Optics for Engineers
Week 7

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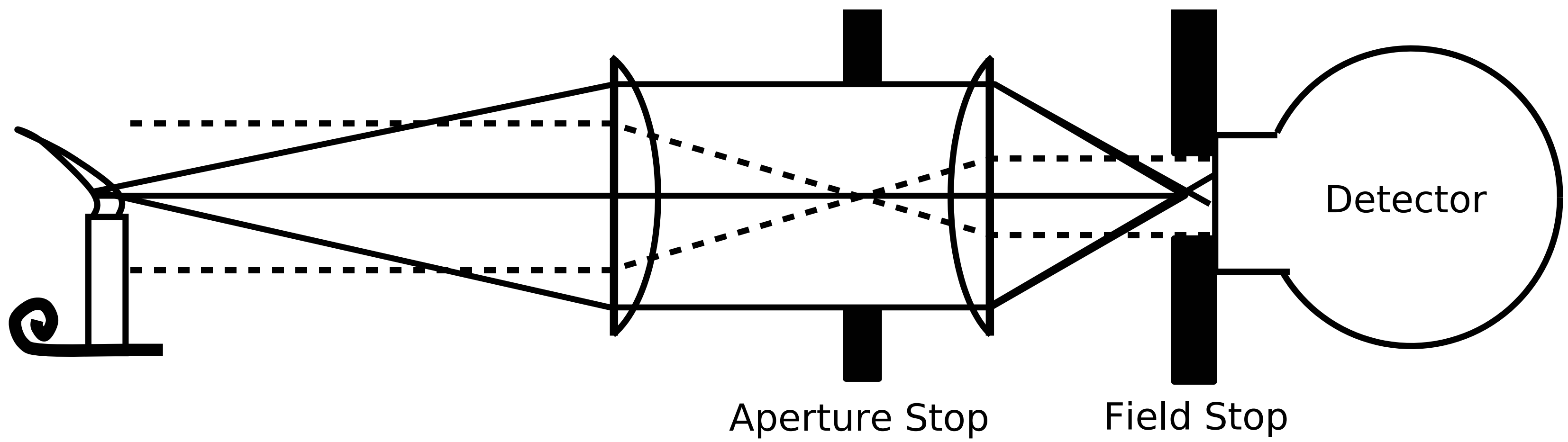
Week 7 Agenda

- Radiometry Concepts and Quantities
- Radiance Theorem
- Spectral Radiometry
- Photometry
- Instrumentation
- Blackbody Radiation
- Illumination
- Polar Bears

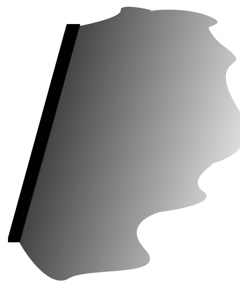
Radiometry



- Power is Proportional to
  - Area of Aperture Stop
  - Area of Field Stop
  - "Brightness" of the Source (Radiance)

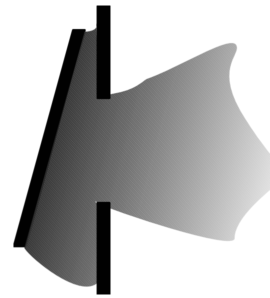
Radiometry and Photometry

Note: "Spectral X:" X_ν, units /Hz or X_λ, /μm.
Rad. to Phot:
683 lm/W $\times\, y(\lambda)$.
$y(555\text{nm}) \approx 1$.



ϕ Radiant Flux, Watt = W
Luminous Flux, lumen = lm

$\partial/\partial A \rightarrow$

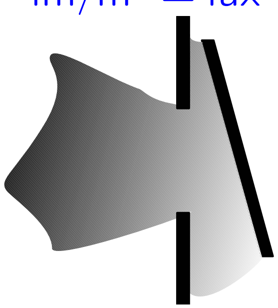


M Radiant Exitance, W/m^2
Luminous Exitance, lm/m^2 = lux = lx

$\downarrow \partial/\partial\Omega \downarrow$ $\downarrow \partial/\partial\Omega \downarrow$

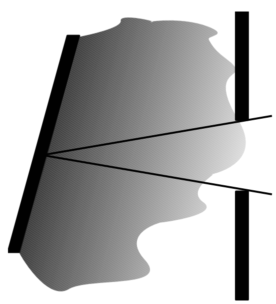
$\leftarrow \cdot/R^2$

E Irradiance, W/m^2
Illuminance, lm/m^2 = lux

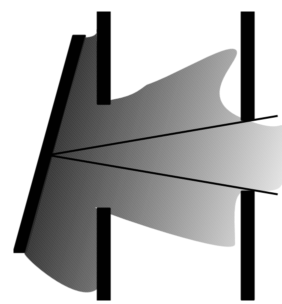


1Ft.Candle = 1lm/ft^2

$\partial/\partial A \rightarrow$



I Radiant Intensity, W/sr
Luminous Intensity, lm/sr = cd



L Radiance, W/m^2/sr
Luminance, nit = lm/m^2/sr

1Lambert = 1lm/cm^2/sr/π
1mLambert = 1lm/m^2/sr/π
1FtLambert = 1lm/ft^2/sr/π

Radiometric Quantities

Quantity	Symbol	Equation	SI Units
Radiant Energy	Q		Joules
Radiant Energy Density	w	$w = \frac{d^3Q}{dV^3}$	Joules/m^3
Radiant Flux or Power	P or Φ	$\Phi = \frac{dQ}{dt}$	W
Radiant Exitance	M	$M = \frac{d\Phi}{dA}$	W/m^2
Irradiance	E	$E = \frac{d\Phi}{dA}$	W/m^2
Radiant Intensity	I	$I = \frac{d\Phi}{d\Omega}$	W/sr
Radiance	L	$lm = \frac{d^2\Phi}{dA\cos\theta d\Omega}$	W/m^2
Fluence	Ψ	$\frac{dQ}{dA}$	J/m^2
Fluence Rate	F	$\frac{d\Psi}{dt}$	J/m^2
Emissivity	ϵ	$\epsilon = \frac{M}{M_{bb}}$	Dimensionless
Spectral ()	$()_\nu$ or $()_\lambda$	$\frac{d()}{d\nu}$ $\frac{d()}{d\lambda}$	()/Hz ()/μm
Luminous Flux or Power	P or Φ		lm
Luminous Exitance	M	$M = \frac{d\Phi}{dA}$	lm/m^2
Illuminance	E	$E = \frac{d\Phi}{dA}$	lm/m^2
Luminous Intensity	I	$I = \frac{d\Phi}{d\Omega}$	lm/sr
Luminance	L	$L = \frac{d^2\Phi}{dA\cos\theta d\Omega}$	lm/m^2
Spectral Luminous Efficiency	$V(\lambda)$		Dimensionless
Color Matching Functions	$\bar{x}(\lambda)$ $\bar{y}(\lambda)$ $\bar{z}(\lambda)$		Dimensionless

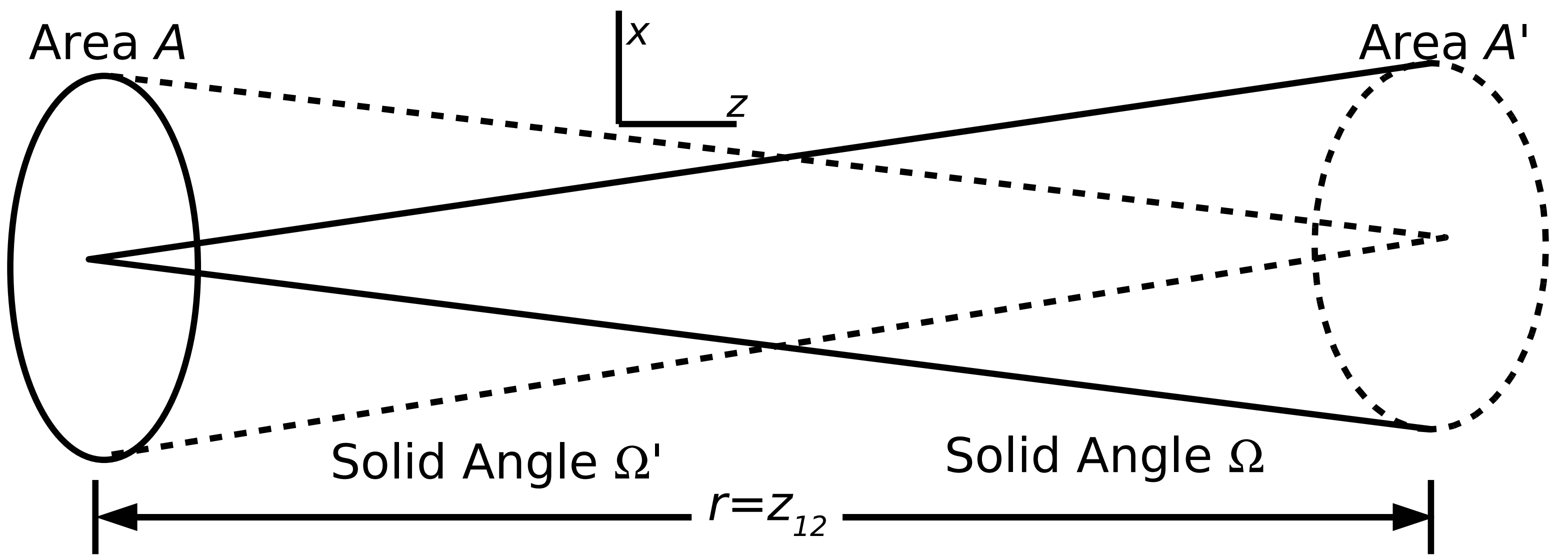
Irradiance

- Poynting Vector for Coherent Wave

$$dS \approx \frac{dP}{4\pi r^2}(\sin\theta\cos\zeta\hat{x} + \sin\theta\sin\zeta\hat{y} + \cos\theta\hat{z})$$

- Irradiance (Projected Area, $A_{proj} = A\cos\theta$)

$$dE = \frac{d^2P}{dA'} = \frac{dP}{\left(4\pi r^2\right)}$$



Irradiance and Radiant Intensity

- Solid Angle

$$\Omega = \frac{A'}{r^2}$$

	·	·
E	I	·

- Intensity from Irradiance (Unresolved Source, A)

$$I = Er^2 \qquad E = \frac{I}{r^2}$$

$$dI = \frac{d^2P}{d\Omega} = \frac{d^2P}{d\frac{A'}{r^2}} = \frac{dP}{\left(4\pi r^2\right)}$$

Intensity and Radiance

- Resolved Source: Many Unresolved Sources Combined

$$I = \int dI = \int \frac{\partial I}{\partial A} dA$$

	$\cdot$	$\cdot$
$\cdot$	I	L

- Radiance

$$L(x, y, \theta, \zeta) = \frac{\partial I(\theta, \zeta)}{\partial A}$$

$$I(\theta, \zeta) = \int_A L(x, y, \theta, \zeta)\, dx dy$$

- On Axis ($x' = y' = 0$)

$$L(x, y, \theta, \zeta) = \frac{\partial I(\theta, \zeta)}{\partial A} = \frac{\partial^2 P}{\partial A \partial \Omega}$$

- Off Axis (Projected Area)

$$L = \frac{\partial I}{\partial A \cos\theta} = \frac{\partial^2 P}{\partial A \partial \Omega \cos\theta}$$

Radiant Intensity and Radiant Flux

	Φ	.
.	I	.

- Flux or Power, P or Φ
- Integrate Intensity

$$P = \Phi = \int\int I(\theta,\phi)\sin\theta d\theta d\zeta$$

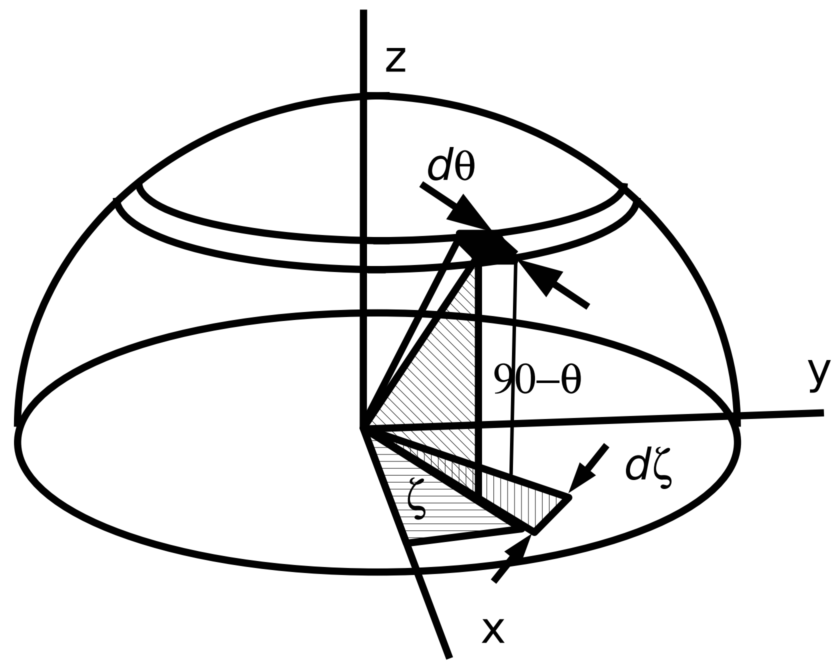
- Constant Intensity

$$P = \Phi = I\Omega$$

$$\Omega = \int\int \sin\theta' d\theta' d\zeta$$

- Solid Angle

$$\Omega = 2\pi\left(1-\sqrt{1-\left(\frac{NA}{n}\right)^2}\right)$$



$$\Omega = \int_0^{2\pi}\int_0^{\Theta}\sin\theta' d\theta' d\zeta = 2\pi(1-\cos\Theta) = 2\pi\left(1-\sqrt{1-\sin^2\Theta}\right)$$

Radiance and Radiant Exitance

	$\cdot$	M
$\cdot$	$\cdot$	L

- Radiant Exitance from a Source (Same Units as Irradiance)

$$M(x,y) = \int\int \left[\frac{\partial^2 P}{\partial A \partial d\Omega}\right] \sin\theta d\theta d\zeta$$

- Radiant Exitance from Radiance

$$M(x,y) = \int\int L(x,y,\theta,\zeta) \cos\theta \sin\theta d\theta d\zeta$$

- Radiance from Radiant Exitance

$$L(x,y,\theta,\zeta) = \frac{\partial M(x,y)}{\partial \Omega} \frac{1}{\cos\theta}$$

Radiance and Radiant Exitance: Special Cases

- Constant L and Small Solid Angle

$$M\left(x,y\right)=L\Omega\cos\theta$$

$$\Omega=2\pi\left(1-\sqrt{1-\left(\frac{NA}{n}\right)^2}\right)\qquad\text{or}\qquad\Omega\approx\pi\left(\frac{NA}{n}\right)^2$$

- Constant L, over Hemisphere

$$M\left(x,y\right)=\int_0^{2\pi}\int_0^{\pi/2}L\cos\theta\sin\theta d\theta d\zeta=2\pi L\frac{\sin^2\frac{\pi}{2}}{2}.$$

$$M\left(x,y\right)=\pi L\qquad\text{(Lambertian Source)}$$

Radiant Exitance and Flux

	Φ	M
.	.	.

- Power or Flux from Radiant Exitance

$$P = \Phi = \int\int M\left(x, y\right) dx dy,$$

- Radiant Exitance from Power

$$M\left(x, y\right) = \frac{\partial P}{\partial A}$$

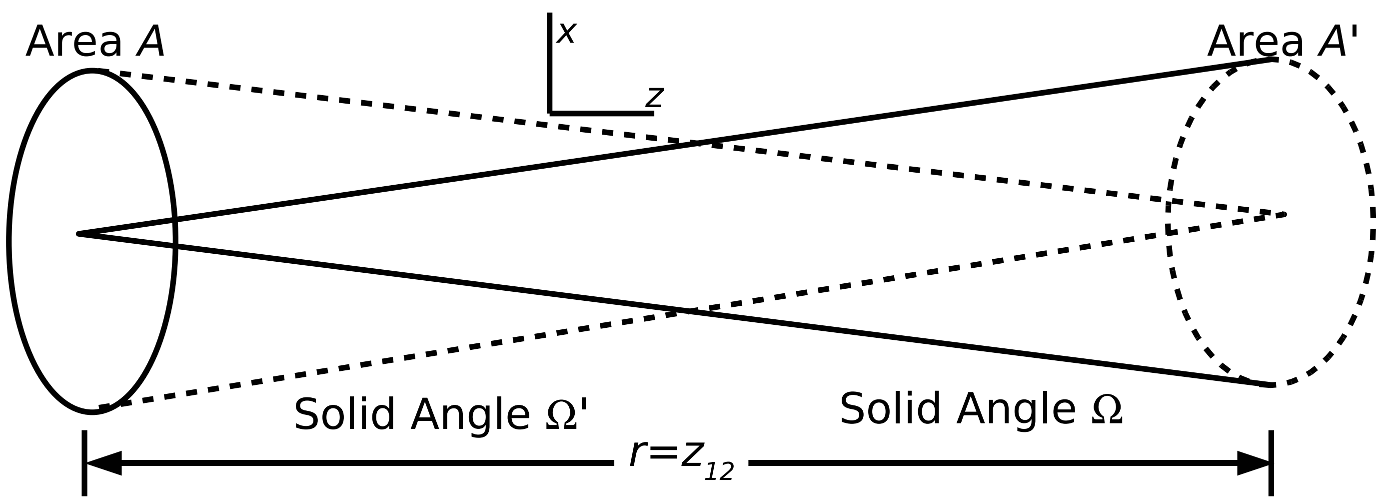
The Radiance Theorem in Air

- Solid Angle two ways

$$\Omega = \frac{A'}{r^2} \qquad \Omega' = \frac{A}{r^2}$$

- Power Increment

$$dP = LdAd\Omega = LdA\frac{dA'}{r^2}\cos\theta \qquad dP = LdA'd\Omega' = LdA'\frac{dA}{r^2}\cos\theta$$



Using the Radiance Theorem: Examples Later

- Radiance is Conserved in a Lossless System (in Air)
- Losses Are Multiplicative
  - Fresnel Reflections and Absorption
- Radiance Theorem Simplifies Calculation of Detected Power
  - Determine Object Radiance
  - Multiply by Scalar, T_{total}, for Loss
  - Find Exit Window (Of a Scene or a Pixel)
  - Find Exit Pupil
  - Compute Power

$$P = L_{object} T_{total} A_{exit\ window} \Omega_{exit\ pupil}$$

Etendue and the Radiance Theorem

- Abbe Invariant:

$$n'x'd\alpha' = nxd\alpha$$

- Etendue

$$n^2 A\Omega = \left(n'\right)^2 A'\Omega'$$

- Power Conservation

$$\int\int\int\int L d^2A d^2\Omega = \int\int\int\int L' d^2A' d^2\Omega'$$

$$\int\int\int\int \frac{L}{n^2} n^2 d^2A d^2\Omega = \int\int\int\int \frac{L'}{n^2} \left(n'\right)^2 d^2A' d^2\Omega'$$

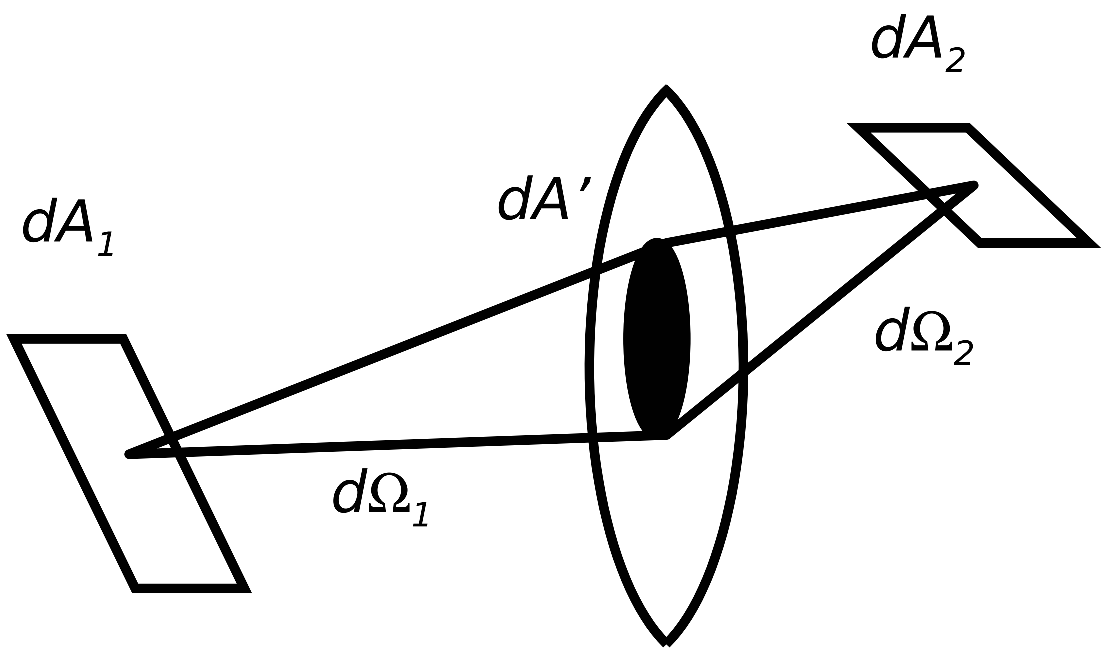
Radiance Theorem: $$\frac{L}{n^2} = \frac{L'}{(n')^2}$$

- Basic Radiance, L/n^2, Conserved (Thermodynamics Later)

Radiance Theorem Example: Imaging

- Imaging Matrix Equation

$$\mathcal{M}_{SS'} = \begin{pmatrix} m & 0 \\ ? & \frac{n}{n'm} \end{pmatrix}$$



Practical Example: Imaging the Moon

- Known Moon Radiance $L_{moon} = 13\text{W/m}^2/\text{sr}$
- Calculation
  - Transmission (0.25), other multiplicative losses (12 Lenses: $\left(0.96^2\right)^{12}$).

$$L = 13\text{W/m}^2/\text{sr}\times 0.25\times 0.96^{24} = 13\text{W/m}^2/\text{sr}\times 0.25\times 0.367 = 1.2\text{W/m}^2/\text{sr}$$

  - Exit Pupil ($NA = 0.1$) and exit window (Pixel: $d = 10\mu\text{m}$).
  - Irradiance, $E = L\Omega$ and Power on a Pixel, $P = EA = LA\Omega$

$$P = 1.2\text{W/m}^2/\text{sr} \times 2\pi\left(1-\sqrt{1-NA^2}\right)\times\left(10\times 10^{-6}\text{m}\right)^2$$

$$P = 1.2\text{W/m}^2/\text{sr} \times 0.315\text{sr} \times 10^{-10}\text{m}^2 = 3.8\times 10^{-12}\text{W}$$

  - If desired, multiply by time (1/30sec) to obtain energy.
    - About One Million Photons (and Electrons)
  - Alternative to Solve in Object Space (Need Pixel Size on Moon)

Radiometry Summary

- Five Radiometric Quantities: Radiant Flux Φ or Power P, Radiant Exitance, M, Radiant Intensity, I, Radiance, L, and Irradiance, E, Related by Derivatives with Respect to Projected Area, $A\cos\theta$ and Solid Angle, Ω.

- Basic Radiance, L/n^2, Conserved, with the Exception of Multiplicative Factors.

- Power Calculated from Numerical Aperture and Field Of View in Image (or Object) Space, and the Radiance.

- Losses Are Multiplicative.

- Finally "Intensity" is Not "Irradiance."

Spectral Radiometry Definitions

- Any Radiometric Quantity Resolved Spectrally
  - Put the Word Spectral in Front
  - Use a Subscript for Wavelength or Frequency
  - Modify Units
- Example: Radiance, L, Spectral Radiance (Watch Units)

$$L_\nu = \frac{dL}{d\nu}\ \mathrm{W/m^2/sr/THz} \qquad \text{or} \qquad L_\lambda = \frac{dL}{d\lambda}\ \mathrm{W/m^2/sr/\mu m}$$

- Spectral Fraction

$$f_\lambda(\lambda) = \frac{X_\lambda(\lambda)}{X} \qquad \text{for} \qquad X = \Phi, M, I, E, \text{ or } L$$

Spectral Radiometric Quantities

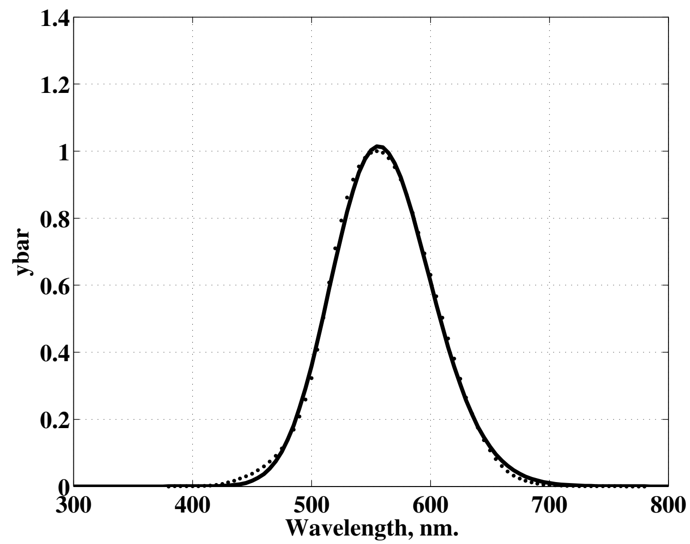
Quantity	Units	$d/d\nu$	Units	$d/d\lambda$	Units
Radiant Flux, Φ Power, P	W	Spectral Radiant Flux, Φ_ν	W/Hz	Spectral Radiant Flux, Φ_λ	W/μm
Radiant Exi- tance, M	W/m^2	Spectral Radiant Exi- tance, M_ν	W/m^2/Hz	Spectral Radiant Exi- tance, M_λ	W/m^2/μm
Radiant Inten- sity, I	W/sr	Spectral Radiant Inten- sity, I_ν	W/sr/Hz	Spectral Radiant Inten- sity, I_λ	W/sr/μm
Radiance, L	W/m^2/sr	Spectral Radi- ance, L_ν	W/m^2/sr/Hz	Spectral Radi- ance, L_λ	W/m^2/sr/μm

Photometry and Colorimetry

- Spectral Luminous Efficiency, $\bar{y}(\lambda)$
- Source Spectral Radiance, $L_{red\lambda}(\lambda, x, y)$
- Eye Response

$$Y(x,y) = \int_0^\infty \bar{y}(\lambda) L_\lambda (\lambda, x, y)\, d\lambda$$

- Four LEDs: Equal Radiance
  - Blue, 400 Appears Weak
  - Green, 550 Appears Strong
  - Red, 630 Moderately Weak
  - IR, 980 Invisible



Lumens

- Power or Radiant Flux (Watts)

$$P = \int_0^\infty P_\lambda(\lambda)$$

- Eye Response

$$Y = \int_0^\infty \bar{y}(\lambda)\, P_\lambda(\lambda)\, d\lambda$$

- Luminous Flux (Lumens, Subscript $_V$ for Clarity)

$$P_{(V)} = \frac{683 \text{ lumens/Watt}}{\max(\bar{y})} \int_0^\infty \bar{y}(\lambda)\, P_\lambda(\lambda)\, d\lambda$$

- Luminous Efficiency

$$\frac{P_{(V)}}{P} = 683 \text{ lumens/Watt} \int_0^\infty \frac{\bar{y}(\lambda)}{\max(\bar{y})} \frac{P_\lambda(\lambda)}{P} d\lambda$$

Some Typical Radiance and Luminance Values

Object	W/m^2/sr		nits = lm/m^2/sr	Footlamberts	lm/W
Minimum Visible	7×10^{-10}	Green	5×10^{-7}	1.5×10^{-7}	683
Dark Clouds	0.2	Vis	40	12	190
Lunar disk	13	Vis	2500	730	190
Clear Sky	27	Vis	8000	2300	300
Bright Clouds	130	Vis	2.4×10^{4}	7×10^{3}	190
	300	All			82
Solar disk	4.8×10^{6}	Vis	7×10^{8}	2.6×10^{7}	190
	1.1×10^{7}	All		$\times 10^{7}$	82

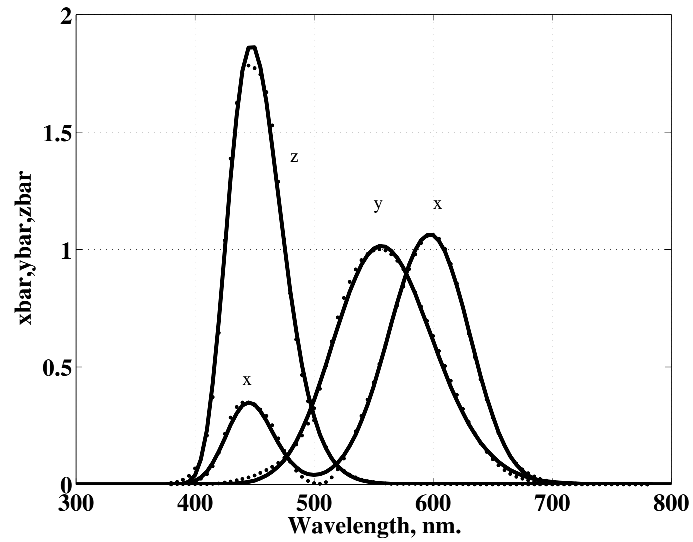
Tristimulus Values: Three is Enough

- X, Y, Z

$$X = \int_0^\infty \bar{x}(\lambda) L_\lambda(\lambda) d\lambda$$

$$Y = \int_0^\infty \bar{y}(\lambda) L_\lambda(\lambda) d\lambda$$

$$Z = \int_0^\infty \bar{z}(\lambda) L_\lambda(\lambda) d\lambda$$



- Example: 3 Lasers

Krypton $\lambda_{red} = 647.1\text{nm}$

$$X_R = \int_0^\infty \bar{x}(\lambda) \delta(\lambda - 647.1\text{nm}) d\lambda = \bar{x}(647.1\text{nm})$$

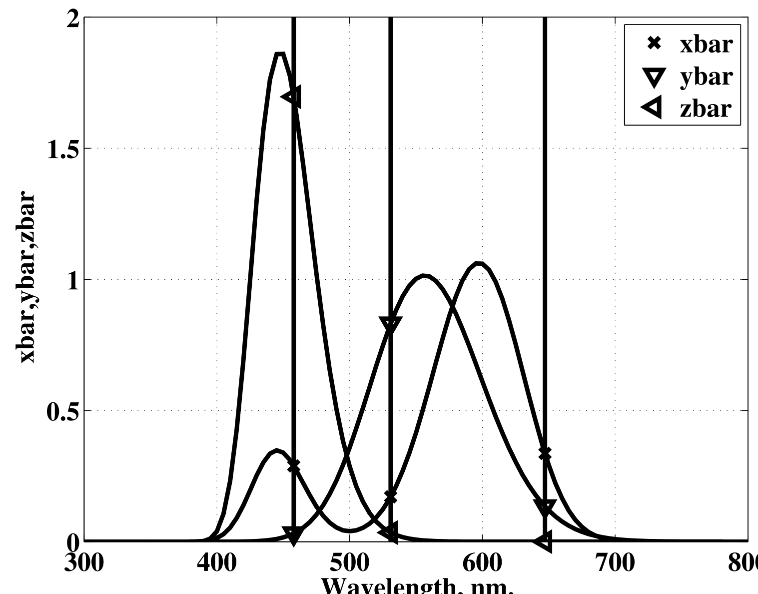
$$X_R = 0.337 \quad Y_R = 0.134 \quad Z_R = 0.000$$

Krypton $\lambda_{green} = 530.9\text{nm}$

$$X_G = 0.171 \quad Y_G = 0.831 \quad Z_G = 0.035$$

Argon $\lambda_{blue} = 457.9\text{nm}$

$$X_B = 0.289 \quad Y_B = 0.031 \quad Z_B = 1.696$$



Chromaticity Coordinates (1)

- Three Laser Powers, R, G, and B, Watts
- Tristimulus Values

$$\begin{pmatrix} X \\ Y \\ Z \end{pmatrix} = \begin{pmatrix} X_R & X_G & X_B \\ Y_R & Y_G & Y_B \\ Z_R & Z_G & Z_B \end{pmatrix} \begin{pmatrix} R \\ G \\ B \end{pmatrix}$$

- Chromaticity Coordinates (Normalized X, Y

$$x = \frac{X}{X+Y+Z} \qquad y = \frac{Y}{X+Y+Z}$$

- Monochromatic Light

$$x = \frac{\bar{x}(\lambda)}{\bar{x}(\lambda)+\bar{y}(\lambda)+\bar{z}(\lambda)} \qquad y = \frac{\bar{y}(\lambda)}{\bar{x}(\lambda)+\bar{y}(\lambda)+\bar{z}(\lambda)}$$

Chromaticity Coordinates (2)

Monochromatic Light (Boundary)

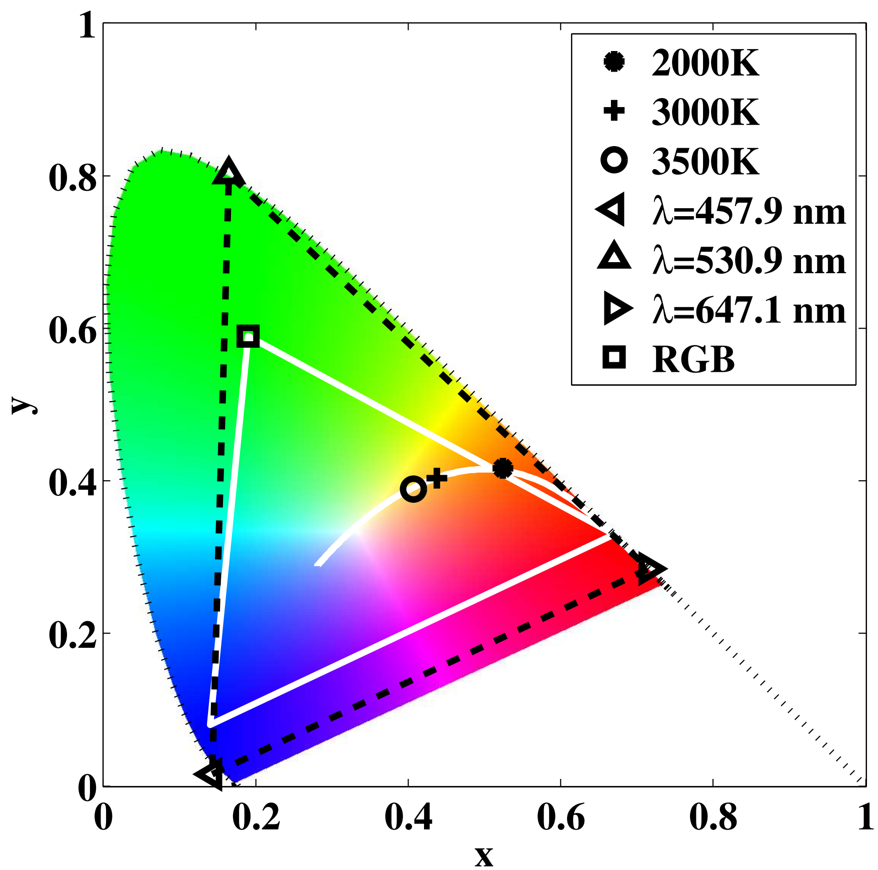
$$x = \frac{\bar{x}(\lambda)}{\bar{x}(\lambda) + \bar{y}(\lambda) + \bar{z}(\lambda)}$$

$$y = \frac{\bar{y}(\lambda)}{\bar{x}(\lambda) + \bar{y}(\lambda) + \bar{z}(\lambda)}$$

3 Lasers (Triangles) and Their Gamut (Black - -)

Phosphors (White)

Thermal Sources (Later)

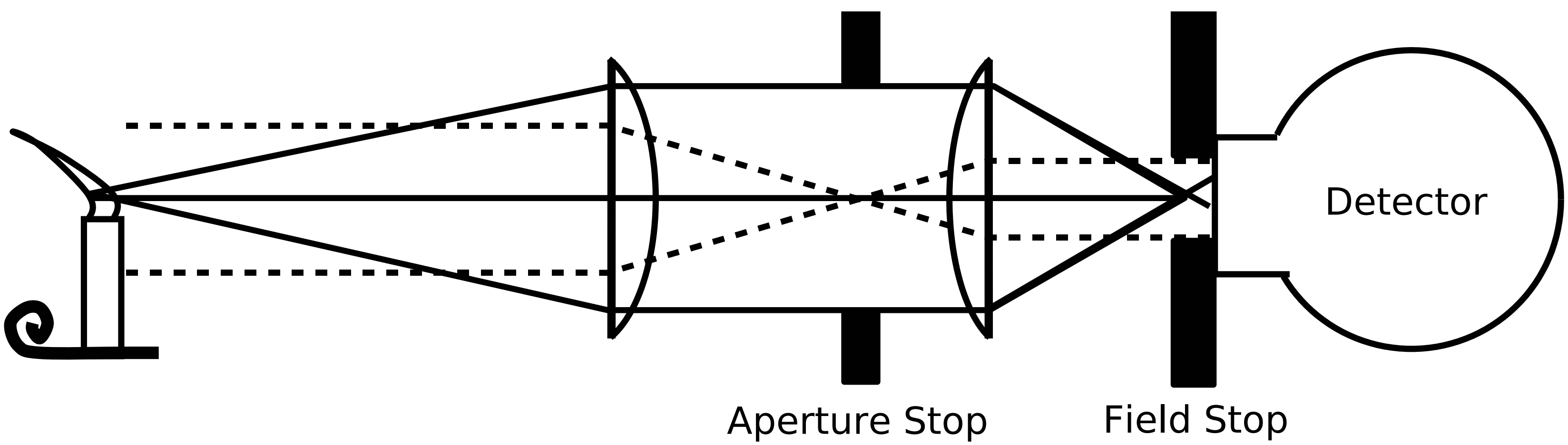


The Radiometer or Photometer

- Aperture Stop
- Field Stop
- Measured Power

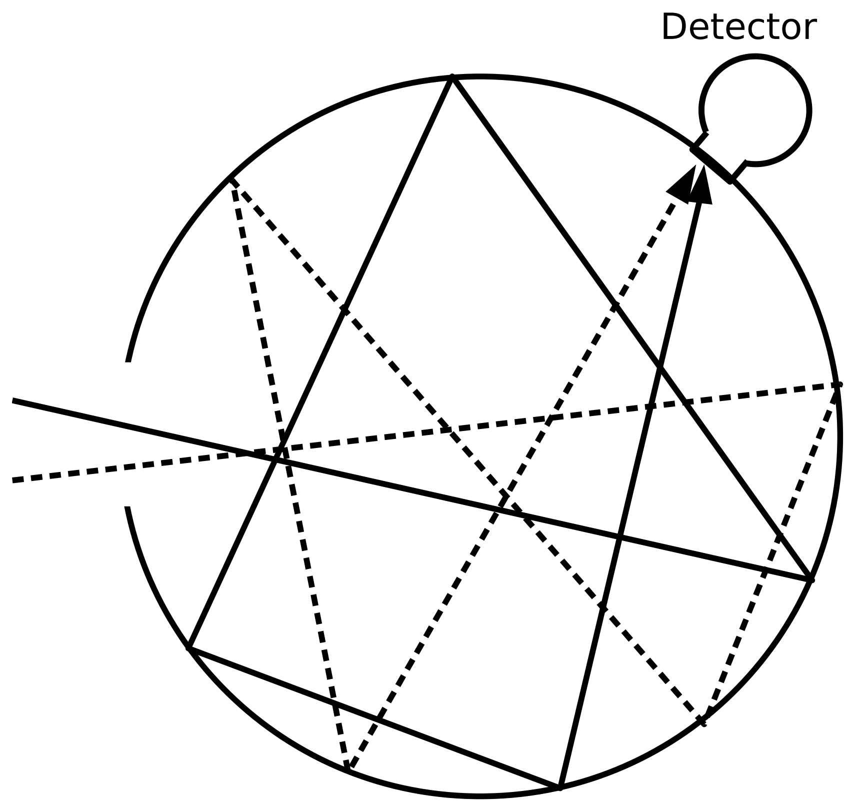
$$P = LA\Omega = LA'\Omega'$$

- Adjustable Stops (Match FOV)
- Sighting Scope?
- Calibration Required
- Spectrometer on Output?
- Spectral Filter?
- Photometric Filter?
- Computer
  - Radiance, $P/(\Omega A)$
  - Luminance (All Units)
  - Spectral Quantities



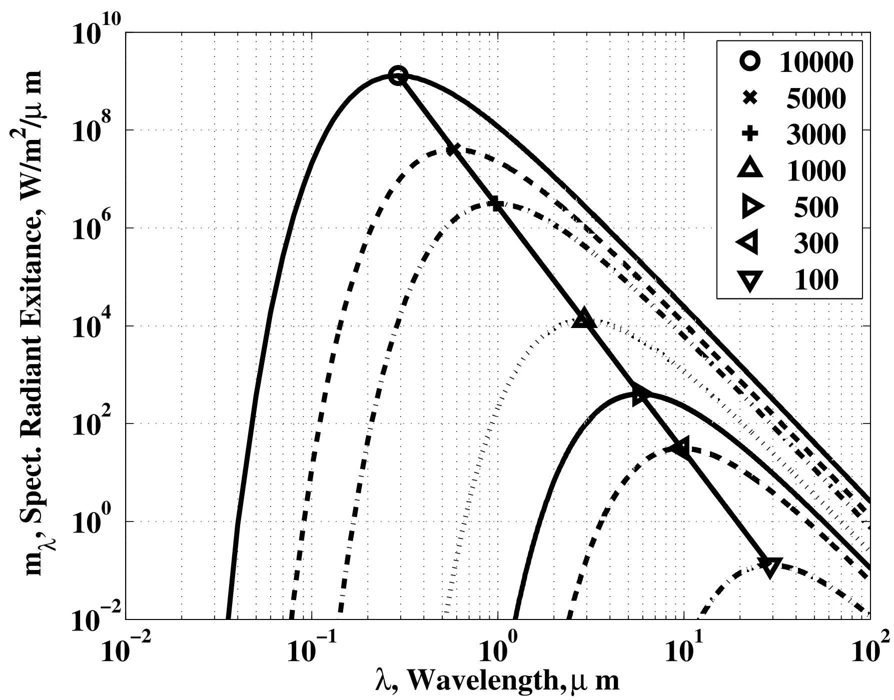
Integrating Sphere

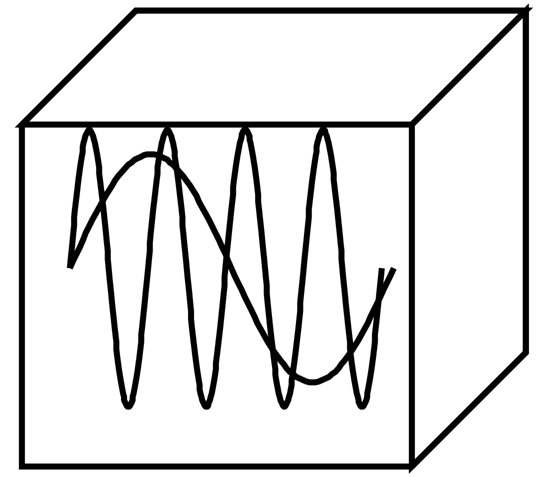
- Power from Intensity
- Integrate over Solid Angle
  - Goniometry
    - Information–Rich
    - Time–Intensive
    - Integrating Sphere
      - Easy
      - Single Measurement
- Applications
  - Wide–Angle Sources
  - Diffuse Materials
- Variations
  - Two Spheres
  - Spectroscopic Detector
  - More

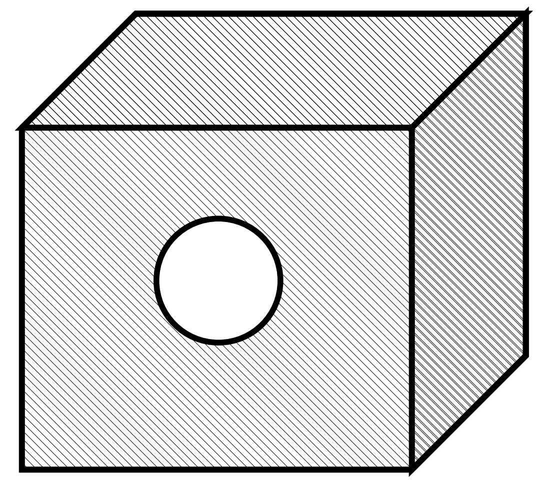


Blackbody Radiation Outline

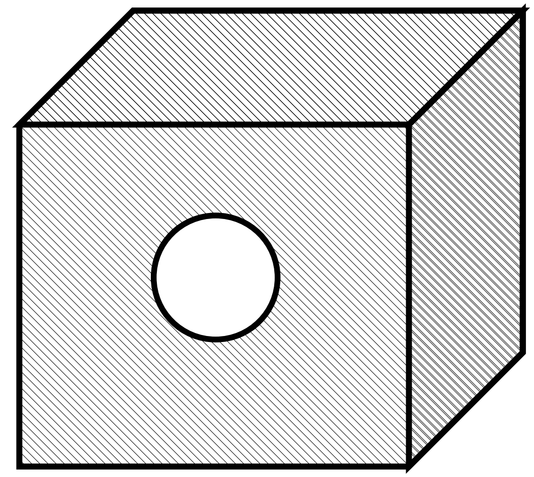
- Background
- Equations, Approximations
- Examples
- Illumination
- Thermal Imaging
- Polar Bears, Greenhouses







The Planck Equation



- Fractional Linewidth

$$|d\nu/\nu| = |d\lambda/\lambda|$$

$$M_\lambda = \frac{dM}{d\lambda} = \frac{d\nu}{d\lambda}\frac{dM}{d\nu} = \frac{\nu}{\lambda}\frac{dM}{d\nu} = \frac{c}{\lambda^2}\frac{dM}{d\nu}$$

- Planck Law *vs.* Wavelength

$$M_\lambda(\lambda, T) = \frac{2\pi hc^2}{\lambda^5}\frac{1}{e^{h\nu/k_BT} - 1}$$

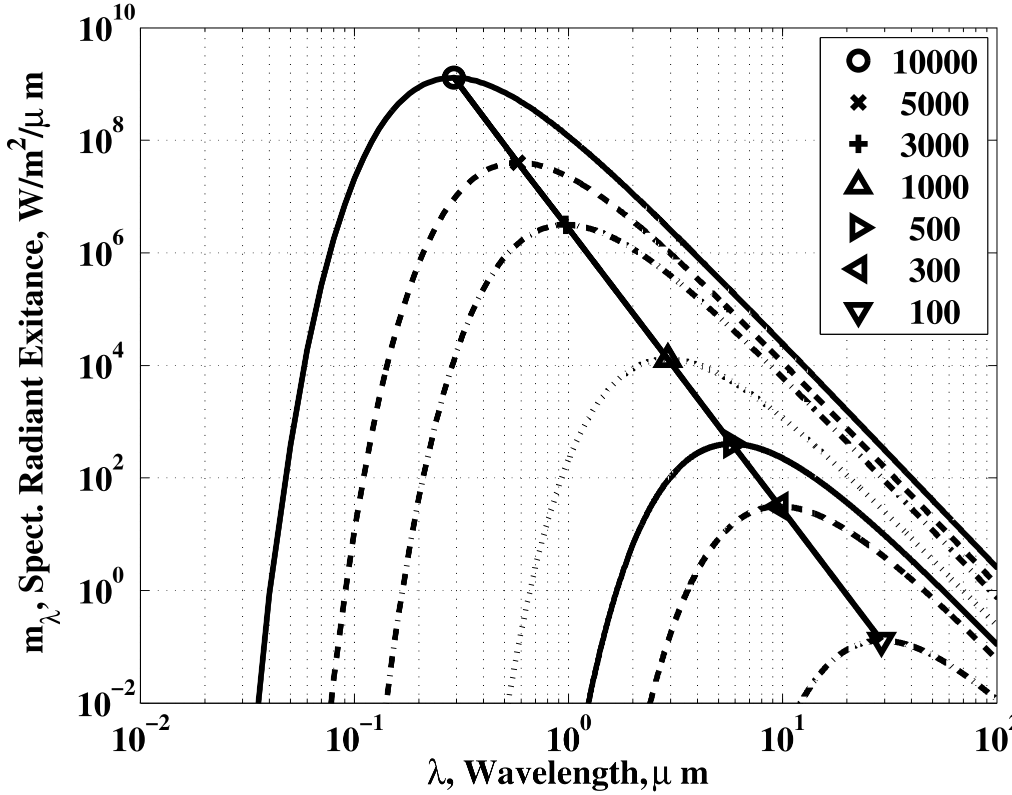
Useful Blackbody Equations

- Wien Displacement Law

$$\lambda_{peak}T = 2898\mu\text{m}\cdot\text{K}$$

- Stefan–Boltzmann Law

$$M\left(T\right) = \frac{2\pi^5 k^4 T^4}{15h^3c^2} = \sigma_e T^4$$



- Stefan–Boltzmann Constant

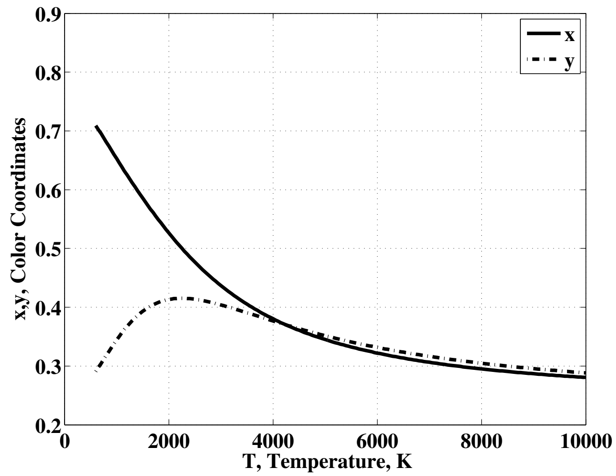
$$\sigma_e = 5.67032\times10^{-12}\text{W/cm}^2/\text{K}^4 = 5.67032\times10^{-8}\text{W/m}^2/\text{K}^4$$

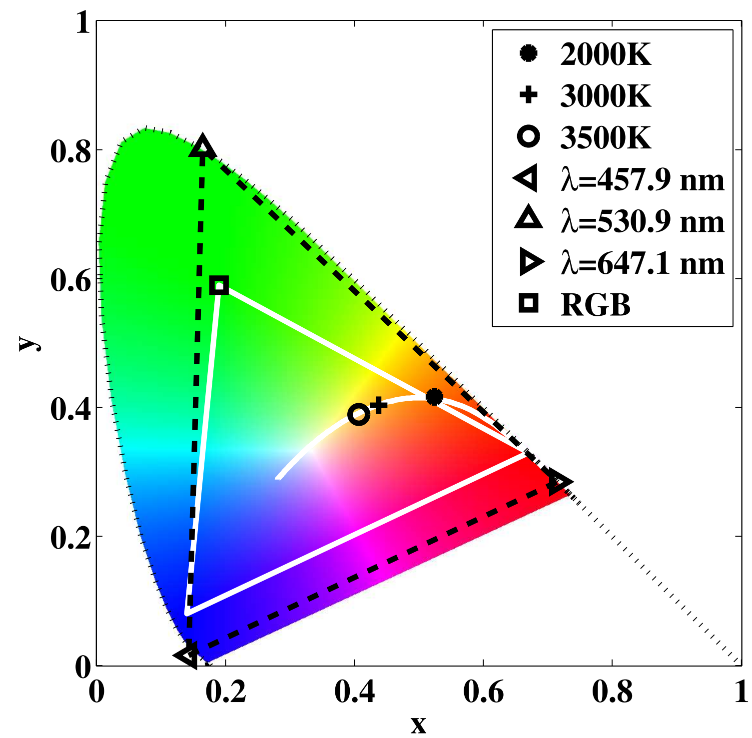
Chromaticity Coordinates of Blackbody

- Chromaticity

$$x = \frac{X}{X+Y+Z}$$

$$y = \frac{Y}{X+Y+Z}$$

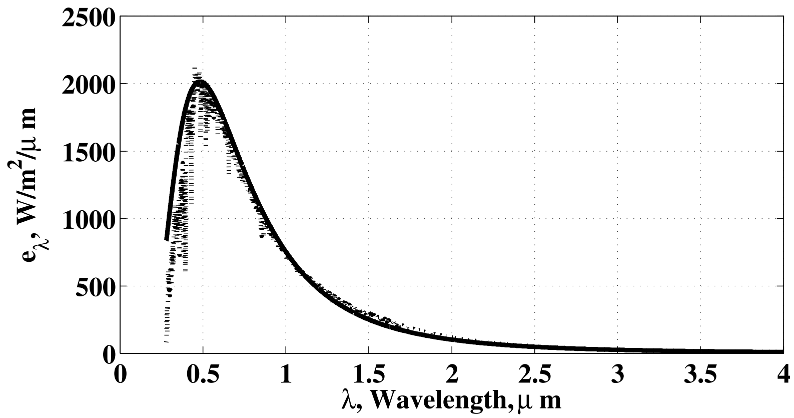




Solar Spectrum

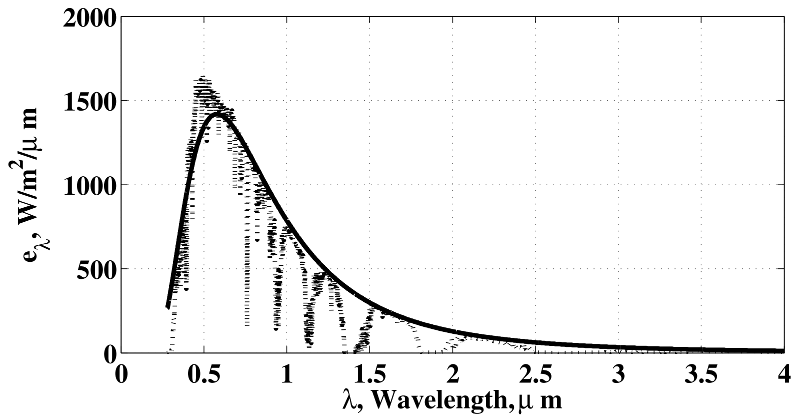
- Exo–Atmospheric
  - 6000K, 1480W/m^2

$$E_\lambda(\lambda) = 1560\text{W/m}^2 \times f_\lambda(\lambda, 6000\text{K})$$



- Sea Level
  - 5000K, 1000W/m^2

$$E_\lambda(\lambda) = 1250\text{W/m}^2 \times f_\lambda(\lambda, 5000\text{K})$$



Constants are higher than total irradiance to account for absorption in certain regions of the spectrum.

Outdoor Radiance

- Sunlit Cloud

$$E_{incident} =$$

$$1000\text{W/m}^2$$

$$5000\text{K}$$

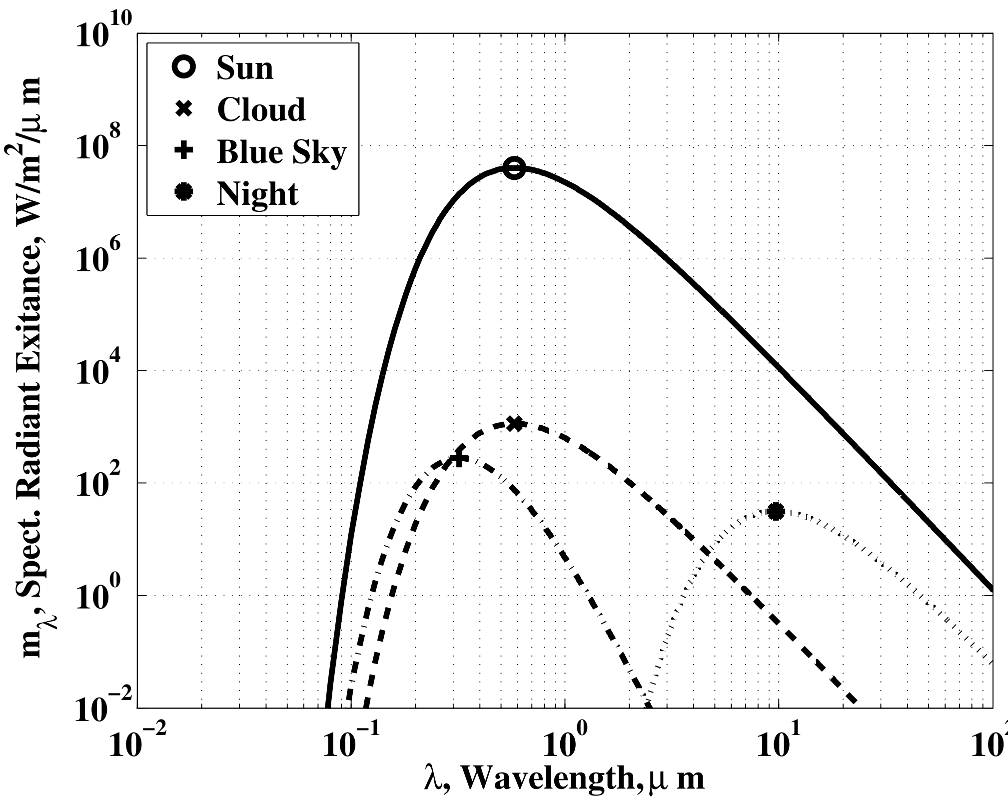
  - White Cloud

$$M_{cloud} = E_{incident}$$

  - Lambertian

$$L_{cloud} = M_{cloud}/\pi$$

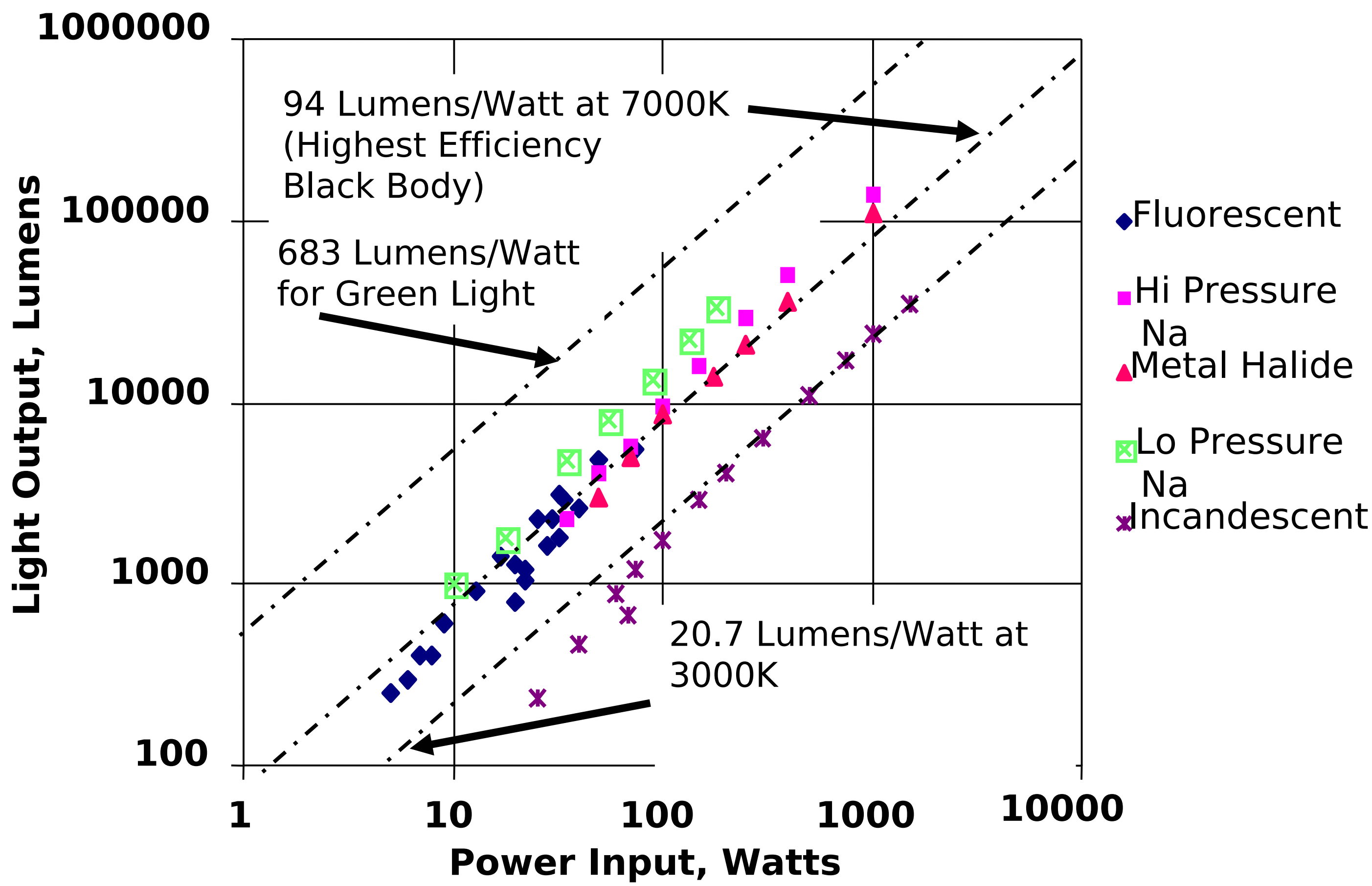
- Blue Sky: $M_\lambda = M_{sky} f_\lambda (5000\text{K}) \times \lambda^{-4}$



- Radiant Exitance, M of Object Surface Illuminated with E

$$M_\lambda(\lambda) = R(\lambda)\, E_\lambda(\lambda)$$

Illumination



Thanks to Dr. Joseph F. Hetherington

IR Thermal Imaging

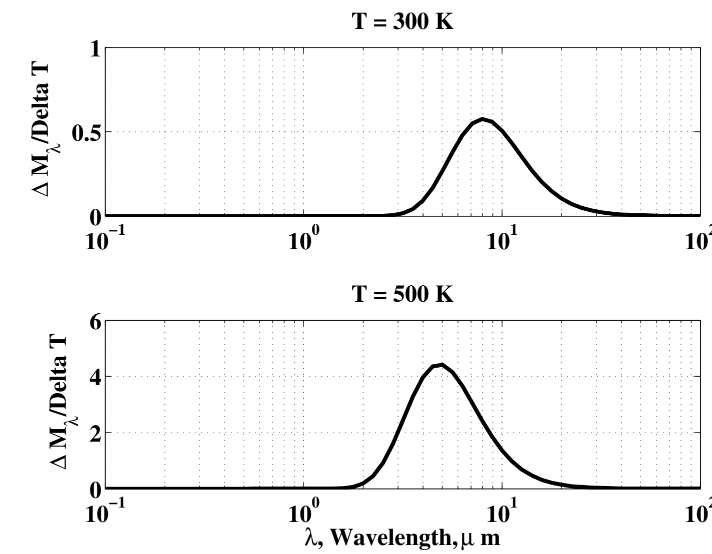
- Infrared Imaging

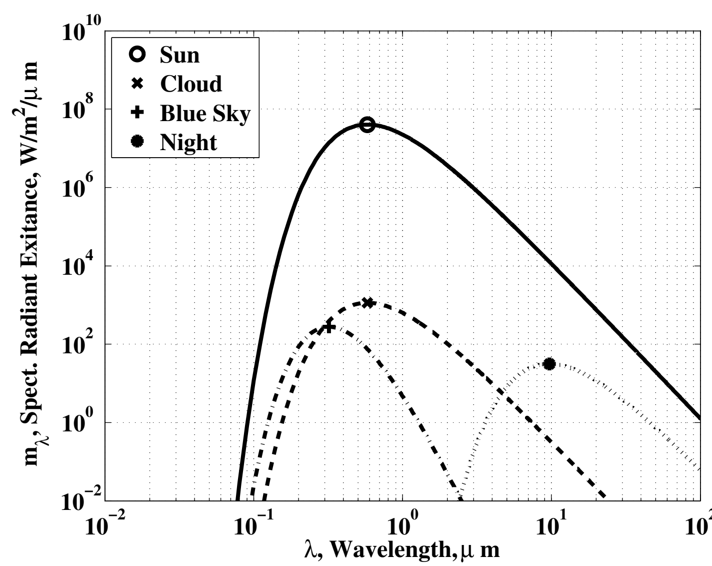
$$\int_{\lambda_1}^{\lambda_2} \rho\left(\lambda\right) \epsilon\left(\lambda\right) M_\lambda\left(\lambda, T\right) d\lambda$$

  - ρ is Responsivity
  - ϵ is Emissivity
- Maximize Sensitivity

$$\frac{\partial M_\lambda\left(\lambda, T\right)}{\partial T}$$

- Work in Atmospheric Pass Bands
- Shorter Wavelength for Higher Temperatures
- Hard to Calibrate (Emissivity, etc.)
- Reflectance and Emission

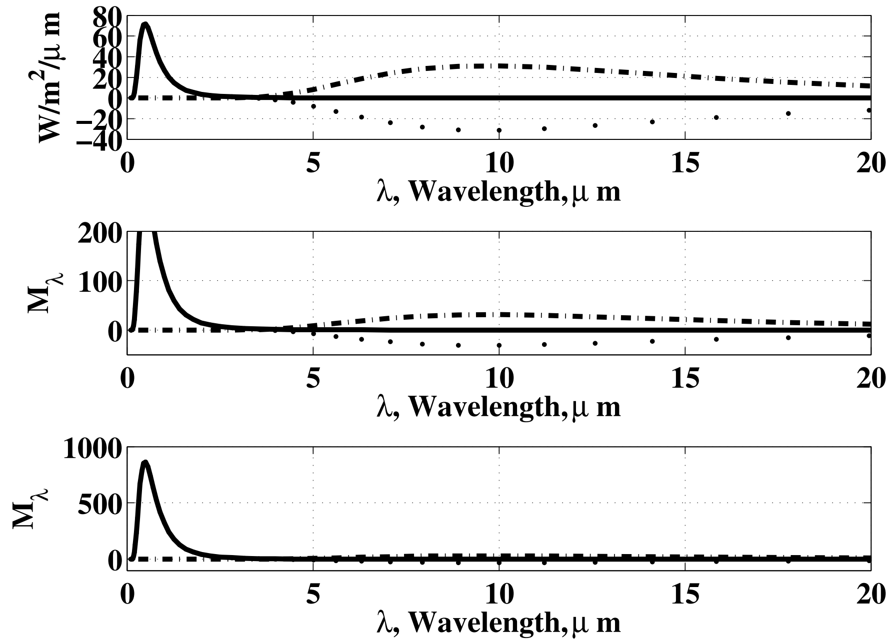




Polar Bears

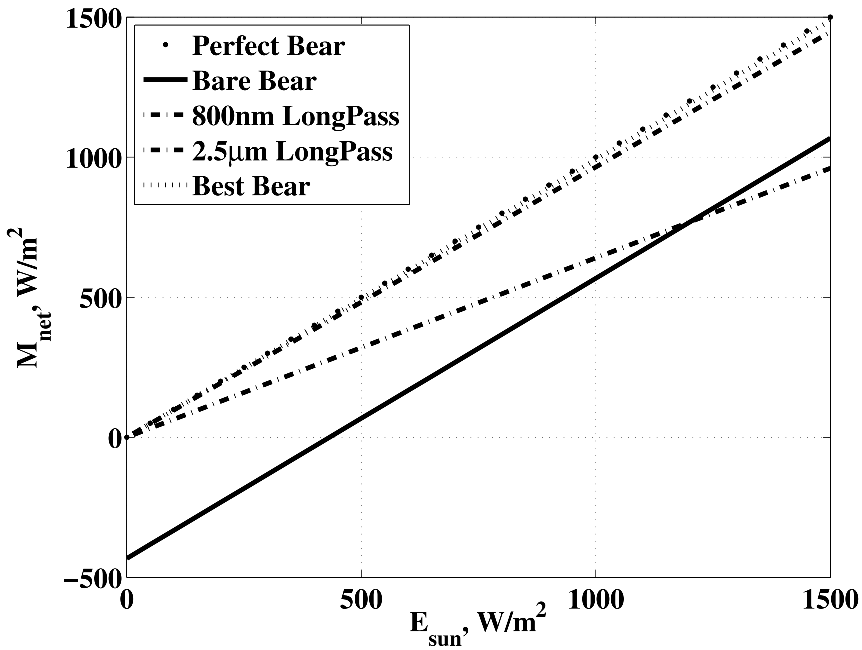
- Thermal Equilibrium
  - Heating by Sun
    - ∗ High Temperature
    - ∗ High Radiance
    - ∗ Small Solid Angle
  - Cooling to Surroundings
    - ∗ Body Temperature
    - ∗ Low Radiance
    - ∗ $\Omega = 2\pi$
  - Extra Heat from Metabolism
- Short–Pass Filter
  - Pass Visible

$E_{incident} = 50\text{W/m}^2$, 200W/m^2 600W/m^2, top to bottom



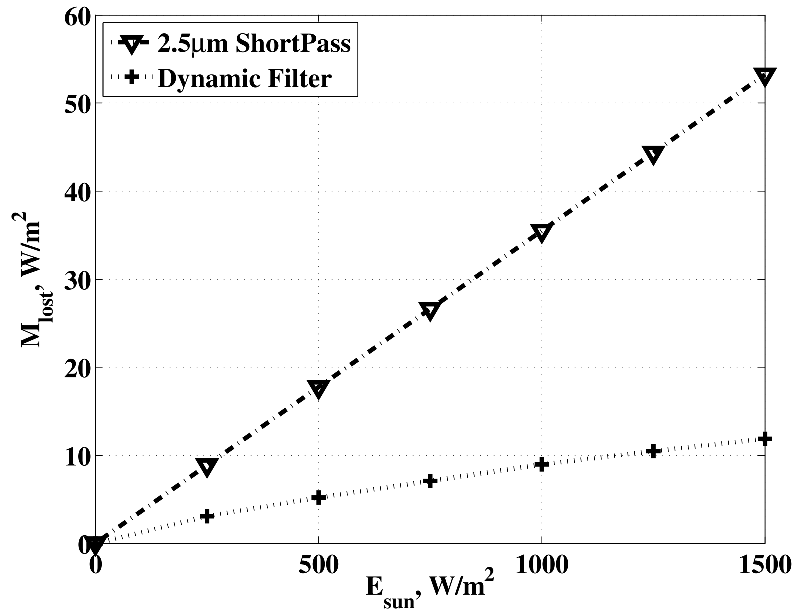
Heating (—), Cooling (- -), Net (··)

Wavelength Filtering



- Bare Bear: No Filter
- 800nm Short Pass
- 2.5nm Short Pass Like Glass
- Best Bear is a Dynamic Filter

- Net Cooling for Two Best



- Dynamic Filter Follows Zero Crossing of Net M_λ
- Perfect: No Cooling (Impossible: Nothing Out Means Nothing In)