A Queuing Model of Multi-hop Wireless Ad Hoc Network with Hidden Nodes

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Abstract

Hidden nodes are a basic problem that can potentially influence any multi-hop wireless networks where nodes cannot hear each other. Even though the hidden node problem is well known, its effects have not been quantified so well in a comprehensive manner up to now. This paper presents a novel framework for the queuing analysis towards getting a quantitative insight into the impact of hidden nodes on the performance of multi-hop wireless ad hoc networks by which the fundamental system limits can be obtained. Analytical expression for the probability of packet collision is first derived and using an iterative process, an M/G/1/K queuing model is developed to determine the mean packet delay, and the maximum throughput, based on IEEE 802.11 DCF model. Results are validated via simulation with statistical analysis.

I. Introduction

In recent years, reconfigurable wireless networks, more specifically, wireless ad hoc networks have gained increasing attention from military, commercial and academic fields. In order to design effective routing, admission control or topology control algorithms, it is important to have the queuing theoretic analysis of the performance limitations of ad hoc networks subject to arbitrary routing and traffic conditions. However, due to the complicate interactions between entities of different layers in wireless, especially, multi-hop wireless networks and the lack of efficient framework and formal models to characterize such interactions, the fundamental performance limit analysis turns out to be a difficult problem.

The performance of the IEEE 802.11 MAC standard for ad hoc networks has been the subject of numerous analysis. However, until recently there had been little work focused on the hidden node problem. The objective of this paper is to present a generalized queuing model for Markov arrivals considering the hidden node problem; this is the first such model to be proposed and has been well validated via simulation under the range of traffic conditions.

The analysis is based upon a bounded network of M/G/1/K queues, where K represents the max queue length at each node and N represents the number of potentially interfering nodes. For multiple-hop systems N represents the number of nodes within two-hops of either a sender or receiver. It represents the effective number of interfering nodes accounting for only that portion of traffic generated by the two-hop away neighbors destined for a one-hop neighbor. An important property of the model is underscored by the earlier reference to ”potentially interfering” nodes. Each of the N nodes may be in any state of a given time, namely, any node may be busy or idle. The analysis assumes only Markov arrivals at each node that are pairwise and collectively independent.

A key point and novel aspect of the M/G/1/K model is that the complexity level normally encountered is reduced by effectively restoring the independence between service times and packet inter-arrivals through the underlying model in [7]. Thus, direct stead-state analysis is possible using iterative techniques to estimate the model parameters for different traffic intensity levels.

The remainder of this paper is organized as follows: Section II describes the system model which encompasses models for the back-off algorithm, service time distribution and the M/G/1/K formulation. Section III validates the system model by comparing the analytical and simulation results. Finally, section IV concludes the paper.

II. System Model

This section describes the basic methodology and components of the M/G/1/K queuing analysis. The main parameters are identified and applied to each of the system entities—the back-off algorithm, the service time distribution and, finally, the queuing model. In the present analysis the states of each node are considered independently and then coupled through an iterative process in order to evaluate the system state. Assuming the independent and identically distributed (IID) arrival processes, uniform traffic distribution and uniform node positions it is possible to evaluate the expected value of the global service time by determining the busy probability, transition probabilities and mean waiting times.

In the system each node alternates between busy and idle periods. During a busy period the node executes the RTS/CTS protocol, transmits its data and receives an acknowledgment. If it has multiple frames to send, it may contend and transmit more than one during the same busy period. After each transmission a node goes into back-off following DIFS; other nodes continue counting down the back-off time according to IEEE 802.11 standards [2]. The number of busy nodes that have at least one frame to send may vary from one contention period to another. Each busy node becomes idle when there are no more frames to send. A packet is reserved in the queue on arrival if at the instant of arrival, the node is non-empty.

One of the basic parameters that must be evaluated is b_0, the steady-state probability that a node is busy or approximately non-empty. The state probabilities depend on the traffic rate λ. Packet inter-arrivals are assumed to include packets originating at a given node and those routed through the node. Based on the Markov assumption the inter-arrival times are IID with exponential distribution.

An iterative process that is dependent on the busy probabilities is used to evaluate the system behavior. Values are estimated through using an initial guess and iterative correction. The process is outlined as follows:

- **Step 1:** Initialize b_0. These must be known to determine the collision probability for the back-off algorithm.
- **Step 2:** Calculate the collision probability, c and the corresponding transmission attempt probability, τ.
- **Step 3:** Given c and τ, evaluate the packet service rate, μ, where $\frac{1}{\mu}$ is the mean packet service time.
- **Step 4:** Given μ and λ, find all the state probabilities in $M/G/1/K$ queuing system. This process will result in a new value of b_0 - the steady-state busy probability.
- **Step 5:** Repeat Steps 2, 3 and 4 until the difference between a new and previous value for b_0 is small.

The remainder of this section provides detailed explanation of the loop steps which reflect three system entities.

A. The Back-off Algorithm

The signal propagation delay is very small in wireless networks -about 1μsec in our simulations - so it is neglected in the analysis. It is also assumed that packets collide with constant equal probability, c. This modeling is based on the analytical work in paper [7]. In [7], the collision probability c was derived for the saturated network case in which every node is fully connected to each other (one-hop case) and always has a packet to transmit at any given time. The model introduced in [4] provides the collision probability for saturated multi-hop wireless networks. In [8] and [5], the model in [7] is also extended for the general case for only one-hop scenarios. G/G/1 queuing model where the queue length is not involved is considered in [8] but M/G/1/K is considered in this paper and in [5] to take the queue length into consideration. Evaluation of collision probabilities is based on the node model in this modeling whereas in [8] it is based on the system model which consists of N node model. Unlike that in [8], an iterative process is used in order to get more accurate estimates of the collision probabilities. The most unique contribution in this paper is that the analysis is extended first to the general traffic case and then to the case of hidden node problem by obtaining an approximate expression for collision probabilities in multi-hop wireless networks.

With probability c the transmission is collided and with probability $1-c$, it is successful. Hence, the number of transmissions per packet is modeled as geometrically distributed with probability of success $1-c$. The average back-off window in the saturated case is given by [7]:

$$W_{backoff} = (1-c)\frac{W}{2} + c(1-c)\frac{2W}{2} + \cdots + c^M(1-c)\frac{2^M W}{2} + c^{M+1}\frac{2^M W}{2}$$

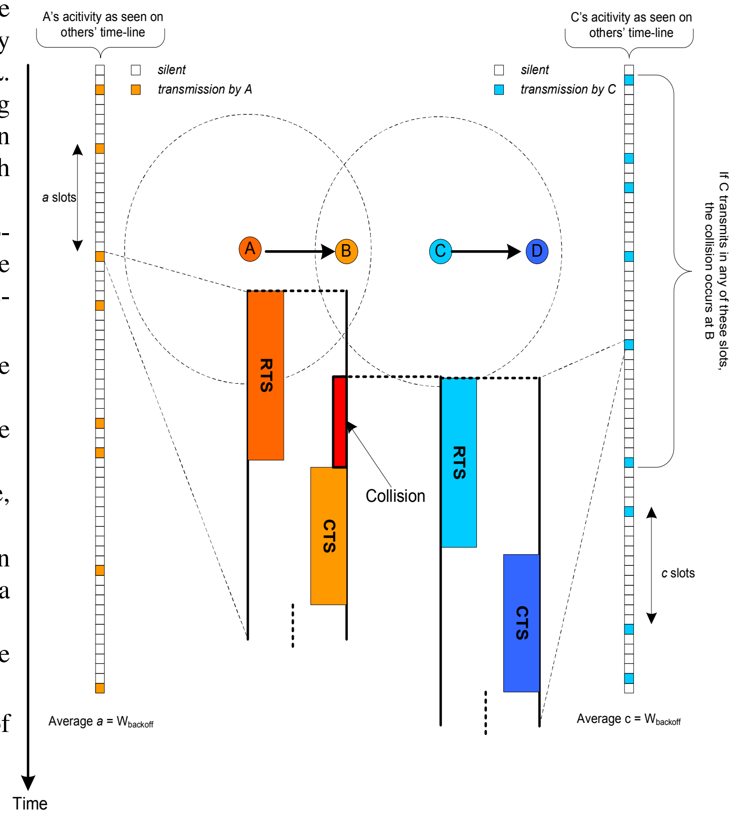

Fig. 1. Packet collision due to a hidden node

$$= \frac{1-c-c(2c)^M}{1-2c}\frac{W}{2} \quad (1)$$

where W is the initial back-off window size and M is the number of retransmission attempts before the back-off window size reaches its maximum size - thereafter, the window size remains unchanged until it is reinitialized to W for a new packet.

Consider now the probability that when node A begins transmission, it collides with another node B. Since A's back-off timer is suspended whenever B is transmitting, it appears to A that B's transmission occupies only one slot (the first slot of B's transmission). On A's time-line, B thus appears to be silent except for every $W_{backoff}$th slot, as illustrated in Fig.1.

Assuming there are sufficiently many other nodes so that A and B's transmissions are not synchronized, then A could begin transmission anywhere along this time-line, so its probability of colliding with B is $1/W_{backoff}$. The probability that A collides with any of the other nodes can therefore be approximated as $1-(1-1/W_{backoff})^{N-1}$,i.e.,

$$c = 1 - \left(1 - \frac{1-2c}{1-c-c(2c)^M}\frac{2}{W}\right)^{N-1} \quad (2)$$

The next task is to derive the collision probability for the general case. A packet is reserved in the queue on arrival if the node is non-empty. It is said before that the probability that the node is busy or non-empty (node utilization factor) when an arbitrary arrival occurs is b_0. Hence, for any arbitrary packet, with probability $1-b_0$, the back-off window is 0 and with probability b_0, it is reserved in the queue. Because the average back-off window size is $W_{backoff}$, the probability that a node attempts a transmission in an arbitrary slot is given by $b_0/W_{backoff}$.

Following the arguments of [7] and considering the fact that only busy nodes can actually collide with packets from other busy nodes, the conditional collision probability is given by:

$$c = 1 - \left(1 - b_0 \frac{1-2c}{1-c-c(2c)^M} \frac{2}{W}\right)^{N-1} \quad (3)$$

All nodes in the wireless networks are fully connected so far, i.e., no any hidden node is considered. As a last step, the hidden node problem is taken into considerations of the derivations. This problem is illustrated in Fig.1. In this topology, node C cannot hear node A's transmissions. However, if node C transmits while node A is transmitting, node A's packet will collide. Node C is the hidden node in this case. Note that this logical topology may in fact represent more complex networks.

Given that A is transmitting a packet to B in a slot, in which slot does the hidden node C begin to transmit in order to cause a collision at B? The answer is the same slot for the case of one-hop scenario. In the multi-hop networks, the answer is any slot in the range of $2 \times R_{hidden}$ where R_{hidden} is found as the ratio of the time interval from the starting moment of the RTS packet transmission until the moment the CTS transmission starts to slot time. It is doubled since C can transmit for a collision in a slot before or after the slot A is transmitting. This is shown in Fig. 1.

Next, the competition faced by active communication is quantized. In this paper the destinations are assumed to be picked randomly from the direct neighbors, hence the traffic can be regarded as uniformly distributed, too. Also, it is assumed that nodes are uniformly distributed through the network. Here three kinds of interference set are defined. Level-1 interference set is the set of nodes which are neighbor of both source and destination including the destination. Level-2 interference set is the set of nodes which are neighbor of destination only, not of the source. Level-3 interference set is the set of nodes which are two-hop neighbors of destination only. The competitors of the nodes affected by the communicating nodes in multi-hop network are not just all the nodes in the networks as in the single hop case, given the multi-hop scenario there exist the following observations:

- All transmissions originated by nodes in level-1 and level-2 interference set will be deferred by the ongoing communication.
- Only the transmission originated by nodes in level-3 interference set toward nodes in level-2 interference set will be deferred by the ongoing communication.

Based on the above observations, not all the two hop neighbors will compete the same channel with the active communication. The number of nodes in level-1 interference set is N_1, N_2 in level-2 and N_3 is the weighted two hop neighbors according to second observation above in level-3 interference set, and more information about the number of nodes in interference sets is given in [4]. The total number of interfering nodes N for a given communication is;

$$N = N_1 + N_2 + N_3 \quad (4)$$

For a source-destination communication, the possible competition for it comes from the nodes N_1 in level-1 interference set, the hidden nodes, $N_2 + N_3$ in level-2 and level-3 interference sets because for a given transmission, the collision occurs due to the direct neighbors or the hidden nodes which are neighbors of destination and destination's neighbors. For example, in 4-node topology shown in Fig.1 given that A is transmitting B, N_1 is 1 which is node B, N_2 is 1 which is node C and N_3 is 1 which is node D. If D had one more neighbor which is not neighbor of C, then the weighted two-hop neighbors, N_3, would be 0.5 since D can transmit to other neighbor which is not in Level-2 interference set. Analysis is carried out for a network which has uniformly distributed nodes. In the light of discussion so far, the collision probability is given by:

$$c = 1 - \left(1 - b_0 \frac{1}{W_{backoff}}\right)^{N_1} \left(1 - b_0 \frac{2R_{hidden}}{W_{backoff}}\right)^{N_2+N_3} \quad (5)$$

Note that the collision happens only during RTS packet transmission. However, in multi-hop networks, there are collision possibilities during DATA packet transmission. In the analysis, these possibilities are not considered because of the simplicity of the analysis and very small occurrence of these collisions.

After determining the collision probability, it is easy to find the transmission probability τ using the Discrete-Time Markov Chain (DTMC) model in [1]. Each node attempts to transmit when its back-off counter reaches zero. The transmission probability τ can be found by considering the summation of the total back-off stage probabilities weighted by the collision probabilities. Note that τ and c are same for all nodes in the network because of the assumption that every node has the same average number of neighbors.

B. The Service Time Distribution

In the next part of the analysis it is necessary to find the the distribution of the back-off window size. Here m is the maximum back-off stage and can have a value larger or smaller than M. Here m can be at most $M+1$. The back-off window size random variable W_i is uniformly distributed at each stage:

$$W_i \sim \begin{cases} U(0,(2^i W - 1)) & 0 \le i \le M \\ U(0,(2^M W - 1)) & i > M \end{cases} \quad (6)$$

where W is the initial back-off window size and $U(a,b)$ shows the uniform distribution between a and b. The aggregate back-off window size random variable $\mathcal{W}_n$ depends on the collision probability c:

$$\mathcal{W}_n = \sum_{i=0}^{m} c^i W_i \quad (7)$$

Consider the following probabilities and random variables as given in [1] for one-hop scenarios: $P_{tr,1}$ is defined as the probability that at least one transmission occurs in a given slot time for the Level-1 interference set. Since N_1 stations contend to access the medium and each station transmits with probability τ, $P_{tr,1}$ is given by:

$$P_{tr,1} = 1 - (1-\tau)^{N_1}$$

$P_{tr,2}$ is defined as the probability that at least one transmission occurs in any given slot time of $2R_{hidden}$ for the Level-2 and

Level-3 interference set. Since N_2+N_3 stations contend to access the medium and each station transmits with probability τ, $P_{tr,2}$ is given by:

$$P_{tr,2} \quad = \quad 1-(1-2R_{hidden}\tau)^{(N_2+N_3)}$$

The probability $P_{s,1}$ that an occurring transmission is successful in the Level-1 interference set is given by the probability that a station is transmitting and the remaining N_1-1 stations remain silent, conditioned on the fact that at least one station transmits:

$$P_{s,1} \quad = \quad \frac{N_1\tau(1-\tau)^{N_1-1}}{1-(1-\tau)^{N_1}}$$

The probability $P_{s,2}$ that an occurring transmission is successful in the Level-2 and Level-3 interference set is given by the probability that a station is transmitting and the remaining (N_2+N_3-1) stations remain silent in at least $2R_{hidden}$ slots, conditioned on the fact that at least one station transmits:

$$P_{s,2} \quad = \quad \frac{(N_2+N_3)\tau(1-2R_{hidden}\tau)^{N_2+N_3-1}}{1-(1-2R_{hidden}\tau)^{N_2+N_3}}$$

T_s is the average time that the medium is sensed busy due a successful transmission and T_c is the average time that the medium is sensed busy by each station when a collision occurs and σ is the duration of an empty slot. The values of T_s and T_c depend on the channel access mechanism. Assuming that all stations use the same channel access mechanism, T_s and T_c are defined as follows, assuming the RTS/CTS access mechanism is employed:

$$\begin{aligned} T_s &= DIFS+RTS+SIFS+CTS+SIFS+ \\ &\quad H+E[P]+SIFS+ACK+\sigma \\ T_c &= DIFS+SIFS+RTS+CTS \end{aligned}$$

Where $H=MAC_{hdr}+PHY_{hdr}$, and $E[P]$ is the average length of the frame. The average length of a slot time, T_{slot}, can be determined as follows:

$$\begin{aligned} T_{slot} &= (1-P_{tr,1})(1-P_{tr,2})\sigma+ \\ &\quad P_{tr,1}(1-P_{tr,2})\left[P_{s,1}T_s+(1-P_{s,1})T_c\right]+ \\ &\quad P_{tr,2}(1-P_{tr,1})[P_{s,2}\tfrac{T_s-(DIFS+RTS+SIFS)}{R_{hidden}}+ \\ &\quad (1-P_{s,2})\sigma]+ \\ &\quad P_{tr,1}P_{tr,2}T_c \end{aligned} \tag{8}$$

Note that T_{slot} is determined by considering the point of view of a node in the Level-1 interference set. Since every node has the same number of neighbors and it becomes a source in Level-1 interference set when transmitting to a destination, T_{slot} is same for all nodes. The term $(1-P_{tr,1})(1-P_{tr,2})$ shows the portion of the slot time where there is no transmission from any interference sets so the slot time is considered empty with probability $(1-P_{tr,1})(1-P_{tr,2})$. With probability $P_{tr,1}(1-P_{tr,2})$ there is interference from only Level-1 interference set. The probability $P_{tr,2}(1-P_{tr,1})$ leads to deferment due to Level-2 and Level-3 interference sets. $P_{s,2}$ of the probability $P_{tr,2}(1-P_{tr,1})$ is successful transmission probability in that the deferring node in Level-1 interference set receives the CTS packet the time $(DIFS+RTS+SIFS)$ after the packet transmission starts. During the time $(DIFS+RTS+SIFS)$ the backoff counter in the deferring node counts down the R_{hidden} many empty slots, hence, it is scaled to one slot by dividing by R_{hidden}. Collision in multi-hop cases happens with probability of $(1-P_{s,2})$. Only nodes in Level-2 and Level-3 interference sets senses the collision in multi-hop scenarios unlike one-hop scenarios. Therefore the nodes in Level-1 set see the channel empty. Note also that collision only occurs between RTS frames and T_c is different from that in paper [1] because the CTS timeout effect is considered.

Finally, the service time for an arbitrary frame is determined based on the parametric model of 802.11:

$$B \quad = \quad T_s+(\mathcal{W}_n T_{slot}) \tag{9}$$

Note that while a packet arrives to an idle node at a moment where other nodes have non-empty queues but are in back-off, it will be transmitted right away if the channel is idle for DIFS time. In the analysis, the occurrence of such cases are disregarded since this possibility is very small. As verified in section-III, this consideration results in reasonably close results.

C. M/G/1/K Queuing model

Let λ be the rate at which packets arrive in a Poisson process. The distribution function and the mean for the service time of a message are denoted by $B(x)$ and b, respectively. We can find them easily from equation 9. The maximum number of packets that can be accommodated in the system (including the one in service) at any time is given by $K<\infty$. Those packets that arrive when K packets are already present in the system are blocked and disappear.

The throughput γ of the system is given by in terms of the blocking probabilities P_B:

$$\gamma=\lambda(1-P_B) \tag{10}$$

We define:

$$\rho=\lambda b=\frac{\lambda}{\mu} \tag{11}$$

as the offered load of the system, and distinguish it from the carried load:

$$\rho'=\gamma b=\rho(1-P_B) \tag{12}$$

of the system, which is the fraction of the time that the server(node) is busy. In other words, ρ' is the probability that the server is busy at an arbitrary time; this probability is also called the node busy probability, b_0, used in the previous analysis.

Let P_k be the probability that there are k messages present in the system at an arbitrary time, where $k=0,1,2,\ldots,K$. How to find P_k is given in [3] and [6] and P_B is equal to P_K.

We are interested in two system performance metrics which are throughput and MAC delay

Throughput: The throughput of the system in $M/G/1/K$ queueing system is given by (10). From the back-off algorithm we know that if a packet is collided m times, it is dropped. Considering this probability, the throughput of the overall system is:

$$\Gamma=\frac{\lambda}{\pi_0+\rho}\cdot\left(1-c^{m+1}\right) \tag{13}$$

802.11 Parameters	Values
PYH Layer Specification	DSSS
Channel Transmission Rate	11Mbits/sec
CW_{min}	32
CW_{max}	1024
m	4

TABLE I

IEEE 802.11 System Parameters Values

MAC Delay: The average MAC delay for a successfully transmitted packet is defined to be the time interval from the time the packet is at the head of its MAC queue ready to be transmitted, until an acknowledgment for this packet is received. If a packet is dropped because it has reached the specified retry limit, the MAC delay time for this packet will not be included into the calculation of the average MAC delay. The average packet MAC delay, provided that this packet is not discarded, is obtained easily by:

$$E[D] \quad = \quad T_s + \frac{\sum_{i=0}^{m}\left(c^i - c^{m+1}\right)\frac{W_i}{2}}{1-c^{m+1}} \tag{14}$$

where $\left(1-c^{m+1}\right)$ is the probability that the packet is not dropped and $\frac{c^i - c^{m+1}}{1-c^{m+1}}$ is the probability that a packet that is not dropped reaches the i stage.

III. Model Validation and Results

The theoretical analysis presented in this paper was validated using the OPNET simulator. For each set of experiments the number of nodes and packet lengths were fixed. The network topology is designed in the hexagon shape such that there are totally 6 nodes and every node has 2 neighbors. The system parameter values are given in the IEEE 802.11 MAC layer implementation in OPNET, given in table-I.

The comparison between the analytical and simulation result of MAC delay for 1024 bytes packet sizes when the average neighbor of each node is 2 is shown in Figure-2. Figure-2 also shows that the simulation validates the analysis under the full spectrum of traffic load. Each simulation point represents the average of 10 independent steady-state replications. Figure-3 shows the comparison of the analytical and simulation throughput results versus the traffic load. Analytical results match the simulation results very closely.

IV. Conclusion

This paper represents an important advance in the analysis and design of effective multi-hop wireless ad hoc networks considering hidden node problem. A novel queuing theoretic model based on the M/G/1/K queue and parametric service model for IEEE 802.11 DCF using RTS/CTS was solved for wireless networks. The applications for the type of queuing model developed in this work include routing, admission control and scheduling.

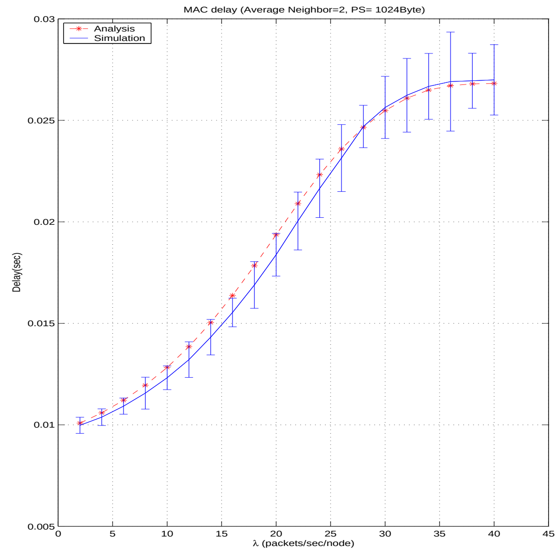

Fig. 2. MAC Delay vs Load

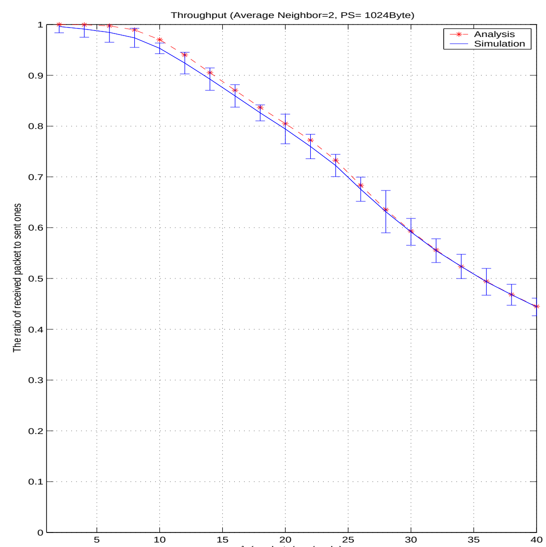

Fig. 3. Throughput vs Load

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