Limiting the Propagation of Localization Errors in Multi-hop Wireless Networks

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Abstract

This paper concerns a study of the process of localizing the nodes of a multi-hop wireless networks, i.e., of having the node computing their coordinates with respect to a suitable reference system. We consider networks where the nodes perform measurements of distance and angle of arrival from nodes within their transmission radius. We describe a simple localization protocol, termed Range-Based Centroid (RBC), that starting from a single node (the beacon) with given coordinates localizes all the network nodes with reasonable accuracy. We then propose a new localization protocol that achieves greater accuracy by containing the propagation of the localization error as the process progresses away from the beacon. We quantify the improvements of the proposed protocol, termed MEC^2 *(for* Minimum Enclosing Circle Containment*) by simulations. In the considered scenarios, MEC*^{2} *keeps the localization error within* 21% *of the nodes' transmission radius, with* 20-30% *improvements over RBC.*

1 Introduction

A number of applications for wireless multi-hop (or *ad hoc*) networks require nodes to be aware of their own physical location. From the implementation of geographic ad hoc routing to the dissemination of sensed data from specific location in wireless sensor networks (WSNs), the network nodes need to know their own coordinates with respect to an absolute or relative system.

Whenever centralized positioning methods such as GPS or manual placement in known positions are not viable solutions for *node localization*, protocols must be defined and implemented that provide the nodes with the needed coordinates.

Several techniques for localizing ad hoc nodes have been proposed in the literature. A coarse taxonomy divides the multiplicity of these protocols into two major classes: *range free* solutions and solutions that are based on inter-nodal distance and/or angle of arrival (AoA). The first kind of solutions is particularly suitable for networks with resource constrained nodes (such as WSNs) since additional, expensive hardware for ranging and angle measurements is not required.

Range-free protocols are often depending on intensive message exchange, which can be extremely energy demanding. Moreover, the produced localization is not accurate. The *localization error*, defined as the misplacement between a node estimated coordinates and its actual position, often exceeds 50% of the node transmission radius, even with a large number of beacons throughout the network.

When accuracy is important, solutions based on measuring the distance (range) between nodes, and/or the angle of arrival have been proposed. Either by using the strength of the wireless signal, or, when available and in suitable scenarios, more sophisticated techniques (e.g., TDoA, ToA, AoA, ultrasound signals, smart antennas), ranging and angle information together with multi-lateration techniques are used to infer node position. In particular, it has been shown that the combined use of range and AoA information reduces message exchange and the localization error is often less than 50% of the transmission range [1]. An additional benefit of the use of range and AoA is the decreased number of beacons in the network. Simple protocols are en-

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abled by the knowledge of these two measures, in which *a single* node (called the source, the beacon, or the sink) starts the localization process by broadcasting its own coordinates. Nodes that compute their own coordinates based on the ones received by the beacon perpetuate the process in a wave-expanding fashion. However, the non-negligible error in the measurements leads to an increasing localization error while the process progresses away from the beacon.

Despite the host of localization protocols recently proposed for ad hoc and sensor networks, few works are concerned with the study and the definition of techniques for limiting the propagation of the localization error in a multi-hop setting. A method for error containment has been proposed by Niculescu et al. in [1]. Nodes compute the variance among many successive measurements of range and AoA and propagate the corresponding covariance matrix. This matrix is used to increase the precision of the computation of a node's coordinates. A drawback of this method, however, stems from the many pairwise transmissions needed to obtain an accurate estimation of the variance of range and angle measurements. Furthermore, matrix manipulation at the node can become quite computationally demanding. This makes this method not viable for networks with resource-constrained nodes, such as WSNs.

In this paper, we are concerned with defining an efficient method for limiting the propagation of the localization error in multi-hop networks with one beacon. We start by defining and testing a very simple localization algorithm, termed the *Range-Based Centroid* (RBC) method. A node receives coordinates from its own neighbors (nodes in its transmission range) and based on the sole knowledge of measures on range and AoA from them it estimates its own coordinates with respect to each one of them. The node's coordinates are computed as the centroid of the estimated coordinates. Although effective in keeping the localization error below 30% of the nodes transmission radius, RBC applies no methods for limiting the propagation error.

In this paper we define and study an error-containing technique that enhances RBC by simply using information about error measurements. In particular, upon receiving a neighbor coordinates, knowing technology-dependent bounds on the possible errors on range and AoA estimates, a node is able to determine the area that contains its real position. We approximate this area with a circle. At this point we can take advantage of the fact that the node exact coordinates must lie in the intersection of all the circles corresponding to different neighbors. The node's estimated coordinates are the center of the minimum circle containing the points of intersection of each pair of circles. We term this technique MEC^2, for *Minimum Enclosing Circle Containment.* (The details of this method are given in the next section.)

In this paper we define the RBC and MEC^2 schemes. The performance of the two protocols is compared (via ns2-based simulations) with respect to the localization error they produce, both network-wide, and as a function of the distance (in hop) of a node from the beacon. We observed that MEC^2 is effective in reducing the localization error produced by RBC. In the considered scenarios the average localization error never tops 21% of the transmission radius.

The protocols RBC and MEC^2 are described in the following section. Simulation results are shown in Section 3. Final remarks are given in Section 4.

2 RBC and MEC^2

The Range-Based Centroid (RBC) solution for node localization works as follows. There is an initial set up phase in which each node gets to know its hop distance h from the beacon, the identity of its neighbors at distance $h-1$ from the beacon and the measurements of range and AoA from these neighbors. (We assume that each node is equipped with a compass, which allows consistent measurements of the AoA.) The beacon starts the localization process by broadcasting its (exact) coordinates to its one hop neighbors (layer 1 nodes). Based on the (possibly faulty) measurements on range and angle, each neighbor of the beacon is able to compute its own (estimated) coordinates, which in turn are broadcast to the nodes that are two hops away from the beacon (layer 2 nodes). The generic node V at level h receives estimated coordinates from its u neighboring nodes one hop closer to the beacon (upstream neighbors). By using the measurements to each of such neighbors i and the coordinates (x_i, y_i) they sent, V computes its estimated coordinates (x_V, y_V) as the average over the estimated coordinates (x_V^i, y_V^i) with respect to each neighbor (centroid, as in [2]), i.e., for a node V in the plane, $(x_V, y_V) = \frac{1}{u}\sum_{i=1}^{u}(x_V^i, y_V^i)$.

Clearly, the localization error increases while the localization process progresses away from the beacon, given that a node h hops away from the beacon computes its coordinates based on faulty measurements and on imprecise coordinates (error propagation).

Based on RBC we propose a localization method that effectively contains the propagation of the localization error experienced by RBC. We assume that each node is aware of an upper bound on measurement errors on the real distance d between two nodes (ε_d, expressed as a percentage of d), and on the AoA α (ε_α, an absolute value). These two bounds allow a node to compute the area in which its real coordinates can be found.

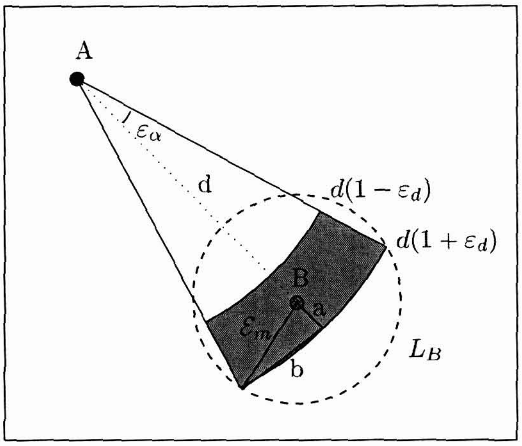

Figure 1. $\mathcal{E}_m$ and corresponding localization area

Let us consider two nodes A and B as in Figure 1, where A is the beacon. The shaded area is where B can be located based on the known bounds on the measurement errors and A's coordinates. We approximate this area with a circle L_B centered on the unknown real position of B. We term L_B the *localization area* of node B. The radius $\mathcal{E}_m$, called *measurement error radius*, is obtained as the norm of the vectorial sum of the errors on distance and angle, and represents the error due to measurements.

$$\mathcal{E}_m = \left\| \vec{d}\,\varepsilon_d + 2\vec{d}\,sin\frac{\varepsilon_\alpha}{2} \right\| \tag{1}$$

(A derivation of Equation (1) can be found in Appendix A.) Together with its estimated coordinates node B transmits also its measurement error radius to its neighbors in layer 2. These neighbors (possibly) receive multiple coordinates and radii, and compute their own coordinates and a *localization radius*, which is a function of the measurement errors and the received coordinates. This radius bounds the localization error from above. (In the case of neighbors of the source the localization radius coincides with the measurement error radius.) More specifically, a generic node V that is h hops away from the beacon receives coordinates (x_i, y_i) and localization radii $\mathcal{E}_i$ from all its u neighbors in layer $h-1$. From each of the received coordinates, given the measurement to the neighbors, V computes its *possible estimated coordinates* (x_V^i, y_V^i). At this point, instead of averaging on the estimated coordinates (as in RBC), node V considers (x_V^i, y_V^i) as the center of a circle L_i' whose radius $\mathcal{E}_i'$ is given by the localization radius $\mathcal{E}_i$ of neighbor i increased by the measurements error radius $\mathcal{E}_m$ (see Figure 2).

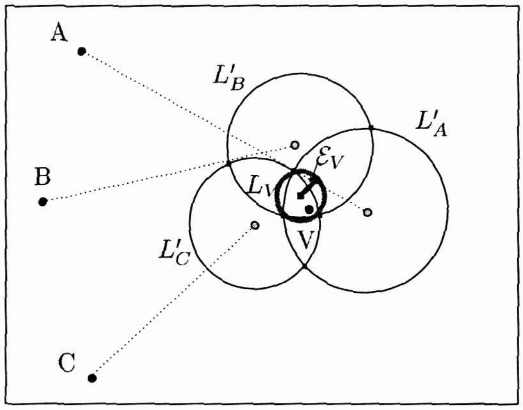

Figure 2. Localization area in MEC[2]

Each of the circles L_i', $i = 1, \ldots, u$, covers an area where the actual coordinates of node V are surely to be found. It is clear that V resides in the intersection of such circles. Node V's final estimated coordinates are therefore computed as the center of the smallest circle that contains all the points common to the intersection of the bigger circles (minimum enclosing circle, or MEC).

The MEC (thicker circle in Figure 2) is V's localization area L_V, whose radius $\mathcal{E}_V$ is sent by V to all its neighbors in layer $h+1$ along with its estimated coordinates.

We notice that the complexity of computing the MEC of a set with $O(u)$ points is $O(u)$, i.e., there is only quite a limited added complexity with respect to the simple centroid computation.

3 Simulation Results

We implemented RBC and MEC[2] by using the CMU wireless extension to the network simulator *ns2*. We consider scenarios where $n = 400$ and 600 nodes are scattered randomly and uniformly in a square area with side $L = 1400$m. The nodes' transmission radius is set to 250m

Error on distance has been modeled by adding to the Euclidean distance d between two nodes a percentage of d randomly chosen in $[-\varepsilon_d, \varepsilon_d]$, $\varepsilon_d \in [5\%, 50\%]$. Errors on the angle are modeled by adding to the actual angle α a value randomly chosen in $[-\varepsilon_\alpha, \varepsilon_\alpha]$ where $\varepsilon_\alpha \in [5^\circ, 45^\circ]$.

The points in the figures below are obtained by averaging over 100 network topologies for each n.

In our first set of experiments we are interested in studying how the localization error grows as the localization process progresses away from the beacon (i.e., the average localization error per layer.)

Figures 3 to 6 display the localization error per layer when the number of nodes is 400 and 600, for the MEC^2 and RBC schemes, respectively. In all our experiments, we have observed that the error on the angle affects localization precision much more than ranging errors. For AoA errors of 30 degrees or more the localization precision is independent on the distance error altogether, and these errors become noticeable only when the measurements on the angle produce small errors. As a consequence, we show "worst-case" pictures for the average localization error per layer where the ranging error is the maximum considered (50%) and the error on the angle varies from 5 (best case, little angle error) to 45 degrees (important measurement error). On the other side of the spectrum, we also show curves for the cases concerning little errors for distance (5%) and angle (from 5 to 15 degrees). (Each curve in the figures is labeled with the associated distance and angle measurements errors.)

We observe that in all cases, there is an expected increase of the error with increasing distances of nodes from the beacon. The only exception is for the nodes in the first layer (beacon's neighbors), whose error is bigger than for the nodes in the second layer. This is because the beacon's neighbors estimate their coordinates based only on the beacon's coordinates and estimated range and angle, and hence they cannot apply either RBC or MEC^2. (This justifies why for both RBC and MEC^2 the localization error for nodes one hop from the beacon is the same.)

In Figure 3 we observe the proportional dependence of localization precision on the angle measurement error. When the distance error is 50% and the angle error is 45 degrees, the error can be as high as 44% of the nodes transmission range ($n = 600$). The same error increases up to 50% in less denser networks ($n = 400$), for those nodes that are 7 hops away from the beacon (Figure 4).

The higher the density, the lower the average localization error per layer. In the case of RBC each node has more coordinates to average on, and in the case of MEC^2 a node estimates its coordinates

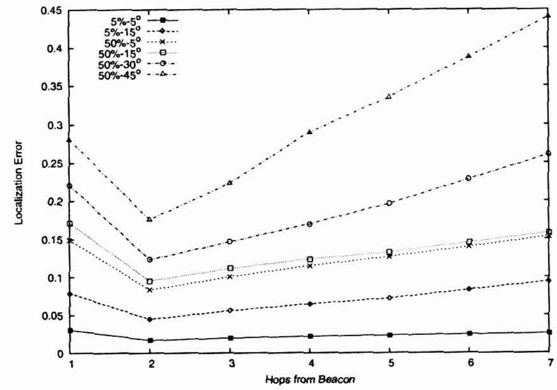

Figure 3. MEC^2: Error per layer ($n = 600$)

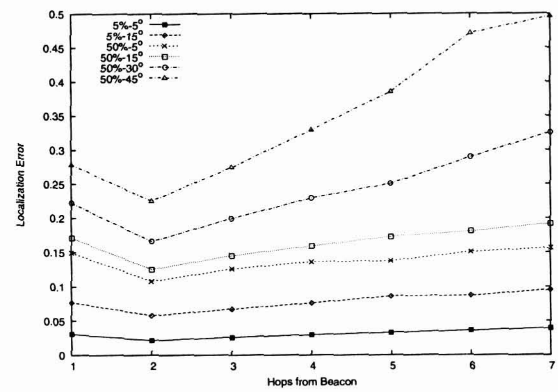

Figure 4. MEC^2: Error per layer ($n = 400$)

as the center of the intersection of a circles enclosing more points. These scenarios concern pretty high measurement errors. Whenever the technology allows a node to estimate range and especially the angle of arrival more accurately (under 5-10% and 15 degrees, respectively) the localization error is much lower (it never tops 13% for nodes 7 hops away from the beacon and in network with lower densities).

Figures 5 and 6 show the same results as the previous figures for the case of RBC. We observe that MEC^2 is effective in containing the propagation of the localization error. RBC leads to error which are 41% higher than those obtained by using MEC^2 ($n = 600$) and higher than 26% ($n = 400$).

Figure 7 and Figure 8 show the average localization error network-wide when $n = 400$ and 600, respectively, and for the best and worst error measurement combinations among those considered. With RBC-5% (MEC^2-5%) we indicate for RBC (MEC^2) with distance error equal to 5% and angle error equal to 5 degrees. RBC-50% (MEC^2-50%) indi-

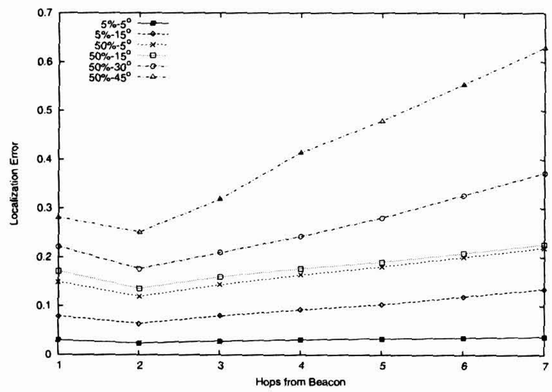

Figure 5. RBC: Error per layer ($n = 600$)

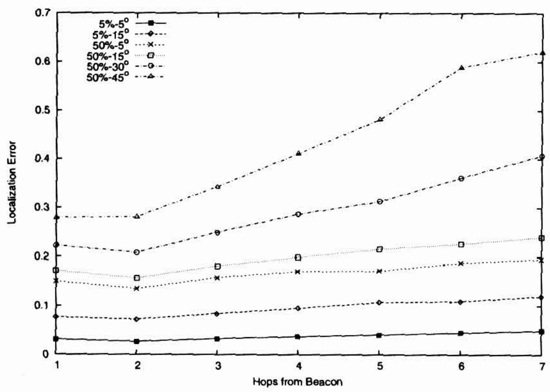

Figure 6. RBC: Error per layer ($n = 400$)

cates RBC (MEC^2) with distance error equal to 50% and angle error equal to 45 degrees.

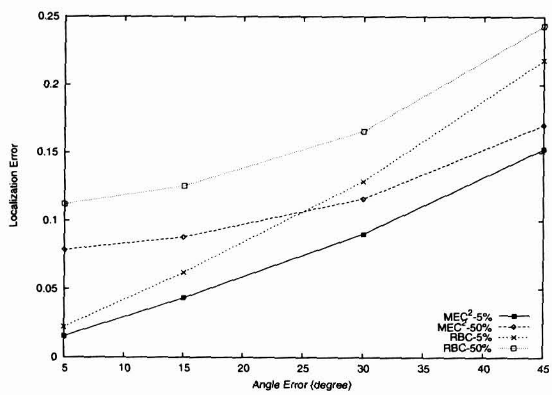

Figure 7. Localization error network-wide ($n = 600$)

We observe that for networks with 600 (400) nodes, the average localization error obtained by

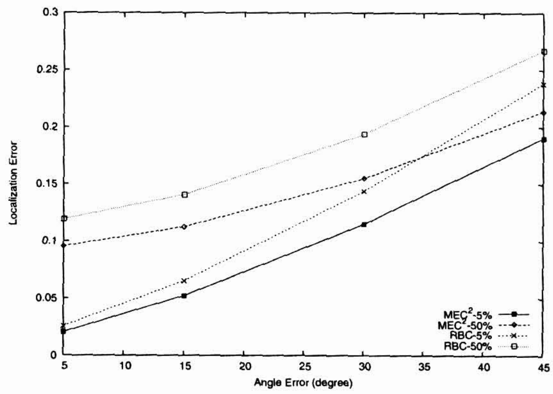

Figure 8. Localization error network-wide ($n = 400$)

MEC^2 is always below 17% (21%) of the nodes transmission radius. This includes the case with the worst measurement errors for both distance and angle we have considered. The localization error becomes negligible (always below 5%) when both measurement errors are relatively small (distance errors below 10% and angle errors $\leq$ 15 degrees). RBC produces an average localization error that can be as high as 27% of the nodes transmission radius. Although this compares favorably with previous localization protocols, we consider the 30% improvement of MEC^2 quite remarkable.

4 Conclusions

We have considered the problem of characterizing the localization error in networks with one beacon and nodes that are aware of the distance and of the AoA from their neighbors. We have also proposed a simple and effective method (MEC^2) for containing the error propagation as the localization process progresses away from the beacon. Simulation results confirm that the combined use of range and AoA information is effective in containing the localization error within the 27% of a node's transmission radius, and that MEC^2 further lowers the error by a factor of 0.3.

Future work includes providing an analytical model for the localization error propagation, investigating the impact of multiple beacons on localization accuracy, evaluating the toll imposed by RBC and MEC^2 on resource constrained nodes (WSNs scenarios) as well as comparisons with other error-containing localization protocols.

5 Acknowledgments

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Appendix A

We show how $\mathcal{E}_m$, the measurement error radius defined in Section 2, is obtained for the single node B. Consider Figure 1 where d, ε_d, and ε_α are defined as in Section 2. Values for a and b are easily computed as follows: $a = d\varepsilon_d$ and $b = 2d\varepsilon_+ \sin(\frac{\varepsilon_\alpha}{2})$, where we define $\varepsilon_\pm = (1 \pm \varepsilon_d)$. The Law of Cosines applied to the triangle $\triangle ab\mathcal{E}_m$ yields to $\mathcal{E}_m^2 = a^2 + b^2 - 2ab \cdot \cos\beta$, where $\cos\beta = \cos(\frac{\pi}{2} - \frac{\varepsilon_\alpha}{2}) = \sin(\frac{\varepsilon_\alpha}{2})$. In the following we indicate $\sin(\frac{\varepsilon_\alpha}{2})$ with sin. By substitution we obtain:

$$\begin{aligned}
\mathcal{E}_m &= \sqrt{d^2\varepsilon_d^2 + 4d^2\varepsilon_+^2 \sin^2 - 4d^2\varepsilon_d\varepsilon_+ \sin^2} \\
&= d\sqrt{\varepsilon_d^2 + 4\varepsilon_+^2 \sin^2 - 4\varepsilon_d\varepsilon_+ \sin^2} \\
&= d\sqrt{\varepsilon_d^2 + 4\varepsilon_+ \sin^2} = dk_1, \qquad (2)
\end{aligned}$$

where k_1 is a constant that depends on the measurement errors. Equation (2) defines $\mathcal{E}_m$ as a function of the real distance d. However, node B only knows the measured distance d_m. Such an expression for $\mathcal{E}_m$ should therefore be defined as a function of d_m, provided that, as it happens by using d, the localization area L_B includes B's real position. This is guaranteed by finding an upper bound on d which is a function of d_m. We notice that d_m is clearly $\geq d\varepsilon_-$ (see Figure 1). The needed upper bound is thus $d \leq \frac{d_m}{\varepsilon_-}$. Since $d \leq R$ (the nodes transmission radius) from Equation (2) we obtain:

$$\mathcal{E}_m \leq \min(Rk_1, d_m\sqrt{\left(\frac{\varepsilon_d}{\varepsilon_-}\right)^2 + \frac{4\varepsilon_+}{\varepsilon_-^2}\sin^2}).$$

Therefore, the needed value of $\mathcal{E}_m$ is $\min(Rk_1, d_m k_2)$.