A family of switching control strategies for the reduction of torque ripple on the direct torque and flux control for induction motors

E. Galvan, J.M. Carrasco, R. Ortega, G. Escobar and A.M. Stanković

Abstract—**In this paper we are interested in the design of switching control strategies which, as pursued in classical Direct Torque Control (DTC), are aimed to directly regulate two outputs: torque and flux amplitude. A criterion in terms of the error and/or the prediction in one–step–ahead on these outputs is proposed to design the switching sequence. As a result, a control vector, i.e., the switch position, is directly selected without the requirement of an auxiliary space vector or other modulation technique. We consider two types of criterion: quadratic and absolute value. Finally, experimental results following these two approaches are presented and compared with respect to the classical DTC.**

I. Introduction

As was established in [13] DTC strategy is in principle a sliding mode technique, and thus, as in most sliding mode feedback control approaches, this technique entirely overlooks the physical properties of the "plant" and, in particular, ignores its energy dissipation characteristics. It is well known that one of the main drawbacks of sliding mode strategies is their need of a sufficiently high switching frequency to guarantee good performance. This condition is in many cases difficult to fulfill, for instance when the switching devices are limited in frequency because of the high power rating of the drive. This constraint evidently limits the performance of the system, as perfect tracking towards the desired references cannot be guaranteed any longer. It also favors the increase of the unavoidable "chattering" phenomena where high frequency oscillations may now excite unmodeled (high order) system modes. However, it is also well known that this type of control shows considerable robustness with respect to parameters uncertainties (except for those parameters intervening in the definitions of sliding surfaces). Unfortunately, this disregard of system information is one of the main reasons for the performance degradation in a low switching frequency operation.

We propose to include information of the evolution of the system to overcome such drawbacks and improve the performance of the system when the switching frequency is limited to small values. For this, we propose the use of a switching strategy based on the minimization of a given criterion in terms of the prediction of the output deviation. This strategy is in turn used to directly generate the switching control vector. A first order approximation is employed to approximate the predicted errors. The switching policy has to select a control vector that will remain valid (in an average sense) over the sampling period. In other words, the policy needs to prevent the controller from switching unnecessarily to a signal that might be the correct one at the sampling instant, but not the proper one during the whole sampling period. This will entail the reduction of unnecessary excursions of the state trajectories and thus reduce the unwanted "chattering". This improvement is made even more evident when the ratio between the motor time constant and the sampling period is small. Following the classical DTC philosophy, the proposed strategies are aimed to generate the switching control vectors which should directly control the outputs torque and flux amplitude while guaranteeing regulation towards their desired references.

The paper is organized as follows: in Section 2 the model and control objective are presented. Section 3 introduces the technique based on a quadratic criterion, and explores its one–step–ahead prediction, together with the first switching strategy. Section 4 revisits the classical DTC and describes its relationship with a quadratic criterion. By proposing a quadratic criterion and advocating the usual assumptions made in DTC (fast switching frequency and a reduced model for the induction machine) we are able to recover and visualize the look–up table commonly used to implement DTC. This description allows us to propose modifications to the classical DTC to include information on the one–step–ahead predictions. Section 4 presents a second criterion based on the absolute value of the predicted errors. In Section 5 a discussion on the flux estimator is presented as, well as some guidelines about parameter tuning. Finally, experimental results are presented in Section 6 to demonstrate the performance improvements over the classical DTC that are obtained with proposed strategies.

II. Problem formulation

The state–space representation of the motor dynamics is usually given in terms of the stator currents i_s, rotor fluxes and rotor speed. This choice of coordinates stems from the fact that the control objective is typically expressed –for instance in FOC– in terms of regulation of rotor flux amplitude and rotor speed [8], [12]. In the basic version of DTC [7], [2], the objective is to directly control torque (hence the name DTC) and the amplitude of the stator flux. It is therefore more convenient to represent the model in stator flux coordinates λ_s as given by the following expressions

$$\begin{aligned} \frac{d}{dt}i_s &= -(\mu\mathcal{I}_2 - w\mathcal{J})i_s + \left(\frac{R_r}{L_r}\mathcal{I}_2 - w\mathcal{J}\right)\frac{\lambda_s}{L_s\sigma} + \frac{u}{L_s\sigma} \\ \frac{d}{dt}\lambda_s &= -R_s i_s + u \end{aligned}$$

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$$\frac{d}{dt}w = \frac{1}{D_m}\left(-R_m w + \tau - \tau_L\right), \ \tau = i_s^T \mathcal{J} \lambda_s \qquad (1)$$

where R_r and R_s are the resistance values and I_2 is the 2×2 identity matrix; L_r, L_s are self–inductances L_{sr} is the mutual inductance; $\mathcal{J} = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}$ is a rotation matrix and we have defined the parameters $\sigma \triangleq 1 - \frac{L_{sr}^2}{L_s L_r}$, $\mu \triangleq \frac{R_s L_r + R_r L_s}{L_r L_s - L_{sr}^2}$ with the leakage factor σ satisfying $0 < \sigma < 1$; D_m is the rotor moment of inertia, R_m the friction coefficient, $w = \dot{\theta}$, θ is the rotor position, τ_L is the load torque assumed an unknown constant and τ is the electrical torque.

In contrast with PWM–based approaches, control vectors u are restricted here to the discrete set $\mathcal{U} = \{U_0, ..., U_7\}$ described in Table I for the stationary $(\alpha\beta)$ reference frame[1], with V_s the constant voltage provided by the source. Moreover, in the actual control implementation these vectors are selected at a given sampling time $T > 0$.

$No.$	u_1	u_2
U_0	0	0
U_1	$2V_s/3$	0
U_2	$V_s/3$	$V_s/\sqrt{3}$
U_3	$-V_s/3$	$V_s/\sqrt{3}$
U_4	$-2V_s/3$	0
U_5	$-V_s/3$	$-V_s/\sqrt{3}$
U_6	$V_s/3$	$-V_s/\sqrt{3}$
U_7	0	0

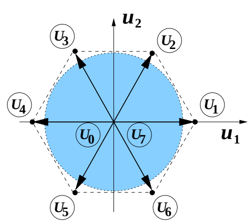

TABLE I

ADMISSIBLE CONTROL VECTORS

Under these conditions, the *control objective* can be established as follows: Design a sequence of vectors $u(kT) \in \mathcal{U}, \ k \in \mathcal{Z}_+$, such that the outputs

$$y_1 = i_s^T \mathcal{J} \lambda_s \quad (= \tau) \quad , \quad y_2 = |\lambda_s|^2 \quad (= \beta^2) \qquad (2)$$

where $|\cdot|$ is the Euclidean norm, are regulated (close) to some constant desired values y_1^* and y_2^*, respectively.

The time derivatives, which will be used later, are computed as follows

$$\dot{y}_1 = (\mathcal{J}\eta)^T \left(u - n_p w \mathcal{J} \lambda_s - \mu \lambda_s\right) \qquad (3)$$

$$\dot{y}_2 = 2\lambda_s^T (u - R_s i_s) \qquad (4)$$

where $\eta \triangleq \left(\frac{\lambda_s}{L_s \sigma} - i_s\right)$, and it is clear that the motor dynamics (1) with outputs as defined in (2) has relative degree $\{1, 1\}$, which is defined in all regions of the state space where $|\lambda_s|, |\lambda_r| \neq 0$.

Since the objective of the controller is to drive y_1 and y_2 to constant values, it is clear that the zero dynamics will play a central role. It has been shown [13] that that the zero dynamics associated to the outputs $(\tau, |\lambda_s|)$ is *input–to–state stable* and indeed periodic.

The restriction to constant references does not amount to a significant loss of generality. Indeed, in most applications y_2^* is typically a constant that changes occasionally (in ramp–wise manner or following a static profile of step changes combined with ramps) for flux weakening operation or optimal energy transfer. Also, y_1^* is slowly varying (relative to the electrical dynamics) as it represents the torque reference usually generated by a PI external speed/position control loop. Nevertheless, in some cases a term can be added to the proposed algorithms to compensate for changes in the reference.

III. SWITCHING STRATEGIES FOLLOWING A QUADRATIC CRITERION

Consider the quadratic functions in the increments described by $H_1 = \frac{1}{2}\tilde{y}_1^2$ and $H_2 = \frac{1}{2}\tilde{y}_2^2$ which might be combined in a single weighted "energy" function as follows

$$H(\tilde{y}_1, \tilde{y}_2) = w_1 H_1(\tilde{y}_1) + w_2 H_2(\tilde{y}_2) \qquad (5)$$

where w_1 and w_2 are positive cost gains, and we used the notation $\tilde{y}_i \triangleq y_i - y_i^*, \ \ i \in \{1, 2\}$

A simple criteria can be obtained by taking the time derivative of $H(\tilde{y}_1, \tilde{y}_2)$ yielding

$$\dot{H}(\tilde{y}_1, \tilde{y}_2,) = w_1 y_1 \dot{y}_1(u) + w_2 \tilde{y}_2 \dot{y}_2(u) \qquad (6)$$

We then evaluate $\dot{H}(\tilde{y}_1, \tilde{y}_2)$ for every $U \in \mathcal{U}$ and select the U that makes it as negative as possible. Following this technique a control vector which is valid for that time instant would be generated. This algorithm would present a good solution if an arbitrarily fast switching frequency would be used to implement it. However, the switching frequency is limited in practice to low values due to physical constraints. In addition, we observed in our experimentation that a vector satisfying a criterion (like the quadratic above) at a given instant of time is not always the optimal selection, and may lead to a considerable performance degradation when the switching frequency is limited to low values, or more specifically, when the ratio between the motor time constant and the sampling period is small.

As noted before, the controller will be implemented digitally, so a regular switching frequency with sampling time T is assumed. Thus, $H_k(\tilde{y}_{1,k}, \tilde{y}_{2,k})$ is used to denote the value of the energy function at k^{th} sampling time.

Our idea is to prevent the controller to switch unnecessarily to a control signal which may be the correct one at the sampling instant, but not the optimal one during the whole sampling period. Thus, we consider not only the information at a given instant, but also introduce information about the evolution of the system trajectories in one–step–ahead[2], and then to select a control vector that will keep the outputs as close as possible to their respective references during the whole period (or at least up to the next sample time). As our first approach, we propose to minimize the one–step–ahead sample of the energy function above, that is, given

$$H_{k+1} = \frac{w_1}{2}\tilde{y}_{1,k+1}^2 + \frac{w_2}{2}\tilde{y}_{2,k+1}^2 \qquad (7)$$

[1]The admissible controls in the stationary $(\alpha\beta)$ reference frame are two dimensional vectors pointing to the vertices of a hexagon centered at the origin, plus a zero voltage point which, following standard convention, is repeated twice.

[2]Throughout the paper we will use indistinctly the terms one–step–ahead and predicted when referring to the $(k+1)$ sample of a given quantity.

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find the controller u according to

$$u = \arg\min_U \{H_{k+1}(\tilde{y}_{1,k+1},\, \tilde{y}_{2,k+1}),\, U \in \mathcal{U}\}$$

Since the predicted error is not available, we propose then to estimate it according to a first order approximation as follows

$$\tilde{y}_{i,k+1} = \tilde{y}_{i,k} + T\dot{\tilde{y}}_{i,k} \quad (8)$$

for easy of presentation we will consider that $\dot{y}_1^* = \dot{y}_2^* = 0$.

Notice that due to the relative degree condition established in the previous section, the controller u appears explicitly in $\dot{y}_{i,k}$, and thus $H_{k+1}(\tilde{y}_{1,k+1},\, \tilde{y}_{2,k+1})$ is a function of u, as shown below

$$\begin{aligned} H_{k+1} \;=\; & \frac{w_1}{2}\left(\tilde{y}_{1,k}^2 + 2T\tilde{y}_{1,k}\dot{y}_{1,k}(u) + T^2\dot{y}_{1,k}^2(u)\right) + \\ & \frac{w_2}{2}\left(\tilde{y}_{2,k}^2 + 2T\tilde{y}_{2,k}\dot{y}_{2,k}(u) + T^2\dot{y}_{2,k}^2(u)\right) \end{aligned}$$

The terms $\tilde{y}_{1,k}^2$ and $\tilde{y}_{2,k}^2$ do not provide any information for comparison purposes, since they are independent of u. Hence, after neglecting terms $\tilde{y}_{i,k}^2$, extracting common factor $2T$ and factorizing with respect to $\dot{y}_{i,k}$, we obtain the following more convenient expression for the controller strategy

$$\begin{aligned} u \;=\; & \arg\min_U \left\{ w_1\left(\tilde{y}_{1,k} + \frac{T}{2}\dot{y}_{1,k}(U)\right)\dot{y}_{1,k}(U) + \right. \\ & \left. w_2\left(\tilde{y}_{2,k} + \frac{T}{2}\dot{y}_{2,k}(U)\right)\dot{y}_{2,k}(U),\, U \in \mathcal{U} \right\} \quad (9) \end{aligned}$$

We will denote this strategy as the *Quadratic* controller, since it is based on the quadratic criterion.

Based on the form of (7), we may also consider an energy function which involves not only the predicted error, but a convex combination of the actual and the predicted error

$$\begin{aligned} H_c \;=\; & \frac{w_1}{2}\left(\delta_1\tilde{y}_{1,k} + (1-\delta_1)\tilde{y}_{1,k+1}\right)^2 + \\ & \frac{w_2}{2}\left(\delta_2\tilde{y}_{2,k} + (1-\delta_2)\tilde{y}_{2,k+1}\right)^2 \end{aligned}$$

This yields the switching control strategy described by

$$\begin{aligned} u = \arg\min\nolimits_U & \left\{ w_1\left(\tilde{y}_{1,k} + \tfrac{T}{2}(1-\delta_1)\dot{y}_{1,k}(U)\right)\dot{y}_{1,k}(U) + \right. \\ & \left. w_2\left(\tilde{y}_{2,k} + \tfrac{T}{2}(1-\delta_2)\dot{y}_{2,k}(U)\right)\dot{y}_{2,k}(U),\, U \in \mathcal{U} \right\} \quad (10) \end{aligned}$$

where it is required

$$0 \le \delta_1 < \frac{1}{2}\,,\;\; 0 \le \delta_2 < \frac{1}{2}$$

to guarantee stability of the imposed discrete dynamics $(\delta_i y_{i,k} + (1-\delta_i)y_{i,k+1})$, $(i \in \{1,2\})$. Notice that the *Quadratic* controller previously presented can be recovered from this strategy for $\delta_1 = \delta_2 = 0$.

It is interesting to notice that, for an arbitrarily small sampling period $T << 1$, the *Quadratic* strategy reduces to

$$u = \arg\min_U \{w_1\tilde{y}_{1,k}\dot{y}_{1,k}(U) + w_2\tilde{y}_{2,k}\dot{y}_{2,k}(U),\, U \in \mathcal{U}\} \quad (11)$$

which is of special interest since it is closely related to the standard DTC strategy as shown below.

A. DTC revisited

In this section we rederive the classical DTC strategy using the *output regulation subspaces (ORS)*, a tool introduced in [10] and we show that the classical DTC is of the form (11) under certain simplifications and for some defined weights w_1 and w_2. First, we proceed to obtain a reduced approximation of the model (1) based on the standard assumptions made in DTC derivation, that is, we consider an unloaded machine ($\tau_L \cong 0$) with speed close to zero ($w \cong 0$) and negligible stator resistance ($R_s \cong 0$), which implies a small stator current $i_s(t) \cong 0$. Under these considerations, the *reduced model* is defined as follows

$$\frac{d}{dt}i_s = \frac{R_r}{L_r}\frac{\lambda_s}{L_s\sigma} + \frac{u}{L_s\sigma} \quad , \quad \frac{d}{dt}\lambda_s = u \quad (12)$$

The time derivatives of the outputs (2) are computed now as

$$\dot{y}_1 = \frac{1}{L_s\sigma}(\mathcal{J}\lambda_s)^T u \quad , \quad \dot{y}_2 = 2\lambda_s^T u \quad (13)$$

We now use the concept of *output regulation subspaces* (see [10] for further details) which are defined as the subspaces in the input space where the time derivatives of the outputs are zeroed, and thus, we may define as many subspaces as there are outputs. In the case of the reduced model for the induction machine, this corresponds to the perpendicular lines intersecting at the origin[3] as shown in Fig. 1. Expressions for such lines are

$$(\mathcal{J}\lambda_s)^T u = 0 \Rightarrow u = c_1\lambda_s \quad , \quad \lambda_s^T u = 0 \Rightarrow u = c_2\mathcal{J}\lambda_s$$

where c_1 and c_2 are arbitrary constants. The directions of these lines are λ_s and $\mathcal{J}\lambda_s$ for $\dot{y}_1 = 0$ and $\dot{y}_2 = 0$, respectively. These straight lines induce a partition of the input space into four quadrants, assigning to each quadrant a unique combination of signs for the time derivatives as shown in Fig. 1.

The DTC switching logic can be derived as follows. First, the input space is divided into six sectors having each a control vector exactly at the middle as shown in Fig. 1, and assign to each sector the same identification number as the vector contained in it. Second, consider that the flux vector is contained in the sector number n as shown in Fig. 1; thus the separating lines cannot go further than the shadowed area. Consider that we want to increase y_1. From Fig. 1, we observe that there are actually three choices, vectors $n+1$, $n+2$ and $n+3$, but if the vector is below the vector n but still in sector k, then instead of vector $n+3$ the other choice would be vector n. In any case, for vectors $n+1$ and $n+2$, $\dot{y}_1 > 0$. Now, if at the same time we want to increase y_2 the only possible solution would be vector $n+1$. Following with this idea leads to Table II, referred in [2] as *Switching Table A*[4]

The switching policy is then reduced to search the sector to which vector λ_s belongs; based on the sign of the errors, table (II) is used to select an appropriate control vector. These simple derivations motivate the qualifier "direct" in DTC because we can –in principle– *directly* regulate the behavior of $y_1 = \tau$ and $y_2 = |\lambda_s|^2$.

[3] In the case of the complete model, these subspaces are lines crossing at a point different from the origin and not necessarily perpendicular to each other. See [13] for further details.

[4] To complete the description of DTC two *hysteresis loops* around the torque and flux norm errors are typically added.

$\tilde{y}_1, \tilde{y}_2$	$<0, <0$	$<0, >0$	$>0, <0$	$>0, >0$
Control Vector	U_{k+1}	U_{k+2}	U_{k-1}	U_{k-2}

TABLE II
SWITCHING TABLE ALGORITHM FOR THE CLASSICAL DTC

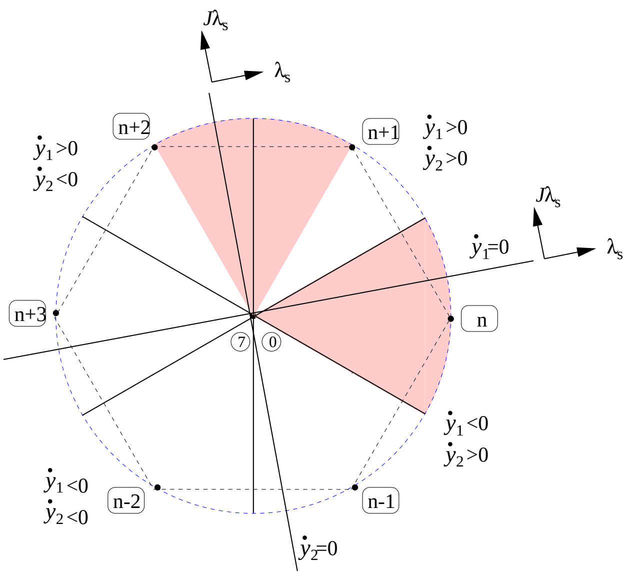

Fig. 1. Classical DTC algorithm

It is then clear, according to this partition, that we can always pick a vector u from the available ones, that make negative definite both products $\tilde{y}_1\dot{y}_1$ and $\tilde{y}_2\dot{y}_2$, simultaneously. Notice that in the standard derivation of DTC as described above, vectors close to the line $\dot{y}_1 = 0$, i.e., vectors n and $n+3$, are neglected and the ones placed farther are preferred. It turns out that the preferred ones are at the same time the closest to the perpendicular line $\dot{y}_2 = 0$. Roughly speaking this means that, DTC strategy is giving more priority to the torque regulation, as its time derivative is forced to take the largest absolute value. Hence, according to our criterion (11), for these cases where more than one vector u makes negative simultaneously $\tilde{y}_1\dot{y}_1$ and $\tilde{y}_2\dot{y}_2$, we can always define a set of gains w_1 and w_2 to decide for the vector that minimizes (11) giving the corresponding priority to the torque output.

An immediate corollary of the previous discussion is that DTC will provide satisfactory performance for lightly loaded machines at slow speeds, which are the operating regimes where DTC has shown to outperform FOC, see [2] for further details. On the other hand, it is clear that the switching strategies which take into account the full motor dynamics will outperform DTC when operating regimes are different –as it has been confirmed by our experiments.

B. Possible predictive modifications for classical DTC

As stated above, DTC strategy relies on the actual error; however, according to the *Quadratic* strategy (9), we may incorporate the prediction feature into the classical DTC strategy, as explained in what follows.

Notice that the only difference between strategies (9) and (11) is the computation of the "error" signal, that is, while in (11) the errors are represented as $\tilde{y}_{i,k}$, in (9) they are $\left(\tilde{y}_{i,k} + \frac{T}{2}\dot{y}_{i,k}(U)\right)$. At a first sight we may intend to modify the computation of the increments used in classical DTC as

$$\tilde{y}_1^{\text{Modif}} = \tilde{y}_{1,k} + \frac{T}{2}\dot{y}_{1,k}(U) \quad , \quad \tilde{y}_2^{\text{Modif}} = \tilde{y}_{2,k} + \frac{T}{2}\dot{y}_{2,k}(U) \quad (14)$$

with this errors we enter in Table II and extract the control vector U. However, such a modification is not implementable with the information available at that instant of time, because in (14) the control vector U (the one we are searching for) is unknown. We may propose, as an approximation, to consider the previous control U for the computation of (13) used in (14), i.e., to assume that the previous control U is preserved in the next switching interval. Unfortunately, it was observed in our experiments that there is no substantial improvement when this idea is implemented.

Since after a sampling instant both the ORS orientation and outputs errors have changed, we may also think of considering predictions for both. We can estimate the new orientation of the ORS in one–step–ahead by computing a prediction of the fluxes using (12), and use the formulas (13) and (8) for the output errors prediction. This idea has the same drawback as the control vector U needed in both computations is unknown. Again we may propose to use instead the previous control vector, but the experiments showed no major improvements.

Another option to search for the control u would be according to (9) for some w_1, w_2 and using the reduced model output derivatives (13) to reduce the computational effort in the evaluation of the algorithm (9). We refer to this strategy as *DTC–Quadratic modified* since the modification is based on the Quadratic criterion but uses the reduced model that results from standard assumptions made in DTC. In this case the experiments revealed a considerable improvement. We show some of the results in the experimental results section.

IV. SWITCHING STRATEGIES FOLLOWING AN ABSOLUTE VALUE CRITERION

We also considered a criterion based on the minimization of a weighted function of the absolute value of the predicted errors; the task is to find a u that minimizes the weighted function

$$\begin{aligned} H_{k+1}(\tilde{y}_{1,k+1}, \tilde{y}_{2,k+1}) &= w_1|\tilde{y}_{1,k+1}| + w_2|\tilde{y}_{2,k+1}| = \\ &= w_1|\tilde{y}_{1,k} + T\dot{y}_{1,k}| + w_2|\tilde{y}_{2,k+1} + T\dot{y}_{2,k}| \end{aligned}$$

where we have used a similar first order approximation (8) to estimate the one–step–ahead values as before.

An expression for the controller can be derived as follows

$$u = \arg\min_{U\in\mathcal{U}} \{w_1|\tilde{y}_{1,k} + T\dot{y}_{1,k}(U)| + w_2|\tilde{y}_{2,k} + T\dot{y}_{2,k}(U)|\} \quad (15)$$

which is referred as *Absolute value* strategy.

As before, we may extend this criterion by considering the convex combination of the present and the predicted error.

V. Some remarks on the implementation

From the previous analysis it is clear that rotor speed, stator currents and stator fluxes are needed for the evaluation of the control. Notice that the outputs of interest (torque and flux amplitude) can be derived from this set of variables as well. While rotor speed, stator currents and voltages can be measured, stator fluxes should be estimated online using some high performance observer. For this task, we prefer to use the observer proposed in [14] which, as claimed by the authors, exhibits an arbitrary fast convergence. For the sake of completeness, we present here the nonlinear equations describing this observer

$$\begin{aligned}\frac{d}{dt}\hat{i}_s &= -(\mu\mathcal{I}_2 - w\mathcal{J})i_s + \left(\frac{R_r}{L_r}\mathcal{I}_2 - w\mathcal{J}\right)\frac{\hat{\lambda}_s}{L_s\sigma} + \\ &\quad \frac{u}{L_s\sigma} + \frac{\xi_2}{L_s\sigma}\left(\frac{R_r}{L_r}\mathcal{I}_2 - w\mathcal{J}\right) \\ \frac{d}{dt}\hat{\lambda}_s &= -R_s i_s + u + \xi_1 L_r\left(\frac{R_r}{L_r}\mathcal{I}_2 - w\mathcal{J}\right)\end{aligned}$$

where $\xi_1 = \xi_1^r\mathcal{I}_2 + \mathcal{J}\xi_1^i$ and $\xi_2 = \xi_2^r\mathcal{I}_2 + \mathcal{J}\xi_2^i$ are two matrix gains, with ξ_1^r, ξ_1^i, ξ_2^2 and ξ_2^i some real constants. In [14] this two matrix gains are expressed as complex gains with $(\cdot)^r$ representing the real part and $(\cdot)^i$ the imaginary part. Following the same ideas for the proof as in [14], and considering a complex representation of the error model, that is, matrix $\mathcal{J}$ is substituted by the complex number $j = \sqrt{-1}$, and using the generalized Routh–Hurwitz criterion [6] (for LTI systems with complex coefficients) we get the following conditions to guarantee convergence of the observer

$$\xi_2^r > 0 \ , \quad \xi_2^r(\xi_1^i\xi_2^i + \xi_1^r\xi_2^r) > \xi_1^i\xi_1^i$$

These conditions suggest a selection of ξ_1^r which is positive, and small values for both imaginary parts (with the same sign). Moreover, as pointed out in [14] these gains are intended to place the poles of the LTI error dynamics. Therefore, as stator current dynamics is faster than stator flux dynamics, we propose to select the gain ξ_2^r larger than ξ_1^r using the ratio between their respective speed of response –in the experimental setup this ratio is about 20.

The torque and flux amplitude outputs can now be estimated as

$$\hat{y}_1 = i_s^T\mathcal{J}\hat{\lambda}_s \ \ (= \hat{\tau}) \ \ , \quad \hat{y}_2 = |\hat{\lambda}_s|^2 \ \ (= \hat{\beta}^2)$$

In the switching strategies presented above the two positive weighting factors w_1 and w_2 are used to give priority to one of the outputs over the other depending on their ratio. Thus, to give approximately the same priority to both outputs, as suggested in [11], we propose to keep similar values for the ratio between the weighting factors and the ratio between rated values of their respective outputs, that is,

$$\frac{w_1}{w_2} = \frac{y_{1,\text{rated}}}{y_{2,\text{rated}}}$$

In the case that more priority is desired for a given output, for instance if we try to mimic the behavior of standard DTC using the weighted energy function, an increase 10% in w_1 should be sufficient.

VI. Experimental results

We implemented the following strategies: *Quadratic* (9), *Absolute value* (15) and the modified version of DTC using the Quadratic criterion (subsection 3-B) in an induction motor controlled by a three–phase inverter, and the results are compared with those obtained with the classical DTC strategy. The motor parameters are: $V_s = 450$V, the number of pole pairs is $n_p = 2$, $J = 0.109$, $R_m = 0.0221 Nm \cdot sec$, $R_r = 0.3733\Omega$, $R_s = 0.3333\Omega$, $L_r = 0.0832$mH, $L_s = 0.0838$mH, $L_{sr} = 0.0795$mH. The sampling frequency is fixed to 20Khz, while the modulating frequency, i.e., the control switching frequency is set to 6.6Khz. The strategies are implemented using a PI speed regulation as an external loop. Several tests were developed under various load and reference conditions, we present here only the most relevant.

In the first experiment we study the start–up operation of the machine having a speed reference of 1500 RPM and zero load torque. Fig. 2 show the transient response of the torque for the different strategies. We observe that the proposed strategies, including the simple modification to DTC, not only show better output response time, but also reduce the chattering compared with the classical DTC strategy. Fig. 3 shows that, as a result of the faster response on torque, the proposed strategies also show faster response on the speed towards its desired reference 1500RPM. In Fig. 4 we present plots of the currents i_α vs. i_β in steady state when the machine has reached 1500RPM. In these plots we observe that the stator currents suffer a larger deformation in classical DTC compared with those in the proposed strategies.

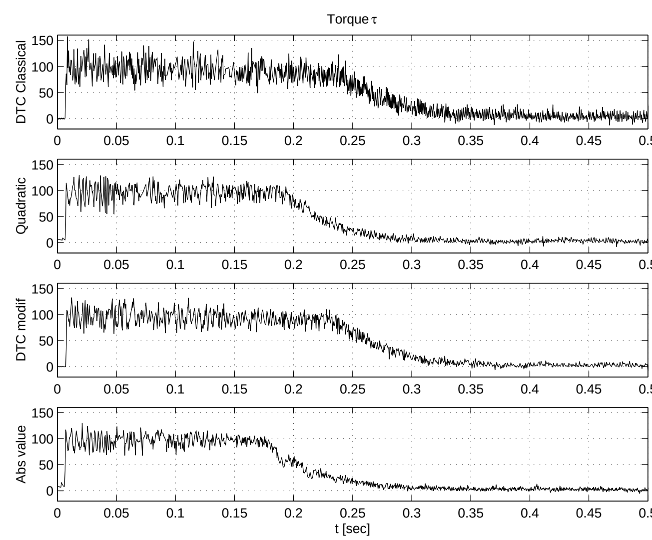

Fig. 2. Torque transient response in the start up from 0 to 1500RPM without load torque. **(From top to bottom)**: Classical DTC, Quadratic based criterion, DTC modified based on the quadratic criterion and Absolute value based criterion.

In the second test, we study the transient response due to a step change in the load torque from 0 to approximately 30 Nm once the machine has reached the speed reference 1500 RPM. For reasons of space limitation we only provide the responses of the torque. As before, Fig. 5 shows that significant chattering is observed in the DTC torque response compared with the re-

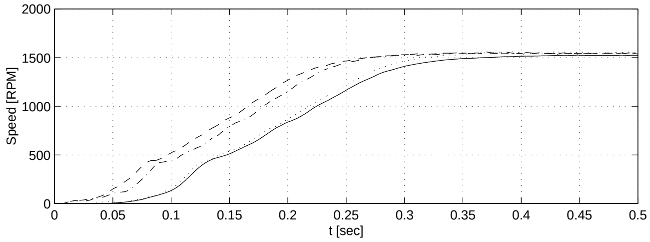

Fig. 3. Rotor speed transient response in the start up from 0 to 1500RPM without load torque: **(–)** Classical DTC, **(- -)** Quadratic based criterion, **(··)** DTC modified based on the quadratic criterion and **(-·)** Absolute value based criterion.

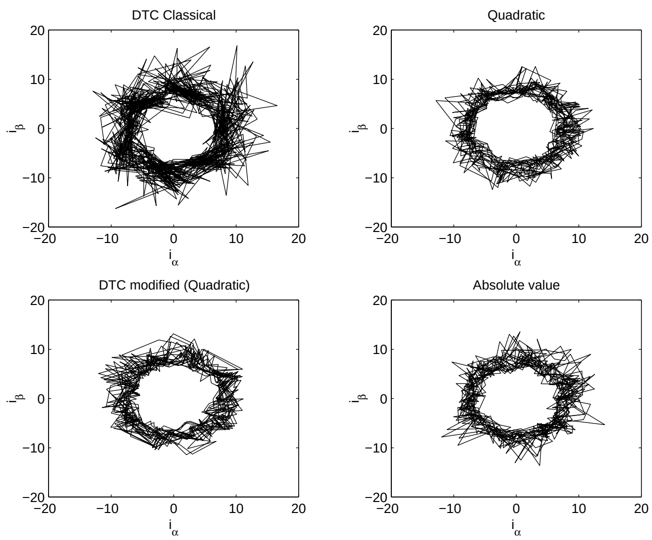

Fig. 4. Stator currents steady state response i_α vs. i_β for a non loaded condition and 1500RPM of rotor speed.

markably reduced one in the Quadratic and the Absolute value based strategies as well as in the DTC–Quadratic modification.

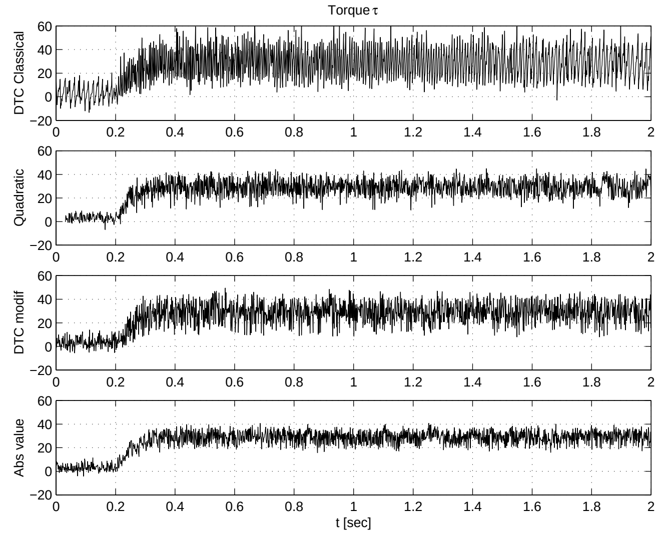

Fig. 5. Torque transient response during a load change from 0 to 20 N-m at 1500 RPM **(From top to bottom)**: Classical DTC, Quadratic based criterion, DTC modified based on the quadratic criterion and Absolute value based criterion.

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