A Queuing Theoretic Model of Ad Hoc Wireless LANs

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Abstract

An M/MMGI/1/K queuing model is developed for the analysis of IEEE 802.11 DCF using RTS/CTS. Results are based on arbitrary contention conditions, namely, collision probabilities, transmission probabilities and contention window sizes vary arbitrarily among nodes contending for channel access. This is fundamentally different from earlier work. Results are presented for the fully-connected case and validated via simulation with statistical analysis. The main contributions are the analysis of DCF and the foundation for the sensitivity analysis. A key element of the model is that complexity normally encountered is reduced by effectively restoring the independence between service times and packet inter-arrivals.

I. Introduction

The performance of the IEEE 802.11 MAC standard for wireless LANs has been the subject of numerous analyses. However, until recently there had been little work focused on the DCF using RTS/CTS. The objective of this paper is to present a generalized queuing model for Markov arrivals; this is the first such model to be proposed and has been well validated via simulation under the range of traffic conditions.

The analysis is based upon a bounded network of M/G/1/K queues, where K represents the max queue length at each node and N represents the number of potentially interfering nodes. For multiple-hop systems N represents the number of nodes within two-hops of either a sender or receiver. It represents the effective number of interfering nodes accounting for only that portion of traffic generated by the two-hop away neighbors destined for a one-hop neighbor. An important property of the model is underscored by the earlier reference to "potentially interfering" nodes. Each of the N nodes may be in any state of a given time, namely, any node may be busy or idle, and a busy node may be in any back-off stage. In contrast to related work [1] [6] [9] [8] [5] [4], no limiting assumption are made that force uniform collision probability, uniform back-off stage distribution or saturation conditions. For the comparison reasons a new model based on [8] is proposed. The analysis assumes only Markov arrivals at each node that are pairwise and collectively independent.

An alternative characterization for the queuing model is the M/MMGI/1/K: The service times are modeled as a Markov Modulated General Independent process. In principle this system is representative of a Phase-Type (PH) service. Hence, comprehensive analysis is well adapted to matrix-geometric techniques. Thee difficulty of this approach, however, is in finding an accurate parametric description of the PH service. For steady-state analysis a general service distribution in which the service-times depend on *local* collision probabilities of the RTS/CTS frames and the distribution of the time to resolve them, which, itself is dependent on the number of busy nodes in contention for the channel.

For single-hop analysis knowledge of node distribution is necessary and sufficient to determine the aforementioned values: a two-dimensional Discrete-Time Markov-Chain (DTMC) that characterizes the back-off stages and collision probabilities associated with each node effectively modulate the general process, thus facilitating the estimation of the needed parameters.

This paper is not "yet another analysis" of IEEE 802.11 throughput and delay. The importance of the M/MMGI/1/K model presented in this paper is that it provides a foundation for the sensitivity analysis and can be extended to model arbitrary network configurations. Thus, it is the basis for a fundamentally different strategy for understanding and improving the effectiveness of practical networks. A key point and novel aspect of the model is that the complexity level normally encountered is reduced by effectively restoring the independence between service times and packet inter-arrivals through the DTMC formulation. Thus, direct stead-state analysis is possible using iterative techniques to estimate the model parameters for different traffic intensity levels. Practical future applications include routing, admission control and scheduling in ad hoc networks.

The remainder of this paper is organized as follows: Section II describes the system model which encompasses two different models for the back-off algorithm, service time distribution and the M/G/1/K formulation. Performance measures are given in section III. Section IV applies simulation and statistical analysis to validate the analytical results. Finally, Section V presents conclusions and discusses future work.

II. System Model

This section describes the basic methodology and components of the M/MMGI/1/K queuing analysis. The main parameters are identified and applied to each of the system entities—the back-off algorithm, the service time distribution and, finally, the queuing model. In the present analysis the states of each node are considered independently and then coupled through an iterative process in order to evaluate the system state. Specifically, the MAC algorithm executes at each node leaving

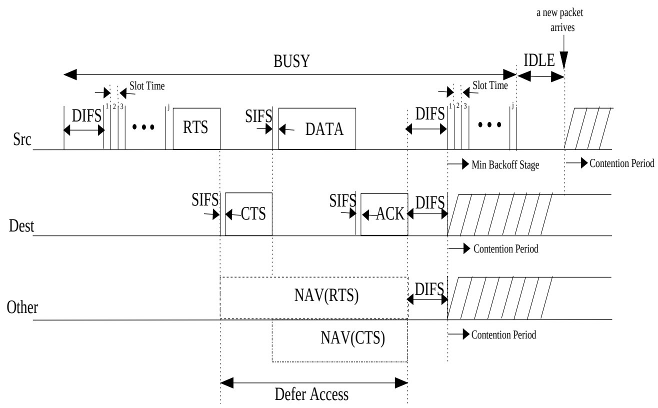

Fig. 1. IEEE 802.11 DCF RTS/CTS procedure

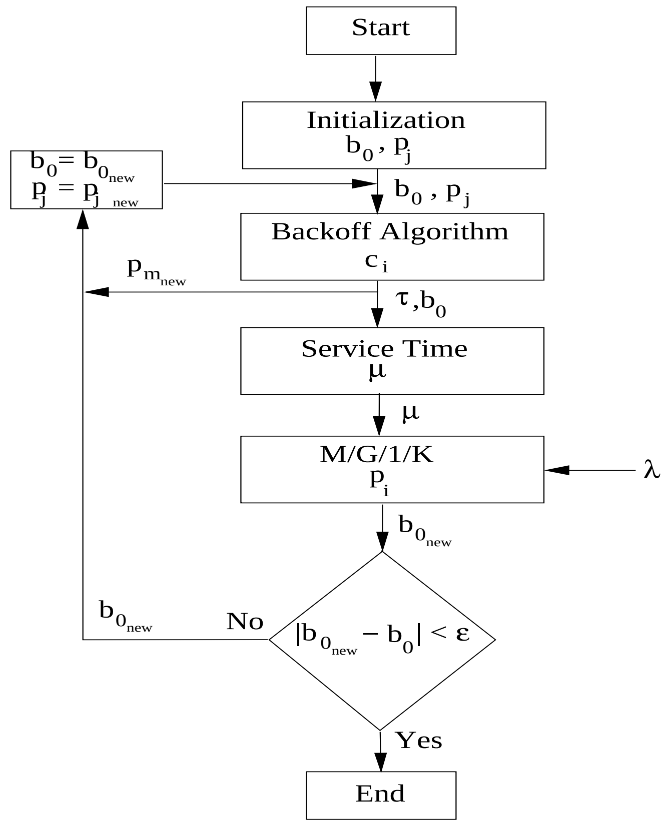

Fig. 2. IEEE 802.11 Organization

each in an arbitrary back-off stage at any time. Due to the independent and identically distributed (IID) arrival processes and assuming uniform traffic distribution and positions it is possible to evaluate the expected value of the global service time by determining the busy probability, transition probabilities and mean waiting times from the DTMC.

Figure-1 illustrates how each node alternates between busy and idle periods. During a busy period the node executes the RTS/CTS protocol, transmits its data and receives an acknowledgment. If it has multiple frames to send, it may contend and transmit more than one during the same busy period. After each transmission a node goes into back-off following DIFS; other nodes continue counting down the back-off time according to IEEE 802.11 standards [2]. Given random packet arrival times and transmission success probability, the busy nodes do not generally share a common Contention Window (CW) at a given time. Moreover, the number of busy nodes that have at least one frame to send may vary from one contention period to another. Each busy node becomes idle when there are no more frames to send. A packet is reserved in the queue on arrival if at the instant of arrival, the node is non-empty. One of the basic parameters that must be evaluated is b_0, the steady-state probability that a node is busy or approximately non-empty. The state probabilities depend on the traffic rate λ. Packet inter-arrivals are assumed to include packets originating at a given node and those routed through the node. This reflects the broader objective of applying the model to generalized wireless ad hoc networks. Based on the Markov assumption the inter-arrival times are IID with exponential distribution.

An iterative process that is dependent on the busy probabilities is used to evaluate the system behavior. Values are estimated through using an initial guess and iterative correction. The process is depicted in Figure-2 and outlined as follows:

- **Step 1:** Initialize b_0 and the probabilities that a node is in any of $m+1$ back-off stages. These must be known to determine the collision probabilities for the back-off algorithm.
- **Step 2:** Calculate the collision probabilities, c_i, for each back-off stage and the corresponding transmission attempt probability, τ.
- **Step 3:** Given c_i and τ, evaluate the packet service rate, μ, where $\frac{1}{\mu}$ is the mean packet service time.
- **Step 4:** Given μ and λ, find all the state probabilities in $M/G/1/K$ queuing system. This process will result in a new value of b_0 - the steady-state busy probability.
- **Step 5:** Repeat Steps 2, 3 and 4 until the difference between a new and previous value for b_0 is small.

The remainder of this section provides detailed explanation of the loop steps which reflect three system entities.

A. The First Modeling of Back-off Algorithm

A core contribution of this modeling is that evaluation of collision probabilities *is not limited by the assumptions* that packets collide with constant equal probability and collisions are pairwise and collectively independent over all transmission attempts [1],[6]. A more accurate characterization is invoked reflecting random variation of back-off stage among nodes and correlation between back-off time and back-off stage which depends on the time the number of collisions already experienced during a given packet transmission attempt. The generalized analysis of collision probability is achieved through a sequential process of conditioning and applying total probability.

The first step is to evaluate the conditional probabilities for two busy nodes in each back-off stage. $\alpha_{ij,k}$ is the conditional probability that the transmission from node 1 results in a collision given node 1 is in stage i and node 2 in stage j, node 1 selects slot k, and there are only two nodes in the system. For notational convenience $k \in (0, (W_i - 1))$, where W_i is the stage i contention window size. $\alpha_{00,k}$ is first considered. The following must hold: $\alpha_{00,1} = 1/W_0$ since if node 1 picks

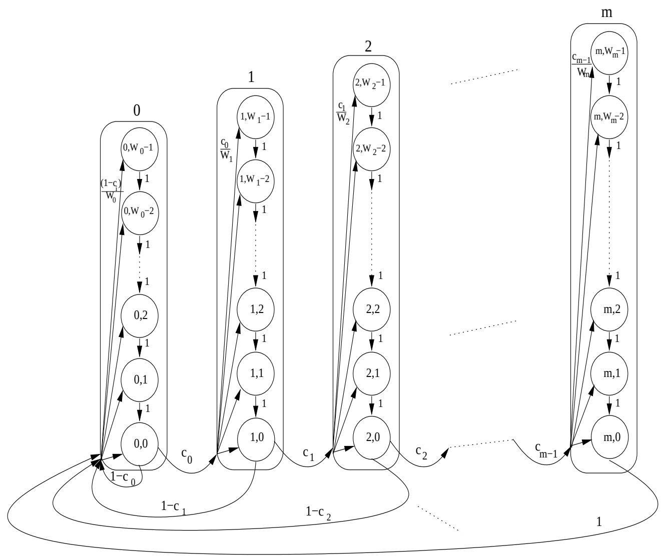

Fig. 3. DTMC model for back-off window size

slot 1, node 2 must also pick slot 1 for a collision to occur. If node 1 picks slot 2, then there are two scenarios that result in a collision: (1) node 2 also selects slot 2, or (2) node 2 first picks slot 1 after which it becomes busy again and chooses slot 1—corresponding to the same time as node 1's initial occupancy of slot 2. Here it is assumed that any transmission attempt does not affect the busy probability, b_0. The conditional collision probability is given by the following:

$$\begin{aligned}\alpha_{00,2} &= \frac{1}{W_0} + \frac{1}{W_0} b_0 \frac{1}{W_0} \\ &= \frac{1}{W_0} + b_0 \frac{1}{W_0^2}\end{aligned}$$

A clear pattern emerges by continuing in this way, thus resulting in the following general expression for $\alpha_{00,k}$:

$$\alpha_{00,k} = \frac{1}{W_0} \sum_{m=0}^{k-1} \binom{k-1}{m} \left(\frac{b_0}{W_0}\right)^m \quad 1 \le k \le W_0 - 1$$

Given an arbitrary back-off stage a node selects a transmission slot randomly from a uniform distribution covering the window size of the stage. Let p_{s_k} be the uniformly distributed random variable that determines the probability that node 1 chooses slot k. Applying the theorem of total probability the first condition is eliminated: $c_{00,2}$ is the probability of a collision between two nodes in stage 0:

$$\begin{aligned}c_{00,2} &= \sum_{k=1}^{W_0} \alpha_{00,k} p_{s_k} \\ &= \sum_{k=1}^{W_0} \frac{1}{W_0} \sum_{l=0}^{k-1} \binom{k-1}{l} \left(\frac{b_0}{W_0}\right)^l \frac{1}{W_0} \\ &= \frac{1}{W_0^2} \sum_{k=1}^{W_0} \sum_{l=0}^{k-1} \binom{k-1}{l} \left(\frac{b_0}{W_0}\right)^l \qquad (1)\end{aligned}$$

Consider the case in which $j = 0$. Let node 1 be in stage 1 with back-off window size 64 and let node 2 be in stage 0 with window size 32. A collision occurs only if both nodes select a slot in the first half of stage 1 which is equivalent to stage 0. Hence, if node 1 selects a slot from the first half of stage 1 the collision probability is equivalent to $c_{00,2}$. This is the probability given by Equation-1. If, however, node 1 picks a slot from the second half of stage 1, then a collision can occur only if node 2 becomes busy. The corresponding collision probability is $c_{00,2}$. The total collision probability for $j = 0$ for the example is:

$$c_{10,2} = \frac{1}{2} c_{00,2} + \frac{1}{2} b_0 c_{00,2}$$

Following with the logic the result $c_{20,2}$ is given by the following, where node 1 is in stage 2, thus, has a back-off window size of 128:

$$\begin{aligned}c_{20,2} &= \frac{1}{4} c_{00,2} + \frac{1}{4} b_0 c_{00,2} + \frac{1}{4} b_0^2 c_{00,2} + \frac{1}{4} b_0^3 c_{00,2} \\ &= \frac{c_{00,2}}{4} (1 + b_0 + b_0^2 + b_0^3)\end{aligned}$$

A pattern emerges that gives the general result for collision probability $c_{i0,2}$:

$$c_{i0,2} = \frac{c_{00,2}}{2^{(i)}} \sum_{k=0}^{(2^i - 1)} b_0^k \qquad (2)$$

To complete the resolution of the first condition consider the case in which $i = 0$. Let node 1 be in stage 0 and let node 2 be in stage j. Node 2 picks a slot and if there is no collision, node 2 goes to stage 0 and chooses a slot from stage 0 until collision occurs. In other words, it is assumed that a node that is at stage j starts from the stage 0 after any number of its subsequent successful attempts. Except for picking the first slot at stage j, the rest is same to $c_{00,2}$. With the help of Equation-(1), the total collision probability for $i = 0$ is:

$$\begin{aligned}c_{0j,2} &= \frac{1}{W_0 W_j} \sum_{k=1}^{W_0} \sum_{l=0}^{k-1} \binom{k-1}{l} \left(\frac{b_0}{W_0}\right)^l \\ &= \frac{W_0}{W_j} c_{00,2} \qquad (3)\end{aligned}$$

Following with the same logic and using Equation-(2) and (3), the general result is given by:

$$c_{ij,2} = \frac{W_0}{W_j} \frac{c_{00,2}}{2^{(i)}} \sum_{k=0}^{2^i - 1} b_0^k \qquad (4)$$

Given the steady-state occupancy probabilities p_j, $j \in (0, m)$ for the $m+1$ back-off stages total probability is used to remove the condition on node 2 resulting in the collision probability $c_{i,2}$ conditioned on node 1 selecting stage i and a second busy node (node 2):

$$c_{i,2} = \sum_{j=0}^{m} c_{ij,2} p_j \qquad (5)$$

Assuming that the probabilities of collisions between different node pairs are independent, then the collision probability $c_{i,n}$ conditioned on node 1 selecting stage i and n busy nodes is given by the following, where N is the total number of competing nodes:

$$c_{i,n} = 1 - (1 - c_{i,2})^{(n-1)} \quad 2 \leq n \leq N \tag{6}$$

The next step is to find the transmission attempt probability, τ_n, for an arbitrary node given n nodes are busy. Let $b(t)$ and $s(t)$ be continuous time stochastic processes representing the back-off time counter and back-off stage associated with an arbitrary node at time t. Taken together $(b(t), s(t))$ is a two dimensional process. Figure-3 relates the values of c_i to the individual node back-off window sizes that are determined by this two dimensional process. Based on the independence between nodes and the exponentially increasing back-off times a DTMC formulation similar to the one first proposed in [1] and later modified in [6] is used to model the process. Referring to Figure-3 if $b_{i,k,n} = \lim_{t\to\infty} P\{s(t) = i, b(t) = k, \texttt{n busy nodes}\}$, where $i \in [0, m]$ and $k \in [0, W_i - 1]$ is the stationary distribution of the Markov chain, then all probabilities $b_{i,k,n}$ can be found using standard Markov analysis.

$$\begin{array}{ll} b_{i,0,n} = c_{i-1} \cdot b_{i-1,0,n} & 0 < i \leq m \\ b_{i,0,n} = \prod_{k=0}^{i-1} c_{k,n} \cdot b_{0,0,n} & 0 < i \leq m \end{array} \tag{7}$$

Equation-7 reflects the condition that a node proceeds to the next back-off stage only if there is a collision in the current stage and the collision probabilities are independent from one stage to the next. The suggested node short retry count $m = 7$ according to the standard [2]. Here m is equivalent to the maximum back-off stage. The MC is irreducible and ergodic, for each $k \in [0, W_i - 1]$ express $b_{i,k,n}$ as follows:

$$b_{i,k,n} = \frac{W_i - k}{W_i} \cdot b_{i,0,n} \quad 0 < i \leq m \tag{8}$$

Equations-(7) and (8) express all $b_{i,k,n}$ values as functions of $b_{0,0,n}$ and of collision probabilities $c_{i,n}$. The normalization condition leads to the following:

$$\begin{aligned} 1 &= \sum_{k=0}^{W_i-1} \sum_{i=0}^{m} b_{i,k,n} \\ &= \sum_{i=0}^{m} b_{i,0,n} \sum_{k=0}^{W_i-1} \frac{W_i - k}{W_i} = \sum_{i=0}^{m} b_{i,0,n} \frac{W_i + 1}{2} \end{aligned} \tag{9}$$

Combining Equation-7 with the normalization condition and applying algebraic manipulation results in the following expression for $b_{0,0,n}$:

$$b_{0,0,n} = \frac{1}{\sum_{i=0}^{m} (\prod_{k=0}^{i-1} c_{k,n}) \frac{W_i+1}{2}} \tag{10}$$

Each node attempts to transmit when its back-off counter reaches zero. The transmission probability τ_n can be found by considering the summation of the total back-off stage probabilities weighted by the collision probabilities. Hence, the probability that a node transmits a packet in a randomly chosen slot is:

$$\tau_n = \sum_{i=0}^{m} b_{i,0,n} = \sum_{i=0}^{m} (\prod_{k=0}^{i-1} c_{k,n}) \cdot b_{0,0,n} \tag{11}$$

Similarly, the steady-state occupancy probabilities $p_{j,n}$ for each back-off stage j are readily determined:

$$\begin{aligned} p_{j,n} &= \sum_{k=0}^{W_j-1} b_{j,k,n} = b_{j,0,n} \cdot \frac{W_j + 1}{2} \\ &= (\prod_{k=0}^{j-1} c_{k,n}) \cdot \frac{W_j + 1}{2} \cdot b_{0,0,n} \end{aligned}$$

Given the individual node busy probability, b_0, the probability that n nodes are busy, β_n, is a random variable with a binomial distribution. Thus, the steady-state occupancy probabilities p_j, conditioned on being in stage j is then determined by total probability:

$$\beta_n = \binom{N}{n} b_0^n (1 - b_0)^{N-n} \tag{12}$$

$$p_j = \sum_{n=2}^{N} p_{j,n} \beta_n \tag{13}$$

B. The Second Modeling of Back-off Algorithm

The following model is proposed in order to compare it with the first model and the simulation results. Unlike the first modeling, the collision probability is limited by the assumption that packets collide with constant equal probability, c. This modeling is based on the analytical work in paper [8]. In [8], the collision probability c was derived for the saturated network case in which every node always has a packet to transmit at any given time. In this paper, it is extended to model the general case by obtaining an approximate expression for collision probabilities. In [9], the model in [8] is also extended for the general case. G/G/1 queuing model where the queue length is not involved is considered in [9] but M/G/1/K is considered in this paper to take the queue length into consideration. Evaluation of collision probabilities is based on the node model in this modeling whereas in [9] it is based on the system model which consists of N node model. Unlike that in [9], an iterative process in Figure-2 is used in order to get more accurate estimates of the collision probabilities in a similar way the first modeling of back-off algorithm uses. In section-IV, both models are compared with the simulation results.

With probability c the transmission is collided and with probability $1 - c$, it is successful. Hence, the number of transmissions per packet is modeled as geometrically distributed with probability of success $1 - c$. The average back-off window in the saturated case is given by [8]:

$$\begin{aligned} \overline{W} &= (1-c)\frac{W_0}{2} + c(1-c)\frac{2W_0}{2} + \cdots + \\ &\quad c^m(1-c)\frac{2^m W_0}{2} + c^{m+1}\frac{2^m W_0}{2} \\ &= \frac{1 - c - c(2c)^m}{1 - 2c} \frac{W_0}{2} \end{aligned} \tag{14}$$

Now consider a network with n busy nodes. A packet is reserved in the queue on arrival if the node is non-empty. It is said before that the probability that the node is busy or non-empty (node utilization factor) when an arbitrary arrival occurs is b_0. Hence, for any arbitrary packet, with probability $1-b_0$, the back-off window is 0 and with probability b_0, it is reserved in the queue. Because the average back-off window size is $\overline{W}$, the probability that a node attempts a transmission in an arbitrary slot is given by $b_0/\overline{W}$.

Following the arguments of [8] and considering the fact that only busy nodes can actually collide with packets from other busy nodes and conditioned on n busy nodes, the conditional collision probability is given by:

$$c_n = 1 - \left(1 - b_0 \frac{1-2c_n}{1-c_n-c_n(2c_n)^m}\frac{2}{W_0}\right)^{n-1} \quad (15)$$

Note that all other parameters can be easily found by replacing $c_{i,n}$ by c_n since all c_i are assumed to be same in the second modeling.

C. The Service Time Distribution

In the next part of the analysis it is necessary to find the the distribution of the back-off window size. Here m is the maximum back-off stage and can have a value larger or smaller than M. The back-off window size random variable W_i is uniformly distributed at each stage:

$$W_i \sim \begin{cases} U(0,(2^iW-1)) & 0 \le i \le M \\ U(0,(2^MW-1)) & i > M \end{cases} \quad (16)$$

where W is the initial back-off window size and $U(a,b)$ shows the uniform distribution between a and b. The aggregate back-off window size random variable W_n depends on the number of busy nodes n and the collision probability $c_{i,n}$:

$$\mathcal{W}_n = \sum_{i=0}^{m}\left(\prod_{k=0}^{i-1} c_{k,n}\right) W_i \quad (17)$$

Consider the following probabilities and random variables as given in [1]: P_{tr} is defined as the probability that at least one transmission occurs in a given slot time. Since n stations contend to access the medium and each station transmits with probability τ_n, $P_{tr,n}$ is given by:

$$P_{tr,n} = 1-(1-\tau_n)^n$$

The probability P_s that an occurring transmission is successful is given by the probability that a station is transmitting and the remaining $n-1$ stations remain silent, conditioned on the fact that at least one station transmits:

$$P_{s,n} = \frac{n\tau_n(1-\tau_n)^{n-1}}{1-(1-\tau_n)^n}$$

T_s is the average time that the medium is sensed busy due a successful transmission and T_c is the average time that the medium is sensed busy by each station when a collision occurs and σ is the duration of an empty slot. The values of T_s and T_c depend on the channel access mechanism. Assuming that all stations use the same channel access mechanism, T_s and T_c are defined as follows, assuming the RTS/CTS access mechanism is employed:

$$\begin{aligned} T_s &= DIFS + RTS + SIFS + CTS + SIFS + \\ &\quad H + E[P] + SIFS + ACK + \sigma \\ T_c &= DIFS + SIFS + RTS + CTS \end{aligned}$$

Where $H = MAC_{hdr} + PHY_{hdr}$, and $E[P]$ is the average length of the frame. Let $T_{slot,n} = (1-P_{tr})\sigma + P_{tr}P_sT_s + P_{tr}(1-P_s)T_c$. Note that collision only occurs between RTS frames and T_c is different from that in paper [1] because the CTS timeout effect is considered.

The conditional service time for an arbitrary frame given n busy nodes is determined based on the parametric model of 802.11:

$$B_n = T_s + (\mathcal{W}_n T_{slot,n})$$

The unconditional service time is found by combining this result with the binomial distribution for the probability of the number of busy nodes given in Equation-12–the mean value of service time b is used to solve the queuing problem in the next section:

$$B = \sum_{n=1}^{N} B_n \cdot \beta_n \quad (18)$$

Note that while a packet arrives to an idle node at a moment where other nodes have non-empty queues but are in back-off, it will be transmitted right away if the channel is idle for DIFS time. In the analysis, the occurrence of such cases are disregarded since this possibility is very small. As verified in section-IV, this consideration results in reasonably close results.

D. M/MMGI/1/K Queuing model

Let λ be the packet arrival rate and denote by $B(x)$ and b the distribution and expected value of packet service time respectively (see Equation-18). The maximum number of packets that can be accommodated at any node in the system (including the one in service) at any time is given by $K < \infty$. Those packets that arrive when K packets are already present are dropped. The initial throughput γ for an arbitrary node can be expressed in terms of the probability of packet loss, which is equivalent to the probability of an arrival find K in the system. Since the arrival process is Poisson this is equivalent to the time average of finding K in the system: P_k.

$$\gamma = \lambda(1-P_K) \quad (19)$$

The traffic intensity or *offered load* is defined as ρ:

$$\rho = \lambda b = \frac{\lambda}{\mu} \quad (20)$$

The offered load is distinguish from the carried load, which is defined as ρ' and accounts for those packets lost due to buffer overrun. Thus, it represents the fraction of time the server is busy. It can also be viewed as is the probability that a server is busy at an arbitrary time; this probability is equivalent to the node busy probability, b_0, from the previous section:

$$\rho' = \gamma b = \rho(1-P_K) \quad (21)$$

Let P_k be the probability that there are k messages in the system at an arbitrary time, where $k = 0, 1, 2, \ldots, K$. From [3] and [7] P_k can be evaluated in terms of the steady-state probabilities. Moreover, this leads to an expression for the value of P_K:

$$P_k = \frac{\pi_k}{\pi_0 + \rho} \quad 0 \le k \le K-1 \tag{22}$$

$$P_K = 1 - \frac{1}{\pi_0 + \rho} \tag{23}$$

Substituting Equation-23 into Equation-21 gives the following expression for the busy probability or node utilization factor:

$$\rho' = \frac{\rho}{\pi_0 + \rho} = b_0 \tag{24}$$

III. Performance

The system performance is measured by three metrics which are MAC delay, end-to-end delay and throughput.

MAC Delay: The average MAC delay for a successfully transmitted packet is defined to be the time interval from the time the packet is at the head of its MAC queue ready to be transmitted, until an acknowledgment for this packet is received. If a packet is dropped because it has reached the specified retry limit, the MAC delay time for this packet will not be included into the calculation of the average MAC delay. The average packet MAC delay, provided that this packet is not discarded, is obtained easily by:

$$E[D|n] = T_s + \frac{\sum_{i=0}^{m}\left(\prod_{k=0}^{i-1} c_{k,n} - \prod_{k=0}^{m} c_{k,n}\right)\frac{W_i}{2}}{1 - \prod_{k=0}^{m} c_{k,n}}$$

$$E[D] = \sum_{n=1}^{N} E[D|n]\beta_n \tag{25}$$

where $(1 - \prod_{k=0}^{m} c_{k,n})$ is the probability that the packet is not dropped and $\frac{\prod_{k=0}^{i-1} c_{k,n} - \prod_{k=0}^{m} c_{k,n}}{1-\prod_{k=0}^{m} c_{k,n}}$ is the probability that a packet that is not dropped reaches the i stage.

Delay: Delay is the time that data packet takes to travel from the sender to the receiver. Finding the mean number in the system, $E[L]$, is straight forward:

$$E[L] = \sum_{k=1}^{K} kP_k = \frac{\sum_{k=1}^{K-1} k\pi_k}{\pi_0 + \rho} + K\left(1 - \frac{1}{\pi_0 + \rho}\right) \tag{26}$$

Based on Little's Law and [7] the mean delay is given by:

$$E[T] = \frac{1}{\lambda}\left[\sum_{k=1}^{K-1} k\pi_k + K(\pi_0 + \rho - 1)\right] \tag{27}$$

Throughput: A packet loss can be defined as a packet that never reaches its destination, or a packet that arrives too late and thus cannot be used to play out the content. It is directly related to throughput. The initial throughput was given by (19). From the back-off algorithm if a packet collides m times it is dropped. Hence, the *adjusted* throughput Γ is given by:

$$\Gamma_n = \frac{\lambda}{\pi_0 + \rho} \cdot \left(1 - \prod_{i=0}^{m} c_{i,n}\right)$$

$$\Gamma = \sum_{n=1}^{N} \Gamma_n \beta_n \tag{28}$$

802.11 Parameters	Values
PYH Layer Specification	DSSS
Channel Transmission Rate	1Mbits/sec
CW_{min}	32
CW_{max}	1024
m	4
σ	20μs
H	6Bytes
DIFS	50μs
SIFS	10μs
RTS	44Bytes
CTS	38Bytes
ACK	38Bytes

TABLE I

IEEE 802.11 System Parameters Values

IV. Model Validation and Results

The theoretical analysis presented in this paper was validated using the OPNET simulator. Space limitation permit a small sample of results. For each set of experiments the number of nodes and packet lengths were fixed. The model assumed an ideal channel and fully connected network, which matched the assumptions of the analytical model. The system parameter values are given in the IEEE 802.11 MAC layer implementation in OPNET, given in table-I.

The comparison between the analytical and simulation result of MAC delay for 1024, 512, 256 and 128 bytes packet sizes where the number of node is 10 is shown in Figure-4, taking into consideration of the same traffic load. In the figures, the first modeling of back-off algorithm is represented by 'Analysis1' and the second one by 'Analysis2'. Figure-5 also shows that the simulation validates the analysis under the full spectrum of traffic load. Each simulation point represents the average of 10 independent steady-state replications. Figure-5 compares the analytical and simulation results in terms of delay with respect to the traffic load. Figure-6 shows the comparison of the analytical and simulation throughput results versus the traffic load. It is observed how the packet size affects the delay, MAC delay and throughput. Figure-7 shows the comparison of the analytical and simulation results for the collision probabilities versus the traffic load for the case of $m = 4$. It can be observed from figures that the first analysis model performs better than the second analysis model in the transition part while the second is better than the first in the saturation part of the traffic load. Both analytical results match the simulation result very closely.

V. Conclusion

This paper represents an important advance in the analysis and design of effective ad hoc networks. It also provides an significant contribution to the performance analysis of IEEE 802.11 wireless LANs. A novel queuing theoretic model based on the M/MMGI/1/K queue and parametric service model for IEEE 802.11 DCF using RTS/CTS was solved. The results are based on the single-hop case. The applications for the type of queuing model developed in this work include routing,

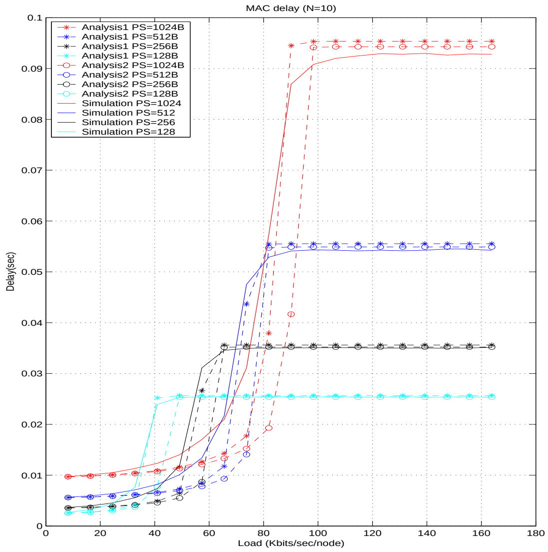

Fig. 4. MAC Delay vs Load

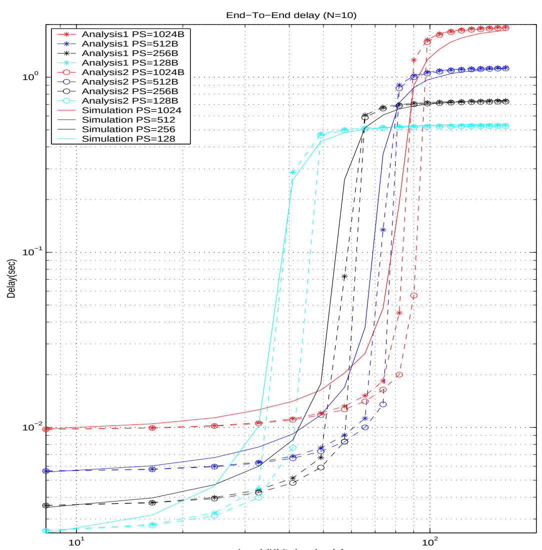

Fig. 5. Delay vs Load

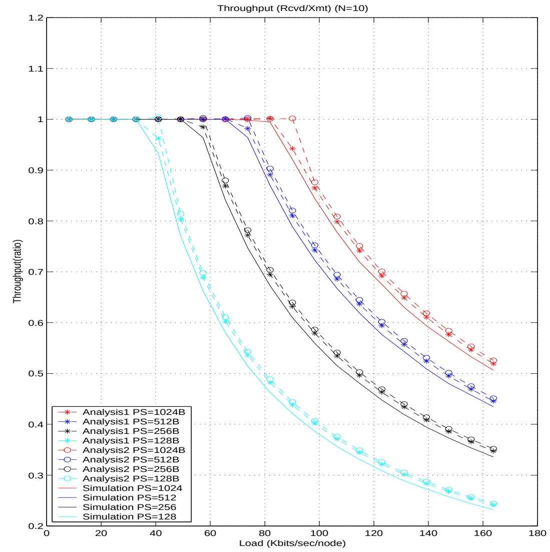

Fig. 6. Throughput vs Load

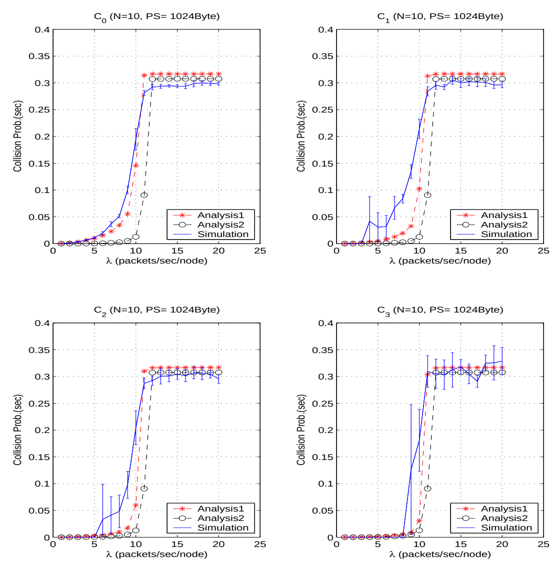

Fig. 7. Collision Probabilities vs Load

admission control and scheduling. To the best of the authors' knowledge this is the first comprehensive queuing analysis of IEEE 802.11 DCF using RTS/CTS that has been well-validated and published in the literature.

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