Energy - conserving Transducers (Ch. 6, Senturia)

↓

two energy domains

1. Parallel - plate capacitor.

(Electrostatic actuator / capacitive sensor)

Electrical-Mechanical Transduction

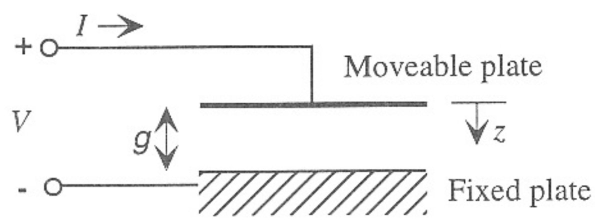
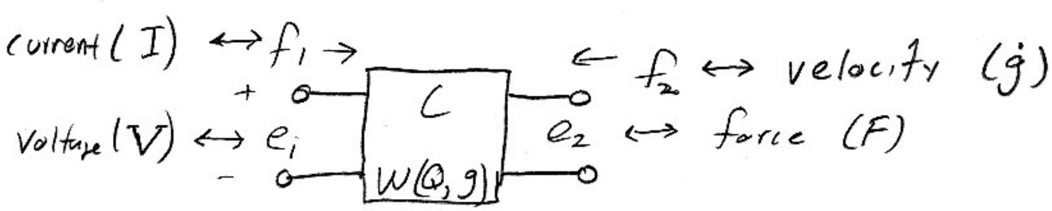


Figure 6.1. A parallel-plate capacitor with a moveable upper plate.

two - port



$$C = \frac{\epsilon A}{g}$$ (neglecting fringing fields)

good when linear size $\gg g$

stored energy, charging the capacitor at fixed gap.

$$w(q) = \int_0^q e(q)\, dq$$

$$w(Q) = \int_0^Q V(Q)\, dQ$$

$Q = CV$ for a linear capacitor,

$$W(Q) = \int_0^Q \frac{Q}{C}\, dQ$$

$$W(Q) = \frac{Q^2}{2C} = \frac{Q^2 g}{2\epsilon A} \quad \left(= \frac{CV^2}{2},\ C \text{ fixed}\right)$$

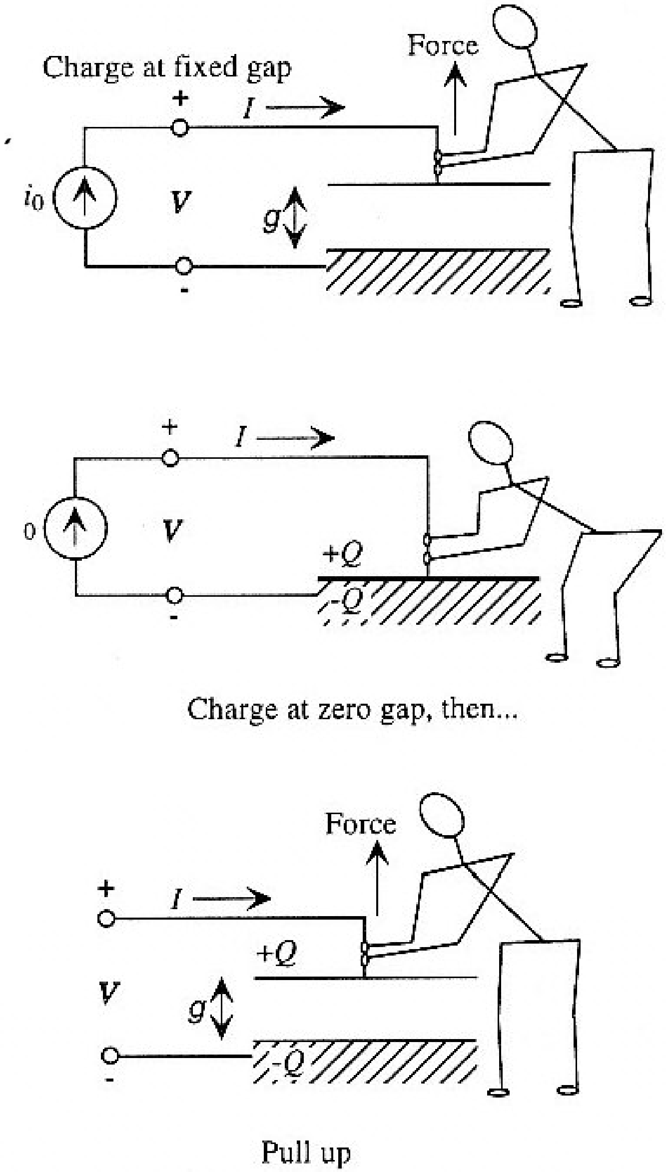


Figure 6.2. Two ways of charging a capacitor.

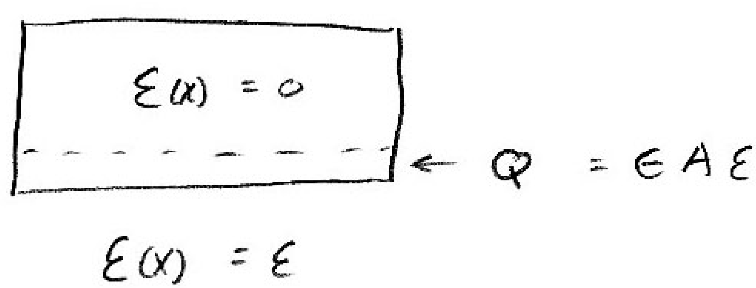
Force on capacitor plates (pulling plates together)

Electric field, $\mathcal{E} = \dfrac{Q}{\epsilon A}$ between plates.

Know, in general

Force $= Q\,\mathcal{E}$

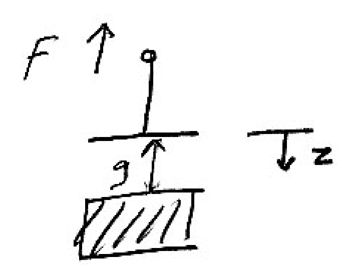
but microscopically $\mathcal{E}(x)$ varies from $\mathcal{E}$ to zero over the thickness of the layer of charge



So, the force is $\frac{1}{2} Q \mathcal{E} = \frac{Q^2}{2 \epsilon A}$

(depends only on charge)

lifting plate



$$W(g) = Fg = \frac{Q^2 g}{2 \epsilon A}$$ (same as before)

F negative
g negative (-z)

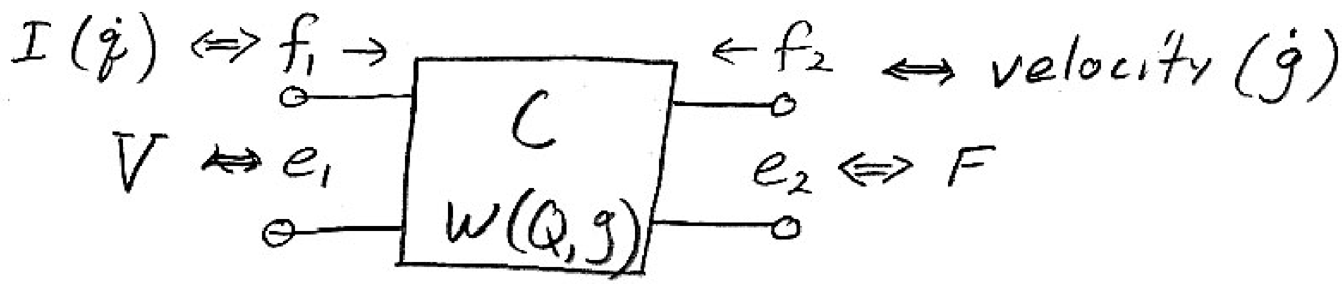
How much energy when $g = 0$?

Add charge to capacitor with no work?

$$C = \frac{\epsilon A}{g} \to \infty \quad \text{as } g \to 0$$

$$V = \frac{Q}{C} \to 0 \quad \text{as } C \to \infty$$

Two - port capacitor



$$W(Q,g) = \frac{Q^2 g}{2\epsilon A}$$

The effort variables are the appropriate partial derivatives of W:

$$dW(Q,g) = F\,dg + V\,dQ$$

$$W(Q,g) = \frac{Q^2 g}{2\epsilon A}$$

$$dW(Q,g) = F\,dg + V\,dQ$$

$$W(Q,g) = \frac{Q^2 g}{2\epsilon A}$$

Senturia has

$$F = \frac{dW(Q,g)}{dg}\bigg|_Q = \frac{d}{dg}\left(\frac{Q^2 g}{2\epsilon A}\right)\bigg|_Q = \frac{Q^2}{2\epsilon A}$$

(as before)

$$V = \frac{dW(Q,g)}{dQ}\bigg|_g = \frac{d}{dQ}\left(\frac{Q^2 g}{2\epsilon A}\right)\bigg|_g = \frac{Qg}{\epsilon A} = \frac{Q}{C}$$

(as expected)

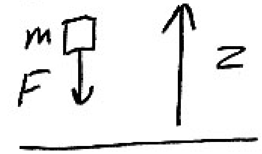
Familiar:

$$F(z) = -\frac{dW}{dz}$$

for gravity

$$W(z) = mgz$$

$$F(z) = -mg$$



?

<u>But</u> why is the force in the $+g$ direction??

Ans.: Because dW was written with F meaning the external force applied to put energy into the capacitor (The electrostatic force is in the opposite direction.

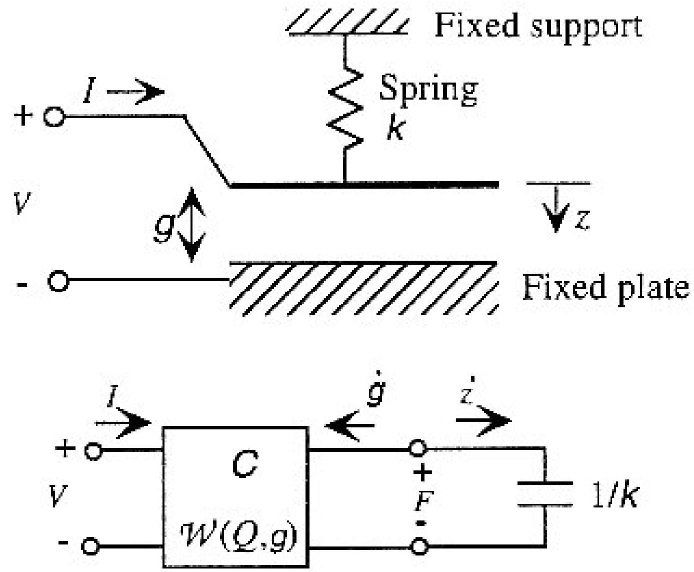


Figure 6.4. The basic electrostatic actuator: a moveable capacitor plate attached to a spring.

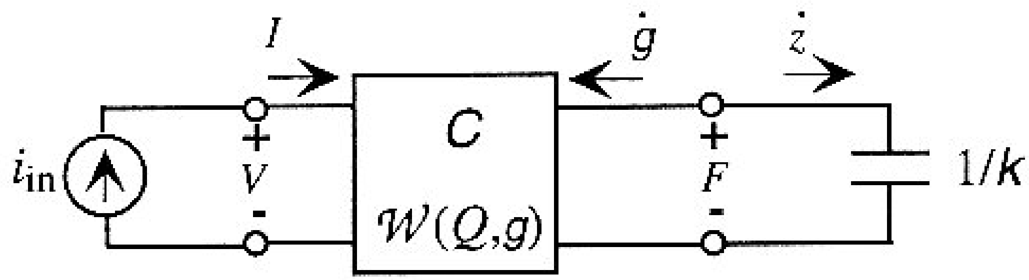


Figure 6.5. Charge control of an electrostatic actuator.

$$Q = \int_0^t i_{in}(t)\,dt + \cancel{Q_0} \rightarrow 0$$ (assume, Senturia)

$$Q = \int_0^t i_{in}(t)\,dt + \cancel{Q_0} \rightarrow 0$$ (assume, senturia)

$$F = \frac{Q^2}{2\epsilon A}$$

$$z = \frac{F}{k}$$ → (senturia's steps?)

$$g = g_0 - z$$

$$g = g_0 - \frac{Q^2}{2\epsilon A k}$$

$$\left(\begin{array}{l} g = 0 \text{ when } g = \dfrac{Q^2}{2\epsilon A k} \\ \text{when } Q = 2\epsilon A k g \end{array} \right)$$

Note: no instability -
see voltage control
next!

$$V = \frac{Qg}{\epsilon A} = \frac{Q\left(g_0 - \frac{Q^2}{2\epsilon A k}\right)}{\epsilon A}$$

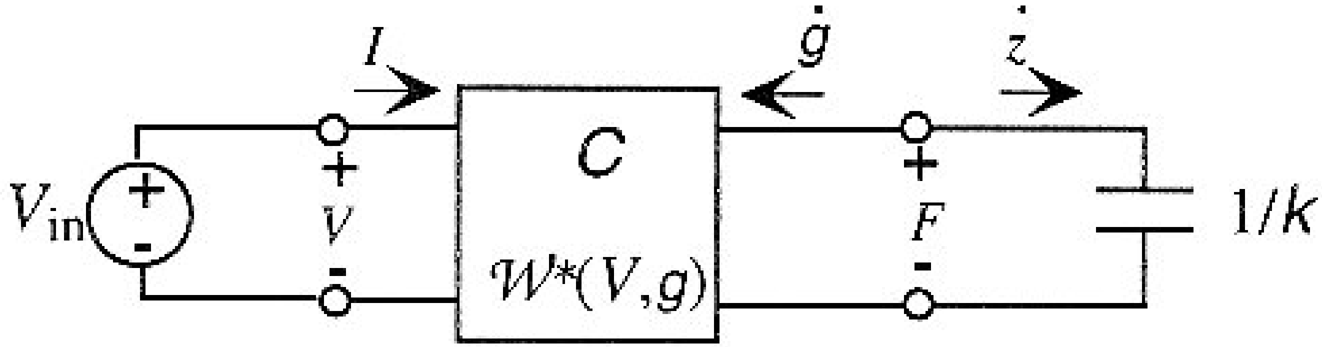


Figure 6.6. Voltage control of an electrostatic transducer.

One approach: we know the force is related to charge, so we can write the charge as a function of gap, then get the force.

$$Q = CV = \frac{\epsilon A V}{g} \quad \text{(note that } C \text{ is not constant!)}$$

$$F = \frac{Q^2}{2\epsilon A} = \frac{\epsilon^2 A^2 V^2}{2g^2 \epsilon A} = \frac{\epsilon A V^2}{2g^2}$$

(Have to interpret direction carefully!)

$$F = \frac{Q^2}{2\epsilon A} = \frac{\epsilon^2 A^2 V^2}{2g^2 \epsilon A} = \frac{\epsilon A V^2}{2g^2}$$

(Have to interpret direction carefully!)

This works as long as we have experience and physical insight into the direction of the force and Q-V relationship.

Senturia notes that to calculate the force under voltage control we can't use $W(Q)$, but must use $W^*(V)$

Remember $F = \left.\frac{dW(Q,g)}{dg}\right|_{Q}$

How do we use W^*?

$$F = \left.\frac{dW}{dg}\right|_Q$$

From before,

$$V = \left.\frac{dW}{dQ}\right|_g$$

$$dW(Q,g) = F\,dg + V\,dQ$$

$$W^*(V) = QV - W(Q)$$

$$W^*(V,g) = QV - W(Q,g)$$

$$dW^*(V,g) = Q\,dV + V\,dQ - dW(Q,g)$$

so

$$dW^*(V,g) = Q\,dV - F\,dg$$

$$Q = \left.\frac{dW^*(V,g)}{dV}\right|_g$$

$$F = -\left.\frac{dW^*(V,g)}{dg}\right|_V$$

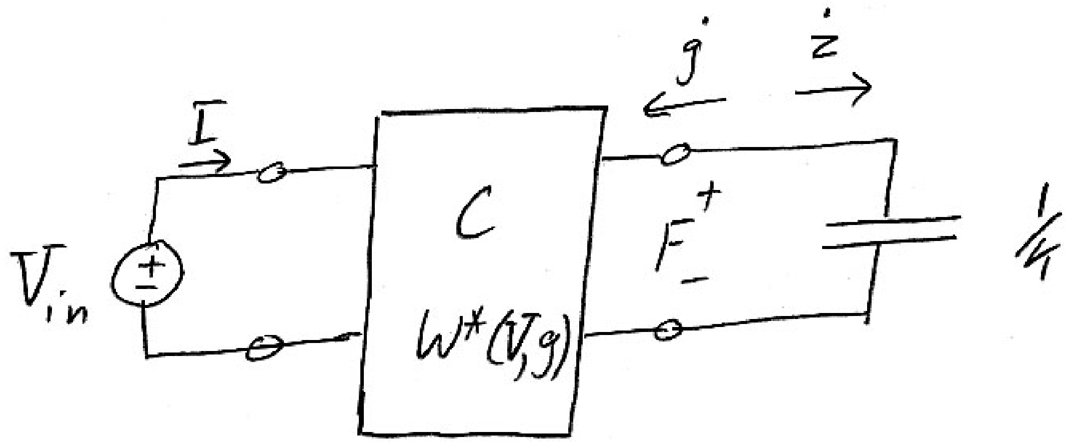
$$Q = \left.\frac{d w^*(V, g)}{dV}\right|_g$$

$$F = -\left.\frac{d w^*(V, g)}{dg}\right|_V$$

Now, $w^*(V, g) = \int_0^V Q\, dV = \int_0^V \frac{\epsilon A}{g} V\, dV$

$$w^*(V, g) = \frac{\epsilon A V^2}{2g} \quad \left(= \tfrac{1}{2} C V^2\right)$$

$$F = -\left.\frac{d w^*(V, g)}{dg}\right|_V = \frac{\epsilon A V^2}{2g^2}$$



$$F = \frac{\epsilon A V_{in}^2}{2g^2}$$ Electrical force

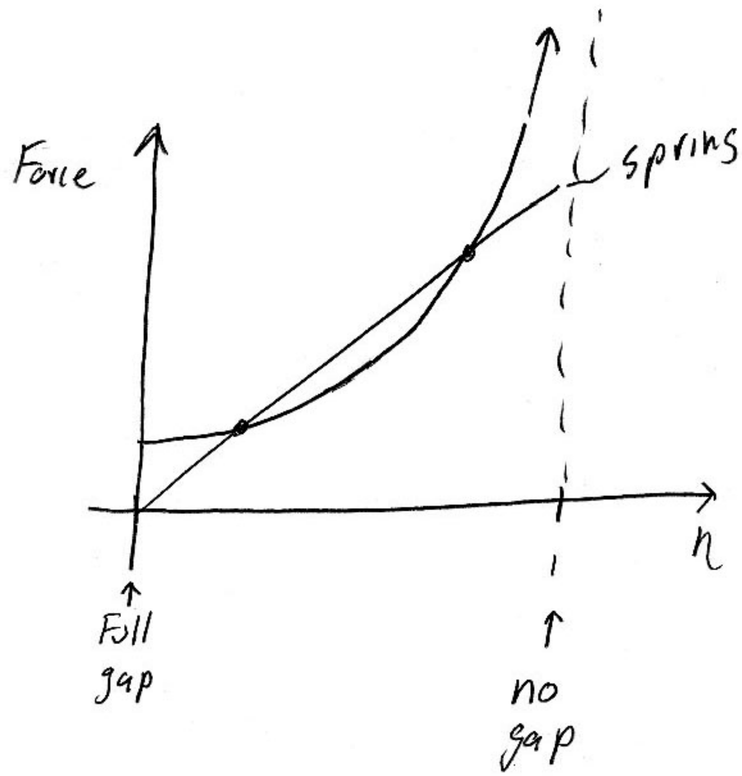
$$g = g_0 - z$$

$$z = \frac{F}{k}$$ Spring force

Set equal

$$g = g_0 - \frac{\epsilon A V_{in}^2}{2kg^2}$$ (Cubic equation)

$$g = g_0 - \frac{\epsilon A V_{in}^2}{2 k g^2}$$ (Cubic equation)



$$\eta = 1 - \frac{g}{g_0}$$

<u>Pull-in</u>

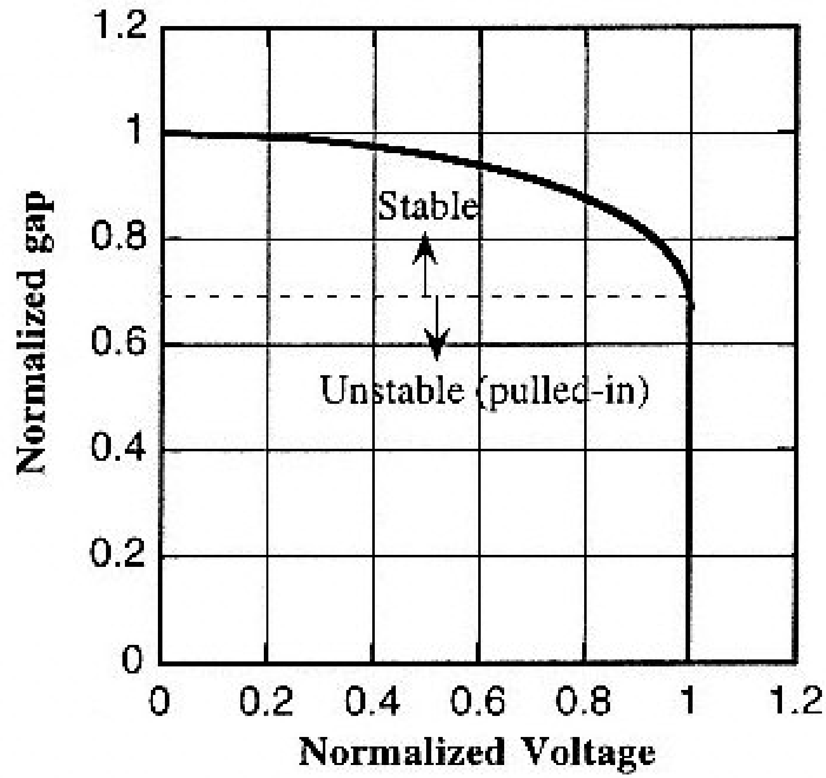


Figure 6.8. Normalized gap as a function of normalized voltage for the electrostatic actuator.

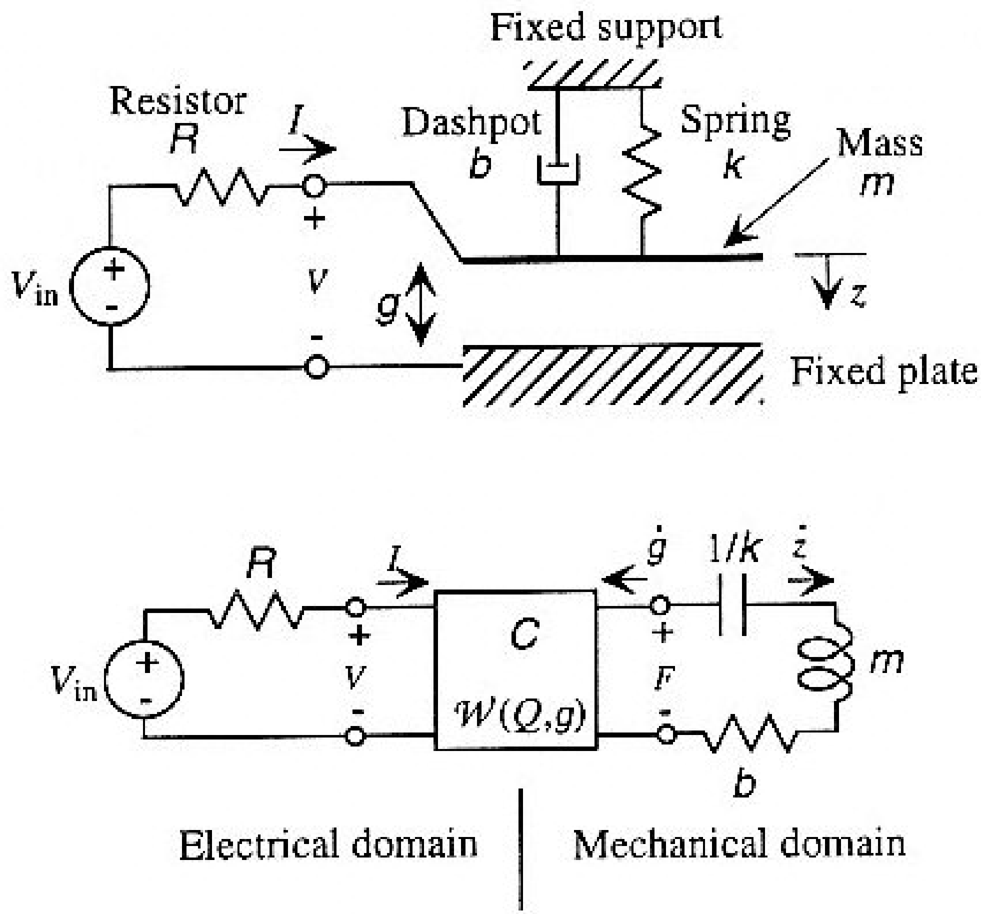


Figure 6.9. A complete electrostatic actuator, with added elements representing the inertia of the moveable element, mechanical damping, and the source resistance of the electrical network.