Optics for Engineers
EECE5646 — Fall 2009
Final Exam

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Name: ______________________________

General Instructions Please DO NOT COLLABORATE with other students or seek help from outside experts. However, you may use any reference book, journal articles, or other readily available resources. Please cite references if you do so.

Note on Academic Honesty: No collaboration is allowed under penalty of failure. Plagiarism and cheating will not be tolerated; they will be dealt in accordance with University policies described in the Student Handbook. All engineering majors should be familiar with the Honor Code of our College of Engineering that is included in the freshman course material, and with professional engineering codes of ethics. If two students' work is suspiciously similar, a penalty may be assessed to both students. If a situation arises in which you are uncertain if cooperation with another student would constitute cheating or some other violation of the honor code, please ask the instructor for guidance and clarification of these rules. Violators will be referred to the Student Court for review, where penalties may include but are not restricted to: zero credit on the work, student placed on probation, submission of information on judgment in the students' permanent record.

Clarification of Questions: Please call or e-mail me if you are confused about the wording of a problem. I will not, of course, give out answers, but I will clarify the wording of the problems, or correct an error in the questions if someone should find one. If someone finds an error or something which is unclear, I will post the question and answer on the web for all to see. Please DO NOT ask each other even the most minor questions. Remember, even *the appearance* that the validity of the test is compromised can have adverse consequences for you and all your classmates.

Suggestions to Improve Your Grade: I love to give partial credit! Make it easy for me. It isn't going to work if it takes me longer to find the answers than it took you to write them.

Draw a figure for each of the problems. Usually in my problems, the first step is to generate a layout of the optical system. I give points for figures.

Work neatly and keep things organized. Include a bit of narrative text explaining the process you are using.

Present your work as clearly as possible. I give partial credit if I can figure out that you know what you are doing. *If the work is sufficiently clear*, I will even give credit for later parts of a problem that are based on an incorrect answer in an earlier part. I do not give credit for putting down everything you know and hoping I will find something correct in it.

You will want to use a computer for some of the problems. You may use any language you like, but make sure that the equations and graphs are presented in such a way that I don't need to look at your code. When I ask for a plot, I am looking for a correctly labeled one, with correct numerical values. A sketch is not sufficient.

Note on Electronic Submissions: I am happy to receive your work electronically. However, if you choose to submit it this way, please observe the following: (1) Please use a single file, preferably a .pdf. It is very tedious to print multiple .m, .jpg, .doc, and other files, and try to assemble them from a number of students. Sometimes I have trouble reading a .doc or especially .docx file, or even worse, sometimes I can read it but it isn't the same as it was written. (2) Be sure your name will print on the page. Once I print all the files, the original names are lost. (3) Please use a filename that will help me keep the documents organized. A good idea would be your initials and the assignment name; "CD-Final.pdf". This will have a positive effect on my mood during the grading process. Thanks.

1 Polarization

Here we consider light scattered from a biological specimen consisting of small isotropic scatterers that reflect $R_{bkg} = 0.3$ of the incident light without changing the state of polarization, and some aligned fibrils that reflect according to the Jones matrix

$$\mathcal{M}_{fib} = \begin{pmatrix} \rho_x & 0 \\ 0 & \rho_y \end{pmatrix}, \tag{1.0.1}$$

where

$$R_x = |\rho_x|^2 = 0.4 \qquad R_y = |\rho_y|^2 = 0.2. \tag{1.0.2}$$

This matrix is multiplied by A, a complex scalar random variable and added to $\mathcal{M}_{bkg}$, so that there is a random phase (and possibly amplitude) relationship between the two types of scatterers. Let us assume that

$$\langle A \rangle = 0 \qquad \left\langle |A|^2 \right\rangle = 1. \tag{1.0.3}$$

1.1 Matrix Multiplication

Now we are going to work with well–defined input fields, so the only random variables are in the matrix of the sample. If

$$\mathbf{E}_{out} = \left(\mathcal{M}_{bkg} + A\mathcal{M}_{fib}\right)\mathbf{E}_{in}, \tag{1.1.1}$$

write the expressions for the output field, $\mathbf{E}_{out}$ and its adjoint, $\mathbf{E}_{out}^{\dagger}$. Then write the general expression for $\mathbf{E}_{out}\mathbf{E}_{out}^{\dagger}$.

1.2 Expectation Values

Now make use of the statistical description of A, and write an equation for the coherency matrix of the output, $\left\langle \mathbf{E}_{out}\mathbf{E}_{out}^{\dagger} \right\rangle$.

1.3 Polarimetry System

Design a system that will allow the input polarization to be linear at $-45°$, $0°$, $45°$, or $90°$, and to measure the same states of the output polarization. Assume you have access to perfect polarizers and waveplates.

1.4 Coherency Matrix

Write equations for the components of the coherency matrix of the output, with an input vector having arbitrary linear polarization with unit power;

$$\mathbf{E}_{in} = \begin{pmatrix} \cos\zeta \\ \sin\zeta \end{pmatrix}. \tag{1.4.1}$$

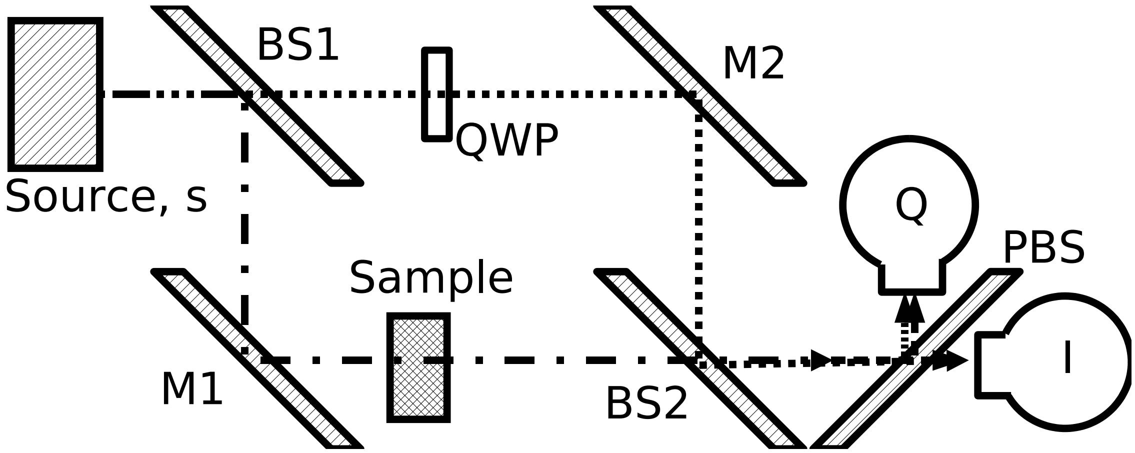

Figure 2.0.1: Optical Quadrature Microscope.

1.5 Stokes Parameters

Plot the Stokes parameters and the degree of polarization as a function of ζ from 0 to 90°.

1.6 Discussion

Discuss the realism of this model. What can be done to make it better?

2 Interference

Here we consider an alternative to the Optical Quadrature Microscope discussed in class and in the homework. Instead of using circular polarization in the reference and linear polarization in the signal, as shown in Figure 2.0.1, we add a quarter–wave plate in the signal arm and use one of the two orthogonal states of circular polarization in the reference and one in the signal.

2.1 Analysis

Show that the polarizers select the in–phase and quadrature signals in the same way as the Optical Quadrature Microscope.

2.2 Polarimetric Imaging

Modify the design so that multiple measurements can be made to determine the Jones matrix of the sample.

3 Diffraction Grating

Here we consider a diffraction grating used as a rear mirror in an Argon Ion laser. The laser operates on several gain lines, two of which are of interest here, centered at 514 nm and 488 nm. On each of these gain lines, there are multiple longitudinal cavity modes under the gain line with sufficient gain to exceed threshold. The cavity length is 0.3 meters.

Suppose that I wish to install the grating at an angle of 20 degrees with respect to the laser beam path for operation at 488 nm, on the first order.

3.1 Grating Spacing

What is the line spacing required for the grating?

3.2 Angle for Green Line

If I now wish to work on the 514–nm line, what is the new angle?

3.3 Nominal Diffraction Pattern

Now if the grating is properly designed for 488 nm, then the phase change across each grating opening will be zero, and there will be a jump of 2π in phase between steps, and thus can be neglected. Suppose that the laser Gaussian beam diameter is 500 micrometers. Assume that there is a band of 10% of the grating spacing between facets, over which no transmission occurs. Using an FFT routine, plot the far–field pattern of the diffracted beam.

3.4 Pattern Change with Tilt

When the grating is tuned to the appropriate angle for 514 nm, then the phase step from one grating facet to the next is still 2π, but there is an extra tilt to each facet, equal to the difference between the two angles. This results in a loss in performance. Plot the far–field pattern under this condition.

4 Compact Disc Player

Here we look at a simple model of a compact disc player. A metal surface has a depression of a certain depth, and a diameter of two micrometers. It is covered with a plastic layer with index of refraction equal to 1.5. A laser beam twice the diameter of this "pit" is incident normally. The beam can be assumed to be a uniform plane wave over the "pit" and, in fact, over the surrounding area, just to make things easy. The laser has a free–space wavelength of 750 nm. Now some of the light will be reflected from the flat surface surrounding the "pit," and some will be reflected from the pit. Both reflected beams will diffract as they leave the surface. For light from the "pit" the phase will be shifted because of the depth of the pit, and this light will interfere with light which is reflected from the surroundings.

4.1 Interference

What is the depth of the pit to produce the best destructive interference on axis in the reflected beam?

4.2 Diffraction Angle

What is the diffraction angle (full width between nulls) of the light reflected from the pit? Remember that the wavelength in the diffraction equation is that in the plastic, and not the free space wavelength.

4.3 Refraction

Here's a little bit of geometric optics. Show that the angle of the diffracted light is air is the same as it would be from a "pit" of the same diameter in air. Use the small–angle approximation here.

4.4 Field from Pit

What is the on–axis scattered field of light from the pit? Remember the relationship between field and irradiance. Call the incident field U_{inc}.

4.5 Field from Surface

What is the on–axis scattered field of light from the surrounding surface?

4.6 Fringe Visibility

What is the ratio of maximum to minimum irradiance?