Northeastern University  
Digital Signal Processing  
ECE 1456, Winter 2003

Due Date: January 28, 2003

Homework Set #3

Reading in P&I'M

- Chapter 1: Section 1.4
- Chapter 6: Sections 6.1 and 6.4 (4.2.9 and 5.1)
- Chapter 9: Sections 9.1 and 9.2 (5.2 and 5.3)

Written Problems in P&I'M

1. Problem 1.11
2. Problem 5.6
3. Problem 5.25
4. Let $x(n)$ be a periodic signal with fundamental period $N$, $X(k)$ be the $N$-point DFT of $x(n)$ and $X_3(K)$ be the $3N$-point DFT of $x(n)$.

   (a) Express $X_3(K)$ in terms of $X(k)$
   (b) Verify the results of part (a) using the sequence

   $$x(n) = \{\cdots, 1, 2, 1, 2, 1, 1, 2\}$$

Matlab Problems

For all DFT operations use the MATLAB commands **fft** and **ifft** as required for all DFT operations. A **help** on the **fft** command yields the following description:

**FFT**  
Discrete Fourier transform.  
FFT($X$) is the discrete Fourier transform of vector $X$. If the length of $X$ is a power of two, a fast radix-2 fast-Fourier transform algorithm is used. If the length of $X$ is not a power of two, a slower non-power-of-two algorithm is employed. FFT($X$,N) is the $N$-point FFT, padded with zeros if $X$ has less than $N$ points and truncated if it has more. If $X$ is a matrix, the FFT operation is applied to each column.

See also IFFT, FFT2, IFFT2, FFTSHIFT.

The **fft** command returns a vector whose first sample corresponds to frequency $\omega = 0$ and whose last sample corresponds to $\omega = 2\pi$. For plotting purposes, it is useful to label the axis in radians rather than sample index. If $X$ is the vector holding the DFT then the appropriate command to plot the magnitude is
>> stem(linspace(0,2*pi*(length(X)-1)/length(X)),length(X)),abs(X))
>> xlabel('Omega (radians)')

Alternately, we might want a graph of the spectrum in the range from $\omega = -\pi$ to $\omega = \pi$. Here we use the \texttt{fftshift} command in the following manner.

>> stem(linspace(-pi,(2*pi*(length(X)-1)/length(X))-pi,length(X)),fftshift(abs(X)))
>> xlabel('Omega (radians)')

Note that an analogous procedure is used to plot the angle. Finally, it may be useful to browse the \texttt{help} screens for descriptions of the \texttt{linspace}, \texttt{length}, and \texttt{fftshift}.

For the remainder of the term, please use the \texttt{fftshift}-based plotting command. Now on to the questions.

1. McLellan, Chapter 1, Basic Sampling Theory, Exercise 1.1: (a)–(d)

2. The analog signal $x_a(t) = \cos(4\pi t) + 2\cos(9\pi t + \pi/6)$ is sampled at a rate of $F_s = 1/T_s = 200$ samples/second to produce a finite duration sequence $x(n) \equiv x_a(nT_s)$ for $n = 0, 1, 2, \ldots, L-1$. Using MATLAB, determine and plot the magnitude of $X(k)$, the $N$-point DFT of $x(n)$ for the following values of $L$ and $N$:
   
   (a) $L = 100$ and $N = 100$ (no padding)
   (b) $L = 100$ and $N = 256$
   (c) $L = 40$ and $N = 40$ (no padding)
   (d) $L = 40$ and $N = 256$

   Explain the differences among these plots.