Energy Conserving Transducers II
Lumped Element Example

What is this??
How do we draw the equivalent circuit?

- Look for elements with shared flows – elements in series electrically.
- Look for elements with shared efforts – elements in parallel electrically.
- Here, the flow (velocity/current) in k₃ is the difference between the flows in M₁ and M₂.
- What about the source?
  - The effort for the element M₂ is the difference between the effort of k₃ and the sum of the efforts of k₂, b₂, and F.
  - F (an effort source) also shares the flow of M₂ with k₂ and b₂.
Two-domain example: (later, 3 domains!)

Magnetic path length $L_m$

$F_{mm} = H_\nu L_m + H_g g$

$F_{mm}$ is the magnetomotive force $\int H \cdot dl = F_{mm} (= nI)$

$H$ is the magnetic field intensity

$B$ is the magnetic flux density
\( B_\nu = \nu H_\nu \)

\( B_g = \nu_0 H_g \)

\[ \text{flux} \ \Phi = \int_S \mathbf{B} \cdot d\mathbf{A} \]

(flux is constant in circuit)

if area is constant (small gap)

\( B_\nu = B_g \)

0, 2, 3 \( \Rightarrow \)

\( H_\nu = \frac{\nu_0}{\nu} H_g \)

\( F_{mm} = \frac{\nu_0}{\nu} H_g L_m + H_g g \)

\( H_g = F_{mm} \left( \frac{\nu_0}{\nu} L_m + g \right)^{-1} \)

\( \nu_0 H_g = F_{mm} \left( \frac{\nu_0}{\nu_0} L_m + g \right) \)
\[ B_j = \nu_0 H_3 = F_{mm} \left( \frac{\nu_0}{g + \frac{\nu_0}{\nu} l_m} \right) \]

\[ \phi = \left( \frac{\nu_0 A}{g + \frac{\nu_0}{\nu} l_m} \right) F_{mm} \]

\[ F_{mm} = R \phi \]

\[ \uparrow \uparrow \text{ displacement (position, } x \text{; charge, } q \text{; Flux, } \phi) \]

\[ \text{effort} \quad \text{Generalized capacitance} \quad (R \rightarrow 4 \text{ or } \frac{1}{2}) \]

\[ \frac{1}{R} = \frac{\nu_0 A}{g + \frac{\nu_0}{\nu} l_m} \]
<table>
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<th>Effort</th>
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<th>Momentum</th>
<th>Displacement</th>
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<td>Torque $\tau$</td>
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<td>...</td>
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</tr>
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</table>
\[ \frac{1}{R} = \frac{V_0 A}{g + \frac{V_0}{\omega} l m} \]

if \( V g \gg V_0 \) l m,
then

\[ \frac{1}{R} = \frac{V_0 A}{g} \equiv C_{\text{mag}} \in \text{magnetic circuit domain} \]

We can connect the electrical and magnetic domains with a gyrator!
\[ \Phi = \frac{1}{n} V \quad \text{(Faraday's law of induction)} \]

\[ F_{\text{mm}} = n I \quad \text{Definition of } F_{\text{mm}} \quad \text{(Appendix B)} \]
Impedance of the capacitor $= \frac{R}{s} = z(s)$

Conductor $\Rightarrow$ impedance transforms to $\frac{\eta^2}{z(s)} = \frac{\eta^2}{R} = s \frac{\eta^2}{R}$

$\Rightarrow$ inductor, $L = \frac{\eta^2}{R}$

or

$w^* = \frac{CV^2}{2} = \frac{F_{mm}^2}{2R} = \frac{1}{2} LI^2$ or $R = \frac{n^2}{L}$

$\uparrow$ generalized mag. domain

why? Explicit representation of magnetic domain for modeling magnetic actuators.

Next: Energy conserving transducers.
The magnetic actuator

\[ B_g = \left( \frac{N_0}{g + \frac{\mu_0 L_m}{\mu}} \right) F_{mm} \]

\[ (F_{mm} = n I) \]

\[ B_{g\omega} = \left( \frac{N_0}{g \omega + \frac{\mu_0 L_m}{\omega}} \right) F_{mm} \]
\[ \phi = 8A \]

\[ \Rightarrow \phi = \left[ \frac{\nu_0 A}{g + \frac{\nu_0}{\nu} L_m} \right] \left( 1 - \frac{x}{X_0} \right) + \left( \frac{\nu_0 A}{g + \frac{\nu_0}{\nu} L_m} \right) \left( \frac{x}{X_0} \right) \right] F_{mm} \]

\[ \phi = \frac{1}{R} F_{mm} \]

\[ W^* (F_{mm}, x) = \frac{F_{mm}^2}{2R} \quad \text{(like } \frac{1}{2} CV^2) \]

\[ F = - \frac{dW^* (F_{mm}, x)}{dx} \Bigg|_{F_{mm}} \]
\[
F = \frac{F_{mm}^2}{2x_0} \left[ \frac{-\nu_0 A}{g + \frac{\nu_0}{u} Lm} + \frac{\nu_0 A}{g + \frac{\nu_0}{u} Lm} \right]
\]

\[
F = \frac{F_{mm}^2}{2x_0} \left[ \frac{-\nu_0 A (g + \frac{\nu_0}{u} Lm)}{(g + \frac{\nu_0}{u} Lm)(g + \frac{\nu_0}{u} Lm)} + \frac{\nu_0 A (g + \frac{\nu_0}{u} Lm)}{(g + \frac{\nu_0}{u} Lm)(g + \frac{\nu_0}{u} Lm)} \right]
\]

\[
F = \frac{F_{mm}^2}{2x_0} \left[ \frac{\nu_0 A (g - g_0)}{g g_0 + \frac{\nu_0}{u} Lm (g + g_0) + \left(\frac{\nu_0}{u} Lm\right)^2} \right]
\]

\[
g_0 \ll g
\]

\[
\frac{\nu_0 Lm}{u} \ll g
\]

\[
I = \frac{F_{mm}^2}{2x_0} \left( \frac{\nu_0 A}{g_0 + \frac{\nu_0 Lm}{u}} \right)
\]
\[ F = \frac{F_{mm}^2}{2x_0} \left( \frac{\nu_o A}{g\nu + \nu o I_m} \right) \]

Constant in \( x \), proportional to square of \( F_{mm} (= n I) \)

\[ F \propto F_{mm}^2 \]

\[ F \propto I^2 \]
If the actuator pulls on a spring

\[ F = F_s = k x_{eq} \]

\[ x_{eq} = \frac{F}{k} = \frac{F_{mm}}{2 k x_0} \left( \frac{\nu_0 A}{J_0 + \frac{\nu_0 h}{2}} \right) \]
\[ X_{eq} = \frac{n^2 I^2}{2\pi x_0} \left( \frac{\omega A}{g_0 + \frac{\omega^2 m}{\omega}} \right) \]

There can add mass + damper

3 energy domains!!
linear transducers:

1. Some transducers are linear
2. Non-linear transducers are linear over some range of operation. Small-signal analysis of electronic circuits, for example.
4 variables, 2 dependent, 2 independent, 4 ways choose the flows to be independent ⇒

\[
\begin{pmatrix}
V \\
F
\end{pmatrix}
= \begin{pmatrix}
Z_{EB} & T_{Em} \\
T_{ME} & Z_{MO}
\end{pmatrix}
\begin{pmatrix}
I \\
\dot{X}
\end{pmatrix}
\]

\begin{align*}
Z_{EB} = \frac{V}{I} & \quad | \dot{X} = 0 \quad \text{Blocked electrical impedance} \\
Z_{MO} = \frac{E}{\dot{X}} & \quad | I = 0 \quad \text{Open-Circuit mechanical impedance}
\end{align*}

Impedance is a generalization of resistance - ratio of effort/flow.
Blocked $\Rightarrow$ $x = 0 \Rightarrow$ no flow

Other domains -

Open circuit $\Rightarrow$ $I = 0 \Rightarrow$ no flow $\Rightarrow$ no flow!

\[ T_{Em} = \frac{V}{X} \bigg|_{I = 0} \quad \text{open circuit electromechanical transduction impedance} \]

\[ T_{ME} = \frac{F}{I} \bigg|_{X = 0} \quad \text{blocked mechanical-electro transduction impedance} \]

If the transducer is energy-conserving - no resistors

If the physical process is energy-conserving

\[ T_{Em} = T_{ME} \]
If reciprocal, linear, generally,

\[ Z_{ms} = Z_{mo} (1-k_e^2) \]

\[ k_e^2 = \frac{T_{em}}{Z_{eb} Z_{mo}} \]

\[ \phi = \frac{T_{em}}{Z_{eb}} \]

\[ \text{can build in spice!!} \]