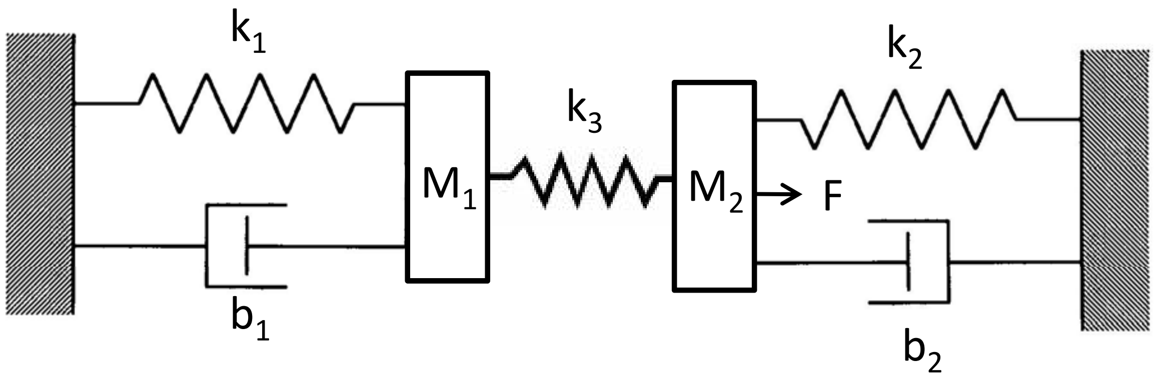
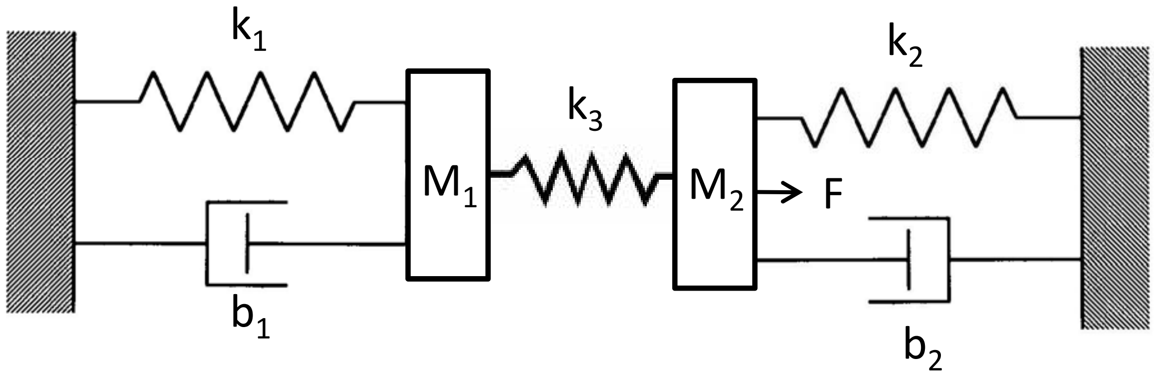
Energy Conserving Transducers II

Lumped Element Example

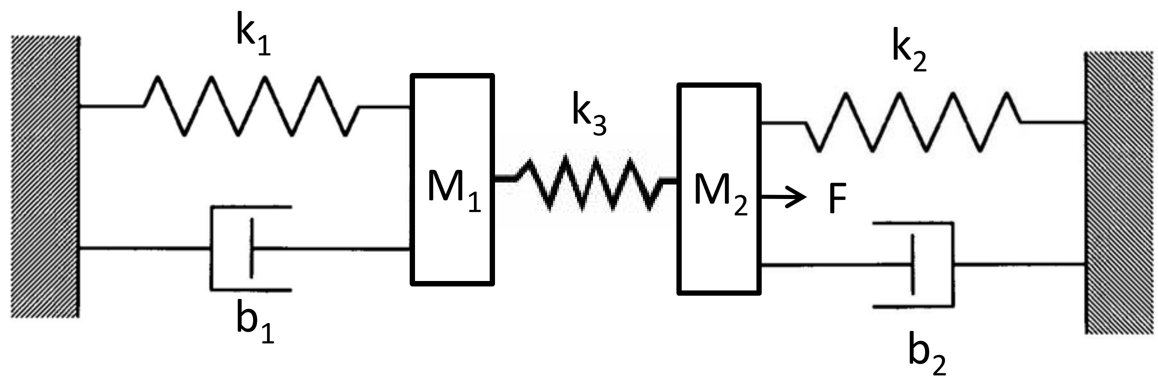


What is this??



How do we draw the equivalent circuit?

- Look for elements with shared flows – elements in series electrically.
- Look for elements with shared efforts – elements in parallel electrically.
- Here, the flow (velocity/current) in k3 is the difference between the flows in M1 and M2.
- What about the source?
    - The effort for the element M2 is the difference between the effort of k3 and the sum of the efforts of k2, b2, and F.
    - F (an effort source) also shares the flow of M2 with k2 and b2.


k1
k3
k2
M1
M2
F
b1
b2

Two-domain example: (Later, 3 domains!)

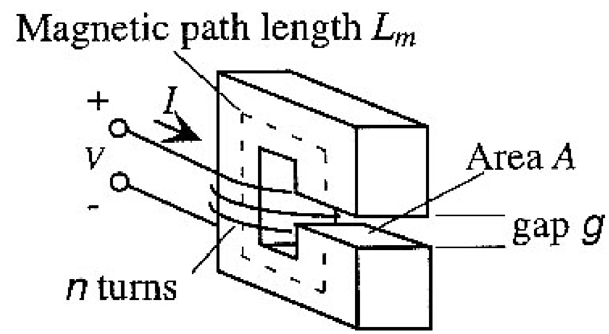


Figure 5.15. An electrical inductor built on a magnetically permeable core.

① $F_{mm} = H_i L_m + H_g g$

F_{mm} is the magnetomotive force $\oint \vec{H} \cdot d\ell = F_{mm}$ (=nI)

mag. circuit

H is the magnetic field intensity

B is the magnetic flux density

② $B_\mu = \mu H_\mu$

③ $B_g = \mu_0 H_g$

flux $\Phi = \int_S B \cdot dA$

(flux is constant in circuit)

if area is constant (small gap)

$B_\mu = B_g$

①, ②, ③ $\Rightarrow H_\mu = \frac{\mu_0}{\mu} H_g$

so $F_{mm} = \frac{\mu_0}{\mu} H_g L_m + H_g g$

$$H_g = F_{mm} \left(\frac{\mu_0}{\mu} L_m + g \right)^{-1}$$

$$\mu_0 H_g = F_{mm} \left(\frac{\mu_0}{\frac{\mu_0}{\mu} L_m + g} \right)$$

$$B_g = \mu_0 H_g = F_{mm} \left(\frac{\mu_0}{g + \frac{\mu_0}{\mu} L_m} \right)$$

$$\phi = \left(\frac{\mu_0 A}{g + \frac{\mu_0}{\mu} L_m} \right) F_{mm}$$

identify

$$F_{mm} = R \phi$$

$\uparrow$ effort

$\uparrow$ generalized capacitance $\left(R \to k \text{ or } \frac{1}{C}\right)$

$\nwarrow$ displacement (position, x; charge, q; Flux, ϕ)

$$\frac{1}{R} = \frac{\mu_0 A}{g + \frac{\mu_0}{\mu} L_m}$$

Table 5.1. Examples of conjugate power variables.

Energy Domain	Effort	Flow	Momentum	Displacement
Mechanical translation	Force F	Velocity $\dot{x}, v$	Momentum p	Position x
Fixed-axis rotation	Torque τ	Angular velocity ω	Angular momentum J	Angle θ
Electric circuits	Voltage V, v	Current I, i	...	Charge Q
Magnetic circuits	Magnetomotive force MMF	Flux rate $\dot{\phi}$	...	Flux ϕ
Incompressible fluid flow	Pressure P	Volumetric flow Q	Pressure momentum Γ	Volume V
Thermal	Temperature T	Entropy flow rate $\dot{S}$	...	Entropy S

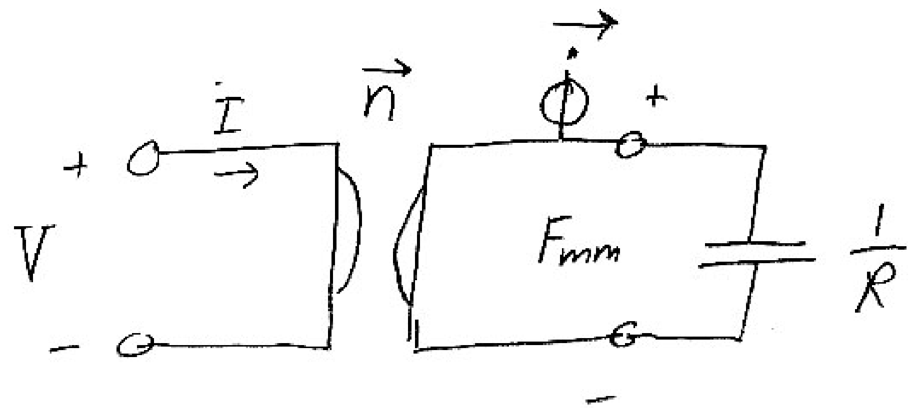
$$\frac{1}{R} = \frac{\mu_0 A}{g + \frac{\mu_0}{\mu} l_m}$$

if $\mu g \gg \mu_0 l_m$,

then

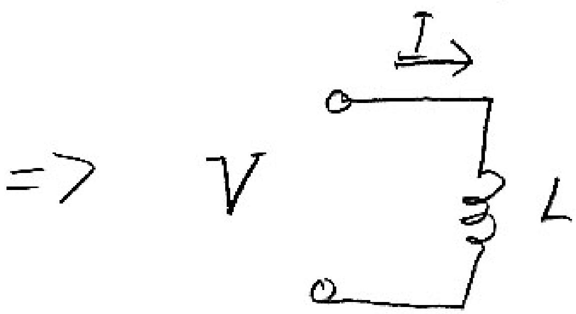
$$\frac{1}{R} \simeq \frac{\mu_0 A}{g} \equiv C_{mag}$$ ← magnetic circuit domain.

We can connect the electrical and magnetic domains with a gyrator!



Φ - flux - displacement

$\dot{\Phi}$ - flux rate - flow



$$\dot{\Phi} = \frac{1}{n} V$$ (Faraday's Law of induction)

$$F_{mm} = n\, I$$ Definition of Fmm (Appendix B)

Impedance of the capacitor $= \frac{R}{s} = Z(s)$

Gyrator $\Rightarrow$ impedance transforms to $\frac{n^2}{Z(s)} = \frac{n^2}{\frac{R}{s}} = s\frac{n^2}{R}$

$\Rightarrow$ inductor, $L = \frac{n^2}{R}$

$L = \frac{n^2}{R}$

or

$$W^* = \frac{CV^2}{2} = \frac{F_{mm}^2}{2R} = \frac{1}{2}LI^2 \quad \text{or } R = \frac{n^2}{L}$$

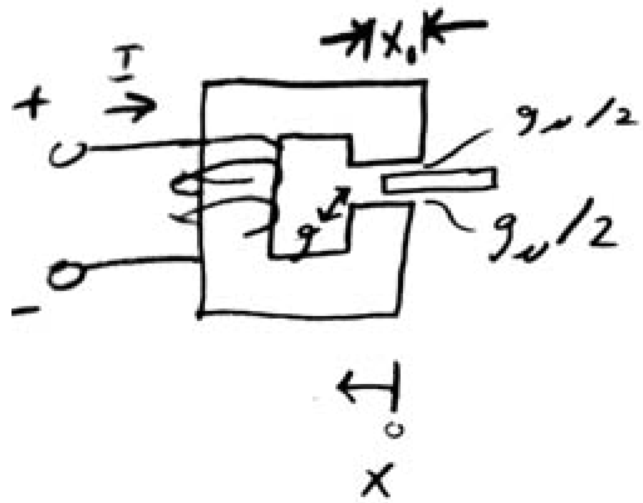
$\left(L = \frac{n^2}{R}\right)$

generalized mag. domain

why? Explicit representation of magnetic domain for modeling magnetic actuators.

Next: Energy conserving transducers.

The magnetic actuator



$$B_g = \left(\frac{\mu_0}{g + \frac{\mu_0}{\mu} L_m} \right) F_{mm}$$

$$\left(F_{mm} = n I \right)$$

$$B_{g_u} = \left(\frac{\mu_0}{g_u + \frac{\mu_0}{\mu} L_m} \right) F_{mm}$$

$$\Phi = BA$$

$$\Rightarrow \Phi = \left[\left(\frac{\mu_0 A}{g + \frac{\mu_0}{\mu} L_m} \right) \left(1 - \frac{x}{x_0} \right) + \left(\frac{\mu_0 A}{g_0 + \frac{\mu_0}{\mu} L_m} \right) \left(\frac{x}{x_0} \right) \right] F_{mm}$$

$$\Phi = \frac{1}{R} F_{mm}$$

$$W^* (F_{mm}, x) = \frac{F_{mm}^2}{2R} \quad \left(\text{like } \tfrac{1}{2} C V^2 \right)$$

$$F = - \frac{d W^* (F_{mm}, x)}{dx} \bigg|_{F_{mm}}$$

$$F = \frac{F_{mm}^2}{2X_0}\left[\frac{-\mu_0 A}{g + \frac{\mu_0}{\mu} L_m} + \frac{\mu_0 A}{g_\mu + \frac{\mu_0}{\mu} L_m}\right]$$

$$F = \frac{F_{mm}^2}{2X_0}\left[\frac{-\mu_0 A \left(g_\mu + \frac{\mu_0}{\mu} L_m\right)}{\left(g + \frac{\mu_0}{\mu} L_m\right)\left(g_\mu + \frac{\mu_0}{\mu} L_m\right)} + \frac{\mu_0 A \left(g + \frac{\mu_0}{\mu} L_m\right)}{\left(g_\mu + \frac{\mu_0}{\mu} L_m\right)\left(g + \frac{\mu_0}{\mu} L_m\right)}\right]$$

$$F = \frac{F_{mm}^2}{2X_0}\left[\frac{\mu_0 A (g - g_\mu)}{g g_\mu + \frac{\mu_0}{\mu} L_m (g + g_\mu) + \left(\frac{\mu_0}{\mu} L_m\right)^2}\right]$$

$$g_\mu \ll g$$

$$\frac{\mu_0 L_m}{\mu} \ll g$$

$$F = \frac{F_{mm}^2}{2X_0}\left(\frac{\mu_0 A}{g_\mu + \frac{\mu_0 L_m}{\mu}}\right)$$

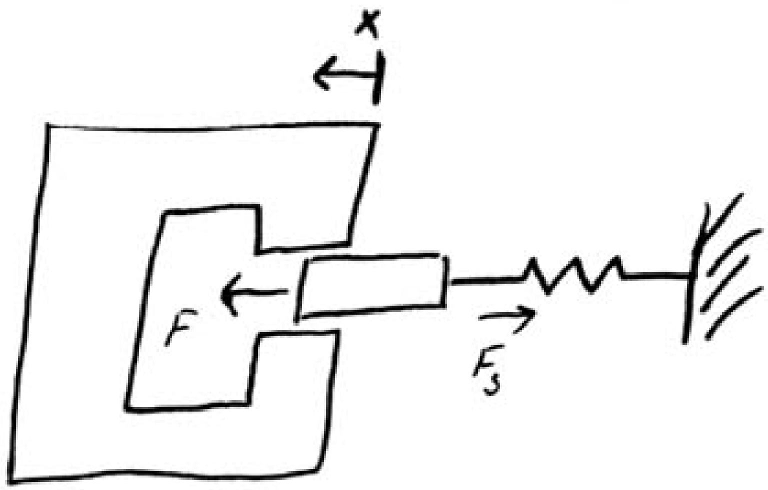
$$F = \frac{F_{mm}^2}{2x_0}\left(\frac{\mu_0 A}{g\mu + \frac{\mu_0 L_m}{\mu}}\right)$$

constant in x, proportional to square of F_{mm} $(= nI)$

$F \propto F_{mm}^2$

$F \propto I^2$

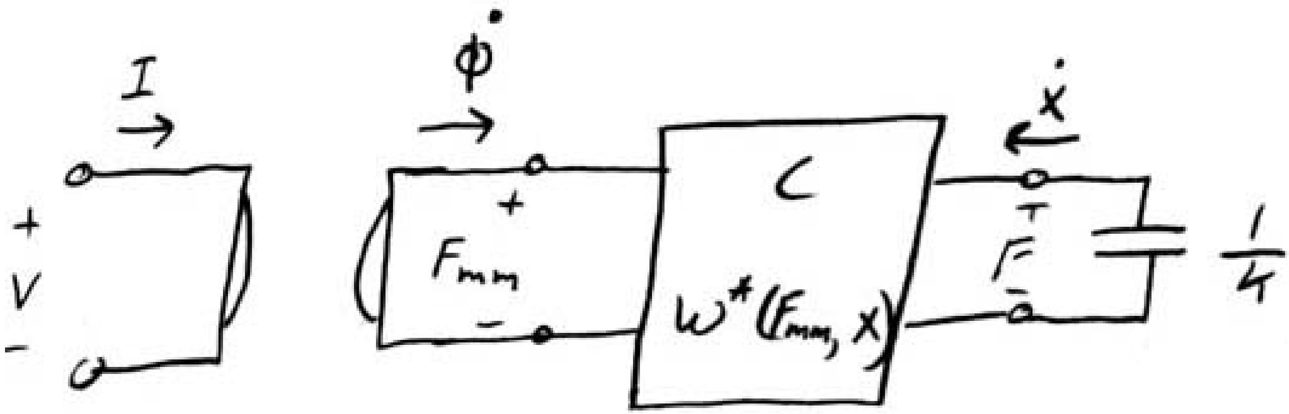
If the actuator pulls on a spring



$$F = F_s = k\,x_{eq}$$

$$x_{eq} = \frac{F}{k} = \frac{F_{mm}^{2}}{2kx_0}\left(\frac{\mu_0 A}{g_u + \frac{\mu_0 L_m}{\mu}}\right)$$

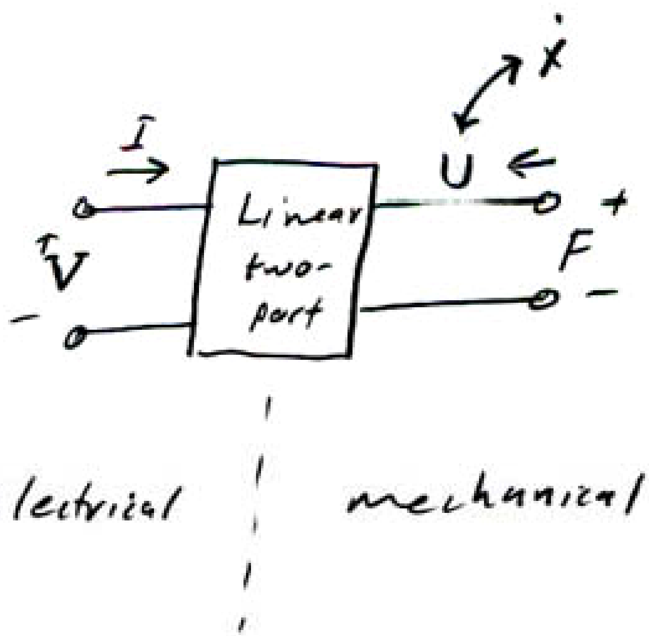
$$X_{eq} = \frac{n^2 I^2}{2 k x_0} \left(\frac{\mu_0 A}{g_0 + \frac{\mu_0 l_m}{\mu}} \right)$$



+ can add mass + damper

3 energy domains!!

Linear transducers:



1. some transducers are linear
2. Non-linear transducers ~ linear over some range of operation. Small-signal analysis of electronic circuits, for example.

4 variables, 2 dependent, 2 independent, 4 ways

choose the <u>flows</u> to be independent =>

$$\underbrace{\begin{pmatrix} V \\ F \end{pmatrix}}_{\substack{\text{dependent} \\ \text{variables}}} = \begin{pmatrix} Z_{EB} & T_{EM} \\ T_{ME} & Z_{MO} \end{pmatrix} \underbrace{\begin{pmatrix} I \\ \dot{X} \end{pmatrix}}_{\text{independent variables}}$$

$Z_{EB} = \frac{V}{I}\Big|_{\dot{X}=0}$ Blocked electrical impedance

$Z_{MO} = \frac{F}{\dot{X}}\Big|_{I=0}$ Open-circuit mechanical impedance

Impedance is a generalization of resistance - ratio of effort/flow.

Blocked $\to \dot{X} = 0 \to$ no flow
open circuit $\to I = 0 \to$ no flow) other domains - no flow!

$$T_{Em} = \left. \frac{V}{\dot{X}} \right|_{I=0}$$ open circuit electromechanical transduction impedance

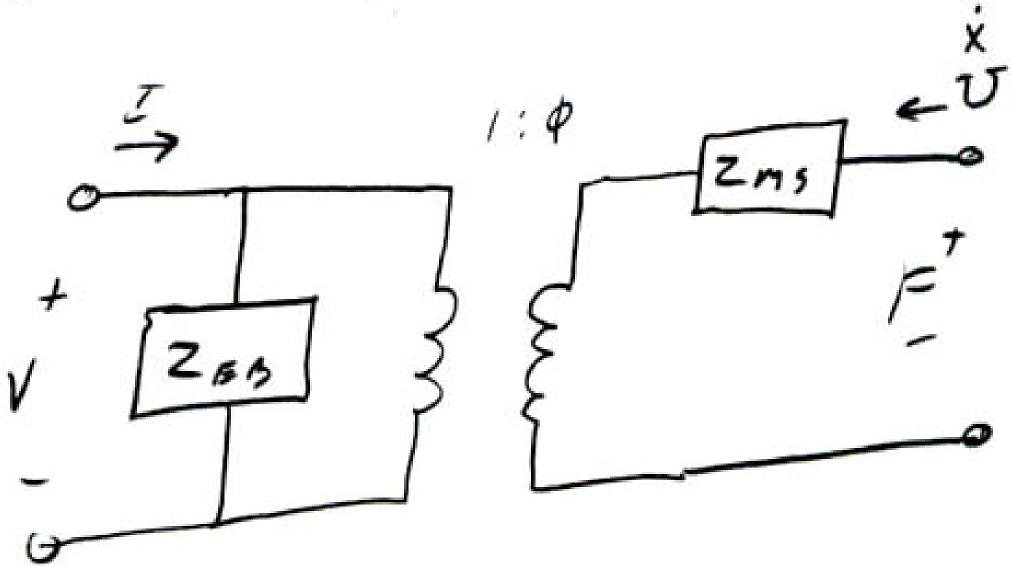
$$T_{ME} = \left. \frac{F}{I} \right|_{\dot{X}=0}$$ Blocked mechanical-electro transduction impedance

If the transducer is energy-conserving - no resistors

If the physical process in energy-conserving

$$T_{Em} = T_{ME}$$

If reciprocal, linear, generally,



← can build in spice!!

$$Z_{MS} = Z_{MO}\,(1 - k_e^2) \qquad \phi = \frac{T_{EM}}{Z_{EB}}$$

$$k_e^2 = \frac{T_{EM}^2}{Z_{EB}\,Z_{MO}}$$