voltage harmonics data for example 8

	Phase 1		Phase 2		Phase 3	
Order	Mag. [V]	Phase [rad]	Mag. [V]	Phase [rad]	Mag. [V]	Phase [rad]
1	488	-0.31416	487	-2.40855	487	1.76278
2	1.464	-0.33161	0.974	1.55334	1.461	-2.79252
3	4.392	-0.40143	2.435	-2.35619	4.383	2.12930
4	2.44	-1.78023	1.948	1.93731	1.461	0.41888
5	2.44	0.13963	2.922	2.21657	2.922	-1.83259
6	0.976	1.53589	0.487	-2.75762	0.487	-1.29154
7	1.464	3.14159	0.487	1.55334	1.461	-0.48869
8	0	0.43633	0.487	-1.91986	0	2.49582
9	1.464	1.76278	0.487	-1.44862	0.974	-1.18682
10	0.488	-0.83776	0	1.81514	0.487	2.49582
11	0.976	-0.22689	0.974	2.19911	0.974	-2.26893
12	0	-0.71559	0	2.87979	0	1.43117
13	0.488	-2.05949	0.487	1.98967	0.487	0.27925
14	0	2.63544	0	2.98451	0	-0.75049
15	0	1.53589	0	2.26893	0	-1.04720

table 7: Current harmonics data for example 8

The data from example9 can be found in **table 8** and **table 9**

	Phase 1		Phase 2		Phase 3	
Order	Mag. [V]	Phase [rad]	Mag. [V]	Phase [rad]	Mag. [V]	Phase [rad]
1	1.01	0	1.045	-2.0944	0.969	2.094395
2	0.0019684	0.163981	0.001813	-2.21981	0.00253	2.199284
3	0.0240583	0.007696	0.021008	-2.09772	0.025819	2.134707
4	0.0126745	-0.05971	0.009376	-1.94493	0.004156	2.054355
5	0.0148953	0.158893	0.014974	-2.11125	0.00549	2.120676
6	0.0097471	-0.12108	0.007888	-2.1795	0.00724	2.209981
7	0.0007461	0.01809	0.003551	-2.21886	0.00178	2.262468
8	0.0001385	0.164138	0.00042	-2.24035	0.000388	2.252073
9	0.0012286	0.013059	0.000241	-2.16101	0.000668	1.932331
10	0.0003256	-0.14289	0.000192	-2.05333	0.000423	2.094095
11	0.001311	0.015543	0.000637	-2.14277	0.000924	2.251078
12	0.0002326	0.086127	0.000187	-2.17632	0.000342	2.165009
13	0.0006082	0.045571	0.000538	-2.21528	0.000618	2.161595
14	0.0001988	-0.16132	0.000138	-2.02197	0.000282	2.012469
15	0.0003106	-0.08452	0.000116	-1.98978	0.00025	2.039833
16	0.000173	0.100815	0.000127	-2.02534	0.000254	2.138595
17	0.0001785	-0.13081	0.000118	-2.1659	0.000236	2.101349

table 8: Load Voltages, input to program in example 9

Order	Mag. [V]	Phase [rad]	Mag. [V]	Phase [rad]	Mag. [V]	Phase [rad]
1	1	-0.58765	0.991026	-2.53716	1.012067	1.605364
2	0	0	0	0	0	0
3	0.1184	1.526478	0.120548	-0.57355	0.116081	3.591153
4	0.057	0.554626	0.057504	-1.0919	0.054242	2.527833
5	0.0485	1.251789	0.047248	-0.93714	0.04767	3.273283
6	0.0498	-0.57066	0.049521	-2.62948	0.052066	1.79634
7	0.0132	0.727752	0.013211	-1.51143	0.013065	2.930918
8	0.003	1.43255	0.003134	-0.90696	0.003138	3.480493
9	0.0085	-0.60783	0.008304	-2.80652	0.00864	1.297217
10	0	0	0	0	0	0
11	0.0066	0.194732	0.00664	-1.9121	0.006692	2.405299
12	0	0	0	0	0	0
13	0.0047	0.335974	0.004691	-1.76492	0.004733	2.436633
14	0	0	0	0	0	0
15	0.004	-0.53208	0.003933	-2.44184	0.003901	1.580216

table 9: Load Currents, input to program in example 9