On the Capacity of Multi-hop Wireless Networks Enabling Dynamic Spectrum Management

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Abstract

A network capacity analysis for Dynamic Spectrum Management (DSM) enabled networks is presented. DSM enables frequency reuse in both time and space when idle. This paper presents a methodology for finding a capacity upper-bound. To this end, a model that characterizes the frequency occupancy and availability is developed based on the network parameters of co-existing networks. The computation of maximum instantaneous capacity is based on the application of the "deferral set" and "boundary zones" concepts introduced in [5] considering the dynamic effect of incumbent systems over their operational frequency intervals. This practical and easy to understand methodology has its application on the design of future DSM networks.

Key Words

Ad hoc, Dynamic Spectrum Management, Capacity upper-bound.

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***Abstract*— A network capacity analysis for Dynamic Spectrum Management (DSM) enabled networks is presented. DSM enables frequency reuse in both time and space when idle. This paper presents a methodology for finding a capacity upper-bound. To this end, a model that characterizes the frequency occupancy and availability is developed based on the network parameters of co-existing networks. The computation of maximum instantaneous capacity is based on the application of the "deferral set" and "boundary zones" concepts introduced in [5] considering the dynamic effect of incumbent systems over their operational frequency intervals. This practical and easy to understand methodology has its application on the design of future DSM networks.**

***Index Terms*— Ad hoc networks, Dynamic Spectrum Management, capacity analysis.**

I. INTRODUCTION

Wireless communications are embracing a period of dramatic changes in the next several years. Their dependence on new ways to access the RF spectrum has been increased by the competition between military, new commercial services and government frequency spectrum usage. There is now a commonly held perception that there is not enough available RF spectrum for new applications; however, the real problem is not a spectrum shortage, but developing reliable Dynamic Spectrum Access (DSA) methods that enable new systems to co-exist with non-cooperative existing users. Based on the work of Mitola on smart and cognitive radios [13], DSA enabled devices are envisioned as policy enabled devices that can sense environmental conditions and opportunistically share frequency band without causing interference to incumbents.

There is significant work currently under way to address these challenges including the Defense Advanced Research Projects Agency (DARPA) who is developing the neXt Generation (XG) program to develop a new generation of spectrum access technology to increase the ability of military to access the spectrum. A series of publicly available RFCs [8], [7] and [9] have been presented in which the vision, approaches, functionalities and policy agile devices of XG are presented. On the regulatory sector, the Federal Communications Commission (FCC) in the Spectrum Task Force Report [6], preliminary data and general observations indicate that portions of the radio frequency spectrum are not in use for significant periods of time. Government agencies are then taking significant steps to remove regulatory barriers and facilitate the development of secondary markets in spectrum usage rights among the Wireless Radio Services. With all this critical mass of work on dynamic frequency spectrum access, it is not too early to question ourselves about the maximum network capacity that DSA enabled networks can achieve. A fully practical methodology is developed to find the capacity of DSA networks is developed based on the the "deferral set" concept introduced by Fang in [4] and the modelling of frequency occupancy of incumbent systems by a queuing system. This methodology have interesting theoretical and practical significance with respect to future DSA system's design—most significant is the parametric model that adapts readily to different DSA implementations. As a result, this model can be used to determine the amount of spectrum that can be assigned to secondary RF spectrum markets.

The rest of the paper is organized as follows: To get a better sense of the amount of frequency that can be reuse, section-II discusses the RF spectrum dynamics modelling. Section-III reviews the problem of finding the capacity of an Ad hoc network with the inclusion of voids in their network topology. This analysis is based on the work developed by Fang and McDonald in [5] for multi-hop wireless networks working with the IEEE 802.11 standard. Section-IV presents a set of conclusions obtained in this work. Finally, section- V describes the next steps in this research.

II. RF SPECTRUM DYNAMICS

A key piece to find the network capacity is first to characterize the frequency occupancy/availability as a function of three different factors that allow reuse: frequency division, time and space.

A. *"Incumbent Systems"*

Incumbent nodes hereafter called primary nodes are the licensees of portions of the RF spectrum. They have the *"right of pass"* to occupy their assigned frequencies. Primary nodes can be characterized by the nature of their communications (i.e. Broadcast, unicast or multicast), topology and by their traffic load. In this work, broadcast and unicast communications are considered for primary nodes.

B. *"Division of the RF Spectrum"*

The RF spectrum is discretized and divided in $n_f + 1$ intervals. n_f intervals are assigned to P_i primary nodes and one unlicensed interval is also present. An interference region with radius R surrounds all primary nodes where DSA nodes, if present, are not allowed to transmit at the same frequency. A total of N DSA nodes are spread uniformly along the primary's network so an average of p_i DSA nodes are inside every primary node interference region.

C. *"Primary Nodes Frequency utilization"*

Based upon the division of the frequency spectrum, a network capacity analysis is done for each frequency interval. At the i^{th} frequency interval, a number of primary nodes going from "0" to P_i are activated. Then, DSA enabled nodes inside any active primary's interference region are banned of transmitting or receiving data; and only the DSA enabled nodes whose transmission range does not intersect with the interference region are able to transmit. Fig. 1 shows the nodes able to communicate under this scheme. The network capacity analysis is done for all nodes able to communicate. As will be shown after, the relationship between the primaries' interference region and DSA node's transmission range has a direct impact on the capacity of the DSA network. The frequency utilization in time can be modelled as a $G/G/m/m$ queueing system with m-server loss system where each state resembles the number of active primary nodes. Fig. 2 shows the queueing $M/M/m/m$ system assuming that the k primary nodes become active according to a Poisson process with rate (λ_i), and the probability distribution of the service time is exponential with mean $(1/\mu_i)$ seconds. The expected state is:

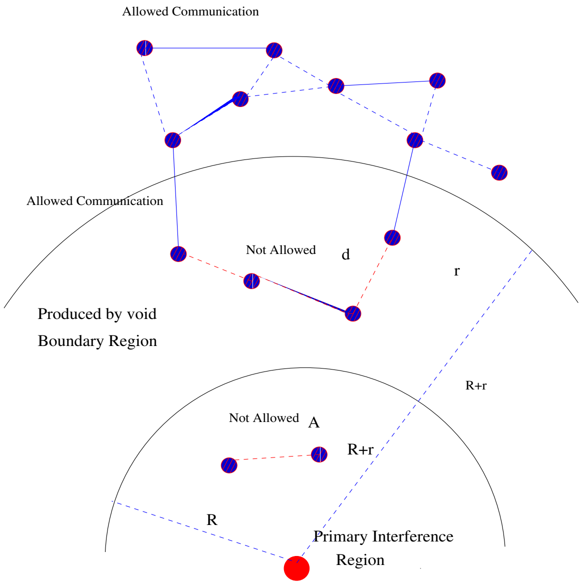

Fig. 1. Boundary zones

$$K_i = \rho(1 - \frac{1}{\sum_{k=0}^{P_i} \rho^{k-P_i}(\frac{P_i!}{k!})}) \tag{1}$$

at the i^{th} frequency interval and the average number of DSA enabled nodes is

$$N_i = N - K_i \times p_i \tag{2}$$

where $\rho = (\frac{\lambda}{\mu})$ and p_i is the average number of DSA enabled nodes inside the interference region of a primary node working at the i^{th} frequency interval.

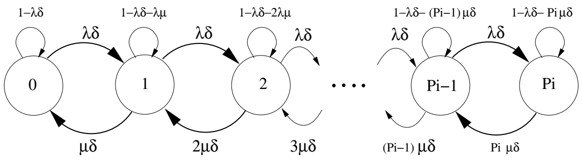

Fig. 2. $M/M/m/m$ Queueing System

Finally, the maximum network capacity is the sum of all computed subnetworks capacities induced by the division of the RF spectrum. To find the capacity of each subnetwork we follow the methodology proposed by Fang and McDonald in [5]. The inclusion of void regions in the network produce a redistribution of the concurrent active links; however, the effects of such voids are modelled as the inclusion of additional boundary zones [5].

III. MULTI-HOP WIRELESS NETWORK CAPACITY

A. *"Network Topology"*

In [14], the network topology is defined by the adjustable vector $P = \{p_1, p_2, ..\}$ that defines the existence of links as a function of the transmission power. For a DSA network, allocated bandwidth should be added; thus, $P = \{p, bw, ..\}$, the bandwidth available at a link is given by $bw(i, j)$. Two nodes are neighbors if the distance between them is less than or equal to the maximum distance (range) for which the signal strength at the receiver exceeds the amount of thermal noise present in the allocated bandwidth. Therefore, bandwidth and transmission power are responsible of defining the network topology.

The topology of a multi-hop wireless network can be represented as a graph $G(N, L)$ that contains a set of nodes N and a set of links L. Each link in L corresponds to an ordered pair of nodes, say (i, j), and indicates i is in range with j meaning that transmission from i can be heard at j. In some situations, node j might be able to hear node i but node i cannot hear node j. In such a case $(i, j) \in L$ but $(j, i)L$. For this work, Ack-type communications are assumed; therefore, for i and j to communicate, both nodes have to be in range with each other and both links will be included in the set L.

B. *“Network Capacity”*

The methodology to be used is based on the collision free sets concept that reflect channel contention mechanism of a multi-hop environment. The analysis assumes arbitrary routing and scheduling algorithm coupled with the characteristics of a contention mechanism of a dynamic frequency allocation and control protocol. The problem of finding the capacity of the DSA network is broken into finding the capacity of several multi-hop wireless networks. For each frequency interval a DSA sub-network topology is induced. The maximum instantaneous capacity is computed for each of the topologies and the total maximum instantaneous capacity is the summation of all maximum instantaneous capacities.

The contention mechanisms used imply that if node i transmits a packet, that packet will be correctly received by node j if and only if:

- 1. There is a link from i to j [i.e. , $(i, j) \in L$] , and
- 2. No other node k for which $(k, j) \in L$ is transmitting while i is transmitting, and
- 3. j itself is not transmitting while i is transmitting.

With these conditions, given an active transmission between a pair of adjacent nodes, all directed neighbors of the destination of the active transmission must defer any transmission in order to avoid collisions. Bertsekas in [1] defines a collision-free set is a set of links that can carry packets simultaneously with no collisions at the receiving ends of the links.
In [4], Fang and McDonald introduce the concept of “deferral set” as the set of nodes and links that can not be active given the activation of link in order to avoid a collision. In that case, the deferral set follows the specifications given by the IEEE 802.11 standard where the contention mechanism force different conditions as those enumerated above. In this work, assuming half duplex communications, both source and destination of the active link must defer any transmission and reception to and from other neighbors.

C. *“Collision Free Set with Maximum Capacity”*

The computation of the collision free set with maximum capacity is a NP problem. To find the number of links in all existing collision free sets corresponds to solving the Maximum Matching Problem that has been shown to be a NP-hard problem. It consists on finding the largest subset of edges included in L such that no pairs of L have a vertex in common. A variant of this problem is the maximum weighted matching problem that could be used to compute the maximum capacity of a network using the maximum Shanon capacity of every link. In this case, a modulation scheme for which the obtained channel capacity is invariant with respect to distance between transmitter and receiver is used.

The Maximum Matching problem is known to be very related to the Maximum Independent Set (MIS) problem, were nodes included in the independent set are not neighbors of each other. Vertices are included in the MIS if and only if no edge exist between them. The MIS problem was one of the first proved to be a NP-complete problem [12] and it is now known that the size of a maximum independent set cannot be approximated even within a factor of $n^{1-o(1)}$ in polynomial time [10]. Robson presented the fastest algorithm to solve MIS problems in $O(2^{n/4})$ in [15].

There exists different techniques to find approximations to the maximum matching problem and it is not the aim of the present work to develop novel approximations to solve it. The technique used in this paper is based on the work done by Erlebach [16] where he proposed an elegant Polynomial Time Approximation Scheme for Maximum Weighted Independent Sets (MWIS) based on the shifting strategy used by Hochbaum [11] and Hunt et al. [3]. The algorithm is the adapted to find a solution of the maximum matching problem where nodes can have different communication ranges. The algorithm relaxes a solution with an approximation within $(1-1/k)OPT$ where k is an integer greater than zero.

D. *“Algorithm”*

It scales all links by the largest range coverage of transmitter and receivers, so that the largest transmission range is 1. The scaling will create different topologies for which several maximum independent sets are induced. The set of links are partitioned on l different levels. The algorithm, also used in [2] shows an efficient way to find an approximation to the maximum matching problem; Along the creation of levels, the plane spanned by all nodes is partitioned in different grids. Links that cross the vertical and horizontal lines that form the grids are erased from any possible solution; in addition, in order to avoid any conflict between local maximum matching from two adjacent grids, only nodes farther than *r/2* are considered. Therefore, at a determined level, the summation of all possible local maximum matching provides the total maximum matching. Grids are displaced horizontally and vertically and the solution and therefore the algorithm claims an approximation $(1-1/k)^2$. With the use of Dynamic programming, the maximum matching coming from smaller levels can be taken into account to find the maximum matching at a lower level (that includes nodes with larger transmission ranges).

E. *“DSA: Theoretical Capacity Upper-bound”*

A DSA network with N nodes independently and uniformly distributed is considered; nodes have a fixed transmission range r and each node has an average n_{avg} neighbors. In section II-C the creation of void regions is discussed. Based on [4], it is known that the higher capacity of ad-hoc networks tents to stay in nodes that are in the boundary zones due to the lower number of neighbors and therefore, less competition for the channels. Following the same philosophy, it is expected that the number of nodes in boundary zones grows with the number of void regions. If nodes are distributed uniformly over a region, the capacity of the network has two components: a)the number of active concurrent communications on the boundary zone, and b)The number of active communications inside the network.

$$MM(G(N,L)) = N_B + d \times Area_I$$

where d is the density of nodes. When opening a void, a new topology represented by the graph $G'(N',L')$, where N' and L' are subsets of N and L respectively, is induced. Also the graph contained inside the primary interference region can be represented by $g'(n',l')$. The new topology has another boundary zone that is created around the void. Hence, the capacity of the network is now given again by the active communications original boundary zone, the number of active communications in boundary zone around the void and finally the number of active communications everywhere excluding the two previous ones.

$$MM(G'(N',L')) = N_B + d \times Area_{II} + N_{B'}$$

As the number $N_{B'}$ is bounded by the maximum number of independent concurrent links around the void, it can be shown from Fig. 3that that number is a function of the R/r ratio and equal to $C = 2\pi/\cos^{-1}(1-(1/(2(R/r+1)^2)))$. Then, it can be concluded that the maximum matching of G' is bounded by:

$$MM(G'(N',L')) = O(MM(G(N,L)) - MM(g(n,l)) + C) \quad (3)$$

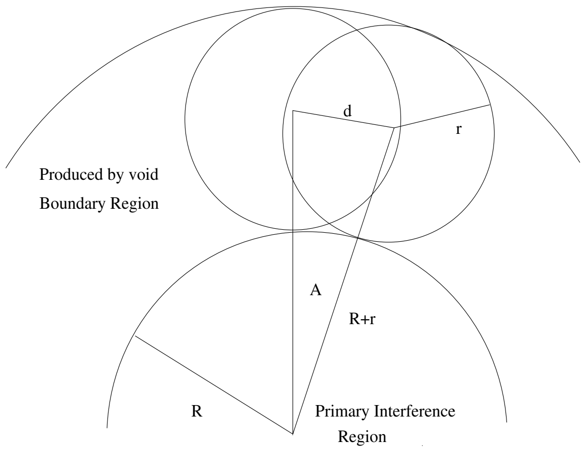

Fig. 3. Maximum number of communications around the void

TABLE I

APPROXIMATION ERROR BETWEEN WHEN APPLYING THE EQUATION (3) ABOVE AND $n_{avg} = 5$

N	$R/r=1$	$R/r=2$	$R/r=3$	$R/r=5$
50	40%	700%	1%	1%
100	5.7%	19.2%	5.5%	30%
150	9.6%	13.9%	13.9%	100%
200	1.6%	0%	20%	14%
300	0.5%	0.4%	9.2%	11%

From the Table I it can be seen that for the case of having $n_{avg} = 5$, it is necessary to have as much as 150 or more nodes if the R/r ratio goes to 2 at the most; then, the primary interference region contains a topology with similar characteristics to the topology when voids are absent. In that case, where the DSA network is large enough to keep all its topology characteristics even in a portion of it, the average number of DSA nodes inside the primary interference region is given by $p_{avg} = (R/r)^2 \times (n_{avg}+1)$ and the average number of neighbors in g' is also n_{avg}. From equation (1) and (2) the average number of DSA nodes is known and from [4] the average area covered by level-1 interference set [1] is about $(\frac{4\pi}{3}+0.068)r^2$, thus the number of non-overlapping level-1 interference set in the i^{th} frequency interval is:

$$\frac{\text{Area}_{\text{network}}}{\text{Area}_1} = \frac{\pi}{\frac{4\pi}{3}+0.068}\,\frac{N - p_{avg}\rho(1-\frac{1}{\sum_{k=0}^{P_i}\rho^{k-P_i}(\frac{P_i!}{k!})})}{n_{avg}+1}.$$

Assuming that the modulation in used by the DSA allows link capacity to be proportional to its bandwidth, the network capacity at the i^{th} frequency interval is bounded by:

$$\frac{BW_i\pi}{\frac{4\pi}{3}+0.068}\,\frac{N - p_{avg}\rho(1-\frac{1}{\sum_{k=0}^{P_i}\rho^{k-P_i}(\frac{P_i!}{k!})})}{n_{avg}+1} \quad (4)$$

and the total capacity of the DSA network, can be bounded by the expression:

$$\sum_{i=0}^{n_f}\frac{BW_i\pi}{\frac{4\pi}{3}+0.068}\,\frac{N - p_i\rho(1-\frac{1}{\sum_{k=0}^{P_i}\rho^{k-P_i}(\frac{P_i!}{k!})})}{n_{avg}+1} \quad (5)$$

Definition *1:* Throughput is defined as the time average of the number of bits per second that can be transmitted by every node to its destination.

Given that only unicast communications are considered, in the network, at any time, there is at most $N/2$ communications in network, in general, the shortest path length is just one hop and the longest path length equals the diameter of the network $\sqrt{N}$, so the average hop count for a communication will be around $\frac{\sqrt{N}}{2}$, then using the previous definition of throughput, the throughput capacity of arbitrary communication will be around :

$$\frac{3\pi}{8(\pi+0.05)(n_{avg}+1)}\sum_{i=0}^{n_f} BW_i(N - p_i\rho(1-\frac{1}{\sum_{k=0}^{P_i}\rho^{k-P_i}(\frac{P_i!}{k!})}))$$

If a common inter-arrival rate λ and service rate μ is assumed for all groups of primary nodes in their respective frequencies as well as a common number of primary nodes, then the DSM network capacity is bounded by the expression:

$$\frac{C_1 \times BW_{TOTAL}}{\sqrt{N}} - \frac{C_2 \times K(\rho,P) \times BW_p}{N\sqrt{N}} \times (\frac{R}{r})^2$$

where P is the number of primary nodes per frequency interval. For a value of $P >= 3$, $K(\rho,P) = \rho \times (1 - \frac{\rho^P e^{-\rho}}{P!})$. n_f is the number of frequency intervals, BW_{TOTAL} is the total bandwidth (assigned and not assigned) BW_p is the bandwidth size for each frequency interval, $K(\rho,P)$ is the average number of active primaries per frequency slot and C_1 and C_2 are constants. Basically it is found that the new capacity upper-bound is the composition of the upper-bound in an ad hoc network working in the total bandwidth size minus the capacity introduced by

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[1] level-1 interference set is the set of direct neighbors of the communication pair

all nodes affected by the instantaneous activation of primary nodes.

IV. "SUMMARY AND CONCLUSIONS"

In this paper, a complete characterization of both RF spectrum dynamics is developed taking into account temporal, frequency and space diversity for the first time. The methodology uses the concepts of "deferral set" and maximum instantaneous capacity and takes into account the effect of boundary regions. Our results show: (1) An extensive analysis and methodology to find an upper-bound for DSA networks regardless the particularities of their implementation. (2) DSA enables significant throughput increase versus the static spectrum management (legacy) case. (3) The capacity obtained depends directly by the total number of active primary communications and the communication range ratio of primary and DSA devices. This information is useful to measure the performance for future DSA designs.

V. "FUTURE WORK"

Finally, future work should investigate several real-world spectrum assignment to primary nodes in order to make an study on the feasibility of DSA systems in a band-per-band basis. These results will provide insight into the dependency of system performance on the assignment employed as well as the class of techniques that should be used under different scenarios of primary nodes' traffic load and topology.

REFERENCES

[1] Dimitri Bertsekas and Robert Gallager, *Data networks*, 2nd ed., Prentice Hall, 1992.

[2] H. Balakrishnan et al., *The distance-2 matching problem and itsrelationship to the mac-layer capacity of ad hoc wireless networks*, IEEE Journal on Selected Areas in Communications **22** (2004), no. 6, 1069–1079.

[3] H. Hunt et al., *Nc approximations schemes for np and pspace hard problems for geometric graphs*, Journal of Algorithms **26** (1998), 238–274.

[4] Yue Fang and A. Bruce McDonald, *Theoretical channel capacity in multi-hop ad hoc networks*, Proceedings of the 13th IEEE workshop on Local and Metropolitan Area Networks (San Francisco, LA), April 25th-28th 2004, pp. 90–100.

[5] Yue Fang and Bruce McDonald, *Theoretical capacity of multi-hop wireless ad hoc networks*, Mobile and Wireless Communications Networks MWCN 2004 (Paris, France), October 2004, pp. 311–322.

[6] Espectrum Policy Task Force, *Spectrum policy task force report*, Technical Report 02-135, Federal Commnications Division, November 2002.

[7] The XG Working Group, *The xg architecture framework*, BBN Technologies, Cambridge, Massachusetts, USA, June 2003.

[8] ______, *The xg vision*, BBN Technologies, Cambridge, Massachusetts, USA, June 2003.

[9] ______, *Xg policy language framework*, BBN Technologies, Cambridge, Massachusetts, USA, June 2004.

[10] J. Hastad, *Clique is hard to approximate within $n^{1-o(1)}$*, Proceedings of the 37th Annual Symposium on Foundations of Computer Science FOCS 1996 (Washington DC), 1996, p. 627.

[11] D. S. Hochbaum and Maass, *Approximation schemes for covering and packing problems in image processing and vlsi*, Journal of ACM **32** (1985), no. 1, 130–136.

[12] R. Karp, *Reducibility among combinatorial problems*, Proceedings on a symposium on the Complexity of Computer Computations (New York), March 1972.

[13] J. Mitola, *Special issue on software radios*, IEEE Communications Magazine, May 1995.

[14] Ram Ramanatham and Regina Rosales, *Topology control of multihop wireless networks using transmit power adjustment*, INFOCOM 2000. Nineteenth Annual Joint Conference of the IEEE Computer and Communications Societies. Proceedings, March 2000, pp. 404–413.

[15] J. M. Robson, *Finding a maximum independent set in time $o(2^{n/4})$*, Technical report, LaBRI, Universit Bordeaux 1, 2001, http://dept-info.labri.u-bordeaux.fr/ robson/mis/techrep.html.

[16] Klaus Jansen Thomas Erlebach and Eike Seidel, *Polynomial-time approximation schemes for geometric graphs*, Proceedings of the twelfth annual ACM-SIAM symposium on Discrete algorithms (Washington, D.C., United States), January 07 - 09 2001, pp. 671–679.