Damping Criteria of Magnetization in Ferromagnetic Ellipsoids

N. X. Sun, *Member, IEEE,* and S. X. Wang, *Member, IEEE*

***Abstract*—The critical damping constant of the magnetization of a uniformly magnetized ferromagnetic ellipsoid can be expressed by a simple formula based upon the small signal solution of the Landau–Lifshitz–Gilbert equation. It is predicted from the damping criteria that the damping condition of the magnetization in a uniformly magnetized ferromagnetic ellipsoid can be controlled by adjusting the damping constant, shape, anisotropy field, and/or external magnetic bias field, which is confirmed experimentally in thin-film geometry.**

***Index Terms*—Damping constant, damping criteria, ferromagnetic resonance, Landau–Lifshitz, soft magnetic films.**

I. INTRODUCTION

THE Landau–Lifshitz damping constants [1] (in short damping constants) of magnetic thin films can substantially change the bandwidth of the permeability spectra of the magnetic thin films. For example, the permeability spectra bandwidth of the CoFeN films with a damping constant of 0.02 is nearly five times of that of the same film if its damping constant is increased to 0.1 [2]. Understanding the magnetization damping mechanisms and the high-frequency performance of magnetic thin films have received more and more attention recently [2]–[6]. However, it is not clear how the damping constant, magnetic body/device geometry, and anisotropy field can be engineered to achieve the optimal high-frequency performance. In this work, the damping criteria of the magnetization in a general ferromagnetic ellipsoid are developed and experimentally confirmed in thin-film geometry.

II. DAMPING CRITERIA DERIVATION

The damping criteria of a general ellipsoid are derived based upon the small signal solution to the Landau–Lifshitz–Gilbert (LLG) equation [7] that is widely used to describe magnetodynamics. The configuration of the Cartesian coordinates, the ferromagnetic ellipsoid, the net external magnetic field, and the saturation magnetization are shown in Fig. 1. Both the saturation magnetization M_s and net external field $H_{\rm net}$ are along one symmetrical axis of the ellipsoid which is parallel to $\hat{z}$ axis, and $\hat{x}$ and $\hat{y}$ axes are along the other two symmetrical axes of the ellipsoid, respectively. When the ac excitation field amplitude is small, the LLG equation can be linearized and solved analytically. The first order susceptibility of the uniformly magnetized ferromagnetic ellipsoid can be expressed by the following tensor [8]:

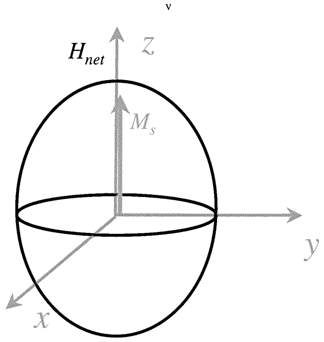

Fig. 1. Configuration of the Cartesian coordinates, ferromagnetic ellipsoid, the net external field, and the saturation magnetization direction at the static state.

$$\overleftrightarrow{\chi} = \begin{pmatrix} \chi_{xx} & -i\chi_a & 0 \\ i\chi_a & \chi_{yy} & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

in which the parameters are $\chi_{xx} = \omega_m(\omega_y + i\alpha\omega)/D$, $\chi_{yy} = \omega_m(\omega_x + i\alpha\omega)/D$, and $\chi_a = -\omega_m\omega/D$. Other relevant parameters above are $\omega_x = \omega_{\rm net} + (N_x - N_z)\omega_m$, $\omega_y = \omega_{\rm net} + (N_y - N_z)\omega_m$, $D = (\omega_x + i\alpha\omega)(\omega_y + i\alpha\omega) - \omega^2 \omega_m = \gamma\mu_0 M_s$, and $\omega_{\rm net} = \gamma\mu_0 H_{\rm net}$, with γ the gyromagnetic constant, and μ_0 the permeability in vacuum.

For an ac excitation field $\overrightarrow{h}(\omega)$ with an arbitrary direction, frequency and amplitude (if the amplitude is small enough), we can get the ac part of the magnetization in the ellipsoid as $\overrightarrow{m}(\omega) = \overleftrightarrow{\chi}(\omega) \cdot \overrightarrow{h}(\omega)$ in the small-signal limit. The critical damping condition of the magnetic ellipsoid can be obtained by the same way as that for a two-pole low pass filter [9]. Letting $s = i\omega$, the denominator of the tensor becomes $D(s) = (1 + \alpha^2)s^2 + \alpha(\omega_x + \omega_y)s + \omega_x\omega_y$. Setting the denominator $D(s) = 0$, we get the relation $\alpha^2(\omega_x + \omega_y)^2 - 4(1 + \alpha^2)\omega_x\omega_y = 0$ when the two roots of this equation are equal, i.e., $s_1 = s_2$. The critical damping constant for a uniformly magnetized ferromagnetic ellipsoid can be obtained from the above equation as

$$\alpha_{\rm crit} = \sqrt{\frac{4\left[\omega_{\rm net} + (N_x - N_z)\omega_m\right] \cdot \left[\omega_{\rm net} + (N_y - N_z)\omega_m\right]}{\left[(N_x - N_y)\omega_m\right]^2}}. \tag{1}$$

Therefore, the magnetization of a ferromagnetic ellipsoid is overdamped when its damping constant is larger than the

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TABLE I
DEMAGNETIZATION FACTORS AND THE CORRESPONDING CRITICAL DAMPING CONSTANTS FOR THE MAGNETIZATION IN SELECTED SHAPES

Shape	Demag. factors			Critical damping constant, $\alpha_{critical}$
	N_x	N_y	N_z	
Spheroid	N_x	N_x	N_z	∞
	N_x	N_y	N_x	$\sqrt{\dfrac{4H_{net}[H_{net}+(N_y-N_x)M_s]}{[(N_y-N_x)M_s]^2}}$
Thin film	0	0	1	∞
	0	1	0	$\sqrt{4H_{net}(H_{net}+M_s)}/M_s \approx 2/\sqrt{\chi_0}$
Long rod	1/2	1/2	0	∞
	1/2	0	1/2	$\sqrt{4H_{net}(H_{net}-M_s/2)}/(M_s/2)$
Sphere	1/3	1/3	1/3	∞

above critical damping constant $\alpha_{\rm crit}$; critically damped when the damping constant is equal to the critical damping constant; and underdamped when the damping constant is less than the critical damping constant.

Magnetization damping criteria of the selected shapes are obtained from the critical damping constant for a general ellipsoid and shown in Table I. For thin magnetic films with an in-plane net magnetic field $H_{\rm net}$, the critical damping constant for the magnetization is $\alpha_{\rm crit} = \sqrt{4H_{\rm net}(H_{\rm net}+M_s)}/M_s$, which can be approximated as $2/\sqrt{\chi_0}$ when the net in-plane magnetic field is much less than the saturation magnetization [2] that is a common situation involving soft magnetic metal films.

As shown in Table I, the critical damping constant $\alpha_{\rm crit}$ is ∞, when magnetization is along one of the axes of revolution symmetry of the spheroid. According to the mathematical definition of damping condition, if $\alpha_{\rm crit} = \infty$, the magnetization is always underdamped. To show how the damping constants can affect the magnetization in a magnetic sphere, which is always underdamped according to Table I, the LLG equation is solved in the small signal limit to get the impulse responses of the magnetization of a magnetic sphere with a typical net field of 10 Oe. These magnetization impulse responses are obtained and shown in Fig. 2 with different damping constants, 0.1, 1, and 10. Mathematically these curves are all underdamped. However, the physical appearance of the magnetization in time domain, when the damping constant is about ten or above, does not show clear ringing, which is the fingerprint of an underdamped signal. Clearly, the physical meaning of the always underdamped magnetization should be interpreted as that an unusually large damping constant, sometimes maybe unphysical for small signal processes (e.g., ~10 or above in the sphere case), is needed to achieve a time domain magnetization curve which looks like critically/overdamped.

III. EXPERIMENTAL VERIFICATION OF THE MAGNETIZATION DAMPING CRITERIA

The damping criteria of magnetization are tested experimentally with a soft magnetic NiFe/FeCoN film, which is a 100-nm-thick FeCoN film with a 2.5-nm-thick $\rm Ni_{80}Fe_{20}$ (wt%) underlayer. This NiFe/FeCoN film shows a high saturation magnetization of ~2.4 T (24 kG), a low coercivity of <80 A/m (1 Oe), a *static* anisotropy field (the anisotropy field measured with a B–H looper) of 1600 A/m (20 Oe), a *dynamic* anisotropy field of 1280 A/m (16 Oe) (the anisotropy field extracted from the ferromagnetic resonance frequency when the magnetic film is longitudinally biased) [2], [10] [16]. The NiFe/FeCoN film is studied by the pulse inductive microwave magnetometer (PIMM) [3] with different dc longitudinal or transverse bias fields and step ac magnetic field perpendicular to the bias magnetic field. The amplitude of the step magnetic field for the PIMM analyses is chosen to be about 200 A/m (2.5 Oe), which is much smaller than the anisotropy field of 1600 A/m [10] [16]. The magnetization oscillation of the longitudinally biased NiFe/FeCoN films is shown in Fig. 3 with different longitudinal bias fields. The damping constants for the longitudinally biased FeCoN films are in the range of 0.01 ~ 0.03, well below the critical damping constant for the FeCoN, $2/\sqrt{\chi_0} = 0.058$ [2], indicating an underdamped magnetization which is consistent with the magnetization ringing shown in Fig. 3.

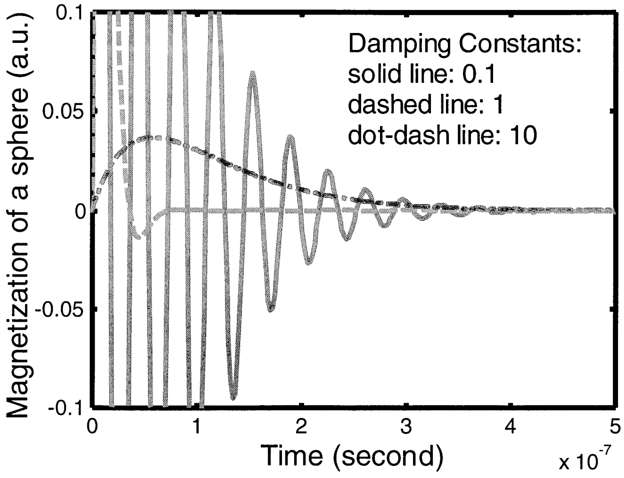

Fig. 2. Typical time domain magnetization of a uniformly magnetized sphere with a net field of 10 Oe with different damping constants of 0.1, 1.0, and 10, as indicated in the figure.

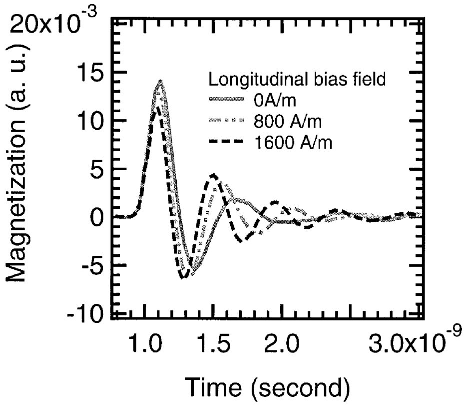

Fig. 3. PIMM signals showing magnetization oscillation of the NiFe/FeCoN film under different longitudinal bias fields, as indicated in the figure.

The magnetization dynamics in the nanosecond range has been tested in thin-film geometry with different alloy compo-

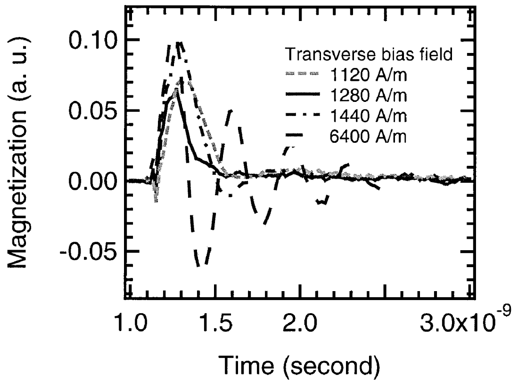

Fig. 4. PIMM signals showing magnetization oscillation of the NiFe/FeCoN film under different transverse bias fields, as indicated in the figure.

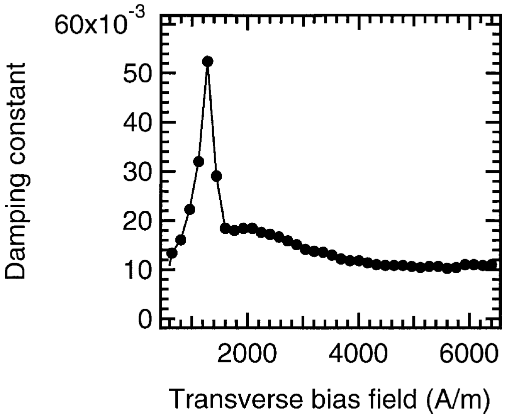

Fig. 5. Damping constants extracted from the PIMM signals of the NiFe/FeCoN film under different transverse bias fields.

sitions, such as Permalloy [3], [11] [17], FeTaN [4], Tb doped Permalloy [6], FeCoN, and NiFe/FeCoN [2]. All these thin films show the clear ringing of the magnetization as a function of time, indicating an underdamped behavior except the Permalloy doped with a certain amount of Tb [6]. Near critically damped magnetization (a slight magnetization ringing is visible) is observed in time domain in 2 at% Tb doped Permalloy with a damping constant of 0.06 [6]. The critical damping constant for the 2 at% Tb doped Permalloy is $2/\sqrt{\chi_0} = 0.063$ with $B_s = 1$ T and an $H_k = 800$ A/m (10 Oe), a little higher than the real damping constant. The 4 at% Tb doped Permalloy shows completely overdamped behavior (no ringing at all) with a damping constant of 0.09 [6]. The critical damping constant for the 4 at% Tb doped Permalloy is 0.075 with a $B_s = 1$ T and an $H_k = 1140$ A/m (14.2 Oe). All these experimental results are consistent with the magnetization damping criteria we proposed for thin-film geometry.

The damping constants for magnetic thin-film materials are generally observed to *decrease* with the increase of net in-plane field, in the Permalloy [3], NiFeMo alloy [5], and FeTaN [4]. This is believed to be related to the low-field effect [12]. The critical damping constant for the magnetization in thin films, however, *increases* with the net in-plane magnetic field. This opposite trend provides the possibility of tuning the magnetization damping behavior in thin films by applying an appropriate magnetic field. When a uniaxial soft magnetic film is biased under different directions, the in-plane net magnetic field can be changed as [13]

$$H_{\text{net}} = H_k + H_{\text{applied}}, \text{ when } H_{\text{applied}}//\text{easy axis};$$
$$H_{\text{net}} = \frac{\left(H_k^2 - H_{\text{applied}}^2\right)}{H_k}, \text{ when } H_k > H_{\text{applied}}//\text{hard axis};$$
$$H_{\text{net}} = H_{\text{applied}} - H_k, \text{ when } H_k < H_{\text{applied}}//\text{hard axis}.$$

Since most of the magnetic thin films show underdamped behavior, we can reduce the net in-plane magnetic field by applying a transverse magnetic field to have a reduced *critical damping field* and an increased damping constant to achieve an under, critical, or even overdamped behavior.

The transversely biased magnetization versus time curves for the NiFe/FeCoN film are shown in Fig. 4 for various transverse bias fields. When the NiFe/FeCoN film is biased with magnetic fields of 1120 and 1440 A/m along the hard axis direction, which are close to the dynamic anisotropy field of 1280 A/m and the net magnetic in-plane field is low, the magnetization is nearly critically damped with slight oscillation. When the transverse bias field is 1280 A/m, which is equal to the dynamic anisotropy field of the film, the magnetization oscillation is no longer seen, indicating over or critically damped behavior. The damping constants for the transverse biased cases are extracted out by using the same algorithm as before [2], and shown in Fig. 5. Clearly the damping constant is the highest at an external bias field of 1280 A/m along the hard axis direction, when the external field compensates the anisotropy field of the film. We should note that when the transverse bias field compensates the anisotropy field of the film, the net in-plane field may be small compared to the step field amplitude of 200 A/m, the damping criteria may not be applicable since the linearization of the LLG equation is no longer valid.

Uniaxial anisotropy and small signal excitations are assumed in this analysis, which are not always true in real magnetic thin films. For example, the anisotropy field of the soft magnetic NiFe/FeCoN film is not a constant at all bias fields, being 1600 A/m at low bias field and 1280 A/m at higher bias field [2]. The anisotropy field the soft magnetic Permalloy ($Ni_{80}Fe_{20}$ wt%) film can be expressed by an addition of an ideal anisotropy field that can be compensated by applying hard axis dc field and an isotropic field, which is nearly a constant in-plane [14]. This never fully compensated isotropic in-plane field, being an *isotropy field* in contrast to the conventional anisotropy field, may explain the fact that critically or overdamped magnetization is not observed in the transversely biased Permalloy film [15].

IV. SUMMARY

The damping criteria for the magnetization in a general ferromagnetic ellipsoid are derived, which are consistent with the experimental results for thin-film geometry. The magnetization damping criteria, albeit phenomenological, can be used as a guidance to achieve optimal high-frequency magnetization behavior in magnetic devices.

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