Brewster angle with a negative-index material

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The demonstration and confirmation of metamaterials with simultaneous negative permittivity and permeability, and thus a negative refractive index, has resulted in a surge of interest in the reflection and refraction phenomena at the interfaces of these so-called negative-index materials (NIMs). We present a systematic study of the Brewster angle, i.e., the angle of incidence at which no reflection occurs, for both TE and TM waves scattering at the interface between two semi-infinite planar media, one of which may be a NIM. Detailed physical explanations that account for the Brewster angle for a plane wave incident upon a NIM are provided under the framework of the Ewald–Oseen extinction theorem, considering the re-emission of induced electric and magnetic dipoles. The conditions under which the Brewster angle exists are concisely summarized in a map of different material parameter regimes. © 2005 Optical Society of America

1. Introduction

There has been growing interest in the study of negative-index materials (NIMs), which have been demonstrated and verified both experimentally and by rigorous numerical simulation. Many of the unique features associated with NIMs were summarized in Veselago’s original paper, e.g., negative phase velocity, reversed Doppler effect, and the prediction of a planar lens. Several other surprising features have been found recently, including amplification of evanescent waves (the “perfect lens”), multiple images, photon tunneling enhancement, nonparallel group-front velocity and group velocity, a negative beam shift upon reflection, and a negative Goos–Hänchen shift upon total internal reflection.

A Brewster angle is an incidence angle for which no energy is reflected for at least one polarization. The geometry considered here is illustrated in Fig. 1, where light is incident from a medium of refractive index $n_1$ into another medium of refractive index $n_2$. Each medium is characterized by a real permittivity $\varepsilon$ and a real permeability $\mu$ (relative to those of free space) of the same sign. Consequently, the refractive index $n = \pm \sqrt{\varepsilon \mu}$ is positive when both $\varepsilon$ and $\mu$ are positive, or negative when both $\varepsilon$ and $\mu$ are negative. In this figure, PIM refers to a conventional dielectric with a positive index of refraction. Note that when light is incident from a PIM to a NIM, as shown in Fig. 1(b), negative refraction occurs, and the refracted wave vector points opposite to the direction of energy flux. For nonmagnetic dielectrics, the Brewster angle results from the fact that an electric dipole cannot radiate along its own axis. The reflected power goes to zero when the electric dipoles induced in the material align with the direction of the reflected wave. Consequently, a Brewster angle exists only for TM waves, when the refracted wave is perpendicular to the reflected wave ($i.e., \theta_1 + \theta_2 = 90^\circ$). When this criterion is combined with Snell’s law, the conventional expression for the Brewster angle is obtained: $\theta_B = \theta_1 = \tan^{-1}(n_2/n_1)$. There are a number of applications and devices that employ the unique property of Brewster’s angle, including polarizers and transmission windows.

In the case of a NIM, the reflected fields consist of radiation from not only electric dipoles but also magnetic dipoles, since a NIM is a magnetic material ($\mu \neq 1$). It has been shown that the Brewster angle may occur for both TE and TM waves when one of the materials is a NIM. However, the criteria for the occurrence of a Brewster angle with a NIM have not been fully addressed. The purpose of the present work is to examine systematically the existence of the Brewster angle at the interface between two semi-
infinite isotropic dielectric–magnetic media, including especially the case of NIMs, with all possible combinations of constitutive parameters. The physical origin is investigated on the basis of the Ewald–Oseen extinction theorem in order to explain, from a microscopic point of view, when the emission from the electric dipoles and that from the magnetic dipoles cancel each other to cause null reflection.

2. Reflection from Dielectric–Magnetic Media

A. Fresnel Coefficients

The starting point for calculating the (power) reflectivity is to evaluate the Fresnel reflection coefficients between medium 1 and medium 2 (see Fig. 1). For a TE plane wave (i.e., s polarized), the reflection coefficient is the ratio of reflected to incident electric field amplitudes. Following, for example, Pendry, it can be shown that

\[ r_s = \frac{E_{r0}}{E_{i0}} = \frac{k_{1z}/\mu_1 - k_{2z}/\mu_2}{k_{1z}/\mu_1 + k_{2z}/\mu_2} \]
\[ = \frac{\sqrt{\varepsilon_1/\mu_1} \cos \theta_1 - \sqrt{\varepsilon_2/\mu_2} \cos \theta_2}{\sqrt{\varepsilon_1/\mu_1} \cos \theta_1 + \sqrt{\varepsilon_2/\mu_2} \cos \theta_2} \]  

(1)

where \( k_{1z} \) and \( k_{2z} \) are the \( z \) components of the wave vectors of the incident and the refracted waves, respectively; \( \theta_1 \) and \( \theta_2 \) are the angles of incidence and refraction and are related by Snell’s law, \( n_1 \sin \theta_1 = n_2 \sin \theta_2 \). Note that \( k_z = n(\omega/c)\cos \theta \), where \( \omega \) is the angular frequency and \( c \) is the speed of light in vacuum. For a TM plane wave (i.e., \( p \) polarized), the ratio of reflected to incident magnetic field amplitudes defines the reflection coefficient as follows:

\[ r_p = \frac{H_{r0}}{H_{i0}} = \frac{k_{1z}/\varepsilon_1 - k_{2z}/\varepsilon_2}{k_{1z}/\varepsilon_1 + k_{2z}/\varepsilon_2} \]
\[ = \frac{\sqrt{\mu_1/\varepsilon_1} \cos \theta_1 - \sqrt{\mu_2/\varepsilon_2} \cos \theta_2}{\sqrt{\mu_1/\varepsilon_1} \cos \theta_1 + \sqrt{\mu_2/\varepsilon_2} \cos \theta_2} \]  

(2)

If medium 2 is a NIM, both \( \theta_2 \) and \( k_{2z} \) become negative as shown in Fig. 1(b), but \( \cos \theta_2 \) remains positive. Because the ratio of permittivity and permeability is positive, the reflection coefficients are real and depend only on the magnitudes of \( \varepsilon \)'s and \( \mu \)'s, with one exception as explained in the following. If \( |n_1/n_2| > 1, \) (i.e., \( \varepsilon_1\mu_1 > \varepsilon_2\mu_2 \)), total internal reflection will occur at incidence angles equal to or greater than the critical angle \( \theta_c = \sin^{-1} |n_2/n_1| \) regardless of the polarization state. When \( \theta_1 > \theta_c \), both \( k_{2z} \) and \( \cos \theta_2 \) become purely imaginary, resulting in complex reflection coefficients (while the magnitude is unity). The requirement that the evanescent wave decay toward positive \( z \) in medium 2 imposes a constraint on the selection of \( k_{2z} \), that is, \( k_{2z} = n_2(\omega/c)\cos \theta_2 = i\beta_2 \), where \( \beta_2 = (\omega/c)(n_2^2|\sin \theta_1 - n_2^2|^{1/2} \) is a positive number no matter whether medium 2 is a PIM or a NIM. When the sign of \( n_2 \) is changed from positive (PIM) to negative (NIM), \( k_{2z} \) remains the same but \( \cos \theta_2 \) changes sign. As a consequence, the reflection coefficient will change to its complex conjugate for a given polarization. Therefore attention must be paid to the signs of \( k_{2z} \) and \( \cos \theta_2 \) when Eqs. (1) and (2) are applied to NIMs.

The power reflectivity is given by \( \rho_p = |r_p|^2 \) and \( \rho_p = |r_s|^2 \), respectively, for a TE wave and a TM wave. Despite the fact that \( \varepsilon \)'s and \( \mu \)'s may be negative, their signs do not have any effect on the reflectivity. The Brewster angle corresponds to the reflection coefficient equal to zero. Because the Fresnel reflection coefficient is a continuous and monotonic function of the incidence angle, the existence of a Brewster angle depends on whether the sign of the reflection coefficient changes between normal incidence and an incidence angle equal to either the critical angle or the grazing angle (90°). Consider the case \( \varepsilon_1\mu_1 < \varepsilon_2\mu_2 \), when there is no total internal reflection. It can be seen from Eqs. (1) and (2) that \( r_p = -r_s \) at normal incidence, and \( r_p = r_s = -1 \) at \( \theta_1 = 90^\circ \). When \( \mu_1/\varepsilon_1 \neq \mu_2/\varepsilon_2 \), either \( r_p \) or \( r_s \) will be positive at \( \theta_1 = 0 \), resulting in a Brewster angle between 0° and 90° for a TE wave if \( \mu_1/\varepsilon_1 < \mu_2/\varepsilon_2 \) and for a TM wave if \( \mu_1/\varepsilon_1 > \mu_2/\varepsilon_2 \). In the extreme case when \( \mu_1/\varepsilon_1 = \mu_2/\varepsilon_2 \), \( r_p = 0 \) at normal incidence. Consequently, a Brewster angle occurs for both polariza-
tions at normal incidence, and the reflectivity (the same for both polarizations) will increase monotonically from 0 to 1 as \(\theta_1\) changes from 0 to 90°. On the other hand, if \(\varepsilon_1/\mu_1 > \varepsilon_2/\mu_2\), both \(k_{2z}\) and \(\cos \theta_2\) are zero at the critical angle \(\theta_1 = \theta_c\), resulting in \(r_p = r_s = 1\). In this case a Brewster angle exists for a TE wave if \(\mu_1/\varepsilon_1 > \mu_2/\varepsilon_2\) and for a TM wave if \(\mu_1/\varepsilon_1 < \mu_2/\varepsilon_2\). When \(\mu_1/\varepsilon_1 = \mu_2/\varepsilon_2\), however, the reflectivity is the same for both polarizations and increases from 0 to 1 when \(\theta_1\) is increased from 0 to \(\theta_c\). In the extreme case when \(\varepsilon_1\mu_1 = \varepsilon_2\mu_2\), \(|\theta_1| = |\theta_2|\), and the reflectivity will be independent of the incidence angle (but dependent on the polarization). If \(\varepsilon_2 = -\varepsilon_1\) and \(\mu_2 = -\mu_1\), however, there will be no reflection at all, and any angle may be considered to be a Brewster angle.

B. Reflected Field Due to Material Dipoles

From the microscopic point of view, the reflected electromagnetic wave at the interface between vacuum and a dielectric is the result of re-emission by the induced electric dipoles in the dielectric medium. When the medium is made of a dielectric–magnetic material such as a NIM, the re-emission of magnetic dipoles also contributes to the reflected field. In the following, the reflection and refraction between two isotropic media is considered for the same coordinates shown in Fig. 1. For simplicity, it is assumed that the upper half of the space \((-\infty < z < 0)\) is vacuum \((\varepsilon_1 = \mu_1 = 1)\) and the lower half of the space \((0 < z < +\infty)\) is occupied by a dielectric–magnetic medium \((\varepsilon_2, \mu_2)\).

A harmonic plane wave of angular frequency \(\omega\) incident from vacuum can be characterized by its electric field \(E_0 = E_0 \exp(ik_0 \cdot r)\) (omitting the common factor \(\exp(-i\omega t)\) here and subsequently). Note that \(k_1 = k_{1x} + ik_{1z}\) and \(k_2 = k_{2x} + ik_{2z}\). The reflected and transmitted electric fields may be expressed as \(E_r = E_0 \exp(ik_k \cdot r)\) and \(E_t = E_0 \exp(ik_k \cdot r)\). The magnetic field can be obtained from the corresponding electric field by use of Maxwell’s equations. A necessary condition is for all the oscillating fields to vary in the \(x\) direction as \(\exp(ik_k x)\), which is the phase-matching condition. By invoking the boundary conditions, which state that the tangential components of both electric and magnetic fields are continuous at the interface, and setting \(k_r = k_1 x - ik_1 z\) and \(k_k = k_2 x + ik_2 z\), one can conveniently obtain the Fresnel reflection coefficients given in Eqs. (1) and (2).

The Ewald–Oseen extinction theorem does not rely on the boundary conditions, nor does it presume that the speed of propagation is determined by the refractive index of the medium. It is based on the summation of retarded fields from induced material-dipole sources, with the requirement that the induced oscillating sources (and consequently the radiated fields) vary as \(\exp(ik_k x)\), which is always satisfied for a linear material. When the lower medium is a nonmagnetic material, it is well known that the extinction theorem can be used to determine the wave vectors \(k_r\) and \(k_k\), to derive the reflection coefficients given in Eqs. (1) and (2), and to obtain the boundary conditions as well.\(^{26}\) Feynman et al.\(^{27}\) and Schwartz\(^{28}\) used the summation of retarded fields as a pedagogical aid to understanding the speed of light in a material. Lai et al.\(^{29}\) applied the Feynman–Schwartz method to deduce the reflection and transmission coefficients. In principle, the Feynman–Schwartz method is the same as the extinction theorem. In the following, the derivation based on the extinction theorem is extended to include magnetic and NIM materials of arbitrary \(\varepsilon_2\) and \(\mu_2\). The results will be used to understand NIM reflection from a microscopic perspective, and particularly the conditions for a Brewster angle.

According to the Ewald–Oseen extinction theorem, the transmitted (or refracted) wave is the superposition of the incident wave and all the waves radiated by the induced dipoles. In other words, the field \(E_t\) is assumed to permeate to the lower medium without being affected by the interface and the properties of that medium, i.e., \(E_t = E_0 \exp(ik_k x + ik_k z (z < -\infty))\). The total radiated field \(E_{\text{rad}}\) is the sum of the contribution from all electric dipoles \(E_{\text{rad}}^{(e)}\) and that from all magnetic dipoles \(E_{\text{rad}}^{(m)}\). Hence,

\[
E_{\text{rad}} = E_{\text{rad}}^{(e)} + E_{\text{rad}}^{(m)} = \begin{cases} E_r, & -\infty < z < 0 \\ E_t - E_r, & 0 \leq z < +\infty \end{cases}.
\]

(3)

It is clear from Eq. (3) that the re-emission of all dipoles to the upper half of the space results in the reflected field, whereas the re-emission of all dipoles to the lower half of the space extinguishes the incident field and produces the transmitted field that propagates inside the medium with a wave vector \(k_r\). The radiated fields can be expressed as follows\(^{26}\):

\[
E_{\text{rad}}^{(e)} = \nabla \left( \nabla \cdot \Pi_e \right) - \varepsilon_0 \mu_0 \frac{\partial^2 \Pi_e}{\partial t^2},
\]

(4a)

\[
E_{\text{rad}}^{(m)} = -\mu_0 \nabla \times \frac{\partial \Pi_m}{\partial t},
\]

(4b)

where \(\varepsilon_0\) and \(\mu_0\) are the (absolute) permittivity and permeability of vacuum. Here \(\Pi_e\) and \(\Pi_m\) are the Hertz vectors, which represent the summed retarded fields of point dipoles and can be expressed by the following volume integrations:

\[
\Pi_e(r) = \int_V \frac{P(r')}{\varepsilon_0} G(r - r') \, dr',
\]

(5a)

\[
\Pi_m(r) = \int_V M(r') \, G(r - r') \, dr',
\]

(5b)

where \(P\) is the polarization, \(M\) is the magnetization, and \(G(r - r') = [\exp(ik_0 |r - r'|)]/(4\pi |r - r'|)\).
paring the phase terms, it can be shown that

\[ k / H \times 2 / H \times 2 \]

where \( k_t = | \mathbf{k} | \) and \( k_{z} \) is the \( z \) component of \( \mathbf{k} \). Likewise, evaluation of the integration in Eq. (5b) yields

\[
\Pi_m = \begin{cases}
-\chi_m \mathbf{H}_0 \frac{\exp[i(k_{x}x - k_{z}z)]}{2k_{1z}(k_{z} + k_{1z})} \\
\chi_m \mathbf{H}_0 \left[ \frac{\exp[i(\mathbf{k} \cdot \mathbf{r})]}{2k_{1z}(k_{z} - k_{1z})} + \frac{\exp(i\mathbf{k} \cdot \mathbf{r})}{k_{t}^2 - k_{1z}^2} \right] 
\end{cases}
\]

for \(-\infty < z < 0\)

for \( 0 \leq z < +\infty \)

(6b)

where \( \mathbf{H}_0 = (1/\mu_2)^{\mathbf{k} \times \mathbf{E}_0} \) denotes the amplitude of the transmitted magnetic field. With Eqs. (6a) and (6b) substituted into Eqs. (4a) and (4b), the total radiated field can be expressed in terms of \( \mathbf{E}_0 \). From Eq. (3), one obtains a relation between \( \mathbf{E}_0 \) and \( \mathbf{E}_0 \) from the equation for \(-\infty < z < 0\) and another between \( \mathbf{E}_0 \) and \( \mathbf{E}_0 \) (from the equation for \( 0 \leq z < +\infty \)). By comparing the phase terms, it can be shown that \( \mathbf{k} = k_{x} \mathbf{x} - k_{z} \mathbf{z} \) and \( k_{z}^2 = (1 + \chi)(1 + \chi_m)k_{1z}^2 \). Hence, \( \mathbf{k} = k_{z} \mathbf{z} \). The relations between the amplitudes of the electric fields read as

\[
\mathbf{k} \times (\mathbf{k} \times \mathbf{E}_0) - \chi_m \mathbf{H}_0 = \frac{\exp[i(k_{x}x - k_{z}z)]}{2k_{1z}(k_{z} + k_{1z})},
\]

\[
\mathbf{k} \times (\mathbf{k} \times \mathbf{E}_0) = \chi_m \mathbf{H}_0 \left[ \frac{\exp[i(\mathbf{k} \cdot \mathbf{r})]}{2k_{1z}(k_{z} - k_{1z})} + \frac{\exp(i\mathbf{k} \cdot \mathbf{r})}{k_{t}^2 - k_{1z}^2} \right].
\]

(7)

(8)

Note that Eqs. (7) and (8) are valid for both TE and TM waves, even when the lower medium is dissipative. If the effect of loss is included, however, the reflectivity will not be exactly zero. Only lossless media are considered throughout this work. The Fresnel coefficients can be obtained from Eqs. (7) and (8) as shown below.

For a TE wave, because \( \mathbf{E}_0 = E_{0y} \hat{y} \) and \( \mathbf{E}_0 = E_{0y} \hat{y} \), \( \mathbf{k} \cdot \mathbf{E}_0 = k_{1} \cdot \mathbf{E}_0 = 0 \). Using the vector identity \( \mathbf{A} \times (\mathbf{B} \times \mathbf{C}) = (\mathbf{A} \cdot \mathbf{C})\mathbf{B} - (\mathbf{A} \cdot \mathbf{B})\mathbf{C} \), one can see that

\[
\mathbf{k} \times (\mathbf{k} \times \mathbf{E}_0) = -k_{1z}^2 \mathbf{E}_0, \quad \mathbf{k} \times (\mathbf{k} \times \mathbf{E}_0) = -k_{1z}^2 \mathbf{E}_0, \quad \mathbf{k} \times (\mathbf{k} \times \mathbf{E}_0) = -k_{1z}^2 \mathbf{E}_0, \quad \mathbf{k} \times (\mathbf{k} \times \mathbf{E}_0) = -k_{1z}^2 \mathbf{E}_0,
\]

and

\[
\mathbf{k} \times (\mathbf{k} \times \mathbf{E}_0) = -\left( k_{1z}^2 + k_{1z}k_{2z} \right) \mathbf{E}_0.
\]

When these quantities are substituted into Eqs. (7) and (8), the expression of the reflection coefficient \( r_p \) in Eq. (1) is readily obtained. For a TM wave the derivation for

\[
\mathbf{H}_0 = \frac{1}{\mu_0 \omega} \mathbf{k}_x \times \mathbf{E}_0 = \mathbf{H}_0 \hat{y},
\]

\[
\mathbf{H}_0 = \frac{1}{\mu_0 \omega} \mathbf{k}_x \times \mathbf{E}_0 = \mathbf{H}_0 \hat{y},
\]

\[
\mathbf{H}_0 = \frac{1}{\mu_2 \omega \mu_0} \mathbf{k}_x \times \mathbf{E}_0 = \mathbf{H}_0 \hat{y}.
\]

The cross product of Eq. (7) by \( (1/\omega \mu_0) \mathbf{k}_x \) and the cross product of Eq. (8) by \( (1/\omega \mu_0) \mathbf{k}_x \) will give two new equations relating the magnetic field amplitudes. The derivation can be greatly simplified by noting that

\[
\frac{1}{\omega \mu_0} \mathbf{k}_x \times \mathbf{E}_0 = \frac{k_{1z}^2 + k_{1z}k_{2z}}{\epsilon_2 k_{1z}^2} \mathbf{E}_0.
\]

Note that Eqs. (7) and (8) are valid for both TE and TM waves, even when the lower medium is dissipative. If the effect of loss is included, however, the reflectivity will not be exactly zero. Only lossless media are considered throughout this work. The Fresnel coefficients can be obtained from Eqs. (7) and (8) as shown below.
In fact, the results are the same as can be obtained by directly interchanging the following pairs: \( \mu_2 \) and \( \varepsilon_2 \), \( \chi \) and \( \chi_m \), and \( \mathbf{E}'s \) and \( \mathbf{H}'s \), from a TE wave to a TM wave in Eqs. (7) and (8).

As expected, all the results from the Ewald–Oseen theorem are consistent with the solutions obtained by using the boundary conditions and dispersion relations. Moreover, Eq. (7) unambiguously identifies the contribution by the electric dipoles and that by the magnetic dipoles, allowing one to investigate how the Brewster angle, corresponding to \( \mathbf{E}_r = 0 \), emerges as a result of the combined contributions.

The first term in Eq. (7) represents the contribution from electric dipole radiation, \( \mathbf{E}_r^{(e)} \), and the second term represents the contribution from magnetic dipole radiation, \( \mathbf{E}_r^{(m)} \). In the special case of \( \varepsilon_2 = \mu_2 \) (i.e., \( \chi = \chi_m \)), it can be shown that \( \mathbf{E}_r = 0 \) at normal incidence because \( \mathbf{E}_r^{(e)} \) and \( \mathbf{E}_r^{(m)} \) have the same magnitude but opposite signs. At the Brewster angle (\( \theta_1 = \theta_2 \)), the relation \( \theta_1 + \theta_2 = 90^\circ \) does not necessarily hold for a given dielectric–magnetic medium. For a TE wave incident from vacuum, the condition for zero \( \mathbf{E}_r \) is obtained from Eq. (7) as

\[
(\mu_2 - 1)(k_x^2 - k_z^2) = \mu_2(\varepsilon_2 - 1)k_1^2. \tag{9}
\]

It follows that

\[
k_x \cdot k_z = k_x^2 - k_z^2 = \frac{\mu_2(\varepsilon_2 - 1)k_1^2}{1 - \mu_2}, \tag{10}
\]

which implies that for \( \mu_2 \neq 1 \), only if \( \varepsilon_2 = 1 \), the transmitted wave is perpendicular to the reflected wave at the Brewster angle. In this case, \( \mathbf{E}_r^{(e)} = 0 \), since there are no induced electric dipoles. When the medium is a NIM, the refracted wave will be on the same side as the incident wave. By applying Eq. (9) and \( k_x^2 + k_z^2 = k_2^2 = \varepsilon_2\mu_2k_1^2 \), it can be shown that \( k_2 = \mu_2k_1 \), and

\[
k_x \cdot k_z = k_x^2 + k_z^2 = \frac{\mu_2(\varepsilon_2 + 1)k_1^2}{1 + \mu_2}. \tag{11}
\]

Therefore the transmitted wave will be perpendicular to the incident wave if \( \varepsilon_2 = -1 \) (unless \( \mu_2 \) is also \(-1 \), in which case the Brewster angle is not uniquely determined). It should be emphasized that, in the case of a NIM, both the electric dipoles and magnetic dipoles contribute to the reflected field. Interchanging \( \varepsilon_2 \) and \( \mu_2 \) in Eqs. (9)–(11) gives the results for a TM wave incident from vacuum. Some results will be shown graphically in Section 3 for demonstration.

3. **Illustrative Results**

The calculated reflectivity for light incident from vacuum to a dielectric–magnetic medium is plotted in Fig. 2 as a function of the incidence angle \( \theta_1 \). It should be noted that the results are applicable for both a PIM and a NIM. The results are shown for TE waves only, since the results for TM waves will be the same if \( \varepsilon_2 \) and \( \mu_2 \) are interchanged. Figure 2(a) corresponds to \(| n_2 | = 2 \). The Brewster angle for a TE wave exists when \(| \varepsilon_2 | \leq | \mu_2 | \). When \( \varepsilon_2 = \mu_2 \), the reflectivity is independent of polarization and increases from zero at normal incidence to 1 at the incidence angle equal to 90°. When \(| n_2 | = 0.5 \), total internal reflection will occur when \( \theta_1 = \theta_2 \), which is 30° in this case, as shown in Fig. 2(b). In this case, a Brewster angle can be observed for TE waves when \(| \varepsilon_2 | \leq | \mu_2 | \).

To understand the contributions of the electric dipoles \( \mathbf{E}_r^{(e)} \) and magnetic dipoles \( \mathbf{E}_r^{(m)} \), corresponding to the first and second terms in Eq. (7), to the reflected field \( \mathbf{E}_r^{(e)} \), consider again a TE wave incident from vacuum. All the electric vectors are parallel to the \( y \) direction but are in general complex numbers. Figure 3 shows the magnitudes \(| E_{r0}^{(e)} |\), \(| E_{r0}^{(m)} |\), and \(| E_{r0} |\) normalized with respect to \( E_{r0} \) (the phase of \( E_{r0} \))...
is set to zero) for incidence into a PIM. It can be seen from Fig. 3(a) that \( E_{r0}^{(e)} = 0 \) and that only the induced magnetic dipoles contribute to the reflected wave when \( \varepsilon_2 = \varepsilon_1 = 1 \). A Brewster angle arises from the induced magnetic dipoles being aligned along the direction of the reflected wave and can be found to be \( \theta_B = \tan^{-1}(n_2/n_1) \) and, at this angle, \( \theta_1 + \theta_B = 90^\circ \). In Fig. 3(b), \( \varepsilon_2 = 0.5 \) and \( \mu_2 = 8 \) are chosen to give the same refractive index \( n_2 = 2 \). However, both induced electric and magnetic dipoles contribute to the reflected wave. The contribution from the induced magnetic dipoles \( E_{r0}^{(m)} \) becomes zero at \( \theta_1 = \tan^{-1}(n_2/n_1) \), whereas that from the induced electric dipoles \( E_{r0}^{(e)} \) is always nonzero. Furthermore, \( E_{r0}^{(e)} \) and \( E_{r0}^{(m)} \) are in phase when \( \theta_1 < \tan^{-1}(n_2/n_1) \) but are \( 180^\circ \) out of phase when \( \theta_1 > \tan^{-1}(n_2/n_1) \). As a consequence, the Brewster angle is pushed toward a larger incidence angle when \( E_{r0}^{(e)} = -E_{r0}^{(m)} \).

The situation is different for a NIM, because the induced electric and magnetic dipoles always coexist and contribute to the reflected wave. For comparison, Fig. 4 shows the results for a NIM with the same absolute values of \( \varepsilon \) and \( \mu \) as the corresponding cases in Fig. 3. The resulting \( E_{r0} \) and \( \theta_B \) are exactly the same for Fig. 3(a) and Fig. 4(a) and for Fig. 3(b) and Fig. 4(b). However, compared with Fig. 3, the values of \( |E_{r0}^{(e)}| \) and \( |E_{r0}^{(m)}| \) in Fig. 4 are much greater and never reach zero. Note that the phases of \( E_{r0}^{(e)} \) and \( E_{r0}^{(m)} \) are always reversed and that a Brewster angle occurs when their magnitudes are equal. Figure 4 clearly demonstrates the microscopic origin of the Brewster angle, or zero reflectivity, in a NIM as a cancellation of the superimposed fields radiated by...
the induced electric and magnetic dipoles. The cancellation of the radiated fields would never occur along the axes of either the electric dipoles or the magnetic dipoles in a NIM. Interestingly, in the case shown in Fig. 4(a) for \( \varepsilon_2 = -1 \), the field radiated by the induced magnetic dipoles \( E_{r0}^{(m)} \) is independent of the incidence angle. By setting \( \varepsilon_2 = -1 \) in Eqs. (7) and (8), it can be shown after some manipulations that \( E_{r0}^{(m)} \) is equal to \((\mu_2 - 1)/\mu_2 + 1 \) times \( E_{r0} \). In this case, the incidence and refraction angles satisfy \( \theta_1 - \theta_2 = 90^\circ \) at the Brewster angle, indicating that the transmitted wave is perpendicular to the incident wave.

It should be noted that when both \( \varepsilon_2 = \mu_2 = -1 \) (not shown in the figure), \( E_{r0}^{(e)} \) and \( E_{r0}^{(m)} \) have the same magnitude (which approaches infinity) but opposite signs, as can be seen from Eq. (7). Hence, the resulting reflected fields cancel each other for any incidence angle. This special case has been investigated extensively in the study dealing with NIMs. The microscopic interpretation of the zero reflectivity in this case lies in the cancellation of two extremely large fields radiated by different types of induced dipoles. An alternative view is that the re-emission of the electric dipoles and magnetic dipoles from any volume element in the medium cancel out in the upper space.

When total internal reflection occurs, \( k_z \) becomes purely imaginary, and there will be an evanescent wave inside medium 2. The radiated field amplitude \( E_{r0} = |E_{r0}| e^{i\phi_0} \) will be complex. Figure 5 shows the magnitude (normalized by \( E_{r0} \)) and the phase for the radiated fields, considering the separate as well as the combined contributions of the electric and magnetic dipoles. The incidence is a TE wave from vacuum to a PIM of \( \varepsilon_2 = 2 \) and \( \mu_2 = 0.125 \). When \( \theta_1 \leq \theta_c = 30^\circ \), the magnitudes vary similarly to those shown in Fig. 3(b) without total internal reflection. Note that \( E_{r0}^{(e)} \) and \( E_{r0}^{(m)} \) are in phase at small \( \theta_1 \), until the angle corresponding to \( E_{r0}^{(e)} = 0 \), and \( 180^\circ \) out of phase afterwards. Beyond the critical angle the phases are more complex, but the combination of \( E_{r0}^{(e)} \) and \( E_{r0}^{(m)} \) makes \( |E_{r0}| = 1 \). If the medium is changed to a NIM (not shown in the plot) with \( \varepsilon_2 = -2 \) and \( \mu_2 = -0.125 \), \( E_{r0}^{(e)} \) and \( E_{r0}^{(m)} \) will be \( 180^\circ \) out of phase when \( \theta_1 \leq \theta_c \). Although the separate contributions are quite different, the resulting \( |E_{r0}| \) does not change. The resulting \( \phi_0 \) is the same when \( \theta_1 \leq \theta_c \) but changes sign when \( \theta_1 > \theta_c \); i.e., \( \phi_0 \) will vary from 0 to \( 180^\circ \) when \( \theta_1 \) is changed from \( \theta_c \) to \( 90^\circ \) for a NIM.

Another example is shown in Fig. 6 for a NIM with \( \varepsilon_2 = 1 \) and \( \mu_2 = 0.25 \). The results for \( \theta_1 \leq \theta_c = 30^\circ \) are similar to those shown in Fig. 4(a) without total internal reflection. The magnetic dipole contribution is a constant with a phase of \( 180^\circ \) regardless of the incidence angle. It is worth discussing the case for a PIM with \( \varepsilon_2 = 1 \) and \( \mu_2 = 0.25 \), though not shown in the figure, in which case there are no induced electric dipoles and \( E_{r0} = E_{r0}^{(m)} \). The resulting magnitude is the same as with a NIM, and the resulting phase is also the same when \( \theta_1 \leq \theta_c \) but changes to a negative sign when \( \theta_1 > \theta_c \). Note that reflection measurements cannot distinguish the separate dipole contributions. Measurement of the power reflectivity cannot determine whether the second medium is a PIM or NIM. However, if total internal reflection occurs (for example, in an attenuated total reflectance arrangement), the phase of the reflectance coefficient allows a distinction between a PIM and a NIM of the second medium: \( \phi_{r0} \geq 0 \) for a NIM, and \( \phi_{r0} \leq 0 \) for a PIM. This can be done by, say, ellipsometric measurements.

Return to Eqs. (1) and (2) for the general case, where both medium 1 and medium 2 are lossless dielectric–magnetic with arbitrary values of \( \varepsilon 's \) and \( \mu 's \). By setting \( r_s = 0 \) or \( r_p = 0 \), one obtains the following expressions of the Brewster angle: \( \theta_B = \sin^{-1}\left[\sqrt{(1 - Y)/(1 - XY)}\right] \) for a TE wave and \( \theta_B = \sin^{-1}\left[\sqrt{(Y - 1)/(Y - X)}\right] \) for a TM wave, where \( X \) and \( Y \) are given by the following equations:

\[
X = \frac{\mu_2}{\varepsilon_1}, \quad Y = \frac{\varepsilon_2}{\mu_1}
\]
in the shaded regions (I) and (IV). Similarly, a Brewster angle exists for a TE wave in regions (II) and (III). Total internal reflection will occur in regions (II) and (IV), where $X > 1$. The line $X = 1$ corresponds to the case $|\theta_1| = |\theta_2|$, when the reflectivity is independent of the angle of incidence. The line $Y = 1$ corresponds to the cases when the reflectivity is independent of polarization and is zero at normal incidence. The normal reflectivity increases as $Y$ is either reduced or increased from 1 and becomes unity as $Y$ approaches 0 or infinity. At the crossing point $X = Y = 1$, the reflectivity is zero for both polarizations, regardless of the angle of incidence, though light will be refracted through a PIM–NIM or NIM–PIM interface. The two curves for $X = Y$ and $XY = 1$ correspond to conditions under which the refracted wave will be perpendicular to the reflected wave (at PIM–PIM and NIM–NIM interfaces) or the incident wave (at PIM–NIM and NIM–PIM interfaces).

4. Conclusion

This work provides a derivation based on the Ewald-Oseen extinction theorem to show that recombination of the radiated fields by the induced electric dipoles and magnetic dipoles is responsible for the reflected wave, and that the cancellation of the radiated fields will result in a zero reflectivity and hence a Brewster angle upon reflection by a NIM. A Brewster angle can exist not only for a TM wave but also for a TE wave, and that the cancellation of the radiated fields by the induced electric dipoles and magnetic dipoles is responsible for the reflected or incident wave at the Brewster angle. A Brewster angle can exist not only for a TM wave but also for a TE wave, and that the cancellation of the reflected wave at the Brewster angle on a Brewster angle or occurs at any angle (if $Y = 1$).

The various possibilities for the Brewster angle can be concisely summarized in a map of different material parameter regimes shown in Fig. 7 in terms of $X$ and $Y$. A different regime map containing similar information was provided in the work of Henderson et al., who considered the reflection and transmission between two dielectric–magnetic materials with all positive values of constitutive parameters without discussing NIMs. Based on the discussion given above, a Brewster angle can be found for a TM wave in the shaded regions (I) and (IV). Similarly, a Brewster angle exists for a TE wave in regions (II) and (III). Total internal reflection will occur in regions (II) and (IV), where $X > 1$. The line $X = 1$ corresponds to the case $|\theta_1| = |\theta_2|$, when the reflectivity is independent of the angle of incidence. The line $Y = 1$ corresponds to the cases when the reflectivity is independent of polarization and is zero at normal incidence. The normal reflectivity increases as $Y$ is either reduced or increased from 1 and becomes unity as $Y$ approaches 0 or infinity. At the crossing point $X = Y = 1$, the reflectivity is zero for both polarizations, regardless of the angle of incidence, though light will be refracted through a PIM–NIM or NIM–PIM interface. The two curves for $X = Y$ and $XY = 1$ correspond to conditions under which the refracted wave will be perpendicular to the reflected wave (at PIM–PIM and NIM–NIM interfaces) or the incident wave (at PIM–NIM and NIM–PIM interfaces).
incidence of certain polarized plane waves. The conclusions, coming along with the occurrence of a Brewster angle in the case of a NIM, may be a useful supplement to the understanding of the material’s electromagnetic behavior, and may help the development of potential applications of NIMs in advanced technologies.

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References