JOURNAL OF APPLIED PHYSICS **99**, 08B316 (2006)

A quantitative model for the nonlinear response of fluxgate magnetometers

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(Presented on 1 November 2005; published online 25 April 2006)

An analytical nonlinear hysteresis model was developed to describe the hysteresis loops of different magnetic materials, which was capable of fitting Permalloy films as an example. As a case study this model was applied in the sensitivity analysis of the fluxgate magnetometers where accurately capturing the nonlinear behavior of the magnetic core is crucial. Straightforward analytical equation of the output voltage signal as well as its harmonic decomposition was obtained for the fluxgate magnetometer as a function of hysteresis loop parameters. Sensitivity values predicted by the model were found to be consistent with the measured values published previously. The model allows the harmonic content distribution in the output signal of the fluxgate magnetometer to be studied analytically as a function of various device parameters, and thereby very useful in the selection of core materials in fluxgate devices. © *2006 American Institute of Physics*.
[DOI: 10.1063/1.2170061]

INTRODUCTION

Historically, analysis of the sensitivity of fluxgate magnetometers has been quite involved, and a straightforward analytical equation was not available.[1–6] Gordon *et al.* presented a fluxgate sensitivity analysis based on a simplified, linearized hysteresis model in 1965.[1] Marshall proposed a nonlinear polynomial hysteresis model.[2] Primdahl presented a theoretical analysis of the fluxgate output based on an actual hysteresis curve of a material reproduced by the means of a coordinate measuring table.[3] Burger segmented the hysteresis loop of the core into arbitrarily small linear segments over which the slope was assumed to remain constant.[4] In 2000, Deak *et al.* proposed a dynamic response model based on the Landau-Lifshitz-Gilbert equation for a single-crystal fluxgate core.[5] Detailed analysis of the fluxgate mechanism was carried out in 1970 by Primdahl.[3] It was shown that the output signal of the fluxgate magnetometer is given by the following expression:

$$V = -2NAH_{\mathrm{SIG}}\frac{d\mu_a}{dH}\frac{dH}{dt} = -2NAH_{\mathrm{SIG}}\frac{1}{(1+D\mu)^2}\frac{d\mu}{dH}\frac{dH}{dt}, \tag{1}$$

where N is the number of windings in the pickup coil, A is the cross-sectional area of the winding, H_{SIG} is the measured dc field, H is the modulation (excitation) field, D is the demagnetizing factor that takes into account the internal field of the magnetic core (discussion on magnetic core geometries and internal fields is presented in Ref. 7). As it is well known, μ as a function of H ($\mu = dB/dH$) is quite nonlinear, especially near the saturation fields of H. Hence, as suggested by Eq. (1) the response must also be nonlinear with respect to H. Our contribution has been in modeling μ as a function of H analytically over the range of H where μ behaves nonlinearly. As a result we were able to show analytically, for example, the effects of the core hysteresis squareness S on sensitivity where S is a measure of nonlinearity in μ defined as B_R/B_S (B_R is the remnant flux density and B_S is the saturation flux density).

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MAGNETIC HYSTERESIS MODELING

As implied in Eq. (1) we need to incorporate the nonlinearity in the fluxgate response via a functional dependence of μ on H near saturation. Thus, we need to model the magnetization process near saturation and then proceed in using Eq. (1) to obtain the response of the sensor. In this section, we present a model for the hysteresis curve including the nonlinear behavior and in the following section the fluxgate response is calculated for a typical case of interest.

Inspired by the "S" shape of the arctangent functions,[8] we propose the following equation to describe the hysteresis loop of magnetic materials:

$$B(H_{\mathrm{EXT}}) = \frac{2B_S}{\pi}\tan^{-1}\left(\frac{H \pm H_C}{H_T}\right), \tag{2}$$

where B_S is the saturation flux density, H is the external field, H_C is the coercive field, and H_T is a threshold field that has to be overcome in order to saturate the sample magnetically. H_T is analogous to an internal local uniaxial anisotropy field, for example, see Fig. 1.

If H_T is assumed to be constant and uniform throughout the sample, B as a function of H is plotted in Fig. 1 for a Permalloy film where $B_S = 1$ T, $H_T = 4.7$ A/m, and $H_C = 28$ A/m. In this case the fitting is performed on a dc hysteresis loop for illustration purposes. The actual ac hysteresis loop at the excitation frequency may be significantly different and the curve fitting would have to be performed at the specific excitation frequency of interest. The calculated curve is superimposed with the actual hysteresis curve of the sample measured by vibrating-sample magnetometer (VSM). In this case, the difference between the measured and the calculated curves is about 15% near saturation. While Eq. (2)

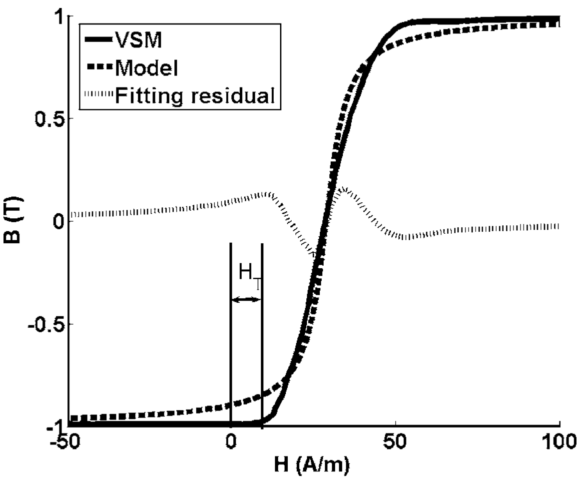

FIG. 1. Permalloy magnetization curve fitting, showing large fitting errors near saturation.

provides a simple analytical equation for modeling hysteresis loops with only one fitting parameter H_T, it suffers from inadequate fitting near saturation. The nonuniform distribution of H_T of the following form:

$$H_T = a + b\frac{\Gamma/2}{(\Gamma/2)^2 + (H - H_0)^2}, \tag{3}$$

where a, b, Γ, and H_0 are a set of fitting parameters for a scaled Lorentzian function is shown in Fig. 2. For most practical films the nonuniform distribution in H_T can be explained in terms of local variations of internal fields from site to site. The modified hysteresis model with both Eqs. (2) and (3) was applied to fit the Permalloy hysteresis as shown in Fig. 2. In this case, maximum error of 5% occurs near saturation.

MODEL OF FLUXGATE RESPONSE

The derivative of the differential permeability μ with respect to the applied field is also the second derivative of the total flux density through the core with respect to the applied field. Consequently, by computing the first two derivatives of Eq. (2) and substituting the results into Eq. (1) we can perform simulations of the fluxgate magnetometer to study how different the parameters of the design affect the ultimate sensitivity, linearity, and other specifications.

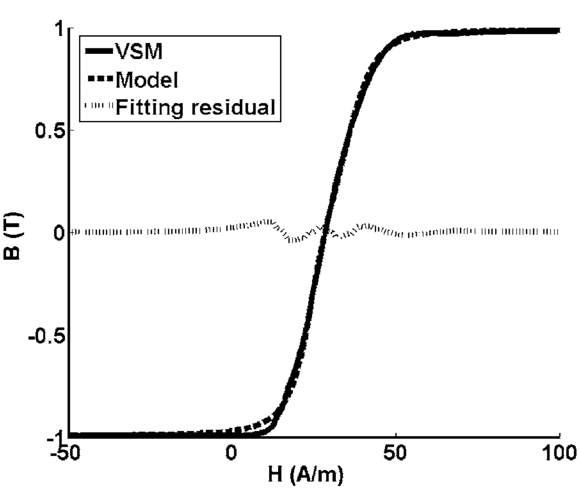

FIG. 2. Improved Permalloy magnetization curve fitting with the modified hysteresis model that incorporates both Eqs. (2) and (3).

In certain cases, such as very thin cores with in-plane easy axis, the demagnetizing term in Eq. (1) may be negligible. In such cases the output voltage equation, after second derivative of Eq. (2) is substituted, simplifies to (assuming H_T constant):

$$V(t) = 8NAH_{\mathrm{SIG}}\frac{B_S}{\pi}\frac{H_T(H \pm H_C)}{[H_T^2 + (H \pm H_C)^2]^2}\frac{dH}{dt}. \tag{4}$$

Since fluxgate magnetometers typically measure the second harmonic of the output voltage Fourier series expansion of the output voltage equation is needed. Taking the triangular modulation field waveform with amplitude H_0 and period T_0 for simplicity and initially assuming the coercive field H_C to be equal to zero the following trigonometric Fourier series representation of the output voltage signal is obtained

$$V(t) = 2NAH_{\mathrm{SIG}}\sum_{n=1}^{\infty}\frac{8\pi n B_S}{H_0 T_0}e^{-H_T|2\pi n/H_0|}\sin\left(2\pi\frac{2n}{T_0}t\right). \tag{5}$$

From this equation, second- or higher-order harmonics of the fluxgate output signal are readily obtained. Nonzero values of the coercive field H_C will result in a time shift of the output voltage signal which will in turn only affect the phase of the Fourier series expansion coefficients but not the magnitude. We can immediately confirm that the sensitivity can be increased by increasing number of windings, cross-sectional area of the core, saturation magnetization of the core, and the frequency of the modulation signal. In practice, improvements achieved by varying the parameters listed above are limited. For example, the number of pickup coil windings is limited by the self-capacitance of the pickup coil. And while the geometry of the pickup coil was not explicitly incorporated into the model its effect on overall magnetometer performance needs to be considered for every specific implementation. The sensitivity advantages drawn from increasing the cross-sectional area of the core are countered by the increasing demagnetizing field within the core. The demagnetizing field may become the prevalent factor thus limiting any further improvements in sensitivity by increasing the cross-sectional area. Equation (4) suggests that higher saturation flux density B_S results in higher sensitivity, however, such an increase will also result in increased power consumption due to additional hysteresis losses. Hence, for the low power devices lower saturation flux density may be preferred. These results were also derived in Refs. 1–4. Coercive field of the magnetic core has no influence on the amplitude of the output signal, however, lower coercive field allows for lower modulation field amplitudes. By varying the quantities of Eq. (2) we can study the effects of different parameters of the magnetic core hysteresis on the performance of the magnetometer. In particular, by keeping B_S and H_C constant and varying the distribution of the model parameter H_T we can study the relationship between the squareness of the hysteresis loop of the magnetic core and the ultimate sensitivity of the magnetometer.

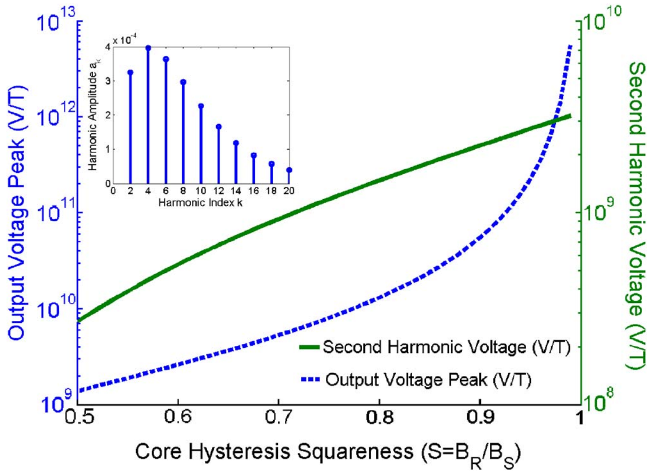

FIG. 3. Output voltage as well as second-harmonic voltage as a function of the magnetic core material hysteresis squareness. Higher-order harmonic distribution in the output voltage signal is also shown.

We have compared the predictions of our model with measured values of sensitivity in Ref. 9. Parameters of the magnetometer with a split pickup coil configuration constructed by Nielsen *et al.* (1995) were used in the simulation resulting in predicted sensitivity value of 15.4 A/T. The measured value reported by the authors was 9.5 A/T. For the four-section coil sensitivity of 9.5 A/T was predicted by our model while the measured value reported in Ref. 9 was 6.1 A/T. The discrepancy in the comparison may be due to the fact that the measurements reported in Ref. 9 were obtained for specific driving and sensing hardware which is currently not incorporated into our model.

In order to study the effects of the core hysteresis squareness on sensitivity the calculated output voltage peak was plotted versus hysteresis loop squareness in Fig. 3 for a measured flux density of 1 nT. The simulation was performed for modulation field amplitude of 60 A/m, frequency of 5 kHz, and core material parameters from Fig. 2. Different squareness values were obtained by varying the model parameter H_T. The harmonic content of the output voltage generated using hysteresis loop in Fig. 2 is shown in a separate plot within Fig. 3. Since the simulation was not restricted to particular core geometry the output voltage was normalized in terms of the number of pickup coil windings and cross-sectional area of the core and the demagnetizing factor was assumed to be zero. The second harmonic of the output voltage, typically used in fluxgate magnetometers is also shown in Fig. 3 as a function of the core hysteresis squareness.

An increase in the output voltage amplitude as well as the second-harmonic voltage amplitude of the fluxgate magnetometer is observed in Fig. 3 as the squareness of the magnetic core increases. Also, the harmonic distribution plot in Fig. 3 suggests that higher-order harmonics, such as fourth and sixth, may result in higher sensitivity than the second harmonic in this case. This may be due to the fact that increased hysteresis loop squareness, such as in Fig. 2, shifts harmonic content to higher orders. As Eq. (1) suggests, the amplitude of the output voltage signal is proportional to the second derivative of the flux density through the core with respect to the applied field, hence increased squareness will result in higher output voltage amplitude. This observation suggests that any modeling efforts in the fluxgate magnetometers should be targeted at reproducing the nonlinear behavior of the magnetic core during transition into saturation. Furthermore, as Eq. (5) suggests, the amplitudes of higher-order harmonics are closely related to the squareness of the hysteresis loop corners, suggesting that the easy axis orientation of magnetic materials with uniaxial anisotropy, such as Permalloy, is well suited for fluxgate applications. However, while high squareness results in high output volts/tesla ratio, in low-field magnetometry sensor noise may become the limiting factor. In this case, the Barkhausen noise produced by domain-wall motion in a core with easy axis orientation may be undesirable. A magnetometer core with hard axis orientation may be preferred due to lower noise levels associated with domain-wall rotation.

Theoretical sensitivity curve presented in Fig. 3 may not achievable in practice due to the limitations introduced by the noise in the magnetic core of the material as well as noise introduced by the peripheral electronics (noise mechanisms in fluxgate magnetometers are analyzed in Refs. 5 and 10). However, the relationship between the magnetic core parameters and the ultimate sensitivity is important to consider in design optimization. Equation (5) suggests that response of the magnetometer is linear over a broad range of input signals as long as $H_{\mathrm{SIG}} \ll H$ condition is satisfied.

CONCLUSIONS

An analytical nonlinear magnetic hysteresis model was developed, which is able to fit the hysteresis loops of diverse materials with a maximum error of 5%–6%. By applying the model in fluxgate sensitivity analysis a simple and straightforward output signal equation was obtained. Fourier analysis of the output signal equation was performed allowing second- or higher-order harmonics of the output signal to be calculated. The ability of the model to reproduce the nonlinearity in the response of magnetic materials is crucial in magnetic materials selection, as well as in design and optimization of magnetic devices such as fluxgate magnetometers where capturing the nonlinear behavior of the magnetic core is particularly important.

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