On the Capacity of Dynamic Spectrum Access Enabled Networks

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Abstract— **A network capacity analysis for multi-hop wireless networks enabling Dynamic Spectrum Access (DSA) is presented. DSA techniques enable frequency reuse in both time and space without causing destructive interference to incumbents. This paper presents a methodology for finding a theoretical capacity upper-bound of DSA enabled networks based on both: Incumbent's frequency occupancy and topology information. This practical and easy to understand methodology is envision to help on the design of future DSA Systems under simple ownership with non interference easement or the commons regime.**

I. Introduction

The wireless communications ”industry” is poised for signifcant technical advances that will dramatically impact the number of wireless ”users”, applications, services and devices. In large part the expected dramatic change has been hampered by the regulation and management of radio spectrum. Today, policy changes are needed to open the doors to make certain frequency assignments more flexible by permitting dynamic spectrum allocation (DSA). Fast switching frequence agile radios and new policy-based algorithms and protocols are needed to support DSA and ”spectrum management” (consider the analogy to mobility management). The concept of ”spectrum shortage” is better expressed as ”inefficient spectrum ulization.”

The objective is to develop reliable policy-based DSA methods that support new systems and co-exist with incumbant (non DSA) systems. Congitive radios and other smart mobile devices based on the work of Mitola [10] will enable designers and providers of public and private wireless systems and services to achieve this objective. Given new, forward-thinking policies for spectrum allocation and use the usable capacity of wireless networks will achieve significantly higher utilization, this supporting greater capacity. DSA enabled devices are envisioned as policy-driven systems that sense enviromental attributes and opportunistically share frequency bands without causing interference to incumbent systems.

In anticipation of regulatory change, The Defense Advanced Research Projects Agency (DARPA) has been leading the development of a new ”wireless architecture.” The on-going project is is the ”Next Generation Wireless (XG) Program.” A series of available RFCs [6], [5] and [7] pose the DARPA vision, approach, and technical functionality regulatory policy required of XG frequency agile devices. The Federal Communications Commission (FCC), which is the government agency responsible for spectrum allocation and regulation in the U.S. is taking steps toward removing the current regulatory barriers. The FCC's objective is to facilitate the development of secondary markets in spectrum usage rights among the many competing Wireless service providers.

The research on DSA has hit a critical mass; however regulatory barriers have prevented much implementation. As the dawn of DSA approaches, one of the significant questions that arise is how effective can it become? Re-stated: what is the maximum theoretical capacity a wireless network can achieve using DSA enabled devices? In this paper a practical practical methodology for answering this question is developed based on the “deferral set” concept introduced by Fang in [3] and the a model for the presense and attributes of incumbant or legacy devices whose FR spectrum demand is effectively a queuing system. The research presented in this paper has both theoretical and practical significance, which will contribute to future DSA system design. Most significant is the parametric model that is readily adaptable to future DSA implementations. Hence, this model provides an important marketing tool that determines the amount of spectrum that can be assigned to secondary markets.

The remainder of this paper is organized as follows: Section-II a model for the characterization of RF spectrum occupancy dynamics. The main theme of this paper is presented as the problem of finding the capacity of a reconfigurable multihop wireless network (RWIN), e.g. an ad hoc network in Section-III. The capacity of DSA enabled networks is determined for fixed channel and frequency reuse cases. Finally, the section presents analysis of a theoretical upper-bound for multi-hop wireless networks utilizing IEEe 802.11 for MAC based on techniques developed in [4]. Conclusions and discussion of future research are presented in Section-IV.

II. Spectrum Dynamics

Characterization of spectrum occupancy dynamics is the first step in the analysis of DSA enabled wireless network capacity. Three factors that allow for frequency reuse, namely, frequency (band), time and space are needed for a generalized analysis. The remainder of this section presents a model characterizing the discrete division of the RF spectrum. Next the concept of the ”interference region” is introduced in order to define a policy for spectrum based on whether a node uses DSA or operates in a fixed frequency band (FSA). Finally, a model is developed to characterize the frequency utilization of FSA nodes and determine the maximum network capacity of a heterogenous network of FSA nodes operating in the same space with DSA enabled nodes.

A. Division of the RF Spectrum

Groups of licensed users occupy only a portion of the RF spectrum. The spectrum is discretized and divided in $n_f + 1$ frequency intervals or bands. In a given geographic region a n_f bands are assigned to $P_i \leq n_f$ licensed "users", hereafter refered to as *primary nodes*; the $n_f - P_i + 1$ remaining frequency bands are unassigned.

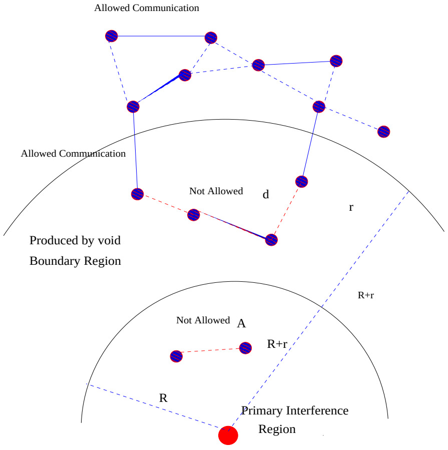

Fig. 1. Exclusive Primary Occupancy Approach

B. Interference Regions

Every primary node is associated with an *interference region* that can be approximated as a circular space of radius R given an omnidirectional antenna and a free-space (like) environment. Two policies are possible for DSA: (1) Exclusive frequency occupancy for primary nodes, or (2) interefece tolerant frequency re-use. Using the first policy DSA nodes located within an interference region *are not* permitted to transmit on the primary's assigned frequency band if the primary is transmitting or receiving in that band. According to the second policy DSA enabled nodes *are* permitted to transmit on the primaty's assigned frequency band subject to the constraint that it's signal power at the primary does not cause destructive interference. Assume that N DSA nodes are distributed uniformly over a network of primary's such that there are an average of P_i DSA nodes are located within each primary node's interference region.

C. Incumbent's Frequency utilization

Based on frequency allocation a capacity analysis is undertaken for each assigned frequency band. At the i^{th} frequency interval, a number of primary nodes going from 0 to P_i can be active and any of both approaches listed above can be taken. The network capacity analysis is done for all nodes able to communicate. The primary interference region and DSA transmission range has a direct impact on the capacity

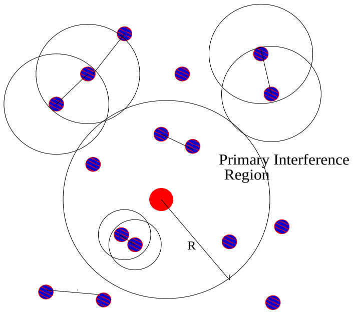

Fig. 2. Interference Tolerance Approach

of the DSA network. Having independent distributions for the activation and service time for all P_i nodes, the frequency utilization can be modelled as a $G/G/m/m$ queueing system with m-server loss system where each state resembles the number of active primary nodes. Assuming that the k primary nodes become active according to a Poisson process with rate (λ_i), and the probability distribution of the service time is exponential with mean $(1/\mu_i)$ seconds. The expected state is:

$$K_i \quad = \quad \rho\left(1 - \frac{1}{\sum_{k=0}^{P_i} \rho^{k-P_i} \left(\frac{P_i!}{k!}\right)}\right) \tag{1}$$

at the i^{th} frequency interval and the average number of DSA nodes is

$$N_i \quad = \quad N - K_i \times p_i \tag{2}$$

where $\rho = (\dfrac{\lambda}{\mu})$ and p_i is the average number of DSA nodes inside the interference region of a primary node working in the i^{th} frequency interval.

Finally, the maximum network capacity is the sum of all computed subnetworks capacities induced by the division of the RF spectrum. To find the capacity of each subnetwork we follow the methodology proposed in [4]. In the case of exclusive primary nodes frequency occupancy the inclusion of void regions produce a redistribution of the concurrent active links. On the other hand in the case of interference tolerance, DSA enabled nodes will reduce their transmission powers so water filling techniques through power control can be applied.

III. MULTI-HOP WIRELESS NETWORK CAPACITY

A. Network Topology

The network topology is defined by the connectivity among nodes therefore the power and bandwidth transmission for each link are important. The vector $P = p_u, bw_u$, where p_u is node's u transmit power allocated for $bw(u,v)$ establish the communication between nodes u and v. Two nodes are neighbors if the distance between them is less than or equal to the maximum distance (range) for which the signal strength at the receiver exceeds the amount of thermal noise present in the allocated bandwidth.

The topology of a multi-hop wireless network can be represented as a graph $G(N,L)$ that contains a set of nodes N and a set of links L. Each link in L corresponds to an ordered pair of nodes, say (i, j), and indicates that transmission from i can be heard at j. In some situations, node j might be able to hear node i but node i cannot hear node j. In such a case $(i, j)\ L$ but $(j,i) \in L$. For this work, we consider bi-directional communications.

B. Network Capacity

The methodology to be used is based on the concepts of collision free sets which reflect channel contention mechanism of a multi-hop environment. The analysis here is based on arbitrary routing and scheduling algorithm coupled with the characteristics of a contention mechanism of a dynamic frequency allocation and control protocol. The total maximum instantaneous capacity is the sum of all maximum capacities over all frequency intervals. In this multi-access scheme, if node i transmits a packet, that packet will be correctly received by node j if and only if:

- 1. There is a link from i to j [i.e. , $(i, j) \in L$], and
- 2. No other node k for which $(k, j) \in L$ is transmitting while i is transmitting, and
- 3. j itself is not transmitting while i is transmitting.

With these conditions, given an active transmission between a pair of adjacent nodes all directed neighbors of the destination of the active transmission must defer any transmission. Using the definition of collision-free sets from [1], a collision-free set is a set of links that can carry packets simultaneously with no collisions at the receiving ends of the links. Different contention mechanisms can be applied if the distance in hops between active transmissions is increased. As links are considered bidirectional the deferral set with respect to an active link is defined as the group of nodes and links which are one hop away the communication pair.

C. The Maximum Matching Problem

The computation of the collision free set with maximum capacity is a NP problem. To find the number of links in all existing collision free sets corresponds to solving the Maximum Matching Problem that has been shown to be a NP-hard problem. The Maximum Matching problem consists on finding the largest subset of edges included in L such that no pairs of L have a vertex in common. If each of the links has a different capacity depending on distance between nodes or characteristics of the medium, the problem of finding the number of concurrent communications becomes the maximum weighted matching problem.

There exist different techniques to find approximations to the maximum matching problem and it is not the aim of the present work to develop novel approximations to solve it. The technique used in this paper is based on the work done by Erlebach [11] where he proposed an elegant Polynomial Time Approximation Scheme for Maximum Weighted Independent Sets (MWIS) based on the shifting strategy used by Hochbaum [9] and Hunt et al. [2].

The algorithm is the adapted to find a solution of the maximum matching problem resulting in a solution with an approximation within $(1-1/k)OPT$ where k is an integer greater than zero. The set of links are partitioned on different levels and with the use of Dynamic programming, the maximum matching coming from smaller levels can be taken into account to find the maximum matching at a lower level (that includes nodes with larger transmission ranges).

D. DSA: Theoretical Capacity Upper-bound

A DSA network with N nodes independently and uniformly distributed is considered; nodes have a fixed transmission range r and each node has an average n_{avg} neighbors. In section II-C the creation of void regions is discussed. Based on [3], it is known that the higher capacity of ad-hoc networks tents to stay in nodes that are in the boundary zones due to the lower number of neighbors and therefore less competition for the channels. Expecting the same pattern, the number of nodes in boundary zones grows with the number of void regions. If nodes are distributed uniformly over a region, the capacity of the network has two components: a)the number of active concurrent communications on the boundary zones, and b)The number of active communications inside the network. c) The number of active communications inside the interference region for the case of using the tolerant interference approach (as this value is independent of the previous two, it will be considered later on this analysis).

$$MM(G(N,L)) = N_B + d \times Area_I$$

where MM is maximum matching d is the density of nodes and $Area_I$ is the area surrounded by the boundary zone. When opening a void, a new topology represented by the graph $G'(N', L')$, where N' and L' are subsets of N and L respectively, is induced. Also the graph contained inside the primary interference region can be represented by $g'(n', l')$. The new topology has another boundary zone that is created around the void. Hence, the capacity of the network is now given again by the active communications original boundary zone, the number of active communications in boundary zone around the void and finally the number of active communications everywhere excluding the two previous boundary zones.

$$MM(G'(N', L')) = N_B + d \times Area_{II} + N_{B'}$$

Where $Area_I I$ is the area inside the void; as the number $N_{B'}$ is bounded by the maximum number of independent concurrent links around the void, it can be shown from Fig. 3that that number is a function of the R/r ratio and equal to $C = 2\pi/\cos^{-1}(1-(1/(2(R/r+1)^2)))$. Then, it can be concluded that the maximum matching of G' is bounded by:

$$MM(G'(N', L')) = O(MM(G(N,L)) - MM(g(n,l)) + C) \quad (3)$$

¿From the Table I it can be seen that is not needed to recompute the Maximum matching of the resulting topology after a void appears but it is only necessary to find the maximum matching inside it. For the case of having $n_{avg} = 5$, it is necessary to have as much as 150 or more nodes if the ranges ratio goes to 2 at the most; then, the primary interference region contains a topology with similar characteristics to the topology when voids are absent.

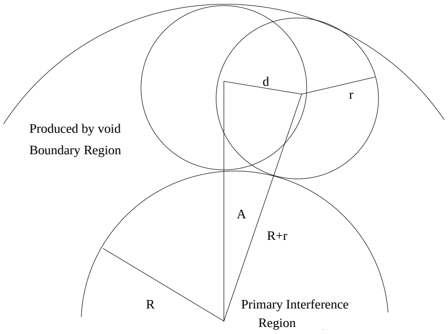

Fig. 3. Maximum number of communications around the void

N	$R/r=1$	$R/r=2$	$R/r=3$	$R/r=5$
50	40%	700%	1%	1%
100	5.7%	19.2%	5.5%	30%
150	9.6%	13.9%	13.9%	100%
200	1.6%	0%	20%	14%
300	0.5%	0.4%	9.2%	11%

TABLE I

APPROXIMATION ERROR BETWEEN WHEN APPLYING THE EQUATION (3) ABOVE AND $n_{avg}=5$

It is assumed that the DSA network is large enough to keep all its topology characteristics even in a portion of it, the average number of DSA nodes inside of one primary interference region is given by $p_{avg}=(R/r)^2\times(n_{avg}+1)$ and the average number of neighbors in g' is also n_{avg}. From equation (1) and (2), and assuming the case of the total exclusive primary frequency approach the average number of DSA nodes is known and from [3] the average area covered by one-hop direct neighbors of the communication pair is about $(\frac{4\pi}{3}+0.068)r^2$, thus the number of concurrent communication sets in the i^{th} frequency interval is:

$$\frac{\text{Area}_{\text{network}}}{\text{Area}_1}=\frac{\pi}{\frac{4\pi}{3}+0.068}\frac{N-p_{avg}\rho(1-\frac{1}{\sum_{k=0}^{P_i}\rho^{k-P_i}(\frac{P_i!}{k!})})}{n_{avg}+1}$$

For the case of the interference tolerance in the primary interference region, the amount of level-1 interference sets increases. Nodes inside the interference region will have smaller communication range that gets smaller as nodes are closer to the primary node. The interference region is divided in n concentric circumferences of radius δ_l such that the transmission range for the nodes in between circumference with radius δ_l and δ_{l+1} has radius $r_{\delta_n}=\delta_{l+1}-\delta_l$. The total number of level-1 interference set is given by:

$$S_{tol}=\sum_{\delta_l=\delta_1}^{\delta_n-1}\frac{2\pi}{\arccos(1-\frac{\Delta\delta_l}{2(\delta_l+\Delta\delta_l)^2})}$$

Assuming that the modulation in used by the DSA allows link capacity to be proportional to its bandwidth, the network capacity at the i^{th} frequency interval is bounded by:

$$BW_i(\frac{\pi}{\frac{4\pi}{3}+0.068}\frac{N-p_{avg}\rho(1-\frac{1}{\sum_{k=0}^{P_i}\rho^{k-P_i}(\frac{P_i!}{k!})})}{n_{avg}+1}+\varepsilon\times S_{tol}) \qquad (4)$$

where ε is "$0''$ if the exclusive frequency occupancy by primaries is follow and "$1''$ if it is the interference tolerance approach. The total capacity of the DSA network, can be bounded by the expression:

$$\sum_{i=0}^{n_f}BW_i(\frac{\pi}{\frac{4\pi}{3}+0.068}\frac{N-p_i\rho(1-\frac{1}{\sum_{k=0}^{P_i}\rho^{k-P_i}(\frac{P_i!}{k!})})}{n_{avg}+1}+\varepsilon\times S_{tol}) \qquad (5)$$

Definition *1:* Throughput is defined as the time average of the number of bits per second that can be transmitted by every node to its destination.

Given that only unicast communications are considered, in the network, at any time, there is at most $N/2$ communications in network, in general, the shortest path length will be just one hop and the longest path length will equal to the diameter of the network $\sqrt{N}$, so the average hop count for a communication will be around $\frac{\sqrt{N}}{2}$.

If a common inter-arrival rate λ and service rate μ is assumed for all groups of primary nodes in their respective frequencies as well as a common number of primary nodes, then the DSA network capacity is bounded by the expression:

$$\frac{C_1\times BW_{TOTAL}}{\sqrt{N}}-\frac{C_2\times K(\rho,P)\times BW_p}{N\sqrt{N}}\times(\frac{R}{r})^2+C_3\times S_{tol}$$

where P is the number of primary nodes per frequency interval. For a value of $P\geq 3$, $K(\rho,P)=\rho\times(1-\frac{\rho^P e^{-\rho}}{P!})$. n_f is the number of frequency intervals, BW_{TOTAL} is the total bandwidth (assigned and not assigned) BW_p is the bandwidth size for each frequency interval, $K(\rho,P)$ is the average number of active primaries per frequency slot and C_1, C_2 and C_3 are constants. Basically it is found that the new capacity upper-bound is the composition of the upper-bound in a ad hoc network working in the total bandwidth size minus the capacity introduced by all nodes affected by the instantaneous activation of primary nodes.

IV. "SUMMARY AND CONCLUSIONS"

In this paper, two approaches for DSA systems are presented: a conservative and a interference tolerance. For them, a complete characterization of both RF spectrum dynamics is developed taking into account temporal, frequency and space diversity for the first time. The methodology presented is simple and easy to understand and validate the results obtained in [8]. This information is useful to measure the performance for future DSA designs.

V. "FUTURE WORK"

Finally, we propose future work to investigate several real-world spectrum assignment to primary nodes. In order to make an study on the feasibility of DSA systems in a band-per-band basis. These results will provide an insight into the dependency of system performance on the assignment employed and provide an insight into the class of techniques that should be used under different scenarios of primary nodes' traffic load and topology.

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