On the Capacity of Multi-hop Wireless Ad Hoc Network, Part I: Maximum Theoretical Channel Capacity

Yue Fang and A. Bruce McDonald, Dept. of Electrical and Computer Eng.,Northeastern University

***Abstract*— This paper presents a novel framework for capacity analysis in multi-hop wireless ad hoc networks by which the fundamental capacity limits can be obtained. Novel concepts such as “deferral set” and “equivalent competitor” are introduced to help characterize the complicate channel contention in multi-hop environments. A parametric node behavior model based on medium access specifications is developed and is used to facilitate the analysis of channel capacity and mean service time during back-logged periods. The analysis is validated by simulations from different aspects. Thus, the framework provides a novel approach of the capacity problem with practical application with respect to effective network design in the areas of routing, admission control, scheduling and topology control.**

***Index Terms*— Ad hoc networks, multi-hop networks, capacity analysis, framework, channel capacity**

I. INTRODUCTION

RECENT years, reconfigurable wireless networks, more specifically, wireless ad hoc networks have gained increasing attention from military, commercial and academic fields. In order to design effective routing, admission control or topology control algorithms, it is important to have the theoretical analysis of the fundamental capacity limitations of ad hoc networks subject to arbitrary routing and traffic conditions with ideal transmission scheduling. However, due to the complicate interactions between entities of different layers in wireless, especially, multi-hop wireless networks and the lack of efficient framework and formal models to characterize such interactions, the fundamental capacity limit analysis turns out to be a difficult problem.

There are two trends to study the capacity of wireless network, namely, the simulation oriented approach and analytical oriented approach. Simulation oriented approach provide intuitive (direct) understanding of the network performance quickly, thus is very useful in the preliminary stage of capacity analysis, when not many thorough data and insights are available. Related work using simulation oriented approach can be found in [21],[5], [6], [20] etc. However, the results from simulation oriented approach are mainly coupled with specific routing algorithms and provide very little information regarding to the fundamental performance limit of wireless ad hoc network, therefore, analytical oriented approach is necessary to fully understand the performance of wireless network, for example, [16], [3], [14] etc. Most of those related literature focused primarily on single-hop scenario or on fully connected networks. Important contributions have been made, however, the failure to capture the essence of the complex cross-layer interactions and characteristics impact induced by multi-hop environments represent significant shortcomings. Recently, important theoretical results have been reported for multi-hop networks: In [14], the capacity of ad hoc networks is presented with results based on randomly selected source-destination pair. A significant shortcoming of this model, however, stems from its lack of parametric network characterization. Although the results illustrate multi-hop performance bounds they are derived from highly abstract models. As such, they lack practical insight that can be applied to design more effective networks. Moreover, the results promote an overly pessimistic vision. Specifically, by failing to account for the benefits of temporal and spatial diversity, coupled with the non-parametric approach a pre-maturely negative tone emerged in the research community. In contrast, although the assumptions in [13] lack practical relevance, the authors illustrates how diversity, namely, the node mobility can improve the network capacity.

The objective of this paper is to present a novel yet promising understanding of the capacity problem of multi-hop wireless network from a new perspective and based on such understanding the “deferral ” framework is developed. Novel concepts, namely, the “deferral set” and “equivalent competitors” facilitate the study in a unique way: from the perspective of the node. This diverges from the more traditional approach of taking the channel-oriented perspective. [18] [19] . The node-oriented approach is more natural and provides a straight-forward and extensible analytical methodology. A key insight is that a node reacts to the contention in the *same* way regardless of whether it originated from a direct neighbor or multiple hops away, while the different network configurations are reflected by a single parameter “equivalent competitors”. Thus the node perspective approach reduces the complexity of multi-hop analysis and unify the seemingly two separate research tasks. Channel capacity as well as the network capacity are investigated based on arbitrary routing and scheduling coupled with the characteristics of IEEE 802.11 distributed coordination function (DCF) [4] in this paper and its sequel; however, the framework is not limited by specific MAC protocols; it can be applied to any other contention-based MAC algorithms such as [2] [17] [12] [7] [24] [9] etc. through appropriate modification of parametric behaviors and re-evaluation of the associated probabilities. The basis of the analysis and the methodology is to reflect the inherent cross-layer interaction and their impact on network performance. Development of accurate, well-validated models will enhance he design and applications for network control, namely, for routing, admission control and scheduling algorithms.

To the best of the authors' knowledge, this is the first complete and fully validated capacity analysis of multi-hop wireless ad hoc networks with broad practical applications. This paper reinstates that the intrinsic cause of the medium access problem in wireless network is the broadcast nature of wireless communications and has been summarized as "path coupling" phenomenon by the authors in their previous work [10]. [1] The spatial reuse of a single radio channel is the underlying cause of the phenomena and are handled at medium access control (MAC) layer. The "deferral " framework quantilically characterizes this phenomena by the "equivalent competitor" and can be utilized to solve the capacity problem for wireless ad hoc network. The most significant contributions of this work are summarized as: First, it presents the first general capacity analysis methodology for multi-hop wireless ad hoc networks. The methodology itself has practical application — most directly in the areas of dynamic topology organization, admission control and scheduling algorithms. Second, it is the first paper that notices and formally addresses the difference between the traditional channel-oriented approach [19] and the more natural and straight-forward node-oriented approach. Third, the detailed MAC specifications are reflected by parametric behavior models that are easily modified and re-evaluated. Moreover, this work completes a "family" of related performance studies of the IEEE 802.11 MAC protocol ([3], [11], [6], [25] etc) and finally, the results provide a more accurate and less conservative estimation of bandwidth capacity.

The remainder of this paper is organized as follows: Section-II provides a detailed review of related literatures regarding to the capacity analysis problem. Section-III presents the system model and framework in terms of node-oriented behavior model based on novel concepts "deferral set" and "equivalent competitors" and Section-IV evaluates the capacity of arbitrary channel for a multi-hop ad hoc network under continuous backlog conditions. Section-V summarize the simulation results used to validated the framework and Section-VI future work is briefly discussed and finally, conclusions are given in Section-VII which summarizes the main them and contributions as well as the limitations of the paper.

II. PREVIOUS ARTS

Network performance is highly coupled with the underlying medium access control protocols, among the many available MAC layer protocols, IEEE 802.11 is selected because it represents a class of access control schemes, even though there are other perhaps "better" protocols, they are just the variations of the widely-used, not-easily-to-die IEEE standard. In the paper, 802.11 DCF with RTS/CTS scheme is used as the underlying MAC layer protocol. A thorough introduction of such protocol can be found in [5]. IEEE 802.11 will be referred as the DCF scheme using RTS/CTS in the rest of this paper in the rest of the paper. In this section, a detail review of related literature, for the understanding of the framework and the capacity analysis problem is provided. Following literatures are discussed according to the order of their appearance and the importance as well as relevance to the work presented here.

Asymptotic throughput of random source-destination pair

In [14], the capacity of ad hoc networks is presented with results based on randomly selected source-destination pair. The main results show that throughput per source-destination pair decreases as $1/\sqrt{n}$ as the number of node per-unit-area increases. Moreover, they claim that given predefined optimal circumstances, the maximum throughput improves on the order of $\sqrt{logn}$ to $\Theta(W/\sqrt{n})$ bps. This paper provided a performance upper bound of wireless network which would be helpful in network design and analysis, however, one of the major shortcoming of this work is the lack of parametric network characterization. In other words, the results are derived from a highly abstract model and can not be applied directly to the analysis and design of more effective networks. Moreover, the results promote an overly pessimistic vision. Specifically, by failing to account for the benefits of temporal and spatial diversity, coupled with a non-parametric approach a pre-maturely negative tone emerged in the research community.

However, it *must not deter efforts* to be challenged or improved through network control, predictive mechanisms, novel transmission schemes and better understanding of cross-layer interactions. For example, although the assumptions in [13] lack practical relevance, the authors illustrate that node mobility can improve the network capacity such that it remains constant as the number of nodes per unit area increases. Although the specific results of Grossglauser and Tse allow for infinite delay and adopts idealized random mobility patterns to minimize the long term average path length, the analysis is significant in that it demonstrates the potential for exploiting diversity. Further the analysis in [1], wherein, a mixed set of mobile and fixed nodes are arranged a subject to random uniform traffic patterns among the static nodes. The authors verify that mobility increases throughput and can do so with a bounded delay. Specifically, in [22], both multiuser diversity and multi-path diversity are exploited in the investigation of network capacity.

Saturation throughput of single hop wireless network

The analysis in [3] and [15] develops an elegant and

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[1] "Path coupling" is defined as the media access contention between nodes distributed along node disjoint paths and is caused by the broadcast nature of wireless communication and can be served as example of contention of multi-hop network.

accurate analytical model for computing the throughput of IEEE 802.11 DCF under single-hop saturation conditions. The key assumptions are fully connected network and the probability p any frame in transmission will end up in collision is constant no matter how many collisions the frame has experienced before. The assumption is a valid approximation under saturation conditions, because all nodes are subject to statistically equivalent conditions over time. This assumption allows a two-dimensional DTMC (Markov Chain) formulation for the back-off procedure at each node. The solution to the DTMC leads to the stationary probability τ that a node transmits in any randomly chosen time slot. The saturation throughput then can be derived as a function of transmission probability τ, which in turn is the function of the network configuration and protocol specifications.

Simulation of end-to-end throughput

In [20] the interaction between the 802.11 MAC and packet forwarding is studied via simulation. Various controlled scenarios are investigated to assess the network throughput. As expected, end-to-end throughput decreased with the size of the network. However, it is also demonstrated that certain traffic patterns are significantly more scalable than others. Thus [20] both validate and refute the capacity limitation from [14], namely that network capacity is highly dependent on network size, traffic patterns, and local radio interactions. Hence it is argued that the locality of the traffic represents the key factor reflecting network capacity and scalability.

Cross-layer interaction in ad hoc networks

In [10], a cross-layer approach is proposed to understand and improve power efficiency and throughput based on the abstract notion of “path coupling”— a phenomena caused by the broadcast nature of wireless communication. It is argued that traffic load along link and node disjoint paths maybe subject to transmission delays and packet loss due to contention with a neighbor with a neighbor for the channel or hidden terminals and suggested that that it will be preferable for MAC-layer to be able to measure the “degree of coupling” and report it to the routing-layer, which is the implication of the “cross-layer approach”.

The review above reveals three irrefutable trends: (1). Given the worst possible scenario wherein there is little or no locality of reference and spatial contention increases with the number of nodes system capacity will go approach zero; this situation is analogous to congestion collapse. The interesting question is whether or not there is an invariant that can characterize the point of collapse with respect to the relevant network parameters. (2) the capacity limitation in [14] is not a fundamental limit — capacity can be improved as a natural side effect of inherently induced diversity, for example, mobility, route diversity and non-uniform traffic patterns. (3) A more flexible and parametric model for capacity analysis is needed to provide accurate performance estimation to facilitate “cross-layer” approach of routing, scheduling algorithm design and admission control.

III. DEFERRAL FRAMEWORK

Previous section illustrates the needs for an accurate, parametric model for network analysis and design, in this section, a novel framework which satisfies the above requirements are introduced — the deferral framework. This section consists two subsections: In subsection-III-A the deferral framework concept is introduced, the components of the framework, the applicable situations for the framework are discussed. In subsection-III-B the definitions and notation used throughout the paper are provided.

A. Framework Overview

The objective of deferral framework is to provide an accurate characterization for the interactions in multi-hop wireless ad hoc network environment thus it is possible to have an universal capacity analysis methodology. In the rest of this subsection three components of the deferral framework are discussed respectively.

The first component of the deferral framework is an abstract model to characterize the interaction in multi-hop wireless network environment. In [10] the authors characterize the interaction conceptually by proposing the “path coupling” concept, however, in the framework, a more concrete (specific) model is needed to quantify the effect of the interaction. The “deferral set” and “equivalent competitor” concepts proposed in this paper facilitate the quantization and hence serve as the abstract model.

The “deferral set” and “equivalent competitor” concepts with respect to IEEE 802.11 are explained as follows: Figure-1 shows a typical 802.11 communication. Two nodes connected by a line means that they are in each other’s transmission range. Suppose the communication is originated by node A toward node B, then solid line represent that there is communication going on that link while dashed line is the deferred links. The words besides each node other than A,B give the reason why the node give up the contention of channel access. Node C and E receive RTS , node D and F got CTS and node G ,K, H, I, J can not get the expected the response from its destination nodes because they have to defer channel access. From Figure-1, it is clear that all the “links” that are less than or equal two hops away from the ongoing communication can not be active simultaneously [10]. Hence the “deferral set” regarding to a specific communication is actually the nodes and links that have to deferred the channel access due to the ongoing communication. However, in terms of the competition, different nodes in the deferral set have different effect. Thus a new concept is needed to reflect the actual competition the ongoing communication will face — the “equivalent competitior”. The “equivalent competitor”

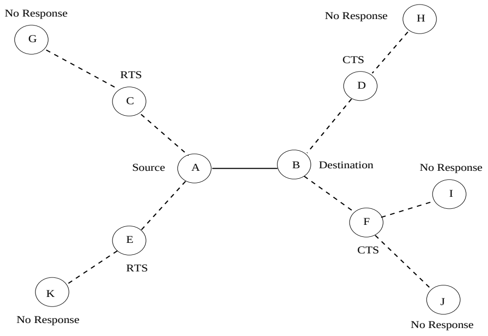


Fig. 1. An example of deferral set

reflect the actual competition imposed on the communication pair in terms of the direct neighbor.

The second component is a mathematical description of the back-off procedure. Normally exponetial back-off scheme is used in contention based MAC algorithm, and the probability of a packet ends up in collision does not depends the number of collision the packet has experienced before, in other words, the back-off procedure is a memory-less process, thus a Markov-chain based back-off model is expected. Based on such model, it is possible to obtain the probability of a node to transmit at arbitrary time. For IEEE 802.11, a two dimensional Markov chain is used to characterize the back-off procedure.

The last component of the framework is the node behavior model which models the possible reactions of the wireless node for every possible case. The model is a finite state machine with the probability and time elapsed for state transition in the node behavior model are obtained by studying the “equivalent competitor” and the specific MAC back-off procedure. As discussed in Section-I, the node’s behavior will be consistent for a specific MAC protocol in either single-hop or multi-hop scenario,thus it is sufficent to have these components to perform capacity analysis in both single and multi-hop wireless networks.

Given the above components of the network, the node/channel capacity for specific MAC algorithm can be obtained by setting the appropriate parameters and re-evaluate the corresponding probabilities of back-off procedure and the node behavior model. The values of the parameters are derived according to the network configuration and the traffic pattern through “deferral set” and “equivalent competitor” concepts.

B. *Definitions and Notation*

In this subsection concepts used in deferral set framework are introduced. Among those concepts, some (definition-3.1-definition-3.4) are well established already, and were put here for the completeness of the work; others (definition-3.5 - definition-3.10) are introduced by the author in order to appropriately characterize the special features of multi-hop wireless networks. Moreover, definition-3.1 to definition-3.4 are suitable to all kinds wireless network while the rest definitions will only be needed in *multi-hop* network analysis and are specifically proposed for multi-hop networks employed contention based medium access control protocols.

Definition *3.1:* A link (x, y) from node x to node y is defined as a wireless communications channel over which nodes x and y are able to ”reliably exchange frames without relaying through intermediate nodes. The parameters that affect the availability of a link at a given time include, but are not necessarily limited to, distance, topography, transmission power and frequency, transmit and receive antenna gain, receiver sensitivity, modulation and coding schemes, SNR and a given threshold probability of received symbol error; the set of communications parameters are expressed together in the variable length vector: $\overline{p}$. The indicator variable $l(x, y, t, \overline{p})$ reflects the link status between nodes x and y at time t. If link (x, y) is available, then $l(x, y, t, \overline{p}) = 1$; otherwise link (x, y) is considered unavailable with $l(x, y, t, \overline{p}) = 0$.

Definition *3.2:* A pair of nodes {x, y} with an available link (x, y) are defined as being Communications Active if nodes x and y are engaged in the exchange of a frame (in either direction); otherwise they are defined as being Communications Inactive. The indicator variable c(x, y, t) reflects the communications status between nodes x and y at time t. If {x, y} are Communications Active, then c(x, y, t) = 1; otherwise c(x, y, t) = 0.

Definition *3.3:* The neighbor set $N_x(t)$ of node x at time t is defined as the set of nodes, each of which share an available link with node x at time t. If V(t) is the set of nodes in the network at time t, the Neighbor Set of x is given by the following expression:$N_x(t) = \{y|\forall y \in V(t), l(x, y, t, \overline{p}) = 1\}$

Definition *3.4:* The incident link set $E_x(t)$ of node x at time t is defined as the set of links incident with x and each node in its Neighbor Set. The set is specified by the following expression:$E_x(t) = \{(x, t)|, \forall k \in N_x(t)\}$

Property *3.1:* Definition 3.3 and Definition 3.4 may also be used collectively with respect to sets of nodes, namely, given a set of node $S = \{x_1, x_2, \ldots\}$ at time t, $\cup_i N_{x_i}(t) = N_S(t)$ and $\cup_i E_{x_i}(t) = E_S(t)$. The following relationships follow directly:

$$\begin{aligned} N_{\{N_x(t)\}}(t) \cup N_{\{N_y(t)\}}(t) &= N_{\{N_x(t) \cup N_y(t)\}} \\ E_{\{N_x(t)\}}(t) \cup E_{\{N_y(t)\}}(t) &= E_{\{N_x(t) \cup N_y(t)\}} \end{aligned}$$

Above definitions are well-defined before and listed here for consistency and completeness of the work; next, novel concepts that facilitate the study of multi-hop wireless environment are introduced. Based on the observation that when IEEE 802.11 four way-handshake DCF is employed, all the nodes that are in two hops away from the communicating pair have to defer their access to the channel even though in fact they won’t content for the same channel which is being used at most one of every three consecutive links can be active at a given time the following concepts are defined. For each of the remaining definitions let nodes *x* and *y* be a Communication Active Pair, namely, $y \in N_x(t)$ and $c(x, y, t) = 1$. Each of the following sets are defined with respect to the *Communication Active Pair* $\{x, y\}$ at time t:

Definition *3.5:* Level-1 interference node set $I^1_{Node}(x,y,t)$ of the communication active pair $\{x,y\}$ at time t is defined as the set of nodes $\{k|\forall k \in \{N_x(t) \cup N_y(t)\}, k \neq x, y\}$

Definition *3.6:* Level-1 interference link set $I^1_{Link}(x,y,t)$ of the communication active pair $\{x,y\}$ at time t is defined as the set of links which can not be active simultaneously with (x,y) :$\{(j,k)|\forall(j,k) \in \{E_x(t) \cup E_y(t)\}, (j,k) \neq (x,y)\}$

Definition *3.7:* Level-2 interference node set $I^2_{Node}(x,y,t)$ of the communication active pair $\{x,y\}$ at time t is defined as the set of nodes $\{k|\forall k \in N_{N_x(t) \cup N_y(t)} - \{N_x(t) \cup N_y(t)\}\}$

Definition *3.8:* Level-2 interference link set $I^2_{Link}(x,y,t)$ of the communication active pair $\{x,y\}$ at time t is defined as the set of links which can not be active simultaneously with (x,y): $\{(j,k)|\forall(j,k) \in E_{N_x(t) \cup N_y(t)} - \{E_x(t) \cup E_y(t)\}\}$

Definition *3.9:* Deferral node set $I_{Node}(x,y,t)$ of the communication active pair $\{x,y\}$ at time t is defined as the set of nodes $\{k|\forall k \in \{N_{N_x(t) \cup N_y(t)} \cup N_x(t) \cup N_y(t)\}, k \neq x, y\}$.

$$I_{Node}(x,y,t) = \{I^1_{Node}(x,y,t) \cup I^2_{Node}(x,y,t)\}$$

Definition *3.10:* Deferral link set $I_{Link}(x,y,t)$ of the communication active pair $\{x,y\}$ at time t is defined as the set of links which can not be active simultaneously with (x,y) $\{(j,k)|\forall(j,k) \in \{E_{N_x(t) \cup N_y(t)} \cup E_x(t) \cup E_y(t)\}, (j,k) \neq (x,y)\}$

$$I_{Link}(x,y,t) = \{I^1_{Link}(x,y,t) \cup I^2_{Link}(x,y,t)\}$$

Notice that level-2 interference set may overlap. Therefore, attention should be paid to avoid counting the same link or node twice in different level-2 interference set, for link that is already deferred by one communication can not be deferred again by *simultaneous* communication.

Lemma-3.1 summarizes the channel access status of the links in the network.

Lemma *3.1:* Assume that $c(x,y) = 1$ over a certain time interval $\triangle t$. During the entire interval $\triangle t$ every node $k \in I_{Node}(x,y)$ must defer its channel access on any link $(j,k) \in I_{Link}(x,y)$ over the entire interval: Formally, when $c(x,y) = 1$ during $\triangle t$:

$$\begin{aligned} c(m,n) &= 0 \forall m,n \in \{N_x \cup N_y\} \\ c(m.k) &= 0 \forall m \in \{N_x \cup N_y\}, \forall k \in N_m - \{N_x \cup N_y\} \end{aligned}$$

Lemma-3.1 states that when two arbitrary nodes x and y which are engaged in the exchange of frame have captured the channel, no direct neighbor of either nodes may transmit to any other direct neighbor of either node; moreover, two-hop neighbor of either node may not transmit to or receive from any direct neighbor of either node. That's where the "deferral link set" comes from since no link from the set of "links" within two hops of a node in a communication active pair can be active simultaneously with the link between the communication active pair.

Table-I and Table-II list the notation and terminologies. Table-I lists notations for the novel concepts and the corresponding variables, while Table-II lists the notations for the intrinsic network or protocol parameters. Some of the listed variables are dependent and their relationship will be discussed in the rest of paper when necessary.

Notation	Explanation
x	node x expressed by its id
$N_x(t)$	node x's neighbor set at time t
$E_x(t)$	node x's incident-link set
$I^1_{Node}(x,y,t)$	level-1 interference node set of the communication active pair (x,y)
$I^1_{Link}(x,y,t)$	level-1 interference Link set of the communication active pair (x,y)
$I^2_{Node}(x,y,t)$	level-2 interference node set of the communication active pair (x,y)
$I^2_{Link}(x,y,t)$	level-2 interference Link set of the communication active pair (x,y)
$I_{Node}(x,y,t)$	Deferral node set of the communication active pair (x,y)
$I_{Link}(x,y,t)$	Deferral link set of the communication active pair (x,y)
$Area_1(x,y,t)$	area covered by $I^1(x,y,t)$
$Num_{I^1}(t)$	number of non-overlapping level-I interference set at time t

TABLE I

NOTATION FOR DEFERRAL SET FRAMEWORK

Notation	Explanation
L	The radius of network
N	Number of nodes in the network
n_{avg}	Average number of neighbors of each node
ρ	Node density of the network
N_l	Number of links in the network
ρ_l	Link density of the network
W	CW_{min}
m	maximum back-off stage, $CW_{max} = 2^m W$
δ	propagation delay
H	packet header, $H = PHY_{hdr} + MAC_{hdr}$
r	transmission range

TABLE II

NOTATION FOR SYSTEM PARAMETERS

In this section the deferral framework is presented. The components of the framework and their corresponding responsibility are discussed. Formal definitions and notation are also provided. In next section an example of the application of the deferral set framework is provided to investigate the arbitrary channel capacity in multi-hop IEEE 802.11 wireless ad hoc networks.

IV. CHANNEL CAPACITY ANALYSIS

In this section, as an application example of the "deferral set" framework, the capacity analysis of arbitrary channel in multi-hop wireless ad hoc network is studied. It is assumed that nodes are uniformly distributed in the network and working under saturation condition, in other words, there is always packet ready to send for each node at any time. In this section, to study the channel capacity problem, first the parameters that characterize the network are studied then the probability of transmission and collision are derived according to the back-off scheme model given the characteristics of IEEE 802.11 distributed coordination function with RTS/CTS, finally the node behavior model is created in turn and throughput of each individual node and channel can be obtained.

A. *Network Characterization*

Given the assumption that the shape of the network is circular and nodes are uniformly distributed through the network, it is sufficient to use the number of nodes in the network N with either average number of neighbors each node has n_{avg} or the node density ρ to characterize the network configuration because of the following relation and the radius L and the total number of links of the network N_l can be obtained as follows:

$$\begin{aligned}\rho &= \frac{n_{avg}+1}{\pi r^2}\\ L &= \sqrt{\frac{N}{n_{avg}+1}}\cdot r\\ N_l &= \frac{N\cdot n_{avg}}{2}\end{aligned}$$

More characterizations of the network include the “deferral set” and “equivalent competitor”, both of them are also parameters based on the N and n_{avg}, however, the latter one will depend on the traffic pattern also, in this paper, the destinations are assumed to be picked randomly from the direct neighbors, hence the traffic can be regarded as uniformly distributed, too. In this case, the “equivalent competitor” can be obtained as shown in the next subsection.

B. *Equivalent Competitors*

In this subsection the competition faced by active communication is quantilized. For IEEE 802.11 distributed coordination function with RTS/CTS, due to the four-way handshake scheme, the competitors of, or namely, the nodes affected by the communicating nodes in multi-hop network are not just all the nodes in the networks as in the single hop case, given the multi-hop scenario there exist the following observations:

- All the transmissions originated by nodes in level-1 interference set will be deferred by the ongoing communication.
- Only the transmission originated by nodes in level-2 interference set toward nodes in level-1 interference set will be deferred by the ongoing communication.

Base on the above observations, not all the two hop neighbors will compete the same channel with the active communication, however, in order to have an accurate capacity analysis, it is necessary to have exact competition, hence, in this paper, the “equivalent competitor” concept is proposed to quantilized the competition faced by the active communication in terms of direct neighbors. To derive the estimation of “equivalent competitor” in the network, both level-1 and 2 interference set should be investigated thoroughly to obtain the exact degree of deferral set. Other important parameters includes the average area covered the level-1 interference set and level-2 interference set ($\overline{Area_1(x,y,t)}$ and $\overline{Area_2(x,y,t)}$) and the expected value of level-1 and level-2 interference node set $E[I^1_{Node}(x,y,t)]$ and $E[I^2_{Node}(x,y,t)]$, etc:

Lemma *4.1:* Consider a random active communication pair (A,B), assume nodes are uniformly distributed in the network and each node has n_{avg} number of neighbors, let $E[I^1_{Node}(x,y,t)]$ be the expected value of the size of level-1 interference node set, then

$$E[I^1_{Node}(x,y,t)] \quad = \quad \frac{(\pi+0.98)(n_{avg}+1)}{\pi} \qquad (1)$$

Proof: The proof of lemma-4.1 is straight forward. From Figure-2, the area cover by level-1 interference set, given the Euclidean distance between the two communicating nodes A and B is l can be calculated as:

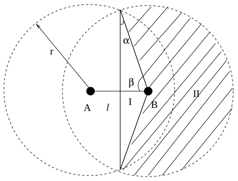


Fig. 2. A typical level-1 interference set

$$\begin{aligned}Area_1(l) &= (\pi+2\alpha)r^2+\frac{l}{2}\sqrt{4r^2-l^2}\\ \alpha &= sin^{-1}(\frac{\frac{l}{2}}{r}) = sin^{-1}(\frac{l}{2r})\end{aligned}$$

Under the assumption that the Euclidean distance between the communicating pair is uniformly distributed between (0,r), then

$$\overline{Area_1} \quad = \quad \int_0^r f(l)Area_1(l)dl = (\pi+0.98)r^2 \qquad (2)$$

$$E[I^1_{Node}(x,y,t)] \quad = \quad \rho\cdot\overline{Area_I}$$

thus yields (1). ∎

Besides direct neighbor, number of two hop neighbors for a communication pair is also an important parameter of capacity analysis. The understanding of it will provide a basis for the calculation of “equivalent competitor”, which is in fact the weighted two hop neighbors according to the traffic patterns. In this paper, the “equivalent competitor” is obtained based on the assumption that traffic is uniformly distributed in the network, i.e, the destination will be random uniformly chosen from the node’s direct neighbors. In the rest of this subsection, first the number of two hop neighbors is studied, then the value of “equivalent competitor” is derived. Lemma-4.2 shows how to calculate two hop neighbors give the distance between the communicating pair is l.

Lemma *4.2:* Consider a random active communication pair (A,B) with coordinate (-l/2,0) and (l/2,0), let E(two hop neighbor) (l) be the expected value of two hop neighbors of (A, B) given the Euclidean distance between A and B is l. E(two hop neighbor) (l) can be evaluated by the following integral:

$$
\begin{aligned}
& E(\text{two hop neighbor})(l) \\
= \;& 4\rho(\underbrace{\int_0^{r+\frac{l}{2}} \int_{\sqrt{r^2-(x-\frac{l}{2})^2}}^{\sqrt{4r^2-(x+\frac{l}{2})^2}} min(\rho V_1(l),1) dD_y dD_x}_{I} \\
& + \underbrace{\int_{r+\frac{l}{2}}^{2r-\frac{l}{2}} \int_0^{\sqrt{4r^2-(x+\frac{l}{2})^2}} min(\rho V_1(l),1) dD_y dD_x}_{II} \\
& + \underbrace{\int_0^{2r-\frac{l}{2}} \int_{\sqrt{4r^2-(x+\frac{l}{2})^2}}^{\sqrt{4r^2-(x-\frac{l}{2})^2}} min(\rho V_3(l),1) dD_y dD_x}_{III} \\
& + \underbrace{\int_{2r-\frac{l}{2}}^{2r+\frac{l}{2}} \int_0^{\sqrt{4r^2-(x-\frac{l}{2})^2}} min(\rho V_3(l),1) dD_y dD_x}_{IV}
\end{aligned} \tag{3}
$$

where

$$
\begin{aligned}
V_1(l) &= \int_{C_3(x)}^{C_5(x)} \int_{D_y-\sqrt{r^2-(x-D_x)^2}}^{\sqrt{r^2-(x-\frac{l}{2})^2}} dxdy \\
& + \int_{C_1(x)}^{C_3(x)} \int_{D_y-\sqrt{r^2-(x-D_x)^2}}^{\sqrt{r^2-(x+\frac{l}{2})^2}} dydx
\end{aligned} \tag{4}
$$

$$
V_3(l) = \int_{C_1(x)}^{C_2(x)} \int_{D_y-\sqrt{r^2-(x-D_x)^2}}^{\sqrt{r^2-(x-\frac{l}{2})^2}} dydx \tag{5}
$$

where $C_i(x), C_i(y), (i = 1, 2 \ldots)$ are the coordinates of the intercept points of the transmission range of node A B and a random node D with coordinates (D_x, D_y) whose Euclidean distance from A(B) is less than or equal to 2r.

Proof: The proof of Lemma-4.2 can be found in Appendix. ■

Corollary *4.1:* The results of lemma-4.2 is used to evaluate the expected number of two hop neighbors in an arbitrary level-II interference set given that the Euclidean distance between node A and B is a random variable with uniform distribution from 0 to r:

$$
\begin{aligned}
E(\text{two hop neighbor}) &= \int_0^r f(l) E(\text{two hop neighbor})(l) dl \\
&= \int_0^r \frac{1}{r} E(\text{two hop neighbor})(l) dl
\end{aligned} \tag{6}
$$

Given Lemma-4.2 and Corollary-4.1, the value of “equivalent competitor” can be obtained according to the following theorem:

Theorem *4.1:* In multi-hop wireless network, **Not ALL** the transmission originated $I^2_{Node}(x,y)$ will affect the ongoing communication c(x,y). In fact, only the communication among $I^2_{Node}(x,y)$ and $I^1_{Node}(x,y)$ compete with c(x,y). The “equivalent competitor” N’(x,y) of c(x,y) reflect the actual competition generated faced by communication c(x,y). The expected value of “equivalent competitor” is the sum of the $I^1_{Node}(x,y)$ and equivalent two hop neighbors $E(N_{eq_2})$. $E(N_{eq_2})$ can be calculated as follows:

$$
\begin{aligned}
& E(N_{eq_2})(l) \\
= \;& 4E(N_{eq_2} \text{ in 1st quadrant})(l) \\
= \;& 4 \int\int_{\text{shades in 1st quad}} min(\rho V(l),1) \frac{\rho V(l)}{n_{avg}} \rho dD_x dD_y
\end{aligned} \tag{7}
$$

and under the assumption that the Euclidean distance between the communication pair is uniformly distributed from 0 to r:

$$
E(N_{eq_2}) = \int_0^r \frac{1}{r} E(N_{eq_2})(l) dl \tag{8}
$$

n_{avg}	$E(N_{eq_2})$	n_{avg}	$E(N_{eq_2})$	n_{avg}	$E(N_{eq_2})$
3	2.1457	4	2.6637	5	3.1817
6	3.6997	7	4.2177	8	4.7357
9	5.2537	10	5.7717	11	6.2897
12	6.8077	13	7.3257	14	7.8437
15	8.3617	16	8.8797	17	9.3977
18	9.9157	19	10.4337	20	10.9517

TABLE III

$E(N_{eq_2})$ FOR DIFFERENT n_{avg}

Table-III shows different $E(N_{eq_2})$ of the communication pair for different n_{avg}s. For a single node, the possible competition for it comes from its direct neighbor and the equivalent two hop neighbor, which is roughly half of $E(N_{eq_2})$, hence the “equivalent competitor” for each node can be expressed as:

$$
N' = n_{avg} + \frac{E(N_{eq_2})}{2} \tag{9}
$$

In this subsection, important parameters that characterize the network are studied, those parameters serve as basis of the capacity analysis, next, another component of the framework, the back-off procedure model are presented.

C. Markov Chain Model

In this subsection the second component of the framework — the back-off procedure model is discussed. In [3] a two dimensional Markov chain to model a single node’s back-off window size with IEEE 802.11 distributed coordination function with RTS/CTS is proposed. This model is improved in the sequel [15] in the sense of back-off limit (Figure-5 in [15]). The key approximation for this model is at each transmission attempt, and regardless of the number of retransmission suffered, each packet collides with constant and independent probability p. However, in this paper, multi-hop network is considered, so the model has to be modified to reflect the multi-hop environment — through "equivalent competitor". Although in this paper the notation used are same as in previous works, the actual value of the probability of collision is derived including characteristics of multi-hop network which never been used in [3] and [15]. Detailed derivations are illustrated in the rest of Section-**??**.

The rest of the subsection illustrate the derivation of the transmission and collision probability: the combination of random variables back-off stage s(t) which corresponding to current contention window size and back-off counter b(t) which corresponding to the slots to be wait before the station can try to transmit again is modeled by a discrete-time Markov chain under the approximation that at each transmission attempt, each packet collides with constant and independent probability p regardless of the number of retransmission. Use the same notation as [3] and [15] [2], the Markov chain can be expressed as following:[3]

$$\begin{cases} P\{i,k|i,k+1\} = 1 & k \in [0, W_i - 2]\ ,\ i \in [0, m] \\ P\{0,k|i,0\} = (1-p)/W_0 & k \in [0, W_0 - 1]\ ,\ i \in [0, m] \\ P\{i,k|i-1,0\} = p/W_i & k \in [0, W_i - 1]\ ,\ i \in [1, m] \\ P\{0,k|m,0\} = 1/W_0 & k \in [0, W_m - 1] \end{cases}$$

Let $b_{i,k}$ be the stationary distribution of the Markov chain, then

$$b_{i-1,0} = p b_{i-1,0}, \quad 0 < i \leq m$$

and since the chain is regular, so for each $k \in (0, W_i - 1)$,[15]

$$b_{i,k} = \frac{W_i - k}{W_i} \begin{cases} (1-p) \displaystyle\sum_{j=0}^{m-1} b_{j,0} + b_{m,0} & i = 0 \\ p b_{i-1,0} & 0 < i \leq m \end{cases}$$

and given that fact that sum of all the probabilities in each state is 1, and the probability of a node to start a new transmission is just the sum of the probability that the node in state $b_{i,0}, i = 0, \ldots, m$,

$$1 = \sum_{i=0}^{m} \sum_{k=0}^{W_i - 1} b_{i,k} = \sum_{i=0}^{m} b_{i,0} \sum_{k=0}^{W_i - 1} \frac{W_i - k}{W_i} = \sum_{i=0}^{m} b_{i,0} \frac{W_i + 1}{2}$$

$$\tau = \sum_{i=0}^{m} b_{i,0} = \frac{1 - p^{m+1}}{1 - p} b_{0,0}$$

Given τ and p and the node state transition model provided in subsection-IV-D, the probability and time needed for a packet to be transmitted successfully can be obtained. Hence, it is possible to calculate the node and channel capacity, the details are illustrated in subsection-IV-E.

D. Node State Transition Model

In order to get the node or channel capacity, a node transition model is needed to obtain the corresponding the probabilities of every possible event, this subsection discuss this component. The state transition model is shown in Figure-3 under the assumption that the node work in saturation condition.

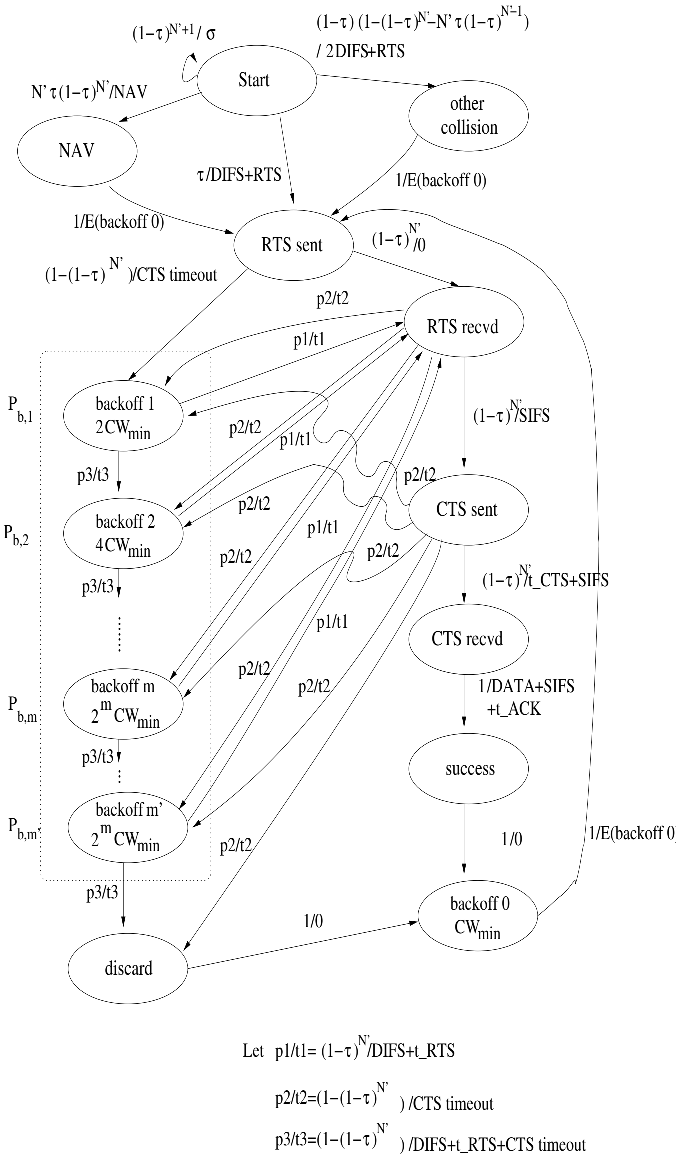


Fig. 3. State state transition model for a specific communication attempt

Figure-3 is a finite state machine based on the characteristics of IEEE 802.11 distributed coordination function, the expression p/t in the figure are the probability and time associated with each transition which are depended on the network configuration. τ is the probability of a node initialize its access to the channel at any arbitrary time obtained from subsection-IV-C while N' is the "equivalent competitor" whose value can be derived follow the investigation shown in subsection-IV-B. Some states in Figure-3 can be explained as follows: "NAV" represent the state that the node hear a valid transmission and adjust its own Network Allocation Vector (NAV) while "other collision" state corresponding to that the node hear a collision which it can not retrieve any information from. $P_{b,i}$

[2] $P\{i_1, k_1|i_0, k_0\} = P\{s(t+1) = i_1, b(t+1) = k_1, |s(t) = i_0, b(t) = k_0\}$

refers the back-off state which the back-off window size is $CW = CW_{min} \cdot 2^i$. The “RTS sent”,“CTS sent” etc states are straight forward hence no explanation will be provided here. Under saturation condition, the node is continuous back-logged, so after transmission attempt is immediately following the completion of each back-off procedure. Note that in different back-off state the corresponding back-off window size is also provided. Given the node transition model, the probability successful transmission, failure are just the probability that the node in the corresponding states in the transition model, more details of the transitions will be investigated in next subsection with the channel capacity analysis.

E. Single Channel Capacity

In the last part of the section, the capacity of arbitrary channel of a continuous back-logged network utilizing IEEE 802.11 DCF with RTS/CTS MAC protocol is studied as an application of the deferral framework. The capacity analysis is conducted based on the following assumptions:

- There is only one basic service set (BSS) in the network, the transmission and interference range of each node is set to be equal to each other.
- During the interested interval, nodes remain in the fixed positions.
- In order to not introduce too much unnecessary randomness, the packet length is assumed to be fixed, the condition can be loosen by using the expect value of the random packet length.
- It is assumed that the channel condition is ideal, namely, collisions are the only source of transmission errors between nodes that are within range of each other.
- Each node maintains a single transmission queue which contains the packet to be transmitted to one of its direct neighbors. One and only one packet can be received at a time.
- Under saturation condition, it is possible to assume that once RTS and CTS are correctly received, the data exchange will succeed. Note this condition will not hold when the network is not saturated, details of this problem can be found in the appendix.

Based on the above assumptions, the following theorem is presented for capacity analysis.

Theorem *4.2:* In multi hop wireless ad hoc network, for an arbitrary communication c(x,y), when a packet comes to the head of outgoing queue, with probability P_s the packet will be successfully transmitted to a neighbor with average delay of $\overline{T}_s$. With probability $P_f = 1 - P_s$ the packet will be discarded following an average delay of $\overline{T}_f$. For the source (transmitting) node the maximum achievable capacity is expressed as follows:

$$\begin{aligned} S &= \frac{\text{E(data exchanged during transmission attempt)}}{\text{E(mean time of transmission attempt)}} \\ &= \frac{\text{E(data exchanged during transmission attempt)}}{P_s\overline{T}_s + P_f\overline{T}_f} \end{aligned} \tag{10}$$

In (10), P_s, P_f depend on the probability of transmission τ which can be obtained from the two-dimensional Markov Chain.

$$\tau = \sum_{i=0}^{SRL} b_{i,0} = \frac{1 - p^{SRL+1}}{1-p} b_{0,0} \tag{11}$$

where $b_{0,0} =$

$$\begin{cases} \dfrac{2(1-2p)(1-p)}{W(1-(2p)^{SRL+1})(1-p) + (1-2p)(1-p^{SRL+1})} \\ SRL \leq m \\ \dfrac{2(1-2p)}{W(1-(2p)^{m+1}(1-p) + (1-2p)(1-p^{SRL+1})} \\ \dfrac{(1-p)}{+W2^m p^{m+1}(1-2p)(1-p^{SRL-m})} \\ SRL > m \end{cases}$$

[15]

As discussed in Section-IV-B, unlike [3] and [15], the overall competition each communicating node faces is reflected by the value of “equivalent competitor” — N'. Thus, the following equations holds:

$$p = 1 - (1-\tau)^{N'} \tag{12}$$

(11) and (12) form a nonlinear system in two unknowns. Numerical techniques can be used to find feasible solutions. Based on those probabilities, the corresponding P_s and P_f can be obtained given the following lemma:

Lemma *4.3:* The probability of successful transmission P_s can be expressed by:

$$P_s = (1 - (1 - (1-\tau)^{3N'})^{SRL+2} \tag{13}$$

similarly, the probability that the transmission will fail P_f is:

$$P_f = (1 - (1-\tau)^{3N'})^{SRL+2} \tag{14}$$

Proof: Study the finite state machine of the node tradition model in Figure-3, after initialization, assume there is always packet ready to transmit, all the nodes in the deferral set will be able to maintain synchronization at the “slot” (σ)level the node, called by node A hereafter, will listen to the channel and chose one the following possible states with the corresponding probability:

- **“NAV”**: Node A enters “NAV” state when one and only one of node A’s neighbor access the channel, thus node A set its NAV and wait for channel contention again with probability $p_a = N'\tau(1-\tau)^{N'}$
- **“Others Collision”**: Node A enters this state when more than one of node A’s neighbor try to access channel and collision happened, node A will wait for channel be idle again to contend for the channel with probability $p_b = (1-\tau)(1-(1-\tau)^{N'} - N'\tau(1-\tau)^{N'-1})$
- **“Reenter”**: Node will stay at the same state “start” when nobody tries to access channel at this slot with probability $p_c = (1-\tau)^{N'+1}$
- **“RTS sent”**: Node A will enter “RTS sent” state when it send out RTS packet no matter the action of other nodes with probability $p = \tau$

Because each node is always back-logged, so besides "Reenter" state, the node will enter a non-reversible transition pattern until either the packet is successfully transmitted or discarded due to consecutive transmission failure and in turn are looped from RTS sent until success or discard state. Moreover, nodes are assumed to stay in fixed position during period of interest thus the transmission can be regarded as successful after the CTS packet is successfully received by the sender. Thus the following lists the possible states of the node A after the first transmission attempt

- **"RTS sent"** :
  - The RTS sent by node A may be successfully received by its destination, say, node B with probability $p_d = \tau(1-\tau)^{N'}$, go to "RTS recvd" state;
  - The RTS sent by node A may be not received by node B due to collision with probability $p_e = 1 - (1-\tau)^{N'}$, go to "back-off" state according to the previous transmission attempts;
- **"RTS recvd"** :
  - After receive RTS, if none of node B's competitor transmits, node B will sent CTS, the probability for such event to happen is $p_f = \tau(1-\tau)^{N'}$, then node A go to "CTS sent" state
  - Otherwise, node B can not send CTS with probability $p_g = 1-(1-\tau)^{N'}$, after CTS timeout, node A will go to "back-off" state according to the previous transmission attempts;
- **"CTS sent"** :
  - The CTS packet will be received by node A with probability $p_h = \tau(1-\tau)^{N'}$, thus go to "CTS recvd" state
  - On the other hand, the CTS packet will not received by node A with probability $p_i = 1-(1-\tau)^{N'}$, then node A will go to "back-off" state according to the previous transmission attempts.
- **"CTS recvd"** : According to the assumptions made in this paper, in this case data exchange will be successful with probability p_j= 1 then node A will go to the "success" state.
- **"success"**: Since the node works under saturation condition, so node will directly enter back-off state with back-off window size CW_{min} and go to "RTS sent" state according to the MAC algorithm.
- **"back-off"**: node A will leave "back-off" state and enter "RTS sent" state directly when the back-off timer reaches zero under saturation condition.

Based on the transition flow, from Figure-3, the probability that data exchange will be completed in i_{th} (i=0,1,..., RL[3]) attempt $p_{b,i}$ will be:

$$
\begin{aligned}
p_{b,0} &= (1-\tau)^{3N'} \\
p_{b,1} &= (1-(1-\tau)^{N'}) + \tau(1-\tau)^{N'}(1-(1-\tau)^{N'}) \\
&\quad +(1-\tau)^{2N'}(1-(1-\tau)^{N'}) \\
&= (1-(1-\tau)^{3N'}) \\
p_{b,2} &= p_{b,1}(1-(1-\tau)^{N'}) + p_{b,1}\tau(1-\tau)^{N'}(1- \\
&\quad (1-\tau)^{N'}) + p_{b,1}\tau(1-\tau)^{2N'}(1-(1-\tau)^{N'}) \\
&= p_{b,1}(1-(1-\tau)^{3N'}) \\
&\vdots \\
p_{b,i} &= p_{b,i-1}(1-(1-\tau)^{N'}) + p_{b,i-1}\tau(1-\tau)^{N'} \\
&\quad (1-(1-\tau)^{N'}) + p_{b,i-1}\tau(1-\tau)^{2N'}(1- \\
&= (1-\tau)^{N'})p_{b,i-1}(1-(1-\tau)^{3N'}) \\
&= (1-(1-\tau)^{3N'})^{i+1} \qquad (i=1,\ldots,RL)
\end{aligned}
$$

Thus P_s and P_f can be obtained as follows:

$$
\begin{aligned}
P_s &= \sum_{i=0}^{RL} p_{b,i} \\
&= (1-\tau)^{3N'} + (1-\tau)^{3N'}(1-(1-\tau)^{3N'}) + \ldots \\
&\quad +(1-\tau)^{3N'}(1-(1-\tau)^{3N'})^{SRL+1} \\
&= (1-\tau)^{3N'} \sum_{j=0}^{SRL+1} (1-(1-\tau)^{3N'})^{j} \\
&= (1-(1-(1-\tau)^{3N'})^{SRL+2} \\
P_f &= (1-(1-\tau)^{3N'})^{SRL+2}
\end{aligned}
$$

■

Next the average time for a packet to be successfully transmitted or discarded are studied. From the state transition model, those time can be obtained by summing up the corresponding time spend at each possible state. Some of them are easy to get, such as "RTS sent", "CTS sent" etc, through the transmission time of the specific packet or inter-frame intervals specified by the protocol, other time, namely, time for node stay at different back-off stage will depend on the network configuration, Lemma-4.4 provides the estimation of the average time node spend at back-off stage n.

Lemma *4.4:* The mean time that a node spend in back-off stage $n(0 \le n \le m)$ is:

$$
\begin{aligned}
E(n) &= \sum_{i=0}^{\infty} C_{i+CW(n)/2-1}^{i} p_1^{CW(n)/2} \\
&\quad (1-p_1)^i \left(\frac{CW(n)}{2}\sigma + i\overline{T}_{no\ decrease}\right)
\end{aligned} \tag{15}
$$

Proof: Study the MAC specification, the possible event that might happen when a node is in its n^{th} back-off stage are summarized in Table-IV.

Combine p_2 and p_3 in Table-IV to be $p_2^{'}$, the mean time E(n) that a node stay in back-off state n is:

$$
\begin{aligned}
p_2^{'} &= 1-p_1 = p_2 + p_3 \\
\overline{T}_{no\ decr} &= (p_2 NAV + p_3(EIFS + RTS))/(p_2+p_3) \\
T_{decr} &= \sigma
\end{aligned}
$$

---

[3] RL refers to the Retry Limit in IEEE 802.11 specification

Event	Probability of the event	Time elapsed of the event
No direct neighbor involves in packet exchange	$p_1 = (1-\tau)^{N'}$	σ
One direct neighbor involves in packet exchange	$p_2 = N'\tau(1-\tau)^{N'-1}$	$\overline{NAV}$
Collision happens	$p_3 = 1 - p_1 - p_2$	EIFS+RTS

TABLE IV

PROBABILISTIC OF EVENTS HAPPENING WHEN NODE IN BACK-OFF STAGE

$$\begin{aligned} E(n) &= \sum_{i=0}^{\infty} C^{i}_{i+CW(n)/2-1} p_1^{CW(n)/2} \\ &\quad (1-p_1)^i (\frac{CW(n)}{2}\sigma + i\overline{T}_{no\ decrease}) \end{aligned}$$

In (15) $\frac{CW(n)}{2}$ is due to the fact that for each back-off stage, the back-off window size is chosen to be random uniformly distributed from [0, CW(n)], thus with mean value $\frac{CW(n)}{2}$. Where $\overline{T}_{no\ decr}$ and T_{decr} represents the mean time elapse between two back-to-back timer decrement attempts when the attempt fails and succeeds respectively. NAV in 15 equals DIFS+RTS+CTS+E[P]+ACK+3SIFS plus relative header lengths. ■

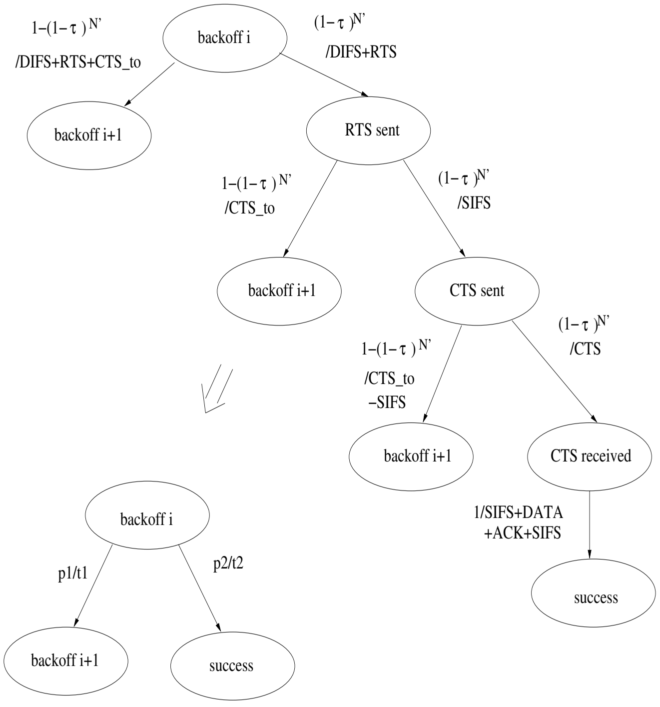
Figure-3 illustrates the finite state machine of the state transitions of a node. However, those transitions are complicated and can be simplified by appropriate combination. Figure-4 extract the transition exclusively from back-off stage i to its possible next stage. In fact, there are only two possible next stages for back-off stage i, back-off stage $(i+1)$ or success $(i \leq RL)$ and only 1 possible stage discard when $i = RL$. Based on this observation, a divide-and-conquer procedure will transform Figure-3 to Figure-5, a simpler finite state machine but summarize the same characterizations shown in subsection-IV-D. Based on Figure-5, the average time $\overline{T}_s$ and $\overline{T}_f$ are presented in Lemma-4.5.

Lemma *4.5:* The average weighted time (by the probability of successful transmission) necessary for a successful transmission is:

$$\begin{aligned} \overline{T}_s &= \sum_{i \in success} P_{s_i} T_{s_i} / P_s \\ \overline{T}_s P_s &= \sum_{i \in success} P_{s_i} T_i \\ &= \sum_{j=0}^{SRL+1} (1-\tau)^{3N'} \cdot (1-(1-\tau)^{3N'})^j (DIFS \\ &\quad + RTS + NAV + jCTS_{to} + \sum_{k=0}^{j-1>0} E(k)) \quad (16) \end{aligned}$$

similarly, the average weighted time (by the probability of unsuccessful transmission) elapsed of failed transmission is:

$$\begin{aligned} \overline{T}_f &= \sum_{i \in fail} P_{f_i} T_{f_i} / P_f \\ \overline{T}_f P_f &= \sum_{i \in fail} P_{f_i} T_{f_i} \end{aligned}$$



$$\begin{aligned} p1 &= 1-(1-\tau)^{N'} + (1-\tau)^{N'}(1-(1-\tau)^{N'}) + (1-\tau)^{2N'}(1-(1-\tau)^{N'}) \\ &= 1-(1-\tau)^{3N'} \end{aligned}$$

$$\begin{aligned} t1 &= ((1-(1-\tau)^{N'})(DIFS+RTS+CTS\_to) + (1-\tau)^{N'}(1-(1-\tau)^{N'})(DIFS+RTS+CTS\_to) \\ &\quad + (1-\tau)^{2N'}(1-(1-\tau)^{N'})(DIFS+RTS+CTS\_to))/\ 1-(1-\tau)^{3N'} \\ &= (DIFS+RTS+CTS\_to) \end{aligned}$$

$$p2 = (1-\tau)^{3N'}$$

$$t2 = DIFS+RTS+3SIFS+CTS+DATA+ACK$$

Fig. 4. State Transition flow of back-off stage i

$$\begin{aligned} &= (1-(1-\tau)^{3N'})^{SRL+2} \cdot (DIFS + RTS \\ &\quad + (RL+2)CTS_{to} + \sum_{j=0}^{SRL} E(j)) \quad (17) \end{aligned}$$

Proof: The proof is straight forward from Figure-5. ■

Now revisit (10) and (13) (16) and (17) in, the capacity of an arbitrary node is obtained. In order to find out the capacity of arbitrary link, notice for each link will transmit packets from both end nodes and the traffic is uniformly assigned to each direct neighbor, thus the capacity can be derived as:

$$S_{link} = \frac{S \cdot 2}{n_{avg}} \quad (18)$$

In this section channel capacity in saturated network is studied as an application of the deferral framework based on the specification of IEEE 802.11 DCF. Channel capacity of other MAC algorithm can also be obtained by modifying the corresponding parameters and/or models and reevaluating the related probabilities.

V. SIMULATION AND SENSITIVITY ANALYSIS

In this section first the analysis of Section-IV is validated by simulation. Simulation results are provided in subsection-V-A, moreover, the sensitivity analysis of the parameters of

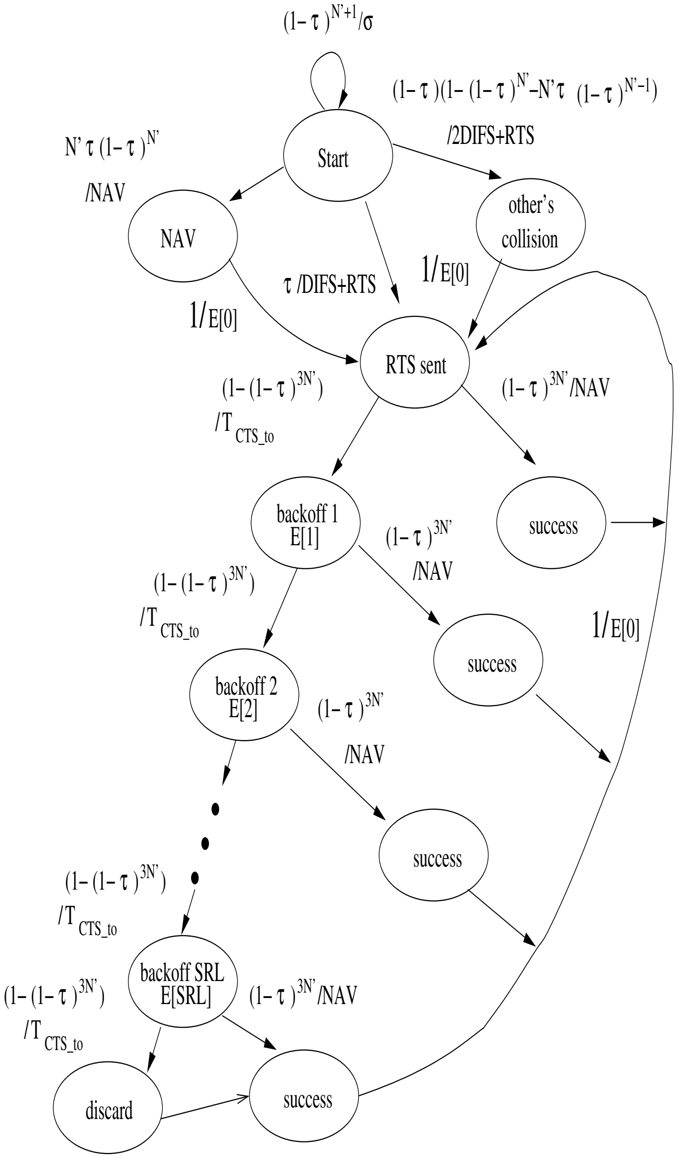


Fig. 5. Transformed state transition model

IEEE 802.11 is performed and corresponding results are listed in subsection-V-B.

A. Model Validation

In this subsection, capacity of arbitrary channel is validated by simulation. The topology considered in this paper consists of a finite number of stations uniformly distributed in a circular space with radius L. The following justified assumptions are made to improve analytical tractability without loss of generality:

- The effect of propagation delay is insignificant with respect to frame transmission time and media access delay; hence, it is negligible in the analysis. This is a very realistic assumption for transmission ranges ≤ 250m.
- Only a single ad hoc network consisting of one BS is considered. Hence, it is assumed there are no interfering BASS—the DHSS spreading sequence is unique.
- Collisions caused by simultaneous transmission are assumed to be the most significant cause of packet corruption.
- Node mobility is assumed negligible with respect to frame transmission time. Hence, transmissions always complete before a mobile receiver moves out of range.

Network simulator (*ns2*) is used in our simulation. The system parameter for analysis and simulation are the default value used by *ns-2*, whose values are listed in Table-V.

Parameter	Value	Parameter	Value	Parameter	Value
CW_{min}	32	CW_{max}	1024	DIFS	50 μs
SIFS	10 μs	m	5	rate	1Mb/s
σ	20 μs	RTS	44 bytes	CTS	38 bytes
ACK	38 bytes	r	250 m	H	6 bytes

TABLE V

SYSTEM PARAMETERS

To validate the model, a single deferral set is picked up from the network regarding to one specific communication, and throughput is measured only for the source and destination node of the specific communication. Different traffic load has been injected to the deferral set until saturation point in reached. See Figure-6 for an example of the procedure. Figure-6 illustrates the throughput under different traffic load of a single link for a deferral set with $n_{avg} = 3$. Then the simulations are run at the saturation point to validate the analytical result. From the figure, the inter-arrival rate of saturation point is set to 10 packets/second. The comparison between the analytical and simulation result is shown in Figure-7. To eliminate the effect of routing, communication are limited between neighbors. Each simulation point is run 15 times and the average of the result is used as the simulation result. Table-VI contains the detail data of Figure-7.

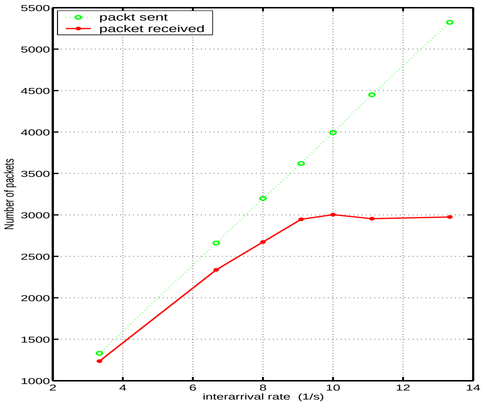


Fig. 6. How to determine the saturation point

Figure-7 shows that the simulation result validates the analysis from sparse network to relatively dense network, the

n_{avg}	Analytical	Simulation	95% CI (h)	95% CI (l)
3	0.025221	0.024781	0.0280	0.0215
4	0.012222	0.010850	0.0131	0.0086
6	0.004346	0.004977	0.0057	0.0042
8	0.002130	0.002860	0.0036	0.0021
11	0.001012	0.001327	0.0016	0.0010
12	0.000834	0.001034	0.0013	0.0008
15	0.000519	0.000619	0.0007	0.0005
18	0.000360	0.000363	0.0005	0.0003

TABLE VI

ANALYTICAL AND SIMULATION RESULT OF ARBITRARY CHANNEL CAPACITY (MB/S)

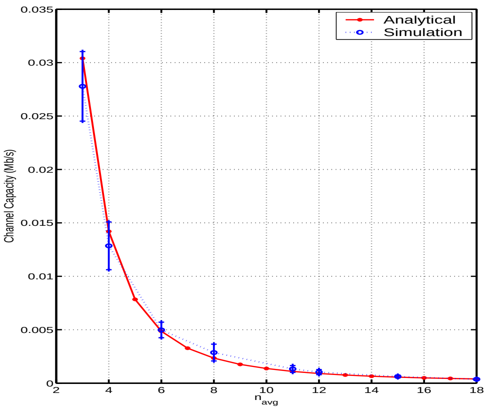


Fig. 7. Analytical and Simulation Result of Arbitrary Channel Capacity (Mb/s)

packet size used simulation in 512 bytes. In order the validate the analysis from more aspects, the mean service time is also recorded. Mean service time in analysis is actually T_s, it is the mean time between a packet become the head of the MAC layer queue and starts to contend for the channel and the time the transmission is successful and next packet comes to the head of the queue. In simulation, this mean service time is just the time between two consecutive ACKs from a specific node. The comparison of analytical and simulation mean service time is shown in Figure-8, this experiment validates the analysis is section-IV. Moreover, intuitively the inter arrival time of the simulation should equal the service delay under saturation condition, where after each packet is served, there is immediately a new packet arrive. Compare the simulation result of mean service time and inter arrival time used in simulation proved this hypothesis.

B. Sensitivity Analysis

Figure-9 illustrates the channel capacity versus different value of Retry Limit (RL) given fixed back-off stage. From Figure-9 bigger Retry Limit degrade the channel capacity because the price of packet retry is the opportunity of transmission of packets from other nodes. The extent of degrade decreases after Retry Limit exceeds back-off stage m because

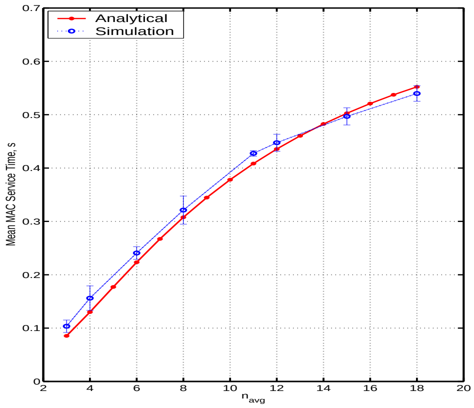


Fig. 8. Model Validation: Mean Service Time

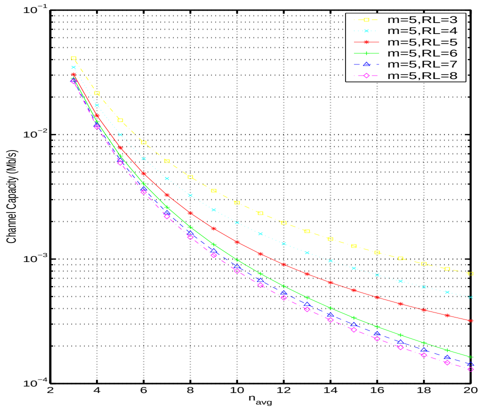


Fig. 9. Statistical Analysis: Channel Capacity of arbitrary link vs Retry Limit, m=5

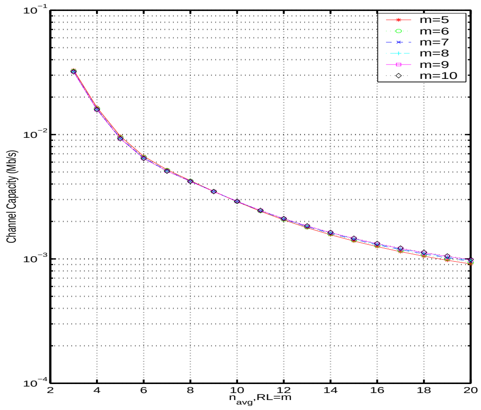


Fig. 10. Statistical Analysis: Channel Capacity of arbitrary link vs back-off stage m, RL=m

part of them will use same back-off window size. On the other hand, Figure-10 shows hat bigger back-off stage will improve the channel capacity. Longer back-off window size implies more randomness of packet transmission hence in turn reduce the possibility of collision. Figure-9 and Figure-10 imply that Retry Limit is a more sensitive parameter than back-off stage, especially when it is less than m.

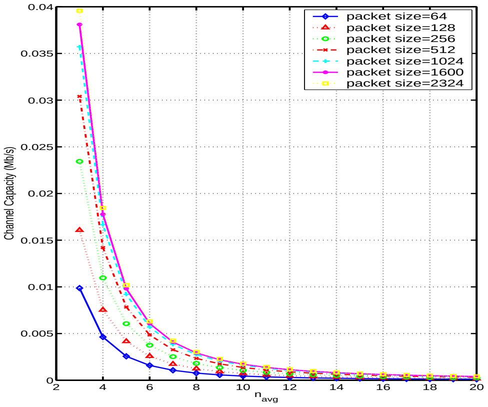


Fig. 11. Channel Capacity of arbitrary link with different packet size

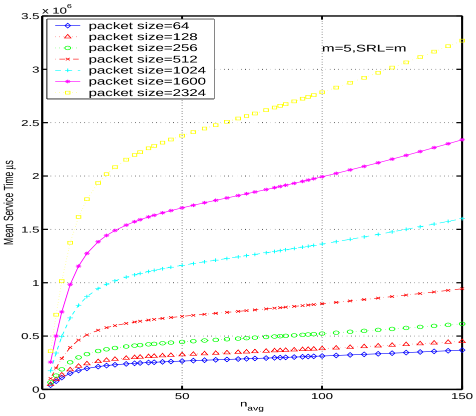


Fig. 12. MAC Service time of different packet size

Figure-11 and Figure-12 depicts analytical result of arbitrary channel capacity and mean service delay of different packet size. A close look at Figure-11 and 12 it can be seen that increase in channel capacity decreases to half each time as the packet size doubles. The mean service delay also has the same property. Moreover, scrutinize the trend of delay increase in Figure-12 it could be seen that when after the rapid growth at the lower end of the n_{avg} (under 20), the delay tends to increase with a constant rate up to some n_{avg}, (i.e. 80) then the rate starts to increase, which implies that there is a stable region for the network, after that, the mean delay will grow with higher rate which makes the network difficult to maintain stable and acceptable performance. (In reality, it is quite possible that those stable points are much lower than the one found here.)

n_{avg}	Size	Analytical	Simulation	CI (h)	CI (l)
3	64	0.007742	0.007851	0.0090	0.0067
4	256	0.009092	0.008684	0.0106	0.0068
18	512	0.000360	0.000363	0.0005	0.0003
6	1024	0.005014	0.005740	0.0066	0.0049
8	1600	0.002632	0.003371	0.0045	0.0022
12	2324	0.001073	0.001483	0.0020	0.0010

TABLE VII

SATURATION THROUGHPUT OF DIFFERENT PACKET SIZE, ANALYTICAL VS SIMULATION WITH 95% CONFIDENCE INTERVAL (MB/S)

VI. FUTURE WORK

In this paper, the capacity of arbitrary channel is studied under some basic assumptions. In the future, there are two categories of the research problems needed to be investigated, first, more realistic parameters should be considered in the framework such as the distribution of the packet length, node mobility, etc; secondly, the network capacity will be studied to provide a more accurate estimation of overall network performance. In this section discusses basic concepts and possible approaches of this two problems.

The deferral framework provided a “equivalent competitor” concept to account for the actual contention in the network. In this sense, other parameters can be incorporated to the “equivalent competitor” by a probabilistic approach. For example, when mobility exists in the network, the “equivalent competitor” N’ will changing according to the mobility when nodes move in and out from the deferral set. Same, the quality of channel will affect the probability of successful data exchange P_s, hence, by modifying the parameters and re-evaluating the probabilities, it is possible to consider more realistic factors under the same framework structure.

Given the arbitrary channel capacity and total number of links in the network, it is easy to obtain the overall throughput of the network, however, network capacity discussed in this paper is a different concept. It is the performance limit the network can provide under ideal situation, it does not depend on traffic pattern or routing scheduling algorithms, etc, but is a intrinsic parameter of the network regarding to specific topology information and medium access control protocol. It provides a performance upper bound anyone can obtained using all the possible schemes.

In this paper, same as the channel capacity, network capacity is also investigated based on the characteristics of IEEE 802.11 DCF. Thus the property that no links that are less than or equal to two hops away from the active link can be active simultaneously still holds. Based on this observation, it is straightforward that there is maximum number of concurrent links can be active in the network, under ideal scheduling, those concurrent links can be utilized to exchange data at their nominal capacity, hence gives a good start point of network capacity estimation. Although the maximum concurrent link can provide the maximum good-put of the network, it can only complete communication between specific source destination nodes pairs and can be reflect the actual performance limit of the network. Generally, any node in the network can try to communicate with any other nodes in the network, which implies that ultimately each link in the network will be utilized for at least once. Similarly, in order to investigate the network capacity, there should be an algorithm that will cover all the links of the network, only under this condition, the performance limit obtained is meaningful.

There are several possible approaches to complete this object and the details of them will be discussed in the second part of the paper.Only the framework is briefly explained in this section. In order to find the network capacity, first an algorithm is needed which is able to find the maximum concurrent links in any given network. Both optimum and greedy sub-optimum algorithm for this problem is discussed in the sequel paper. Then another algorithm is needed to obtain the sequence of concurrent links which covers all the links in the network. There are a lot of possible combinations of the sequel, however, the sequence should have the following properties, at each round, the number of concurrent link should not exceed previous round, otherwise switch them. By doing so, it is guaranteed that each round, the largest possible concurrent list set is picked, and the number of concurrent links of the last round in the sequence can serve as the network capacity because the it is the bottleneck. Details of the network capacity problem can be found in the second part of this paper. In this paper, a simple random algorithm is implemented to approximate the optimal solution in provided. In the random algorithm, each round, one link is randomly picked from the list of all the possible link that can be active. It is clear not a optimal solution, but can provide some insight of approximate number of concurrent links in the network. Table-VIII provides the results of random algorithm and Table-IX is the number of rounds needed to cover all the links in the network.

Nodes	n_{avg}	Concurrent active links	95% confidence interval
432	3	99.4167	1.1125
530	4	87.3750	0.9928
737	6	107.5417	0.7271
952	8	115.0833	0.6971
1312	12	117.8333	1.4337

TABLE VIII

NUMBER OF CONCURRENT ACTIVE LINK IN THE NETWORK BY RANDOM ALGORITHM AND THE 95% CONFIDENCE INTERVAL

VII. CONCLUSION

In the this paper, a novel framework for capacity analysis of wireless ad hoc network is presented. The framework characterize the features of multi-hop wireless network by novel concepts— deferral set and equivalent competitors and used node prospective to study the link (and node) capacity. Simulation Results validates the theoretical analysis and future work are discussed.

Node	n_{avg}	Iterations	95% confidence interval
432	3	44.6667	3.3570
530	4	94.8333	8.6138
737	6	177.5833	11.4250
952	8	347.8750	23.6835
1312	12	779.3333	47.4213

TABLE IX

NUMBER OF ITERATIONS TO COVER ALL LINKS BY RANDOM ALGORITHM AND THE 95% CONFIDENCE INTERVAL

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APPENDIX

The appendix is organized as follows: in the Appendix-A, the proof of Lemma-4.2 is presented, while appendix-B explains why saturation condition assumption is necessary for the capacity analysis.

A. Proof of Lemma-4.2

Proof: Whether or not a node is two hop neighbor of communication pair not only depends on the node's coordinates but also on the existence of one hop node in appropriate location which serves as the relay node from the communication pair to the node of interest . Thus in order to find $E[I^2_{Node}(x,y,t)]$, first the probability that a node with specific coordinates being the two hop node of communication pair should be evaluated and the integral of it over all the possible coordinates (the area of level-2 interference set covered) yield $E[I^2_{Node}(x,y,t)]$.

Figure-14 illustrates a typical case of level-2 interference set, whether or not D is two hop neighbor is determined by the probability of at least 1 one hop node in shaded area V.

The coordinates of node A, B and D are:

A(-$\frac{l}{2}$,0), B($\frac{l}{2}$,0), D(D_x,D_y)

The coordinates of the intersection points as:

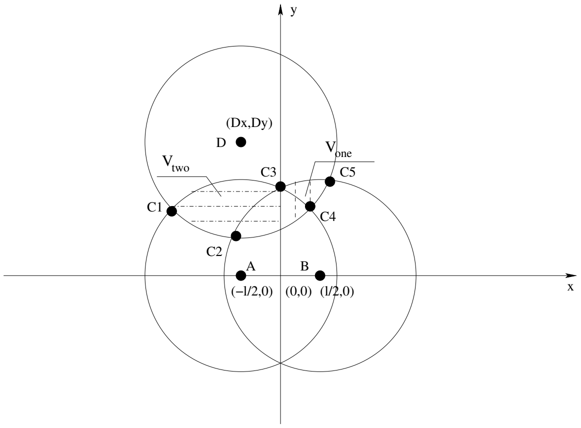

Fig. 13. Determining if D is two hop neighbor

$$\begin{array}{ll}
C_1 & (\dfrac{-A_2-\sqrt{A_2^2-4A_1A_3}}{2A_1},\dfrac{D_x^2+D_y^2-\frac{l^2}{4}-(2D_x+l)C_1(x)}{2D_y}) \\
C_2 & (\dfrac{-B_2-\sqrt{B_2^2-4B_1B_3}}{2B_1},\dfrac{D_x^2+D_y^2-\frac{l^2}{4}-(2D_x-l)C_2(x)}{2D_y}) \\
C_3 & (0,\sqrt{r^2-\dfrac{l^2}{4}}) \\
C_4 & (\dfrac{-A_2+\sqrt{A_2^2-4A_1A_3}}{2A_1},\dfrac{D_x^2+D_y^2-\frac{l^2}{4}-(2D_x+l)C_4(x)}{2D_y}) \\
C_5 & (\dfrac{-B_2+\sqrt{B_2^2-4B_1B_3}}{2B_1},\dfrac{D_x^2+D_y^2-\frac{l^2}{4}-(2D_x-l)C_5(x)}{2D_y})
\end{array}$$

where

$$\begin{aligned}
A_1 &= 4D_x^2+4D_xl+4D_y^2+l^2 \\
A_2 &= 4D_y^2l-2(l+2D_x)(D_x^2+D_y^2-\frac{l^2}{4}) \\
A_3 &= (D_x^2+D_y^2-\frac{l^2}{4})^2+l^2D_y^2-4D_y^2r^2 \\
B_1 &= 4D_x^2-4D_xl+4D_y^2+l^2 \\
B_2 &= -4D_y^2l-2(2D_x-l)(D_x^2+D_y^2-\frac{l^2}{4}) \\
B_3 &= (D_x^2+D_y^2-\frac{l^2}{4})^2+l^2D_y^2-4D_y^2r^2
\end{aligned}$$

Region V can be partitioned to 2 disjoint parts, $V_{one}(l)$ and $V_{two}(l)$. Note here the integral limitations of variable y are based on the assumption that node D resides in the 1st and 2nd quadrant, because of symmetry, if a node D resides in the 3rd or 4th quadrant, the problem can be easily transformed to a 1st or 2nd quadrant position by taking the symmetry regarding to y axis.

$$\begin{aligned}
V_{one}(l) &= \int_{C_3(x)}^{C_5(x)}\int_{D_y-\sqrt{r^2-(x-D_x)^2}}^{\sqrt{r^2-(x-\frac{l}{2})^2}} dydx \\
V_{two}(l) &= \int_{C_1(x)}^{C_3(x)}\int_{D_y-\sqrt{r^2-(x-D_x)^2}}^{\sqrt{r^2-(x+\frac{l}{2})^2}} dydx
\end{aligned}$$

thus

$$\begin{aligned} V_1(l) &= \int_{C_3(x)}^{C_5(x)} \int_{D_y-\sqrt{r^2-(x-D_x)^2}}^{\sqrt{r^2-(x-\frac{l}{2})^2}} dxdy \\ &+ \int_{C_1(x)}^{C_3(x)} \int_{D_y-\sqrt{r^2-(x-D_x)^2}}^{\sqrt{r^2-(x+\frac{l}{2})^2}} dydx \end{aligned}$$

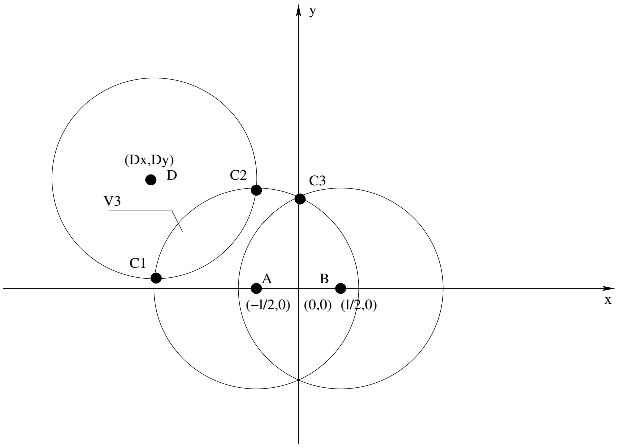

Fig. 14. Another Example of Level-2 interference Set

In some cases which depends on the value of D_x, D_y, the interception may have a different shape, such as shown in Figure-14. Clearly, in those cases, there are just two interception points C_1 and C_2. Thus the integration becomes:

$$\begin{aligned} V_2(l) &= \int_{C_1(x)}^{C_2(x)} \int_{D_y-\sqrt{r^2-(x-D_x)^2}}^{\sqrt{r^2-(x+\frac{l}{2})^2}} dydx \\ &= \int_{C_1(x)}^{C_2(x)} \sqrt{-x^2-lx+(r^2-\frac{l^2}{4})} - D_y \\ &+\sqrt{-x^2+2D_x x+(r^2-D_x^2)}dx \end{aligned} \tag{19}$$

or, if node D resides in the first quadrant,

$$\begin{aligned} V_3(l) &= \int_{C_1(x)}^{C_2(x)} \int_{D_y-\sqrt{r^2-(x-D_x)^2}}^{\sqrt{r^2-(x-\frac{l}{2})^2}} dydx \\ &= \int_{C_1(x)}^{C_2(x)} \sqrt{-x^2+lx+(r^2-\frac{l^2}{4})} - D_y \\ &+\sqrt{-x^2+2D_x x+(r^2-D_x^2)}dx \end{aligned}$$

The probability a node being two hop neighbor equals the probability of at least 1 node in region V can be calculated as:

$$Pr_{2\ hop} = \min(\rho V(l),\, 1)$$

Where $V(l)$ can be $V_1(l)$, $V_2(l)$ or $V_3(l)$.

Then the expectation of the number of two hop neighbors can be calculated by integrating the probability of two hop neighbor over the shaded area (region II) in Figure-16.

Because nodes are uniformly distributed in the network, which in turn means they are also uniformly distributed In the shaded area , so the integrations in different quadrant are symmetry, therefore only calculation of the integration of the first quadrant is needed . The integral region in first quadrant is then divided into 4 disjoint parts:

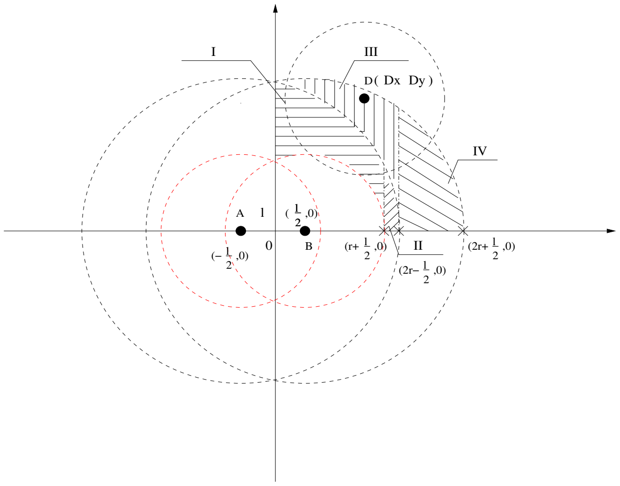

Fig. 15. Calculation of Two Hop Neighbors

$$\begin{aligned} & E(N_{eq_2})(l) \\ = \;& 4E(N_{eq_2} \text{ in 1st quadrant})(l) \\ = \;& 4\int\int_{\text{shades in 1st quad}} Pr_{2\ hop}\rho dD_x dD_y \\ = \;& 4\int\int_{\text{shades in 1st quad}} min(\rho V(l),1)\rho dD_x dD_y \\ = \;& 4\rho(\underbrace{\int_0^{r+\frac{l}{2}} \int_{\sqrt{r^2-(x-\frac{l}{2})^2}}^{\sqrt{4r^2-(x+\frac{l}{2})^2}} min(\rho V_1(l),1)dD_y dD_x}_{I} \\ & + \underbrace{\int_{r+\frac{l}{2}}^{2r-\frac{l}{2}} \int_0^{\sqrt{4r^2-(x+\frac{l}{2})^2}} min(\rho V_1(l),1)dD_y dD_x}_{II} \\ & + \underbrace{\int_0^{2r-\frac{l}{2}} \int_{\sqrt{4r^2-(x+\frac{l}{2})^2}}^{\sqrt{4r^2-(x-\frac{l}{2})^2}} min(\rho V_3(l),1)dD_y dD_x}_{III} \\ & + \underbrace{\int_{2r-\frac{l}{2}}^{2r+\frac{l}{2}} \int_0^{\sqrt{4r^2-(x-\frac{l}{2})^2}} min(\rho V_3(l),1)dD_y dD_x}_{IV} \end{aligned}$$

where V(l) can be $V_1(l)$, $V_2(l)$ and $V_3(l)$ which are also functions of D_x and D_y.

■

B. The Importance of Saturation Condition Assumption

Revisit the last assumption made in subsection-IV-E, "under saturation condition, it is possible to assume that once RTS and CTS are correctly received, the data exchange will succeed". However, this assumption only valid under saturation condition, under other circumstance, collision still can happen after successful exchange of RTS and CTS packet. The following Figure-**??** illustrate one of the situations:

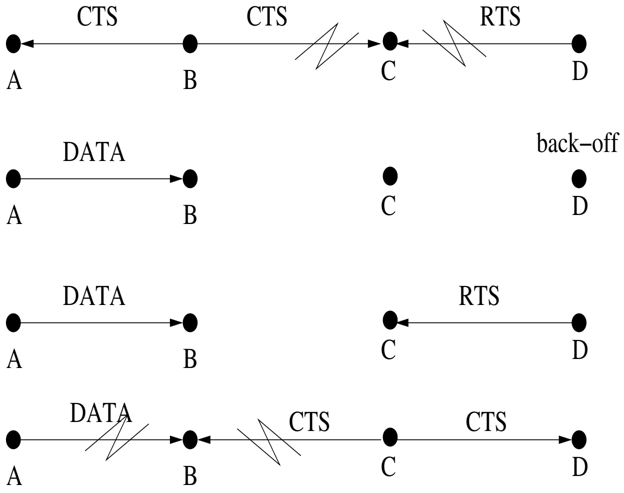

Fig. 16. Collisions under non-saturation situation.

In Figure-**??**, if the network is not saturated, when node B send CTS to node A, it will collide with node D's RTS packet at node C, hence node D will back-off while the data exchange between A and B can proceed. However, since at node C side it does not have any information about the NAV due to the collision of CTS packet, it will reply node D's following RTS, if at that time, the data transmission is still going on, the CTS packet will collide at node B with data packet from node A. In this case, even the exchange of RTS and CTS are successful, it can not guarantee that the data exchange will be complete without collision. However, under saturation case, since all the nodes are continuous back-logged, node C will hear another RTS or CTS originated from other nodes with high probability, hence the assumption in subsection-IV-E can be regard valid.