

On the Capacity of Dynamic Spectrum Access Enabled Networks

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Abstract—A network capacity analysis for multi-hop wireless networks enabling Dynamic Spectrum Access (DSA) is presented. DSA techniques enable frequency reuse in both time and space without causing destructive interference to incumbents. This paper presents a methodology for finding a theoretical capacity upper-bound of DSA enabled networks based on both: Incumbent’s frequency occupancy and topology information. This practical and easy to understand methodology is envisioned to help in the design of future DSA Systems under simple ownership with non interference easement or the commons regime.

I. INTRODUCTION

The wireless communications “industry” is poised for significant technical advances that will dramatically impact the number of wireless “users”, applications, services and devices. Current systems are hampered by regulatory policy that does not account for low or dynamically changing utilization patterns. Thus, leading to sub-optimal RF spectrum utilization. Regulatory changes are needed to make frequency assignments more flexible and leverage frequency utilization patterns by permitting dynamic spectrum allocation (DSA). Frequency agile radios and new policy-based algorithms are needed to support DSA enabled “spectrum management.”

Development of reliable policy-based DSA methods that support new systems co-existing with incumbent (non DSA) systems is needed to increase wireless network capacity. Cognitive radios and other smart mobile devices based on the work of Mitola [11] enable designers and providers of public and private wireless systems to achieve this. Given new, forward-thinking policies, the usable capacity of wireless networks should achieve significantly increased utilization, thus, supporting greater capacity. DSA enabled devices are envisioned as policy-driven systems that sense environmental attributes and opportunistically share frequency bands without causing interference to incumbent systems.

The Defense Advanced Research Projects Agency (DARPA) has been leading the development of a new “wireless architecture.” The project is the “Next Generation Wireless (XG) Program.” A series of RFCs [6], [5] and [7] specify DARPA’s vision, approach, and the technical functionality and required for XG frequency agile devices. The Federal Communications Commission (FCC), which is the government agency responsible for spectrum allocation for public and private use in the U.S. is taking steps toward removing regulatory barriers. The FCC’s objective is to facilitate development of secondary markets in spectrum usage rights.

As the dawn of DSA approaches, one of the significant questions that arise is how effective will it be? Re-stated: what is the maximum theoretical capacity a wireless network can achieve using DSA enabled devices? In this paper a practical methodology for answering this question is developed based in-part on the “deferral set” concept introduced by Fang in [3] and a model for the state and attributes of incumbent devices whose spectrum demand is modelled as a queuing system. The research presented in this paper has both theoretical and practical significance that will contribute to future DSA system design. Most significant is the parametric model that is readily adaptable to future DSA implementations. Hence, it provides an important marketing tool that determines the amount of spectrum that can be assigned to secondary markets.

The remainder of this paper is organized as follows: Section-II presents a model for the characterization of RF spectrum occupancy dynamics. In Section-III the main theme of this paper is presented as the problem of finding the capacity of a reconfigurable multihop wireless network (RWIN), e.g. an ad hoc network. The capacity of DSA enabled networks is determined for conservative and interference tolerant regimes. Finally, using a similar methodology to that developed by Fang and McDonald [4] the section presents analysis of a theoretical upper-bound for multi-hop wireless networks that use IEEE 802.11 for media access. Conclusions of this work are presented in Section-IV.

II. SPECTRUM DYNAMICS

Characterization of spectrum occupancy dynamics is the first step in the analysis of DSA enabled wireless network capacity. Three factors that allow for frequency reuse, namely, frequency (band), time and space are needed for a generalized analysis. The remainder of this section presents a model characterizing the discrete division of the RF spectrum. Next the concept of the “interference region” is introduced in order to define a policy for spectrum based on whether a node uses DSA or operates in a fixed frequency band (FSA). Finally, a model is developed to characterize the frequency utilization of FSA nodes and determine the maximum network capacity of a heterogeneous network of FSA nodes operating in the same space with DSA enabled nodes.

A. Division of the RF Spectrum

Groups of licensed users occupy only a portion of the RF spectrum. The spectrum is discretized and divided into $n_f + 1$ frequency intervals or bands. Each of the n_f bands are assigned to P_i licensed "users", hereafter referred to as *primary nodes*; the remaining spectrum of arbitrary bandwidth is unassigned.

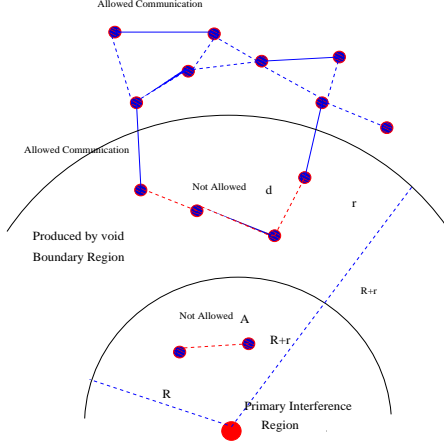


Fig. 1. Exclusive Primary Occupancy Approach

B. Interference Regions

An *interference region* is associated with each primary node. Based on the assumption of omnidirectional antennae and a simple two wave ground propagation channel model the interference region is modelled as a circular space of radius R , which defines the threshold power loss and SNR. Two policies are considered for DSA: (1) Exclusive frequency occupancy for primary nodes, and (2) interference tolerant. Using the first policy DSA nodes located within an interference region *are not* permitted to transmit on the primary's assigned frequency band *if* the primary is transmitting or receiving in that band. According to the second policy DSA enabled nodes *are* permitted to transmit on the primary's assigned frequency band subject to the constraint that its signal power at the primary does not cause destructive interference. Figures-1 and 2 illustrate the "exclusive occupancy" and "interference tolerant" approaches respectively.

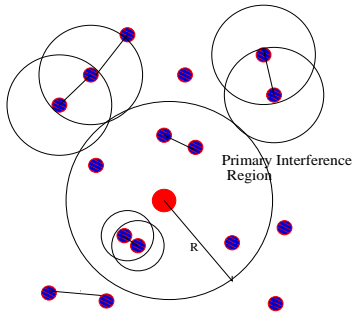


Fig. 2. Interference Tolerant Approach

C. Incumbent's Frequency Utilization

Assume that N DSA nodes are distributed uniformly over a network of primaries such that an average of p_i DSA nodes are located within each primary node's interference region. Capacity analysis is completed for each assigned frequency band by investigating all DSA nodes able to engage in single-hop communications with one or more adjacent DSA nodes. At the i^{th} frequency band the number of *active* primary nodes ranges from 0 to P_i according to traffic characteristics. Assume independence between each primary's activation process and the distribution of its active time. The $G/G/m/m$ m -server loss queueing system provides a convenient model for determining frequency utilization. The system state represents the number of active primary nodes. If the system can be shown to be memoryless and ergodic with independent and stationary increments, the system is Markov and can be characterized according to $M/M/m/m$ steady-state results. The k_i primary nodes activate according to a homogenous Poisson process with activation rate λ_i . The probability distribution of the time any node remains active (transmitting or receiving) is exponentially distributed with mean of $1/\mu_i$ seconds. The spectrum utilization for frequency band i is given by $\rho_i = \lambda_i/\mu_i$. For each of the i frequency bands the expected number of active primaries is K_i and the average number of communication enabled DSA nodes is N_i :

$$K_i = \rho_i \left(1 - \frac{1}{\sum_{k_i=0}^{P_i} \rho_i^{k_i - P_i} \left(\frac{P_i!}{k_i!} \right)} \right) \quad (1)$$

$$N_i = N - K_i \times p_i \quad (2)$$

The maximum network capacity is found by taking the sum of all "subnetwork" capacities, wherein each subnetwork corresponds to a different frequency band. To find the capacity of each subnetwork we follow the methodology proposed in [4]. Exclusive occupancy interference regions cause the redistribution of concurrently active DSA links whenever the primary is active; These regions shall be referred to as *void regions*. For the interference tolerant case DSA enabled nodes will reduce their transmission powers so water filling techniques through power control can be applied [9].

III. MULTI-HOP WIRELESS NETWORK CAPACITY

A. Network Topology

The network topology is defined by the connectivity among nodes. Transmission power and bandwidth are critical factors affecting connectivity that may dynamically change network topologies. The topology of a multi-hop wireless network can be represented as a graph $G(N, L)$ that contains a set of nodes N and a set of links L . Each link in L corresponds to an ordered pair of nodes, say (i, j) , and indicates that transmission from i can be received at j . Only bi-directional links are considered, hence the existence of link (i, j) implies the existence of link (j, i) . The capacity, however, must only be counted once for each pair.

B. Network Capacity

The methodology to find network capacity is based on the concept of collision-free sets, which reflect the channel contention mechanism of a multi-hop environment. A collision-free set, as defined in [1] is a set of links that can carry packets simultaneously with no collisions at the receiving ends of the links. Arbitrary routing and scheduling are assumed along with the characteristics of the contention mechanism. If node i transmits a packet, that packet will be received by node j if and only if:

- 1. There is a link from i to j [i.e. , $(i, j) \in L$].
- 2. No other node k for which $(k, j) \in L$ is transmitting while i is transmitting.
- 3. Node j itself is not transmitting while i is transmitting.

Given an active transmission between a pair of adjacent nodes all other neighbors of either node must defer any transmission. For other contention-based schemes the distance in hops from the transmission that must defer may increase beyond one-hop. The deferral set with respect to an active link is defined as the group of nodes and links which must defer communications until the link is no longer active.

C. The Maximum Matching Problem

The computation of the maximum capacity collision-free set is an NP-complete problem. Finding the number of links in all existing collision-free sets can be transformed into the well-known maximum matching problem that has been shown to be NP-hard. The maximum matching problem consists on finding the largest subset of edges included in L such that no pairs of L have a vertex in common. If each of the links has a different capacity depending on distance between nodes or characteristics of the medium, the problem of finding the number of concurrent communications becomes the maximum weighted matching problem.

A number of algorithms exist for approximating the solution to the maximum matching problem. It is not the aim of this work to develop novel approximation techniques, rather the Polynomial Time Approximation Scheme for Maximum Weighted Independent Sets (MWIS) proposed by Erlebach [12] based on the shifting strategy used by Hochbaum [10] and Hunt et al. [2] is adopted for this work. This algorithm results in an approximate solution within $(1 - 1/k)OPT$ where k is an integer greater than zero. The set of links are partitioned into multiple levels. Using Dynamic programming the maximum matching sets obtained from all higher levels are used to find the maximum matching at the next lower level.

D. DSA: Theoretical Capacity Upper-bound

A DSA network with N nodes independently and uniformly distributed is considered; nodes have a fixed transmission range r and the mean number of neighbors for the N nodes is n_{avg} . In section II-C the creation of void regions is discussed. Based on [3], it is known that the higher capacity nodes of ad-hoc networks tend to be found in the boundary zones due to the lower number of neighbors, and, therefore less channel contention. The number of nodes in boundary zones

grows with the number of void regions. Given that nodes are distributed uniformly over a region, the capacity of the network has two components: 1) the number of concurrently active links in the boundary zones, and 2) the number of concurrently active links within the network; including the number of concurrently active links inside the interference regions when considering the interference tolerant regime.

$$MM(G(N, L)) = N_B + d \times Area_I$$

In the expression given above MM is the maximum matching, d is the number of nodes per square unit and $Area_I$ is the area limited by the boundary zone. When opening a void, a new topology represented by the graph $G'(N', L')$ is induced, where N' and L' are subsets of N and L respectively. The graph contained inside the primary interference region is represented by $g'(n', l')$. The new topology has another boundary zone that is created around the void. Hence, the capacity of the network is now given by the sum of concurrently active links in the original boundary region, around the void and in between the previous two.

$$MM(G'(N', L')) = N_B + d \times Area_{II} + N_{B'}$$

Where $Area_{II}$ is the area inside the void; and the number $N_{B'}$ is bounded by the maximum number of independent concurrent links around the void. It can be shown from Fig. 3 that $N_{B'}$ is a function of the R/r ratio and equal to $C = 2\pi/\cos^{-1}(1 - (1/(2(R/r + 1)^2)))$. Thus, it can be concluded that the maximum matching of G' is bounded by:

$$MM(G'(N', L')) = O(MM(G(N, L)) - MM(g(n, l)) + C) \quad (3)$$

From the Table-I it can be seen that it is unnecessary to

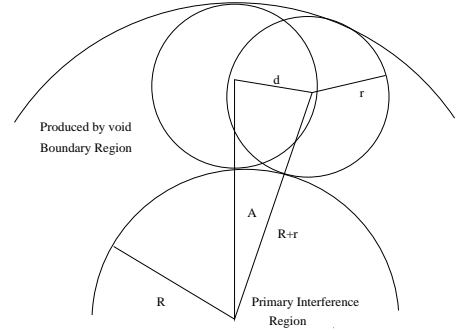


Fig. 3. Maximum number of communications around the void

re-compute the Maximum matching of the resulting topology after a void appears, but it is only necessary to find the maximum matching inside it. For the case of $n_{avg} = 5$, it is necessary to have at least 150 nodes if the ratio R/r goes to 2 at most.

It is assumed that the DSA network is large enough to retain all its topology characteristics even when isolating a portion of it. The average number of DSA nodes inside of one primary interference region is given by $p_{avg} = (R/r)^2 \times (n_{avg} + 1)$

| N | $R/r=1$ | $R/r=2$ | $R/r=3$ | $R/r=5$ |
|-----|---------|---------|---------|---------|
| 50 | 40% | 700% | 1% | 1% |
| 100 | 5.7% | 19.2% | 5.5% | 30% |
| 150 | 9.6% | 13.9% | 13.9% | 100% |
| 200 | 1.6% | 0% | 20% | 14% |
| 300 | 0.5% | 0.4% | 9.2% | 11% |

TABLE I

APPROXIMATION ERROR BETWEEN WHEN APPLYING THE EQUATION (3) ABOVE AND $n_{avg} = 5$

and the average number of neighbors in g' is also n_{avg} . From equations (1) and (2), assuming the total exclusive primary frequency approach, the average number of DSA nodes is known from [3]. The average area covered by one-hop neighbors of the communication pair is approximately $(\frac{4\pi}{3} + 0.068)r^2$, thus the number of concurrent communication sets in the i^{th} frequency interval is:

$$\frac{\text{Area}_{\text{network}}}{\text{Area}_1} = \frac{\pi}{\frac{4\pi}{3} + 0.068} \frac{N - p_{avg} \rho \left(1 - \frac{1}{\sum_{k=0}^{P_i} \rho^{k-P_i} \left(\frac{P_i!}{k!}\right)}\right)}{n_{avg} + 1}$$

For the interference tolerance case, the number of level-1 interference sets in the primary interference region increases. Nodes inside the interference region have shorter communication range that becomes increasingly shorter for nodes that are closer to the primary node. The interference region is partitioned into n concentric rings with radius δ_l such that the transmission range for the nodes in between circumference with radius δ_l and δ_{l+1} has radius $r_{\delta_n} = \delta_{l+1} - \delta_l$. The total number of level-1 interference set is given by:

$$S_{tol} = \sum_{\delta_l=\delta_1}^{\delta_n-1} \frac{2\pi}{\arccos\left(1 - \frac{\Delta\delta_l}{2(\delta_l + \Delta\delta_l)^2}\right)}$$

Assuming that the modulation scheme used by the DSA allows link capacity to be proportional to its bandwidth, the network capacity at the i^{th} frequency interval is bounded by:

$$BW_i \left(\frac{\pi}{\frac{4\pi}{3} + 0.068} \frac{N - p_{avg} \rho \left(1 - \frac{1}{\sum_{k=0}^{P_i} \rho^{k-P_i} \left(\frac{P_i!}{k!}\right)}\right)}{n_{avg} + 1} + \varepsilon \times S_{tol} \right) \quad (4)$$

where $\varepsilon = 0$ in the case of exclusive frequency occupancy by primaries and $\varepsilon = 1$ if the interference tolerant regime is used. The total capacity of the DSA network is bounded by the expression:

$$\sum_{i=0}^{n_f} BW_i \left(\frac{\pi}{\frac{4\pi}{3} + 0.068} \frac{N - p_i \rho \left(1 - \frac{1}{\sum_{k=0}^{P_i} \rho^{k-P_i} \left(\frac{P_i!}{k!}\right)}\right)}{n_{avg} + 1} + \varepsilon \times S_{tol} \right) \quad (5)$$

Definition 1: Throughput is defined as the time average of the number of bits per second that can be transmitted by every node to its destination.

Considering only unicast traffic, in the network, at any time, there is at most $N/2$ concurrent communication sessions in the network. The shortest path length will be one hop and the longest path length is bounded by the diameter of the network \sqrt{N} . Hence, the average hop count for communicating pairs is approximately $\frac{\sqrt{N}}{2}$.

If a common inter-arrival rate λ and service rate μ is assumed for all groups of primary nodes in their respective frequencies as well as a common number of primary nodes, then the DSA network capacity is bounded by the expression:

$$\frac{C_1 \times BW_{TOTAL}}{\sqrt{N}} - \frac{C_2 \times K(\rho, P) \times BW_p}{N\sqrt{N}} \times \left(\frac{R}{r}\right)^2 + C_3 \times S_{tol}$$

where P is the number of primary nodes per frequency interval. For a value of $P \geq 3$, $K(\rho, P) = \rho \times \left(1 - \frac{\rho^P e^{-\rho}}{P!}\right)$. n_f is the number of frequency intervals, BW_{TOTAL} is the total bandwidth (assigned and not assigned) BW_p is the bandwidth size for each frequency interval, $K(\rho, P)$ is the average number of active primaries per frequency slot and C_1 , C_2 and C_3 are constants. It is found that the resulting capacity upper-bound is the difference between the network capacity working with the total bandwidth minus the capacity of the remaining nodes affected by the activation of primaries.

IV. CONCLUSIONS

In this paper, two approaches for spectrum management of DSA systems are presented: a conservative and interference tolerant approach. For Characterization of RF spectrum dynamics is characterized for both case. A novel aspect of the analysis is that it considers temporal behavior, frequency division and spacial diversity for the first time. The methodology is efficient and easy to understand. The results validate Gupta/Kumar [8], which lends credibility to the theoretical models. The importance, however, of the new model is its practical application in performance-based design of future DSA networks.

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