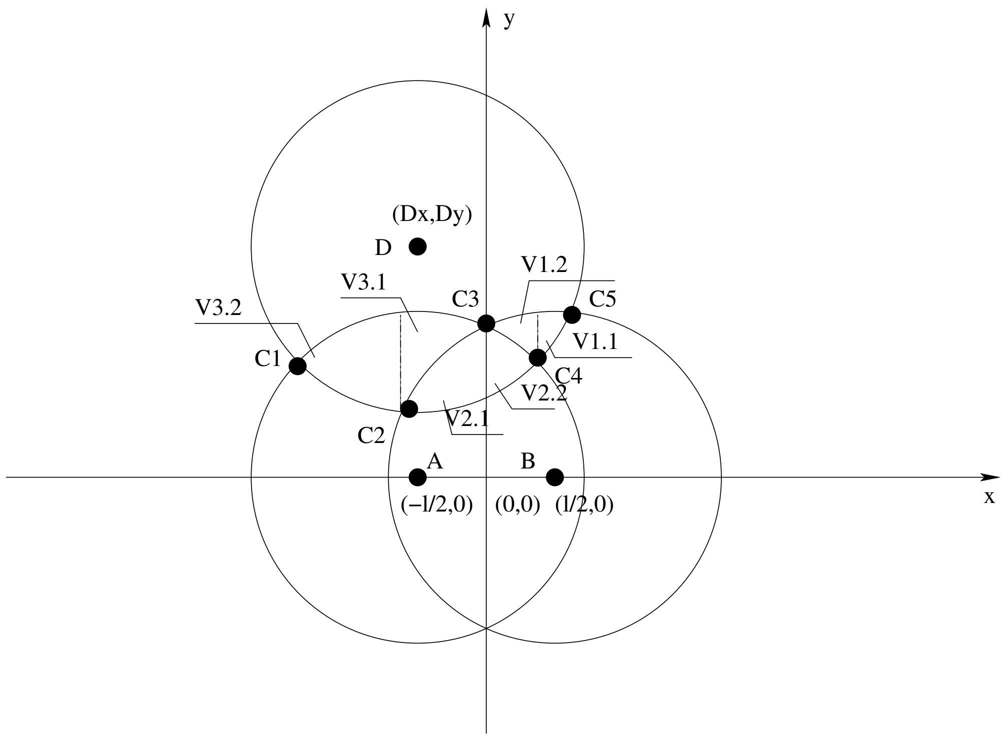

Figure 1: Example

Given Figure-1, it is easy to see that the probability that node D is two hop neighbor of node A or B equals the probability that there is at least 1 node in region V.

- Subproblem 1: How to calculate the probability of at least 1 node in a region given the area covered by the region? Example: Given that 10 nodes are uniformly distributed in an area of $100m^2$, then the probability that at least 1 node in an area of $5m^2$ is

$$10 \text{ node} <---> 100\ m^2,\ 5m^2 <---> \frac{10}{100} \times 5 = 50\%$$
$$\text{probability of \# nodes in the area=density} \times \text{ area /\#}$$

- Subproblem 2: How to calculate the area covered by region V?

  Using Area(id) to denote the circle with id as center with radius r, Region V can be partitioned into 3 disjoint parts

  V.1: Intersection of Area(D) and Area(B)

  V.2: Intersection of Area(D) and Area(A) and Area(B)

  V.3: Intersection of Area(D) and Area(A)

  set up an x,y space, we have the coordination of the nodes:

$$\text{A}(-\tfrac{l}{2},0),\ \text{B}(\tfrac{l}{2},0),\ \text{D}(D_x,D_y)$$

  - Step 1: Finding the coordination of the intersection nodes:

    * for node C1, C4
      $(x - D_x)^2 + (y - D_y)^2 = (x + \frac{l}{2})^2 + y^2$
      $(x + \frac{l}{2})^2 + y^2 = r^2$

$(l+2D_x)x+2D_y y = D_x^2+D_y^2-\frac{l^2}{4}$

$y = \dfrac{D_x^2+D_y^2=\frac{l^2}{4}-(l+2D_x)x}{2D_y}$

$x^2+lx+\dfrac{((D_x^2+D_y^2-\frac{l^2}{4})-(l+2D_x)x)^2}{4D_y^2} = r^2-\dfrac{l^2}{4}$

$(4D_x^2+4D_xl+l^2+4D_y^2)x^2+(4D_y^2l-2(l+2D_x)(D_x^2+D_y^2-\frac{l^2}{4}))x+(D_x^2+D_y^2-\frac{l^2}{4})^2+l^2D-y^2-4D_y^2r^2=0$

let

$$\begin{aligned} A_1 &= 4D_x^2+4D_xl+4D_y^2+l^2 \\ A_2 &= 4D_y^2l-2(l+2D_x)(D_x^2+D_y^2-\frac{l^2}{4}) \\ A_3 &= (D_x^2+D_y^2-\frac{l^2}{4})^2+l^2D_y^2-4D_y^2r^2 \end{aligned}$$

then

$$\begin{aligned} x_{1,2} &= \frac{-A_2\pm\sqrt{A_2^2-4A_1A_3}}{2A_1} \\ y_{1,2} &= \frac{D_x^2+D_y^2-\frac{l^2}{4}-(l+2D_x)x}{2D_y} \end{aligned}$$

* for node C2, C5
$(x-D_x)^2+(y-D_y)^2=(x-\frac{l}{2})^2+y^2$
$(x+\frac{l}{2})^2+y^2=r^2$

$(2D_x-l)x+2D_y y = D_x^2+D_y^2-\frac{l^2}{4}$

$y = \dfrac{D_x^2+D_y^2=\frac{l^2}{4}-(2D_x-l)x}{2D_y}$

$x^2+lx+\dfrac{((D_x^2+D_y^2-\frac{l^2}{4})-(2D_x-l)x)^2}{4D_y^2} = r^2-\dfrac{l^2}{4}$

$(4D_x^2-4D_xl+l^2+4D_y^2)x^2+(4D_y^2l-2(2D_x-l)(D_x^2+D_y^2-\frac{l^2}{4}))x+(D_x^2+D_y^2-\frac{l^2}{4})^2+l^2D-y^2-4D_y^2r^2=0$

let

$$\begin{aligned} B_1 &= 4D_x^2-4D_xl+4D_y^2+l^2 \\ B_2 &= 4D_y^2l-2(2D_x-l)(D_x^2+D_y^2-\frac{l^2}{4}) \\ B_3 &= (D_x^2+D_y^2-\frac{l^2}{4})^2+l^2D_y^2-4D_y^2r^2 \end{aligned}$$

then

$$\begin{aligned} x_{1,2} &= \frac{-B_2\pm\sqrt{B_2^2-4B_1B_3}}{2B_1} \\ y_{1,2} &= \frac{D_x^2+D_y^2-\frac{l^2}{4}-(2D_x-l)x}{2D_y} \end{aligned}$$

* node C3:
  C3's coordination will be $(0,\sqrt{r^2 - \frac{l}{2}^2})$

- Step 2: Integration
  To ease the integration, we partition region V furthermore to 6 disjoint parts.

$$
\begin{aligned}
V1.1 &= \int_{C4(x)}^{C5(x)} \int_{D_y - \sqrt{r^2-(x-D_x)^2}}^{\sqrt{r^2-(x-\frac{l}{2})^2}} dxdy \\
V1.2 &= \int_{0}^{C4(x)} \int_{\sqrt{r^2-(x+\frac{l}{2})^2}}^{\sqrt{r^2-(x-\frac{l}{2})^2}} dxdy \\
V2.1 &= \int_{C2(x)}^{0} \int_{D_y - \sqrt{r^2-(x-D_x)^2}}^{\sqrt{r^2-(x-\frac{l}{2})^2}} dxdy \\
V2.2 &= \int_{0)}^{C4(x)} \int_{D_y - \sqrt{r^2-(x-D_x)^2}}^{\sqrt{r^2-(x+\frac{l}{2})^2}} dxdy \\
V3.1 &= \int_{C2(x)}^{0} \int_{\sqrt{r^2-(x-\frac{l}{2})^2}}^{\sqrt{r^2-(x+\frac{l}{2})^2}} dxdy \\
V3.2 &= \int_{C1(x)}^{C2(x)} \int_{D_y - \sqrt{r^2-(x-D_x)^2}}^{\sqrt{r^2-(x+\frac{l}{2})^2}} dxdy
\end{aligned}
$$

  There is a closed form integration for $\int \sqrt{ax^2 + bx + c}dx$

$$
\begin{aligned}
& \int \sqrt{ax^2 + bx + c}dx \\
= & \frac{2ax + b}{4a}\sqrt{ax^2 + bx + c} - \frac{b^2 - 4ac}{8a}\int \frac{1}{\sqrt{ax^2 + bx + c}}dx \\
& \int \frac{1}{\sqrt{ax^2 + bx + c}}dx \\
= & \frac{1}{\sqrt{a}}ln(2ax + b + 2\sqrt{a}\sqrt{ax^2 + bx + c}) when a > 0 \\
= & \frac{1}{\sqrt{-a}}arcsin(\frac{-2ax - b}{\sqrt{b^2 - 4ac}}) when a < 0, b^2 - 4ac > 0
\end{aligned}
$$

  Then the total area covered by region V is the sum of all 6 parts

  V=V1.1+V1.2+V2.1+V2.2+V3.1+V3.2

- Step 3: the probability of at least 1 node in the region
  The probability of at least 1 node in region V can be calculated as:

  prob= $\min(\rho \times V, 1)$