

2.35 Probability of success.

Suppose that event A has the probability p of at least one occurrence in N independent trials. Find

a) The probability of event A on a single trial.

$$\begin{aligned} P(\text{at least one}) &= 1 - P(\text{none}) \\ &= 1 - \binom{N}{0} p^0 (1-p)^N \\ p &= 1 - (1-p)^N \end{aligned}$$

$$\boxed{P(\text{event A}) = 1 - (1-p)^{1/N}}$$

b) The probability that A occurs at most once.

notes The probability that it occurs at most once is the same as saying that it does not occur at all or occurs just once.

$$\therefore P(\text{none}) + P(\text{once}) = (1-p) + \binom{N}{1} p (1-p)$$

$$\text{answer} = (1-p) + \binom{N}{1} p (1-p)$$

c) The probability of event A on a single trial if $p = 79/81$ and $N = 4$

$$\begin{aligned} P(\text{event A}) &= 1 - (1-p)^{1/N} \\ &= 1 - (1 - 79/81)^{1/4} = \boxed{0.602} \end{aligned}$$