Voronoi Diagram-Based Linear Programming Modeling of Wireless Sensor Networks with a Mobile Sink

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Abstract

Wireless sensor networks (WSNs) are multi-hop wireless networks made up of energy-constrained sensing devices capable of delivering the sensed data to one or more collection points (sinks). In a typical WSN scenario the sink usually keeps stationary among the sensors that are static as well. A recent research trend aims at proposing energy-efficient protocols where the mobility of the sink is exploited for maximizing the lifetime of the network. A Linear Programming (LP) model is used for determining the sink sojourning points (SSPs) and the sojourn times of the sink that maximize network lifetime. Sensor nodes are assumed to be deployed realistically at arbitrary locations and the SSPs are selected among the vertices of the Voronoi Diagram (VD) corresponding to the sensor network topology. Message routing is also determined efficiently using the VD. Experimental results demonstrate the effectiveness of our LP model in determining a sink route and sojourn times along the route that considerably prolong the network lifetime with respect to the case with static sink.

Keywords: Wireless Sensor Networks (WSNs), Voronoi Diagrams (VDs), Linear Programming (LP), Network Lifetime

1. Introduction

Breakthroughs in MEMS technology and in low-power electronics and RF design have enabled the development of low-cost wireless sensor devices that can perform sensing activities and can wirelessly self-configure to form a wireless sensor network (WSN). Typical WSNs applications include environmental monitoring, building surveillance, industrial production control, etc. All these applications concern the delivery of the sensed data to one or more collection points often termed the sinks. Sensor nodes are often deployed in large numbers, and each sensor is characterized by tight resource constraints (especially power) and by being irreplaceable. When the energy of a sensor is depleted it is considered "dead" once and for all. Therefore, one of the major concerns of WSNs research is the design of techniques and protocols for limiting energy consumption and for prolonging the WSN lifetime. Most of the research so far has focused on developing protocols [2] for energy-conserving MAC [14,11,17], and routing [1,3,10,13]. The primary objective in these works is the minimization of the total energy consumed for data communication to and from the sink. A common assumption made in the previous works is that the sensor network has its sinks stationary at fixed positions.

Recently, the mobility of the sink is being considered in order to prolong network lifetime. In Rahul [12] a three-tier low-power sensor network architecture is proposed with mobile sinks. The movement of each of the sinks is modeled as a random walk. Analytical results are presented that concern system performance metrics such as data access rate, nodes buffer size, etc. In Gandham [4], an ILP (Integer LP) model is defined that determines the locations of multiple sinks and a flow-based routing protocol. The model aims at minimizing the total energy consumption instead of maximizing the network lifetime. Zhao [18] uses mobile nodes (called ferries) for transporting the sensed data of nearby sensors to their intended destination. The paper does not address the issue of energy saving at each sensor node as well as the problem of prolonging sensor network lifetime. All the cited papers never answer completely the question of whether and how we can actually control the movement of the sink among the deployed nodes for maximizing the WSN lifetime.

A first clear answer to this question has been recently given by Wang et al. in [15]. The paper shows that specific sink sojourn points (SSPs) can be determined together with sojourn times at those SSPs that maximize the network lifetime. In particular, experimental results show that five-fold improvements can be obtained with respect to the case when the sink is static. That seminal result is based on quite restrictive assumptions: The sensor nodes are positioned in a bi-dimensional grid and the SSPs coincide with the node locations. The contribution of this paper concerns overcoming the limitation of [15] by presenting a LP model capable of determining SSPs and sojourn times for WSNs where the nodes are deployed more realistically, e.g., arbitrarily. In particular, the SSPs are now chosen among the vertices of the Voronoi Diagram (VD) corresponding to the network topology. Furthermore, we propose a VD-based geographical routing that we prove to require low computational complexity and overall low energy cost, which is extremely beneficial for WSNs. Our experimental results demonstrate the effectiveness of our LP model in determining the sink's sojourning sites and sojourn times that considerably prolong the network lifetime with respect to when the sink is static.

2. Preliminaries

2.1. Sink's Sojourning Point (SSP) Discretization

In tackling the problem of modeling a mobile sink, two immediate issues have to be resolved: a) the sites at which the sink will sojourn, and b) the method for data routing from one sensor to another toward the sink. A basic fact about the data communication in a sensor network is that whenever a sensor node changes its position, a large amount of message exchanging overhead will be incurred to inform other nodes of the event so that routing tables are updated. This holds true for the change in the sink's position. This suggests the need for discretization of the sink's position to a small number of suitable sites.

We assume that the sensor network consists of a large number of sensors with arbitrary but known locations and the *double range property* holds, i.e., $R_c/R_s \geq 2$, where R_c and R_s separately denote the transmission range and the sensing range of a sensor node. Consider now a virtual VD graph, generated by the sensors, which is embedded in the sensor network. The VD is one of the most fundamental computational geometry structures and has been applied to a variety of areas [9]. In addition, VD can be constructed relatively fast in $O(n\log n)$ time, where n is the number of generating nodes [16]. The sink only sojourns at certain VD vertices (not all) and nowhere else, i.e., the sink visits one allowable VD vertex determined by the model and sojourns there for some time before it moves to another, and so on until the end of the network's lifetime is reached. A heuristic for constructing the SSP discretization is provided in Table 1. In the paper, discretizing sink's movement with VD and selecting VD vertices as SSP are made out of the considerations given below.

Due to the limited transmission range of wireless sensors, after data is collected at a sensor, it is aggregated and relayed through multiple nodes to reach the sink. The nodes that are the closest to the sink take on the largest traffic load as the last stops of relaying data from all the other nodes to the sink, and thus these nodes consume the most energy and die at the earliest time. An illustration is given in Figure 1. Six data sources have all of their data relayed at one sensor node, SN1, before they are finally sent to the sink. Therefore SN1 unavoidably dies earlier than the other nodes and thus it ends the network's lifetime. In this study, we define the *network lifetime* as the time duration till the first sensor node runs out of initial energy. In contrast, a VD vertex has at least three neighboring generator points, which means if the sink sojourns at a VD vertex there will be at least three neighboring sensor nodes to carry the traffic together. In Figure 2, we give an example where the sink stays at a VD vertex with S1, S2 and S3 being the neighboring sensor nodes (also VD generator points).

The power consumption at a sensor node is an exponential function of the Euclidean distance between the sender and the receiver nodes, i.e. $q=k_1\beta+k_2(\alpha_1+\alpha_2 d^p)$. Here q is the total power consumption for receiving/transmitting k_1/k_2 bits per second. α_1, α_2 and β are the energy consumption factors on the circuits, and d is the Euclidean distance between the transmitting and receiving nodes. p is the path loss exponent, usually $2 \leq p \leq 4$. A VD vertex has the property of being equidistant to its neighboring generator points, e.g., the sink in Figure 2 has equidistance with the three sensor nodes S1, S2 and S3. Therefore at the last stop of data relaying toward the sink, the neighboring sensor nodes of the sink will pay equal tolls by sending the heavy-loaded data to the sink through equal distance. In fact, it can be shown that a VD vertex presents a local minimum sink location of the maximum power expended by the neighboring nodes in relaying data to the sink. Thus, VD vertices are quite appropriate for sink locations since they balance the energy burden of the neighboring sensors.

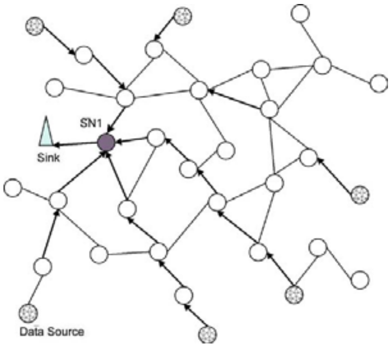

Figure 1 Sensor network data transmission

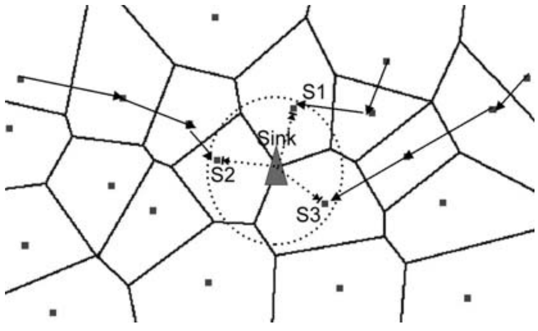

Figure 2. SSP discretization

Another advantage of the VD is that the triangulation method can be used for localization. The triangulation method is an effective and efficient localization method [8]. The neighboring sensor nodes of the sink work perfectly as the three anchors to localize the sink's current position and thus well support the sink's localization process.

In addition, A VD-based greedy forwarding routing mechanism is applied in this model (discussed in the section 2.2). Discretizing SSP with VD vertices works efficiently with the routing method and it can be seen in the later discussion that this also greatly complements the energy cost computation during the analytical modeling process.

Table 1 below provides a heuristic for SSP discretization. The rationale of the heuristic is as follows: in normal cases, a VD vertex has three neighboring generator points, and four or more in degraded cases. This means that in a sensor network with double range property, when the sink is at a VD vertex, it has at least three neighboring sensor nodes. In addition, two VD vertices that share a common edge have two same generator points. So if the sink moves from this vertex to another, two neighboring sensor nodes remain unchanged. This is definitely not an efficient way. To solve this, we use the heuristic in Table 1 to reduce the SSP density. Let g and v be the number of VD generator points and VD vertices respectively. For a VD with $g \geq 3$, $v \leq 2g - 5$. Then with the heuristic, we obtain $|SSP|/|GP| \leq 1$. Here SSP and GP denote two sets: SSP and VD generator points.

Table 1. List of the heuristic for SSP discretization

```
SSP:  set of all possible SSPs, and $ssp_i$ is the element of SSP
VT:   set of VD vertices, and $vt_i$ is the element in VT
$NV_i$:  set of neighboring vt's of the VD vertex $vt_i$
U:    set of forbidden VD vertices, and $u_i$ is the element of U

FOR $i = 1 \leftarrow |VT|$
    IF $\forall\ vt_j \in |NV_i|,\ vt_j \notin SSP$
        $SSP = SSP \cup vt_i$
        $U = U \cup vt_j$ AND $VT = VT \cap vt_j$ for $\forall\ vt_j \in |NV_i|$
    END IF
END FOR
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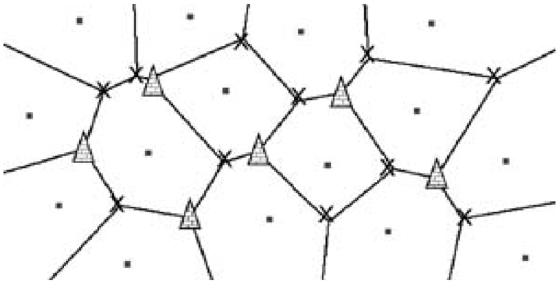

Figure 3. Demonstration of SSP discretization

2.2. Data Routing Method

The routing algorithms in mobile ad hoc networks can be classified into two categories, topology-based and position-based [7]. Compared with the former, position-based routing strategy doesn't need to know the network topology, and thus the overhead to establish and maintain routing tables is avoided. The routing decision is made at each node based on the geographic position of the destination node and that of the forwarding node's neighbors. Position-based routing is thus also called geographic routing. Geographic routing has been studied extensively as a suitable routing protocol in sensor and mobile ad hoc networks [5]. As a simple form of geographic routing, greedy forwarding relays packets to the neighbor of the forwarding node that is closer to the destination than the forwarding node itself. The measurement of forwarding progress can be Euclidean distance or others according to the optimization criteria. Greedy forwarding may not always find a path to the destination if it cannot find a better neighbor than itself. GPSR [5] and GOAFR [6] gave different recovery strategies to solve the problem. In [16] a Bounded Voronoi Greedy Forwarding (BVGF) is proposed. The paper shows that in a sensing covered network as long as the *double-range property* is satisfied, greedy forwarding can always find a routing path between any two nodes. BVGF chooses as the next hop the neighbors closest to the destination and have their Voronoi regions intersected by the line segment joining the source and the destination. An example from [16] demonstrates this routing method in the figure below.

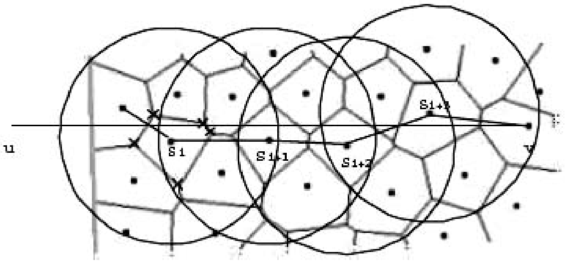

Figure 4. An example of routing with BVGF

Figure 4 shows the BVGF routing path going through four consecutive nodes (s_i - s_{i+3}) from a source *u* to a destination *v*. The communication range of each node is also circled out in Figure 4. In this paper we adopt the BFGF routing method to obtain energy-savings.

3. Mathematical Model

We make the following assumptions in developing the system model:

— Sensor nodes with limited energy are randomly distributed at known locations and remain stationary on a bi-dimensional plane with double the range property

— The mobile sink moves around in the sensor network and makes stops only at VD vertices

— Sensor nodes consume energy only when the sink sojourns at sojourning points. During the time the sink moves from one sojourning point (VD vertex) to another, sensors are in sleep status and do not have energy consumption

— Data transmission and reception are the major energy consuming activities

— Sensor nodes are homogeneous and wireless channels between them are bi-directional, symmetric and error-free

The sensor network is modeled as a graph $G(N, L)$ where N is the set of all the nodes in the two-dimensional plane and L is the set of all links with each link (i, j) representing the directed wireless channel between two adjacent nodes i and j. A node i can communicate directly with the neighboring nodes within its transmission range. Data packets are generated at each sensor node of set N with a constant rate. We allow the sink to move between VD vertices, i.e., the sink only stops at a VD vertex then moves to another. Let S be the set of SSP. If a sensor node i is not within the direct transmission range of the sink, which is at the sojourning point k, then data packets generated at node i must be relayed through multiple hops to reach the sink. The underlying routing method utilized here to relay data from remote sensor nodes to the sink is the revised BVGP as discussed in Section 2. The sink keeps moving in a path to be determined by the model until the maximum network lifetime is reached. Since sensor nodes consume energy only when the sink sojourns at SSPs but not during its movement, the real network lifetime equals the sum of the sink's sojourning times at different sojourning points till the first sensor node runs out of energy. The objective is to maximize network lifetime

The model parameters are:

e_0	Initial energy (J) of each node above
N	Set of sensor nodes in the plane, $N = \{1,\ldots, \lvert GP\rvert\}$
S	Set of SSPs (a subset of Voronoi vertices), $S = \{1,\ldots, \lvert SSP\rvert\}$
e	Energy consumption coefficient for transmitting or receiving one bit (J/bit)
r	Rate at which data packets are generated (bits/sec), same for all nodes
f_{ij}^k	Data transmission rate from node i to node j while sink stays at SSP k (bits/sec)
c_i^k	Power consumption for receiving and transmitting packets at sensor node i when the sink sojourns at SSP k (J/sec)

Considering the data *balance flow* at each sensor node combined with the underlying routing method, the power consumption coefficient is calculated using parameters r and e, for each pair (i, k) of nodes (see [15]).

The decision variables are:

t_k	Sojourn times (secs) of the sink at nodes k ($k \in S$)

The model determines the sojourn times, t_k, of the sink at each SSP $k \in N$ so that the network lifetime is maximized. If the optimal value for a t_k equals zero, the sink does not visit SSP k . Every SSP k, $k \in N$, whose optimal t_k is positive, is visited by the sink for a time duration equal to t_k . The order of visiting these nodes does not matter since the traveling time of the sink between nodes is considered negligible and data generation rate is independent of time. The mathematical formulation is shown below.

$$Max \quad z = \sum_{k \in S} t_k \tag{1}$$

$$\text{subject to:} \quad \sum_{k \in S} c_i^k t_k \le e_0 \quad i \in N \tag{2}$$

$$t_k \ge 0 \quad k \in S \tag{3}$$

4. Numerical Example

We compare two cases in doing the numerical example. I) the Sink remains stationary in one of the SSPs of the network in Figure 5; II). The sink moves in an optimal path determined by the LP model.

Parameter values are set at r = 1bit/sec, e = 0.62μJ/bit, e_0 = 1.35 J.

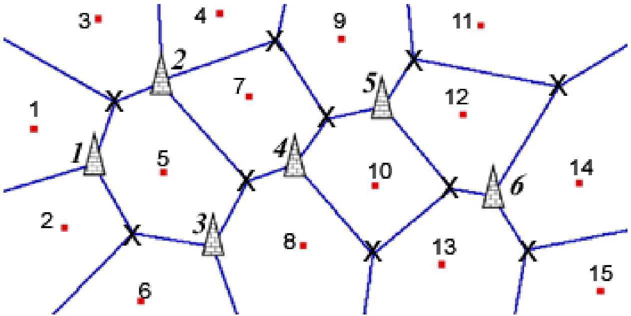

Figure 5. An example with 15 sensor nodes and 6 SSPs

In Figure 5, the SSPs obtained from the discretization are represented by the triangles and they are numbered from 1 to 6. The square dots are sensor nodes numbering from 1 to 15.

To solve Case I, similar as in [15], we use the model (4) below:

$$z_s = \max_k \left\{ \min_i \frac{e_0}{c_i^k} \right\} \qquad i \in N\,, k \in S \qquad (4)$$

The result shows that when the SP #3 is selected, the network has the optimal lifetime of 0.54 MSec. When the network lifetime is reached, all sensor nodes except #8 still have non-zero residual energies left. The lifetime is reached only because of the energy depletion of one node.

Table 2. List of the pseudo code for the c_i^k computation

```
GP:   set of generator points and $gp_i$ is VD generator point i
C :   set of VD cells containing the element $C_i$. $C_i$ corresponds to $gp_i$ in it
$l_{ik}$:   the line segment joining $gp_i$ and $ssp_k$
$RT_{ik}$: routing table from $gp_i$ to $ssp_k$ composed of generator points on the path

STEP1. SSP Discretization;
STEP2. Forming routing tables $RT_{ik}$
FOR k = 1 ← |SSP|
    Get the coordinates of $ssp_k$
    FOR i = 1 ← |GP|
        Get the coordinates of $gp_i$
        $l_{ik}$ ← the line segment in between $ssp_k$ and $gp_i$
        FOR s = 1 ← | C |  AND $C_i \notin C$
            IF $C_j$ is crossed by $l_{ik}$ AND $d_{kj} \le R_c$
            THEN $RT_{ik} = RT_{ik} \cup gp_j$
            END IF
        END FOR
    $RT_{ik} = RT_{ik} \cup gp_j$
    END FOR
END FOR

STEP3. Calculating $c_i^k$
FOR k = 1 ← |SSP|
    Get the coordinates of $ssp_k$
    FOR j = 1 ← |GP|
        Get the coordinates of $gp_j$
            FOR i = 1 ← |GP|
                IF $gp_i \in RT_{jk}$
                THEN $c_i^k = c_i^k + r.e$
            END FOR
    END FOR
END FOR
```

In Case II, we obtain the sink's lifetime about 20% more than the Case I and the sink's sojourning times as in Table 3. The sink sojourns at the SSP #3, #4 and #5 with the longest sojourning time at #3. When the network lifetime is reached, the residual energies of three sensor nodes, i.e, #7, 8 and 10, reach zero simultaneously, which means the energy consumption is more evenly distributed among the sensor nodes. From Figure 5, we can see that sensor nodes #7, 8 and 10 are about in the center position of the network, and this explains the earliest energy depletion on the three nodes.

Table 3. Sink's sojourning times at the six SSPs (seconds)

***k* (SSP)**	**1**	**2**	**3**	**4**	**5**	**6**
t_k (sojourning time	0	0	0.508065	0.072581	0.072581	0

An intuitive explanation of the sink's different sojourning times at different SSPs is as follows. With BVGF, each sensor node that participates in the data relaying between the sink and a source node must lie in one of the VD cells crossed by the line segment joining the sink and the data source point. Therefore, by drawing the straight lines from the sink's position to all the other sensor nodes, we can obtain a picture about how the energy consumption among the network nodes is distributed (see Figure 6 and 7). In Figure 6, the lines radiate more evenly throughout the area while in Figure 7 they tend to have a direction from left to right. The sink will spend more time at the point where the energy consumption could be more evenly distributed among the network nodes. This verify the effectiveness of our approach, i.e., applying a VD-based SSP discretization method combined with a VD-based routing method to tackle the arbitrary network topology in prolonging network lifetime. This can be predicated that with the network size increasing, great improvement on the network lifetime can be achieved.

5. Conclusion

In this paper, we tackle the previously unsolved issue of arbitrary network topology in modeling WSNs with a mobile sink. We propose a Voronoi Diagram-based sink movement pattern in which the mobile sink sojourns at Voronoi vertices during its movement in the sensor deployment area. Through implementing this scheme combined with a Voronoi Diagram-based data routing method, we solve the LP optimization model with relatively low computational complexity. Our experiment demonstrates that we can obtain a satisfactory network lifetime improvement of the WSN with an arbitrary topology by enforcing a Voronoi Diagram-based optimal movement pattern of the sink.

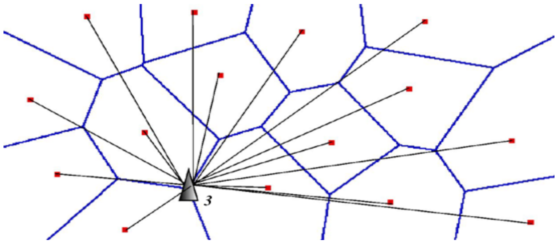

Figure 6. Energy radiation when the sink is at SSP#3

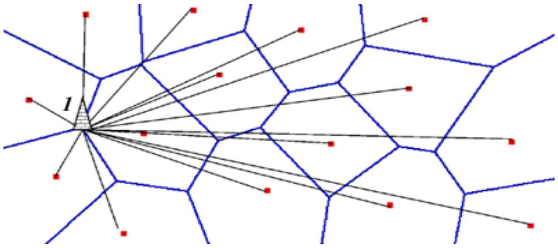

Figure 7. Energy radiation when the sink is at SSP#1

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