DEPARTMENT OF ELECTRICAL AND COMPUTER ENGINEERING

NORTHEASTERN UNIVERSITY

ECEG105 OPTICS FOR ENGINEERS FALL 2007

Homework Set 1: Geometric Optics

Problem 1 — Dogleg.

A beam of laser light is incident on a beamsplitter of thickness, t, and angle, θ, as shown in Figure 1. The index of refraction of the medium on each side is the same; n, and that of the beamsplitter is n'. When I remove the beamsplitter, the light falls on a detector. When I replace the beamsplitter, the light follows a "dogleg" path, and I must move the detector in a direction transverse to the beam. How far?

a. Derive the general equation.

b. Check your result for $n' = n$ and for $n'/n \to \infty$. Calculate and plot *vs.* angle, for $t = 5mm$, $n = 1$, with $n' = 1.5$,

c. and again with $n' = 4$.

Problem 2 — Some Lenses.

a. Consider the thin lens in panel A of Figure 2. Let the focal length be $f = 10$cm. Plot the image distance as a function of object distance from 50 cm down to 5 cm.

b. Plot the magnification.

c. Consider the thin lens in panel B of Figure 2. Let the focal length be $f = -10$cm. Plot the image distance as a function of object distance from 50 cm down to 5 cm.

d. Plot the magnification.

e. In panel C of Figure 2, the lenses are both thin, and have focal lengths of $f_1 = 20$cm, and $f_2 = 10$cm, and the separation is $d = 5$ cm. For an object 40 cm in front of the first lens, where is the final image?

f. What is the magnification?

Problem 3 — Focussing a Laser Beam.

The configuration in panel D of Figure 2 can be used to focus a laser beam to couple it into an optical fiber. We want the sine of the half-angle subtended by the last lens (shaded region) to equal the numerical aperture of the fiber, $NA = 0.4$. Let the second lens have a focal length of 16 mm, and assume that the initial laser beam diameter is $D_1 = 1.4$mm. We want the magnification of the second lens to be $-1/4$.

a. What is the focal length of the first lens?

b. What is the spacing between the two lenses, assuming thin lenses?

c. Now, assume that the lenses are both 5 mm thick, the first is convex–plano and the second is bi–convex. What is the spacing between the vertices of the lenses now? What is the distance to the image point from the second lens vertex? Where should the fiber be placed relative to the second lens? You may use an approximation we discussed in class, and the fact that the lenses are glass.

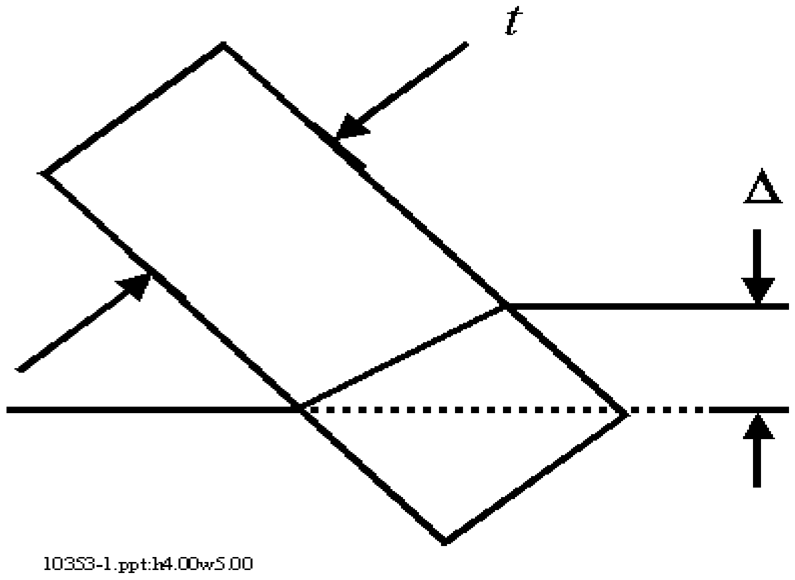

Figure 1: Layout for Problem 1

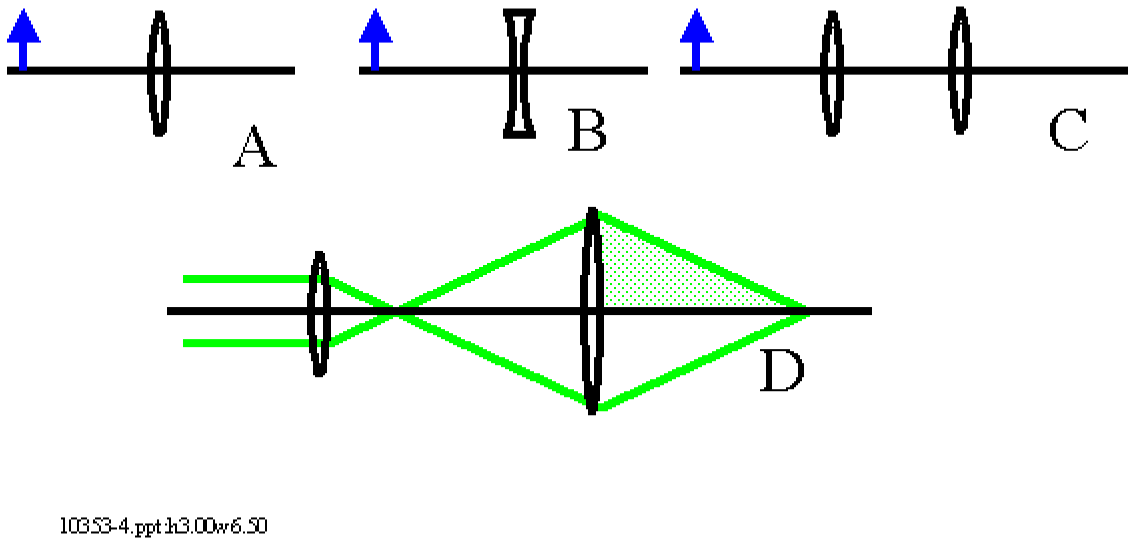

Figure 2: Sample Lenses for Problem 2

d. Use matrices and find the focal length of this pair of lenses and the location of the principal planes. Be sure to specify distances and directions from lens vertices.

Problem 4 — Coaxial Lidar: Pupil and Image Conjugates .

Consider the coaxial Lidar (Laser Radar) system shown in Figure 3. The laser beam is one inch in diameter, and is reflected from a 45–degree mirror having an elliptical shape so that its projection along the axis is a 2–inch circle (This mirror, of course, causes an obscuration of light returning from the target to the detector. We will make good use of this feature.). The light is then refelected by a scanning mirror, to a distant target. Some of the backscattered light is returned *via* the scanning mirror, reflected from the primary mirror to a secondary mirror and then to a lens and detector. The focal lengths are 1 meter for the primary, 12.5 cm for the secondary, and 5 mm for the detector lens. The diameters are, respectively, 8 inches, 2 inches, and one inch. The detector is at the focus of the detector lens, which is separated from the secondary by a distance of 150 cm. The primary and secondary are separated by the sum of their focal lengths.

a. Locate all the optical elements in object space, *i.e.* the space containing the target, and determine their size; the detector, detector lens, secondary lens, primary, obscuration caused by the transmitter folding mirror.

b. Identify the aperture stop and the field stop for a target at a long distance from the lidar.

c. As mentioned above, the folding mirror for the transmitter causes an obscuration of the received light. It turns out that this is a very useful feature. The scanning mirror surface will always contain some imperfections, scratches, and dirt, which will scatter light back to the receiver. Because it is very close, the effect will be strong, and will prevent detection of weaker signals of interest. Now, if we place the scanning mirror close enough, the obscuration and the finite diameter of one or more lenses will block light scattered from the relatively small transmitter spot in the center of the mirror. Find the "blind distance," z_{blind} inside which the scan mirror can be placed.

Problem 5 — Spherical Aberrations.

Here we consider how the focus moves as we use larger parts of an aperture. The lens has a focal length of 4 cm., and the object is located 100 cm in front of it. We may assume a thin lens with a diameter of 2 cm.

a. What is the f–number of this lens?

b. Plot the location of the focus of rays passing through the lens at a height, h, from zero to $d/2$, assuming the lens is biconvex.

c. Repeat assuming it is plano–convex.

d. Repeat assuming it is convex–plano.

e. Repeat for the optimal shape for minimizing spherical aberrations.

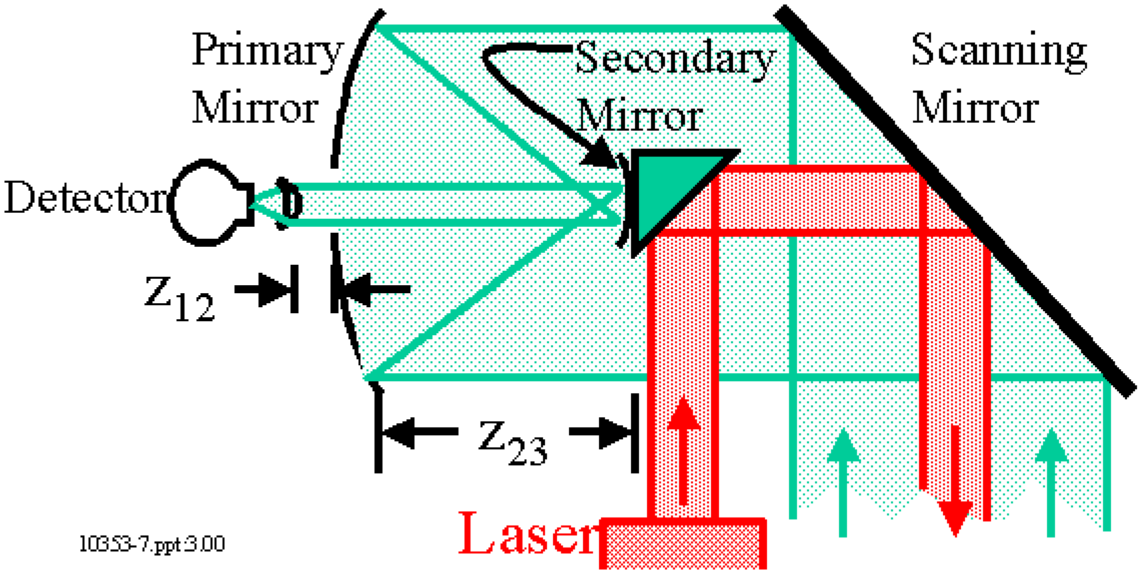

Figure 3: Coaxial Lidar Configuration

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Homework Set 2: Polarized Light

Problem 6 — Polarizer in an Attenuator.

Polarizers can be useful for attenuating the level of light in an optical system. For example, consider two polarizers, with the second one aligned to pass vertical light. The first can be rotated to any desired angle, with the angle defined to be zero when the two polarizers are parallel. The first parts of this problem can be solved quite simply without the use of Jones Matrices.

a. What is the output polarization?

b. Plot the power transmission of the system as a function of angle, from zero to 90 degrees, assuming the incident light is unpolarized.

c. Plot the power transmission assuming vertically polarized incident light.

d. Plot the power transmission assuming horizontally polarized incident light.

e. Now, replace the first polarizer with a quarter–wave plate. I recommend you use Jones matrices for this part. In both cases, the incident light is vertically polarized. Plot the power transmission as a function of angle from zero to 90 degrees.

f. Repeat using a half–wave plate.

Problem 7 — Electro–Optical Modulator.

Suppose that an electro–optical modulator crystal is placed between crossed polarizers so that it's principal axes are at 45 degrees with respect to the axes of the polarizers. The crystal produces a phase change between fields polarized in the two principal directions of $\delta\phi = \pi V/V_\pi$, where V_π is some constant voltage, and V is the applied voltage.

a. Write the Jones matrix for the crystal in the coordinate system of the polarizers.

b. Calculate and plot the power transmission, T through the whole system, polarizer–crystal–polarizer, as a function of applied voltage.

c. Next, add a quarter–wave plate with axes parallel to those of the crystal. Repeat the calculations of part b.

d. What is the range of voltages over which the output power is linear to 1%?

Problem 8 — Laser Cavity.

Consider a laser cavity with a gain medium having a power gain of 2% for a single pass. By this we mean that the output power will be equal to the input power multiplied by 1.02. The gain medium is a gas, kept in place by windows on the ends of the laser tube, at Brewster's angle. This is an infrared laser, and these windows are ZnSe, with an index of refraction of 2.4. To make the problem easier, assume the mirrors at the ends of the cavity are perfectly reflecting.

a. What is the Jones matrix describing a round trip through this cavity? What is the power gain for each polarization? Note that if the round–trip multiplier is greater than one, lasing can occur, and if not, it can not.

b. What is the tolerance on the angles of the Brewster plates to ensure that the laser will operate for light which is P–polarized with respect to the Brewster plates?

c. Now assume that because of sloppy manufacturing, the Brewster plates are rotated about the tube axis by 10 degrees with respect to each other. What states of polarization are eigenvectors of the cavity? Will it lase?

Problem 9 — Depolarization in a fiber .

Consider light passing through an optical fiber. We will use as a basis set the eigenvectors of the fiber. The fiber is birefringent, because of imperfections, bending, etc., but there are no polarization dependent losses. In fact, to make things easier, assume there are no losses at all.

a. Write the Jones matrix for a fiber of length ℓ with birefringence Δn, in a coordinate system in which the basis vectors are the eigenvectors of the fiber. Note that your result will be a function of wavelength.

b. Suppose that the coherency matrix of the input light in this basis set is

$$\begin{pmatrix} a & b \\ b^* & c \end{pmatrix}.$$

What is the coherency matrix of the output as a function of wavelength?

c. Assume now that the input light is composed of equal amounts of the two eigenvectors with equal phases, but has a linewidth $\Delta\lambda$. Provided that $\Delta\lambda$ is much smaller than λ, write the coherency matrix in a way that expresses the phase as related to the center wavelength, λ, with a perturbation of $\Delta\lambda$.

d. Now let's try to average the coherency matrix over wavelength. This would be a hard problem, so let's approach it in the following way: Show the value of b from the coherency matrix, as a point in the complex plane for the center wavelength. Show what happens when the wavlength changes, and see what effect this will have on the average b. Next write an inequality relating the length of the fiber, the linewidth, and the birefringence to ensure that depolarization is "small."

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Homework Set 3: Interference

Problem 10 — Laser Tuning.

Suppose that I place the back mirror of a laser on a piezoelectric transducer which moves the mirror axially through a distance of 10 micrometers with an applied voltage of 1kV, and that the relationship between position and voltage is linear. The laser has a nominal wavelength of 633 nanometers. The cavity length is 20 centimeters.

a. What voltage is required to sweep the cavity through one free spectral range?

b. What is the free spectral range in frequency units?

c. What voltage is required to tune the laser frequency by 50 MHz?

Problem 11 — Interference Filters.

Design a pair of layers for a high–reflection stack for 633–nm light, using $n = 1.35$ and $n = 2.3$. Assume light coming from air to glass.

a. Write the matrix for the layer.

b. Plot the reflection for a stack with 5 pairs of layers. Plot from 400 to 1600 nm.

c. Repeat for 9 pairs.

Problem 12 — Quadrature Interference .

Consider a coherent laser radar in which the signal beam returning from the target is linearly polarized. We place a quarter–wave plate in the local oscillator arm so that this beam becomes circularly polarized. Now, after the recombining beamsplitter, we have a polarizing beamsplitter which separates the beam into two components at $\pm$ 45 degrees with respect to the signal polarization. A separate detector is used to detect each polarization.

a. Show that we can use the information from these two detectors can determine the magnitude and phase of the signal.

b. What happens if the sensitivities of the two detectors are unequal. Explain in the frequency domain, as quantitatively as possible.

c. Suppose now that both beams are linearly polarized, but at 45 degrees with respect to each other. I pass the combination through a quarter–wave plate with one axis parallel to the reference polarization. I then pass the result to a polarizing beamsplitter with its axes at 45 degrees to those of the waveplate, and detect the two outputs of this beamsplitter with separate detectors. Write the mixing terms in the detected signals. Is it possible to determine the phase and amplitude?

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Homework Set 4: Diffraction

Problem 13 — Gaussian Beams.

A helium–neon laser (633 nm) produces a collimated output beam with a diameter of 0.7 mm.

a. If the laser is 20 cm. long, what are the radii of curvature of the two mirrors?

b. How big is the beam at the back mirror?

c. I now wish to launch the beam into a fiber whose core is best matched to a Gaussian beam of 5 micrometer diameter. Figure out how to do this, using one or more lenses in a reasonable space.

Problem 14 — Resolution.

A car approaches along a long straight road. Initially it is so far away that the two headlights appear as one.

a. Using the Rayleigh criterion for resolution, at what distance do we begin to see that they are two separate lights? Use reasonable numbers for the spacing of the headlights and the diameter of the pupil of the eye.

b. Now, consider the fact that we have two eyes, separated by a distance of a few centimeters. Does this change your answer? Why or why not?

Problem 15 — Koehler Illumination .

A common way of illuminating an object for a microscope is to focus the filament of a tungsten lamp on the pupil of the system (which, if the lenses are modeled as thin, means focussing it in the plane of the objective lens). The image plane will then contain the two–dimensional Fourier transform of the filament. The idea, as I have mentioned in class, is that the illumination is thus very uniform. Start with the following code:

```
\% Koehler Illumination
\%
\% By Chuck DiMarzio, Northeastern University, April 2002
\%
\% coherent source
\%
[x,y]=meshgrid(1:256,1:256);
coh=zeros(size(x));
coh(60:69,50:200)=1;
coh(80:89,50:200)=1;
```

```
coh(100:109,50:200)=1;
coh(120:129,50:200)=1;
coh(140:149,50:200)=1;
coh(160:169,50:200)=1;
coh(180:189,50:200)=1;
subplot(2,2,1);imagesc(coh);colormap(flipud(bone));colorbar;
title('Coherent Source');
```

a. Now, take the 2–D Fourier transform and image the magnitude of the result. A couple of hints are in order here. Try "help fft2," and "help fftshift." If you haven't used subplots, try "help subplot," to see how to display all the answers in one figure.

b. The result above is not very uniform, but the prediction was that it would be. See if you can figure out why it doesn't work. You have two more subplots to use. On the third one, show the corrected source, and . . .

c. on the fourth one show the correct image (Fourier transform).

There are different degrees of correctness possible in part b, which will affect the results somewhat. You only need to develop a model which is "correct enough," to show the important features.

Problem 16 — Sectioning with a Confocal Microscope.

Consider a microscope with a digital camera, in which a particular pixel has a sensitivity function given by,

$$E_r = \frac{1}{w_r^2}$$

for

$$-\frac{w_r}{2} < x < \frac{w_r}{2} \quad \text{and} \quad -\frac{w_r}{2} < y < \frac{w_r}{2}$$

and 0 elsewhere, where

$$w_r = w_{r0}\sqrt{\left(\frac{z - z_{r0}}{b_r}\right)^2 + 1}.$$

The parameters in this equation are shown in Figure 4.

Note that a realistic function would be more complicated, but this one illustrates the concepts with sufficient clarity and does not involve messy integrals. Let's be specific about the dimensions:

$$w_{r0} = 5 \text{ micrometers}$$

$$b_r = 25 \text{ micrometers}$$

$$z_{r0} = 4 \text{ millimeters}$$

The goal of this problem is to look at the "sectioning" capability of the confocal microscope. We will look at two different z locations, one near the focus, and one not, and see what component of the total signal comes from each. For comparison, we will begin by doing the same calculations for a conventional bright–field microscope.

(a) Plot the sensitivity as a function of z for a suitable region around $z = z_{r0}$.

Next suppose that the transmitter (source) irradiance (power per unit area) obeys

$$E_t = \frac{P}{w_t^2}$$

where P is the source power. Here w_t is defined in a way similar to w_r above, except that the values may be different.

First consider brightfield imaging, where the transmitter parameters are

$$z_{t0} = 4 \text{ millimeters} \qquad w_{t0} = 1 \text{ millimeter} \qquad b_t = 1 \text{ meter}$$

Note that this means that, to a good approximation, $w_t = w_{t0}$ over the region of interest. Assume furthermore that the source power is one milliwatt.

The region of interest consists of particles which scatter light so that the received signal from one of them is

$$S_1 = E_t \sigma E_r.$$

The area, σ is called the scattering cross–section of the particle. We are interested in relative changes in signal, so we have left out a large number of multiplying parameters.

Now suppose the scatterers are evenly distributed with a density N_v per unit volume.

$$N_v = 10^{18}/m^3 \qquad \sigma = 10^{-12}m^2$$

We need to sum the signal over all the scatterers. We don't know exactly the location of each particle, and even if we did, this would be a tedious process. Fortunately, for large numbers of particles, we can approximate it to a good degree of accuracy by multiplying by the density and integrating. If the range of z is small, we simply need to integrate over the area and multiply by dz. Do this for **(b)** a layer dz thick, centered at $z = z_{r0}$ and again for **(c)** one where z is 50 micrometers beyond the focus, z_{r0}. Let $dz = 1$ micrometer.

Now, let's look at the confocal situation. For this case, set the transmitter parameters equal to those of the receiver. Once again, compute **(d)** the signal from a layer at the focus, $z = z_r 0$, **(e)** a layer 50 micrometers out of focus.

(f) Integrate the signal over z from zero to infinity. Hint; Integrate from $-\infty$ to ∞, because it is easier and the result will be almost the same. Estimate the fraction of the total signal coming from a layer at $z = z_{r0}$ if the thickness is $dz = b_r$.

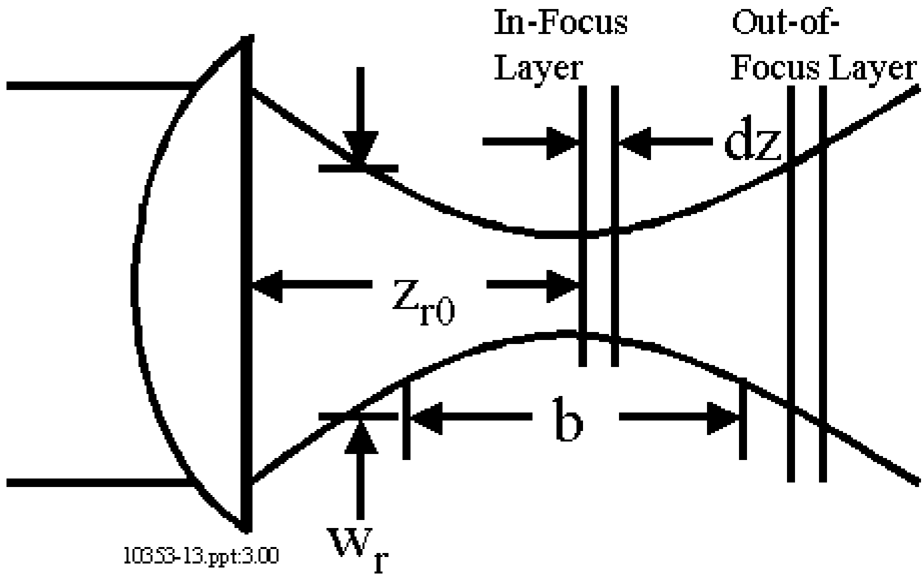

Figure 4: Layout for Problem 1. The drawing shows the receiver beam boundaries. A similar diagram could be drawn for the transmitter, possibly with different numerical values. See the problem text for definition of the labelled distances.

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Homework Set 5: Radiometry and NLO

Problem 17 — Radiation from a Human.

For this problem, suppose that the surface area of a human being is about 1 square meter.

a. What is the total power radiated by a human being at normal body temperature?

b. What is the total power radiated back to the human from surroundings at a temperature of 22C? What is the net radiated power? FYI, a 2kCal/day diet is 92 Watts.

c. What is the power radiated in an infrared wavelength band from 10.0 to 10.1 micrometers? How many photons per second is this?

d. What is the power radiated in a green wavelength band from 500 to 600 nanometers? In what time is the average number of photons equal to one?

Problem 18 — A Laser Radar in Fog .

Consider a large patch of fog sufficiently thick that, seen from the air, it is impossible to see the ground. Suppose that the optical properties of this fog bank are:

$$\mu_s = 1 \times 10^{-6}/m \qquad \mu_s = 7 \times 10^{-4}/m \qquad p(\pi) = 0.2.$$

a. Compute the diffuse reflectance, the scattered radiance divided by the indicent irradiance.

b. Now suppose that embedded in the fog bank is a denser fog, having the same properties, except that;

$$\mu_s = C \times 10^{-6}/m,$$

beginning at a depth z_1 and ending at a depth z_1+t. Develop the equation for the reflectance. Think of special cases which will help you verify that you have done the math correctly.

c. Now calculate the fractional change in the diffuse reflectance as a function of depth, z_1. Assume the thickness is $t = 100$ meters, and evaluate for $C = 2$, $C = 0.5$.