The Effect of Multi-Radio Nodes on Network Connectivity – A Graph Theoretic Analysis

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***Abstract*—We analyze the gain in network connectivity that is obtained by implementing multiple radio interfaces in the nodes. The multi-radio nodes can act as if the network had effectively multiple physical layers. We model such a network topology by a *multigraph* and capture the gain by introducing the novel graph theoretic concept of the *multigraph advantage*. When applied to connectivity, it is the surplus of connectivity over the sum of the individual connectivities, as we put together several graphs to form a "multigraph sum". We prove in a random graph model that this results in a strict super-additive behavior, always yielding multigraph advantage. Moreover, for the most important density range, called moderately dense regime, we prove that the gain grows to infinity with the graph size and the percentage (relative) gain remains constant and does not vanish with growing network size.**[1]

I. Introduction and Motivation

The growing importance of wireless networks is a major and lasting trend in the networking landscape. It also comes hand in hand, however, with the increasing *diversity* of wireless networking solutions and standards. Some of them are already widely used, such as variants of IEEE 802.11 wireless LANs (802.11a, 802.11b, 802.11g), others are emerging, such as IEEE 802.15 personal area networks, Zigbee, Bluetooth, broadband wireless (IEEE 802.16), a variety of sensor networking solutions, etc.

Another well visible trend is that radio interfaces are rapidly getting more and more inexpensive and physically small. It is now technically and economically quite feasible to equip wireless network nodes with several radio transmitters/receivers. This creates an environment where the network effectively has *multiple physical layers*. For example, it is already quite common that a laptop or a PDA has both an IEEE 802.11 card and a Bluetooth interface. Given the tendency of decreasing price and shrinking physical size, it is quite likely that the trend of having multiple radio interfaces will get even more prevalent in the future. Thus, many of the ubiquitous wireless network nodes will likely be capable of operating with multiple physical layers. A few examples, present and future:

- Multi-radio nodes (e.g., laptops with multiple radio interfaces, as mentioned above).
- Multi-channel radio environments that logically act as multiple radios. Note that the IEEE 802.11 standard defines multiple channels that are only partly utilized today.
- Multiple antenna systems that implement several independent channels via sophisticated physical layer techniques, such as beamforming, Space Division Multiple Access (SDMA), cooperative coding, multiple-input/multiple-output (MIMO) systems, etc.
- Combination of radio and infrared interfaces in a node.
- Utilization of low-power wireless technologies, such as RFID solutions with energy harvesting, various wireless sensor platforms, etc.
- Combinations of radio and other potentially possible wireless transmission technologies, such as free space optical transmission, laser beams, etc.

Thus, while the technical *possibility* of multiple physical layers is already quite clear today, it is much less obvious how can it be efficiently *utilized* to gain significant improvement in the overall network performance. Or, from the practical/economical point of view, the ultimate question is: Will the multi-radio network development lead to sufficient performance improvement that justifies the investment?

We believe that to answer this question it is necessary to develop methods that can somehow *quantify* the network performance gain that is obtained via multiple physical layers. The modeling approach should be sufficiently general, too, so that it can capture a variety of important scenarios. To develop such a methodology is the core motivation of this paper. Before discussing our modeling approach, we briefly review the previous work related to multi-radio networks in the next section. We note in advance that our plan is to pursue a somewhat more abstract approach than what is typically found in the networking literature, since we seek an avenue of modeling that is general enough to incorporate many particular scenarios.

II. Previous Work on Multi-Channel/Multi-Radio Networks

The availability of multiple radio interfaces and multiple channel per radio in multi hop networks have raised significant

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interest in the academic and industrial community due to the design and deployment of *mesh wireless networks*, i.e., multi-hop networks where some of the nodes, usually less mobile, provide backbone connectivity to mobile end users [2], [10]. As a consequence, whether related directly to mesh networking or just considering generic multi-radio systems, a good number of papers have recently appeared at premier networking conferences in this area. Although quite different in nature, all works on multi-radio/multi-channel systems strongly suggest and demonstrate the advantage of exploiting the radio diversity provided by multi-channel/multi-radio systems, and in general, of having multiple radio interfaces for considerable performance improvement.

Various solutions have been proposed for single-radio, multi-rate systems. Some technologies, such as IEEE 802.11a/b/g have multiple transmission rates, so that they can accommodate multiple channel conditions, enhance throughput and decrease interference losses. For example, in the work by Awerbuch et al. [4] it is shown that the minimum-hop routing strategy, which is usually deployed in single rate networks, is sub-optimal in multi-rate networks. Therefore, the authors propose a different link selection metric (Medium Time Metric, MTM) that favors shorter, higher throughput and more reliable links instead of the longer links that regular routing would select. Interference-aware topology control and routing is discussed in [23] where single-radio (IEEE 802.11b) nodes are considered. By taking advantage of the multiple channels of the radio, the authors define a way of choosing channels so that the resulting topology is "interference-minimum" among all possible k-connected topologies. For splittable traffic demands, the authors then present the Bandwidth-Aware Routing (BAR) protocol, which is demonstrated to outperform routing on a single-channel topology. Chandra et al. [7] advocates the use of single-radio systems in a multi-LAN environment. A wireless device could connect to multiple wireless LANs simultaneously despite having a single radio card. The paper describes a software layer approach to this problem, called MultiNet, that *virtualizes* a single wireless card and facilitates parallel multiple connection.

The *capacity* of wireless multi-radio, multi channel networks have been addressed, e.g., by Kyasanur and Vaidya in [16]. In networks with n static nodes, each with m radios and c channels, $1 \leq m \leq c$, this work shows that the capacity of the network exhibits bounds that are different from those established by the paper on single-radio/single-channel systems by Gupta and Kumar [11]. In particular, the multi-radio, multi channel bounds depend on the ratio between c and m. A characterization of the achievable rates in a mesh network with orthogonal channels is given in [14] where the authors determine necessary and sufficient conditions for achieving a rate vector in systems where the channels do not interfere with each other. By extending a former solution of theirs [15], the authors derive joint routing and scheduling algorithms for systems that are full duplex and equipped with multiple radios.

Routing for a multi-radio network with static nodes (like a *wireless community network* [24]) has been presented in [8]. The authors propose a new metric for routing in networks with multi-radio nodes, taking into account loss rate and link bandwidth and define a corresponding path metric that also considers interference among links using the same channel. The performance of the new metric is demonstrated via a 23 node testbed with nodes mounting two IEEE 802.11a wireless cards. The same testbed is also described in [5] where general design guidelines are provided for building multi-radio systems. This work also revisits the standard problems in wireless networking in light of having nodes with multiple radio cards. The advantage of using multi-radio nodes has also been demonstrated in [17], where a single-hop wireless system is presented, termed Multi-Radio Diversity (MDR), which uses path diversity to improve loss resiliency. The paper demonstrates the multi-radio advantage by showing a throughput gains up to $2.3\times$ over single radio communication schemes (their testbed is based on nodes supporting multiple IEEE 802.11a wireless cards). Shin et al. [20], [21] advocate the use of multi-radio to increase the capacity and performance of cellular networks. The idea in this case is to exploit the best available link to the base station. The authors envision an environment in which relay networks are dynamically formed so that whenever no acceptable direct links are available from a mobile node to the base station, a multi-hop path can be found. Algorithms for efficient formation of multi-hop relay networks are presented and evaluated for latency, signal overhead, gateway load (network formation) as well as path length and link sharing (relay network functions).

Channel assignment and routing for multi-hop wireless mesh networks is formulated mathematically as a joint optimization problem, by Alicherry et al. in [3]. The authors consider interference constraints, the number of channels per radio, and the number of radios per node. By solving the corresponding mixed integer linear programming model a centralized algorithm is proposed that optimizes the overall network throughput subject to fairness constraints. Distributed channel assignment in multi-hop, multi-radio networks has been explored in [22]. The problem is shown to be *NP*-hard even if the paths between any two nodes are given. A randomized assignment scheme is then proposed (Skeleton Assisted partition FrEe, or SAFE) that maintains network connectivity. A method for interference-aware cannel assignment has been proposed by Ramachandran et al. [19]. By modeling interferences between each mesh multi-radio router via a newly defined *multi-radio conflict graph*, a new channel assignment algorithm is designed that addresses explicitly the interference problem. Simulations and testing on a IEEE 802.11 testbed are performed that yield performance gains in excess of 40% with respect to static (non-interference aware) channel assignment solutions.

In order to ease the application access to multi-radio cards on a single node the Multi-radio Unification Protocol (MUP) has been proposed by Adya et al. [1]. The goal of MUP is coordinating the operations of the different (IEEE 802.11) cards tuned to non-overlapping frequency channels by optimizing local spectrum usage. The advantage of using this kind of software interface has been demonstrated via ns2-based simulations. Results show that TCP throughput and user perceived latency significantly improve under dynamic traffic patterns over realistic topologies.

III. Modeling Approach for Network Topology Analysis

Our fundamental model of the network topology for a network with multiple physical layers is a *multigraph*, possibly with labeled edges. As it is well known, a multigraph differs from an ordinary graph in that multiple edges can connect any pair of vertices. Naturally, the vertices represent network nodes, while the edges represent the links in the various physical layers, generated by the different radio interfaces. The labels on the edges distinguish their affiliation to the various layers.

As graphs have long been used to model network topologies, one may rightfully ask the question at this point: Can a multigraph lead to any *essential* new insight? In what follows we show that this model yields interesting and nontrivial novel problems.

A. The Multigraph Advantage

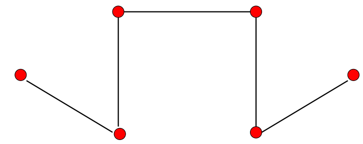
Consider a wireless multi-hop network with omni-directional antennas and let its network topology be represented by the graph G_1 in Figure 1(a). Assume that we equip the same nodes with another physical layer, such as a second radio with beamforming capabilities. This second physical layer generates another network topology, shown by graph G_2 in Figure 1(b). If we put them together, we obtain the topology of the combined system, shown by the multigraph G in Figure 1(c).

Let us call the above merging operation of graphs the **multigraph sum** of G_1 and G_2. The operation will be denoted by $\uplus$. Thus, we obtain G as

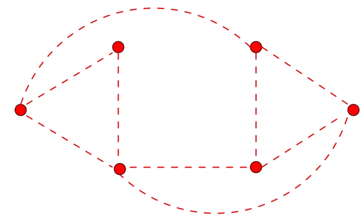
$$G = G_1 \uplus G_2.$$

The operation can be extended to more components in a natural way, so we can take the multigraph sum of any number of graphs or multigraphs: $\biguplus_{i=1}^{N} G_i = G_1 \uplus G_2 \uplus \ldots \uplus G_N$. Note that by assigning labels to the edges, the multigraph sum becomes invertible, that is, we can recover each individual G_i from $\biguplus_{i=1}^{N} G_i$.

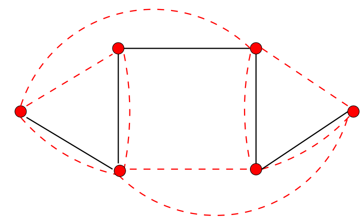
Having introduced the multigraph sum, let us now consider a curious property of it, which we call **multigraph advantage**. We explain it through an important graph parameter, the *edge-connectivity* (for short, we simply call it *connectivity*), denoted by $\lambda(G)$. This is the minimum number of edges that can disconnect the graph, when these edges are deleted. In other words, this is the size of a *minimum cut* in the graph. In case G is a multigraph, the parallel edges are all counted when we consider the size of a cut. The connectivity provides an important characterization of the network topology. For example, it tells how vulnerable is the network to link failures. It also shows how rich is the network in link-disjoint routes.[2] Now one can easily see that



(a) G_1: $\lambda(G_1) = 1$



(b) G_2: $\lambda(G_2) = 2$



(c) $G = G_1 \uplus G_2$: $\lambda(G) = 4$

Figure 1. The multigraph advantage: Edge connectivity

$$\lambda(G_1 \uplus G_2) \geq \lambda(G_1) + \lambda(G_2) \tag{1}$$

always holds. More generally, we have

$$\lambda\left(\biguplus_{i=1}^{N} G_i\right) \geq \sum_{i=1}^{N} \lambda(G_i). \tag{2}$$

The reason is that the size of any given cut in the multigraph sum is just the sum of the sizes of the corresponding cuts in the components. Thus, the connectivity of the multigraph sum cannot be smaller than the sum of the connectivities of the components. In fact, from the above reasoning one might first expect that actually the inequalities (1) and (2) always hold with *equality*. This is, however, not the case. Let us look at the graphs in Figure 1. One can directly check that $\lambda(G_1) = 1$ (Figure 1(a)) and $\lambda(G_2) = 2$ (Figure 1(b)). On the other hand, as shown in Figure 1(c), $\lambda(G) = 4$, which is strictly greater than the sum.

The above small example shows what we call *multigraph advantage*. Regarding connectivity, it means that the connectivity of the multigraph sum can be *strictly larger* than the sum of the connectivities of the components. Thus, the

[2] This is because it guarantees that between any pair of nodes there are at least $\lambda(G)$ disjoint routes. (This fact is a consequence of Menger's Theorem in graph theory.)

network topology has a *quantifiable* extra benefit from taking the multigraph sum, representing multiple physical layers.

One may reasonably ask at this point: Is this behavior just the consequence of specially chosen examples or is it somehow typical? In the following sections we show that it is quite typical.

IV. Multigraph Advantage in a Random Network Model

Mobile wireless networks are often modeled by various types of *random graphs*. In what follows, we first discuss the choice of the considered random graph model, then we formally state the thorems showing that it exhibits significant multigraph advantage. Unfortunately, size limitations do not allow us to include the (lengthy) formal proofs, they will be presented in the journal version of the paper.

A. The Choice of the Random Graph Model

The type of random graph that is most frequently used to model mobile wireless networks is the *random geometric graph* [18]. In the simplest case it is generated by randomly placing points in a planar domain, typically a regular one, such as a square. The points represent the randomly positioned network nodes. Any two points are connected with an edge (representing a network link), whenever their distance is at most a given value r, that stands for the transmission radius.

There are various generalizations of the above simple model. For example, the domain can be irregular and it may be part of a higher dimensional space, possibly with a different metric. To our knowledge, the most general version that still has a geometric flavor is based on pre-metric spaces [9] subsuming all previously studied geometric random graph variants.

On the other hand, the oldest and most studied random graph model is the one in which all edges are chosen *independently* with the same probability [6]. This model is referred to as independent-edge random graph, or Erdős-Rényi random graph, or sometimes Bernoulli random graph. While this model lacks the edge correlations induced by geometry, nevertheless it has a number of important advantages:

- In situations where the transmission radius is comparable with the domain diameter, the main reason for a missing link is not the distance, it is the presence of random obstacles or random variations in the radio propagation. In such a case an independent edge model can be adequate, or even more accurate than a geometric random graph.
- Some studies have found (see [12], [13]) that if the radio propagation model is more realistic, i.e., it takes into account statistical variations around the mean power, then it tends to decrease link correlations and the random graph becomes similar to the indepedent-edge model.
- If the network applies power control, it tends to counterbalance the effect of distance, especially if the distances are not too large. This again points to the applicability of the independent edge model.
- The independent-edge model is much more amenable to mathematical analysis.

The above reasons justify that we choose the independent-edge model for our investigations.

B. Results

We consider the following situation. Let $G_{n,p}$ denote a random graph on n nodes, with edge probability p. Note that typically p is a function of n, i.e., $p = p(n)$, and we are interested in asymptotic properties, when $n \to \infty$.

A well known property (see [6]) is that $G_{n,p}$ is connected asymptotically with probability 1 if and only if

$$p(n) = \frac{\log n + \omega(n)}{n}$$

where $\omega(n)$ is any function that tends to infinity with n. Informally, this means that the edge probability of $\frac{\log n}{n}$ is the minimum requirement for connectivity, as $\omega(n)$ can tend to infinity arbitrarily slowly.

First we investigate the multigraph advantage in a situation when the network is moderately dense in the sense that the average degree is only constant times larger than the minimum needed for connectivity.

Definition 1 *The random graph $G_{n,p}$ is said to be in the* moderately dense regime *if there exists a constant $c > 1$, such that*

$$p(n) = \frac{c \log n}{n}$$

holds.

Now we consider the following situation. Let G_{n,p_1} and G_{n,p_2} be two independently drawn random graphs on the same set of nodes, with edge probabilities p_1, p_2, respectively. Assume that they are both in the moderately dense regime. Let us compare the connectivities $\lambda(G_{n,p_1})$, $\lambda(G_{n,p_2})$ and the multigraph connectivity $\lambda(G_{n,p_1} \uplus G_{n,p_2})$. We can prove the following result about the multigraph advantage regarding connectivity.

Theorem 1 *Let G_{n,p_1} and G_{n,p_2} be independently drawn random graphs in the moderately dense regime, on the same set of nodes. Let their edge probabilities be*

$$p_1(n) = \frac{a \log n}{n} \quad \text{and} \quad p_2(n) = \frac{b \log n}{n}$$

with constants $a, b > 1$. Then there exists a constant $c = c(a, b) > 1$, such that the asymptotic multigraph advantage regarding connectivity is at least $c \log n$. That is,

$$\lambda(G_{n,p_1} \uplus G_{n,p_2}) \geq \lambda(G_{n,p_1}) + \lambda(G_{n,p_2}) + c \log n$$

holds asymptotically with probability 1.

Thus, as shown by Theorem 1, the multigraph advantage is quite significant in the moderately dense regime, for two reasons. First, it tends to infinity as the graphs grow. Second, since it is known [6] that in the moderately dense regime the connectivity of the random graph is $O(\log n)$, therefore, the gain in connectivity is of the same order of magnitude as the connectivity of the component graphs itself. In other words, there is a guaranteed constant percentage of *relative gain*, and this percentage does not vanish as $n \to \infty$.

The actual value of the constant c, as a function of a and b, can also be computed, although somewhat indirectly, as follows. Let $f(\alpha)$ be a function defined for $\alpha > 1$ by

$$f(\alpha) = \alpha\eta_\alpha$$

where η_α is the unique solution in $(-1, 0)$ of the equation

$$1 + \alpha x = \alpha(1 + x)\log(1 + x).$$

With this, c can be expressed as

$$c = f(a + b) - f(a) - f(b).$$

One can also prove that the function f is strictly superadditive, so we get $c > 0$.

If the density of the random graphs are not restricted to the moderately dense regime, then we cannot guarantee a constant percentage gain in connectivity. Nevertheless, we can still prove that there is a nonzero multigraph advantage. This is made precise in the following theorem.

Theorem 2 *Let G_{n,p_1} and G_{n,p_2} be independently drawn random graphs that are connected asymptotically with probability 1, but otherwise their density can be arbitrary. Then*

$$\lambda(G_{n,p_1} \uplus G_{n,p_2}) > \lambda(G_{n,p_1}) + \lambda(G_{n,p_2})$$

holds asymptotically with probability 1.

The message of Theorem 2 is that whenever the random graphs have enough density to be connected, then the multigraph advantage is asymptotically *always* present.

V. Conclusion

We have analyzed the effect of multiple radio interfaces on network connectivity. This was captured by the novel graph theoretic concept of the *multigraph advantage*. We have proved in a random graph model that this multigraph advantage is always present, whenever the component graphs are connected. Moreover, for the most important density range, called moderately dense regime, we proved that the gain grows to infinity with the graph size and the percentage (relative) gain remains constant and does not vanish with growing network size. The proofs and supporting simulations will be presented in the journal version of the paper.

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