Optics for Engineers
Week 6

Charles A. DiMarzio
EECE–4646
Northeastern University

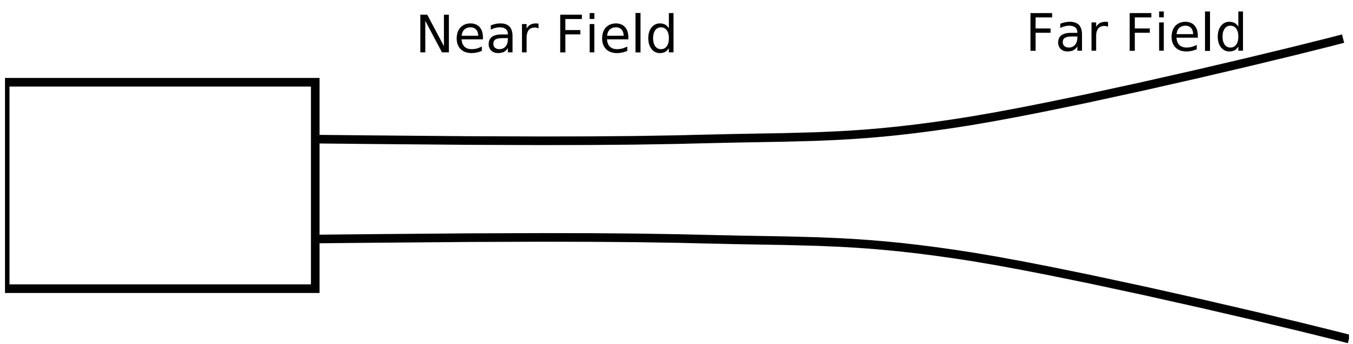
Mar 2024

Week 5 Agenda

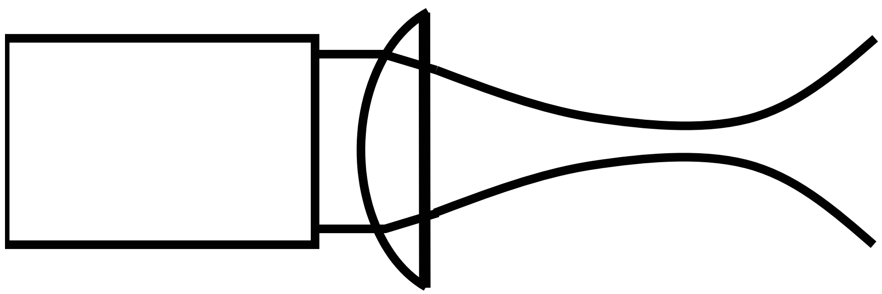
- Aperture Size and Spot Size
- Fourier Optics
- Image Resolution
- Diffraction Gratings
- Gaussian Beams
- Laser Cavities
- Holography and Phase Conjugation

Diffraction

Collimated Beam: Divergence in Far Field

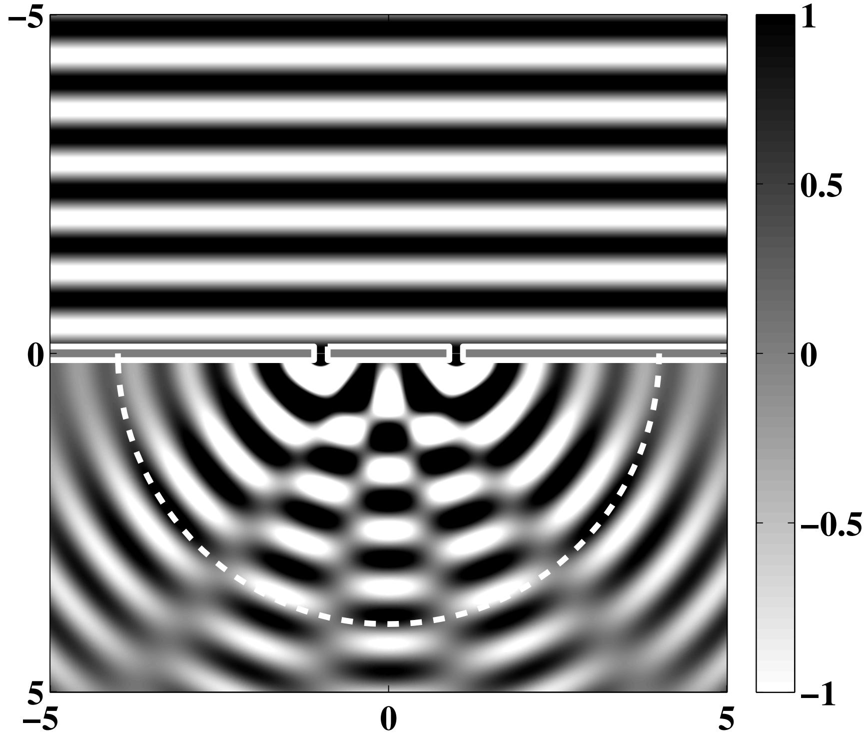


Focused Beam: Minimum Spot Size and Location

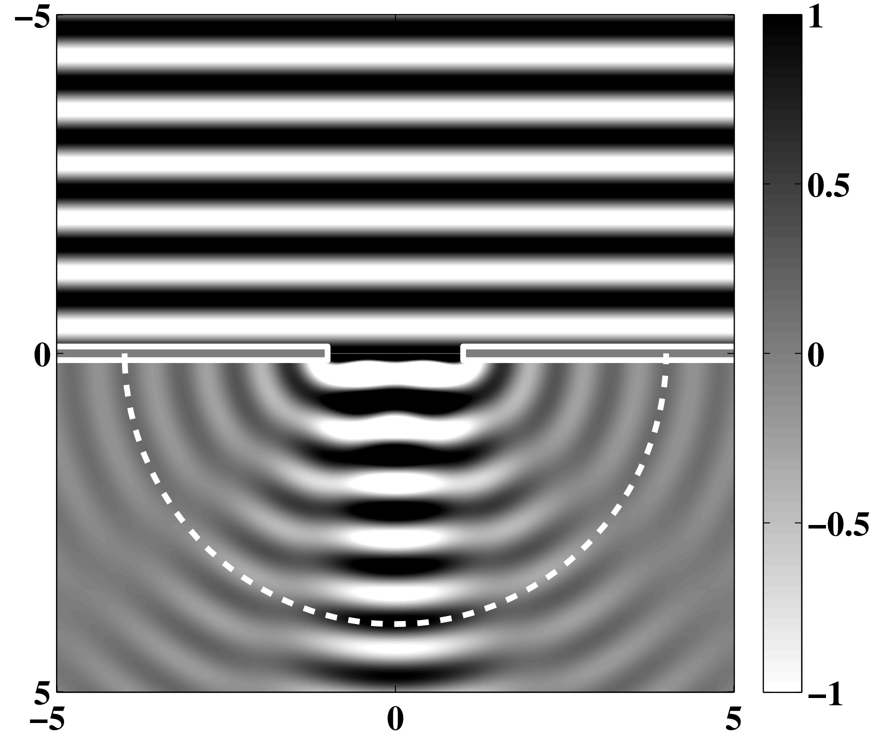


$$\alpha = C\frac{\lambda}{D}$$

Slit Experiments



A. Two slits

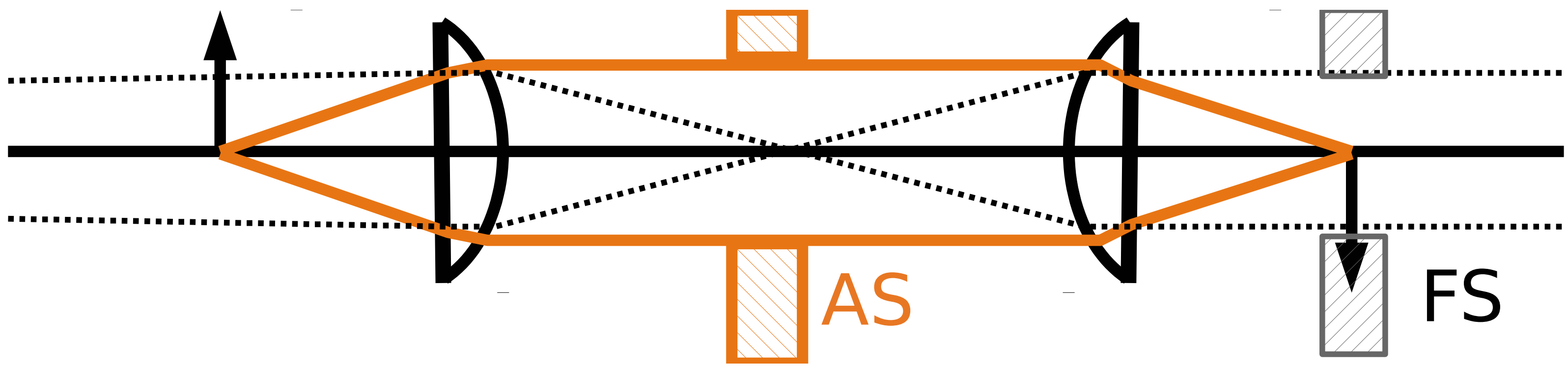


B. Aperture

- Angular Divergence of Light Waves
- Alternating Bright and Dark Regions
- Near– and Far–Field Behavior

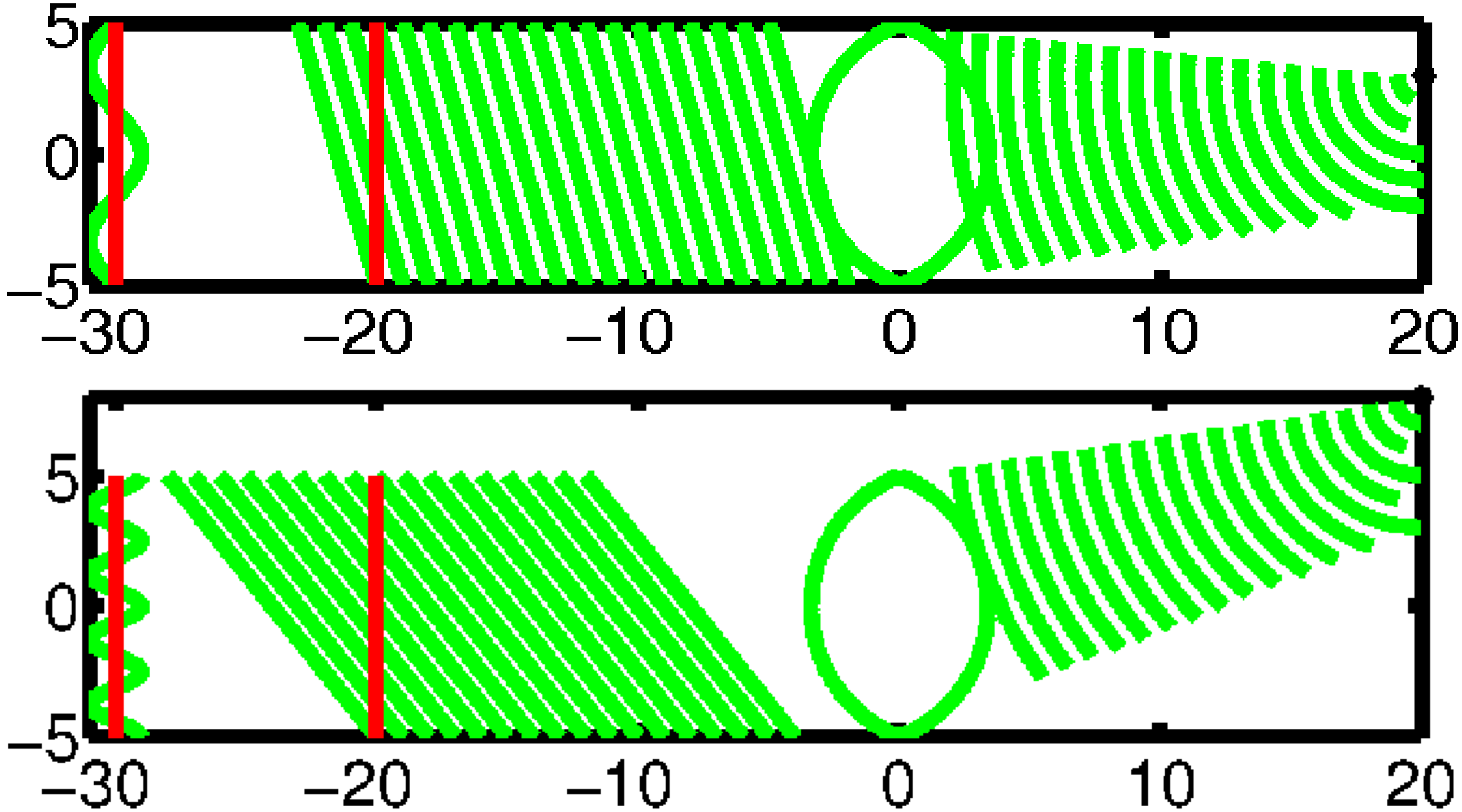
$\lambda = 800$nm. Axis Units are μm

Two–Lens System



- Separation: $f_1 + f_2$
- Magnification $m = \frac{x'}{x} = -\frac{f_2}{f_1}$
- Angle from Axis $\theta' = \frac{1}{m}\theta$
- $u = \sin\theta \quad u_{max} = NA$
- Aperture Diameter $D = 2f_1 \tan\theta = 2f_2 \tan\theta'$
- Fourier Optics: $Epupil \propto FT\left(E_{field}\right)$

Varying Spatial Frequencies



Coordinate Relationships

	Spatial Frequency	Pupil Location	Direction Cosines
Spatial Frequency		$x_1 = \lambda z f_x$ $y_1 = \lambda z f_y$	$u = f_x \lambda$ $v = f_y \lambda$
Pupil Location	$f_x = \frac{x_1}{\lambda z}$ $f_y = \frac{y_1}{\lambda z}$		$u = \frac{x_1}{z}$ $v = \frac{y_1}{z}$
Direction cosines	$f_x = \frac{u}{\lambda}$ $f_y = \frac{v}{\lambda}$	$x_1 = uz$ $y_1 = vz$	
Angle			$u = \sin\theta\cos\zeta$ $v = \sin\theta\sin\zeta$

Camera Example

- Pixel Pitch: 7.4μm

$$f_{sample} = \frac{1}{7.4 \times 10^{-6}\text{m}} = 1.35 \times 10^{5}\text{m}^{-1} \qquad \text{(135 per mm.)}$$

- Nyquist Sampling: $f_{max} = f_{sample}/2$ at $u_{max} = NA$

$$NA = \frac{2 f_{sample} \lambda}{2} = f_{sample}\lambda \qquad \text{(Coherent Imaging)}$$

- Green light at 500nm,

$$NA = 0.068$$

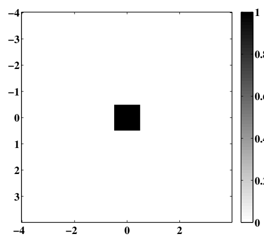
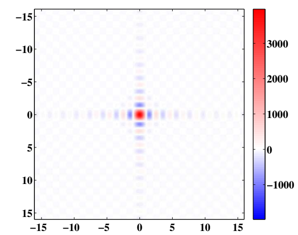
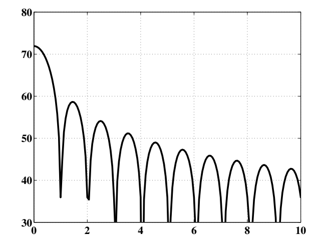
  – Lower NA Acts as Anti–Aliasing Filter

  – Higher NA Allows Aliasing

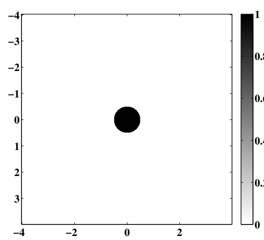
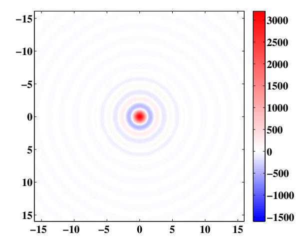
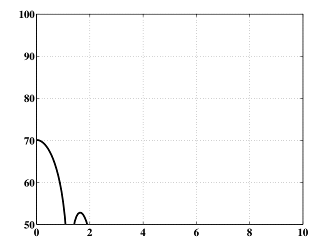
Summary of Common Fraunhofer Patterns

Pupil Pattern | **Fraunhofer Pattern** | **Slice**

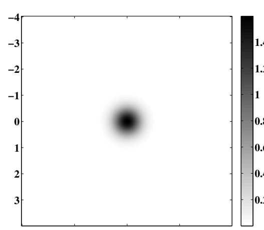
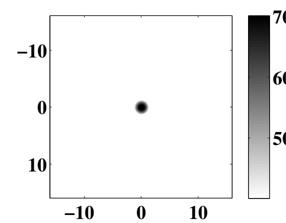
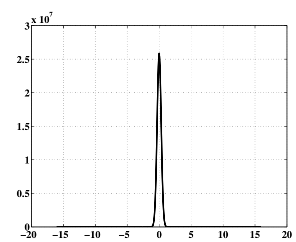
SQUARE:
$d = 2\frac{\lambda}{D}$ (1st Nulls)
$I_0 = \frac{PD^2}{\lambda^2 z_1^2}$
$I_1/I_0 = -13dB$

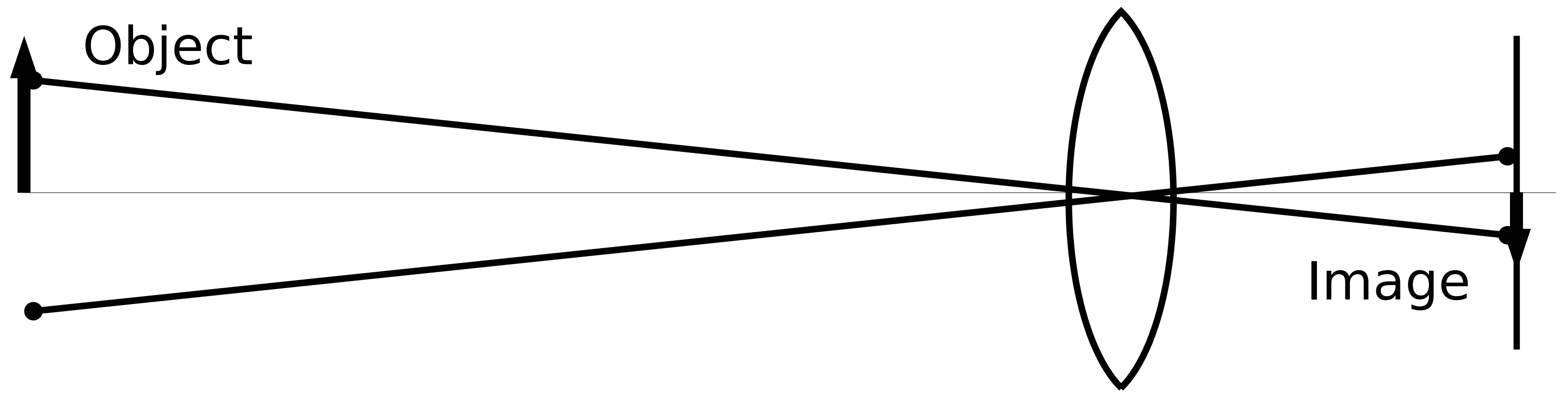
CIRCULAR:
$d = 2.44\frac{\lambda}{D}$ (1st Nulls)
$I_0 = \frac{\pi PD^2}{4\lambda^2 z_1^2}$
$I_1/I_0 = -17dB$

GAUSSIAN:
$d = \frac{4}{\pi}\frac{\lambda}{d_0}$ (e^{-2} Width)
$I_0 = \frac{\pi P d_0^2}{2\lambda^2 z_1^2}$
No sidelobes

Resolution of an Imaging System



- "To Resolve" or "Resolution" Defined

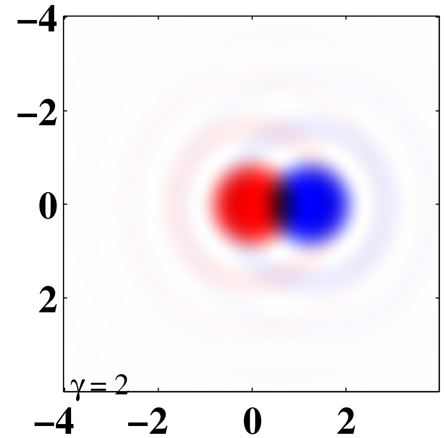
"... to distinguish parts or components of (something) that are close together in space or time; ..."

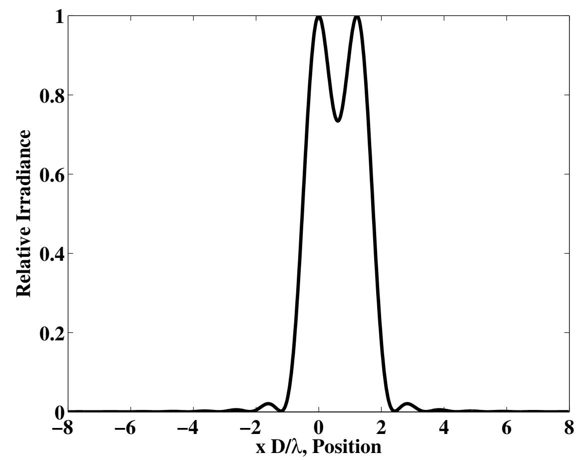
"... the process or capability of rendering distinguishable the component parts of an object or image; a measure of this, expressed as the smallest separation so distinguishable, ..."

- Resolution is Not Number of Pixels or Width of Point–Spread Function

Resolution Analysis

- Diffraction Patterns of Two "Point Objects"
  - Point–Spread Functions
  - From Fraunhofer Diffraction at Pupil
  - Fourier Transforms (See Ch. 11)
- Add
- Set Criterion for Valley
  - Noise Analysis?
  - Contrast?
  - Arbitrary Decision?





A complete and consistent definition would require knowledge we are not likely to have.

Rayleigh Criterion

- Frequently Used, but Arbitrary
- Defined by Nulls of Point–Spread Function
  - Peak of One over Valley of Other
- Produces Inconsistent Valley (Depends on Pattern)
  - Square Aperture

$$\delta = z_1\lambda/D$$

$$\delta\theta = \lambda/D$$

  - Valley Depth

$$2\left[\frac{\sin(\pi/2)}{\pi/2}\right]^2 =$$

$$\frac{8}{\pi^2} = 0.81$$

  - Circular Aperture

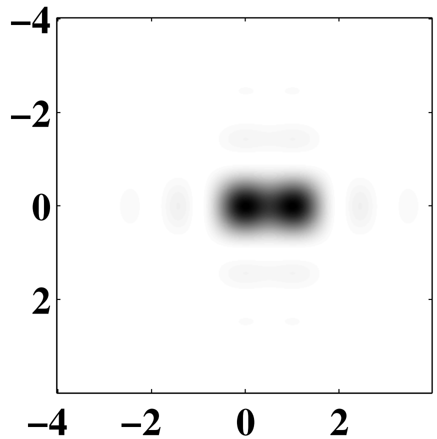
$$\delta\theta = 1.22\frac{\lambda}{D}$$

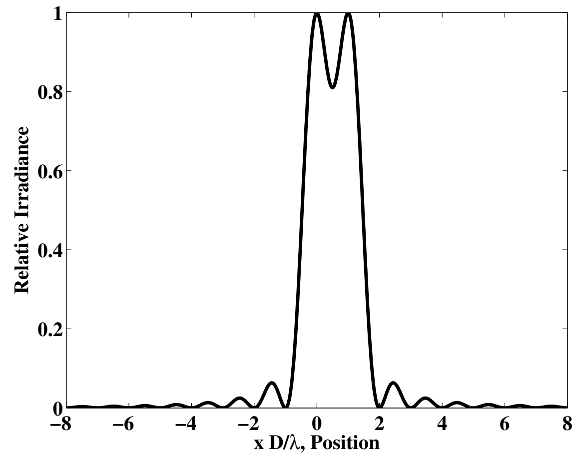
$$\delta = 0.61\frac{\lambda}{NA}$$

  - Valley Depth

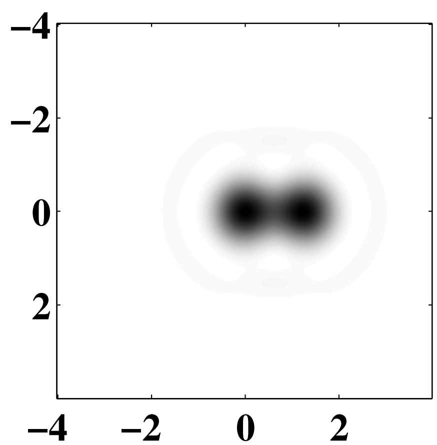
0.73

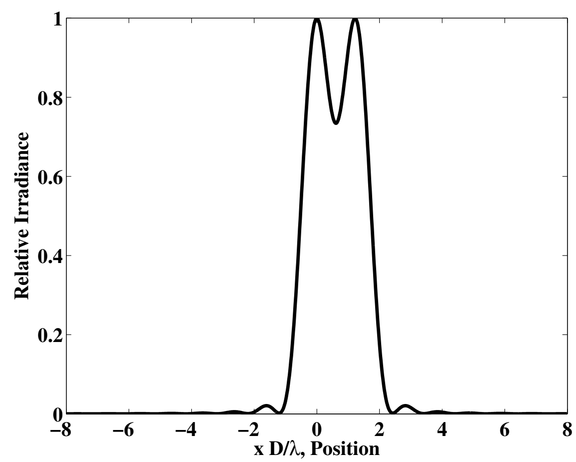
Resolution at the Rayleigh Limit





Square Aperture, λ/D, 81% Valley



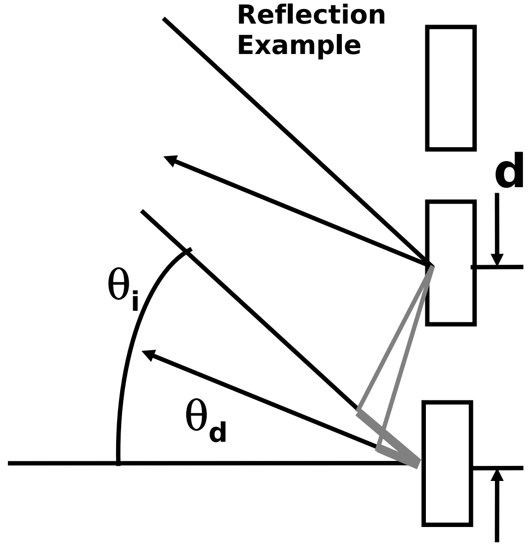


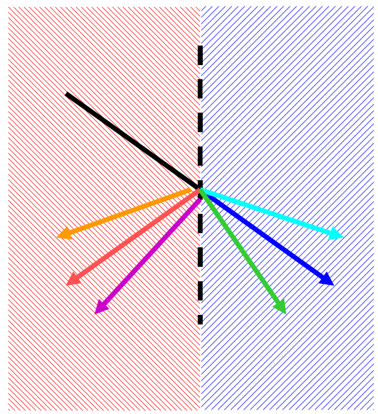
Circular Aperture, $1.22\lambda/D$, 73% Valley

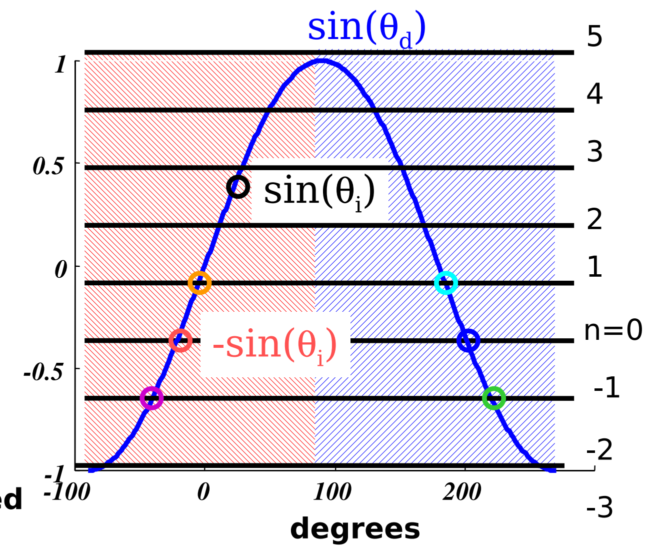
- Issues
  - Noise
  - Contrast
  - Statistics
  - Sampling*
- Other Definitions
  - MTF* (Ch. 11)
  - Any Valley (Sparrow)
  - 81% Valley (Wadsworth)
  - PSF FWHM (Houston)*

* Not really resolution

The Diffraction Grating (1)







- The Grating Equation

$$N\lambda = d\left(\sin\theta_i + \sin\theta_d\right)$$

$$\sin\theta_d = -\sin\theta_i + N\frac{\lambda}{d}$$

The Diffraction Grating (2)

- Grating Equation

$$N\lambda = d\left(\sin\theta_i + \sin\theta_d\right)$$

- Grating Dispersion

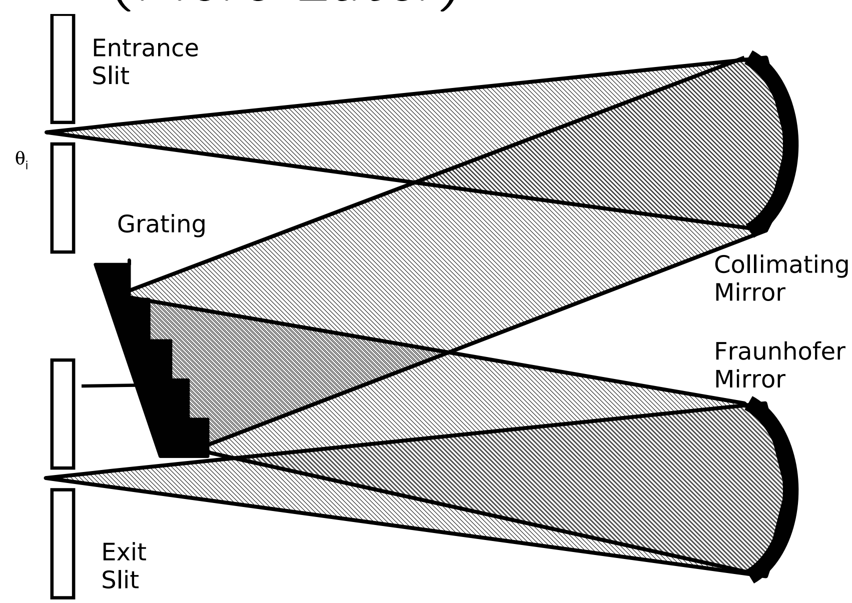
$$\delta\lambda = \frac{d}{N}\delta\left(\sin\theta_d\right)$$

- Applications
  - Monochromator
  - Spectrometer
- Aliasing Issues
  - N to $N+1$

$$\lambda_{N+1} = \lambda_N \times \frac{N}{N+1}$$

  - Maximum Width: Factor of 2

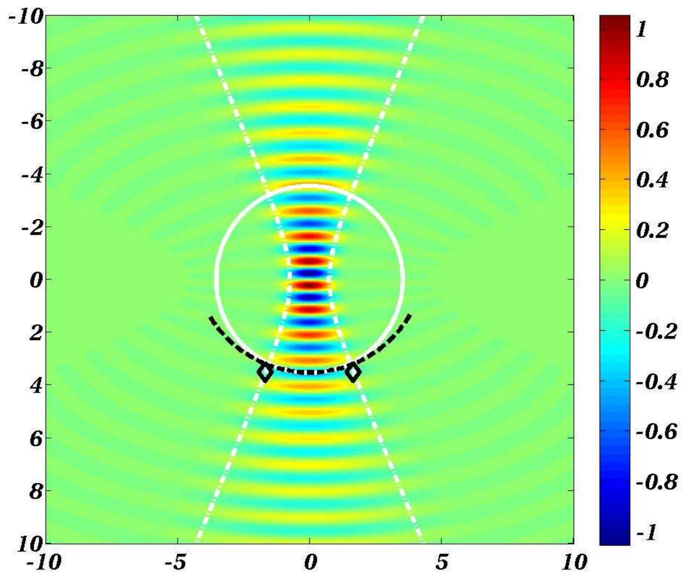
- Anti–Aliasing Filter
  - *e.g.* Colored Glass
  - *e.g.* “Filter Wheel”
- Monochromator (More Later)



Gaussian Beams

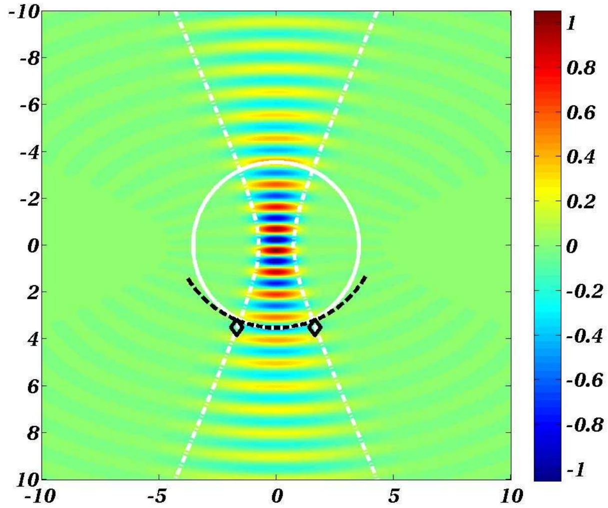
- Applications
  - Many Laser Beams
  - Minimum–Uncertainty
  - Simple Equations
  - Good Approximation
  - Extensible (*e.g.* Hermite–Gaussian)
- Equations
  - Solution of Helmholz Equation
  - Solution to Laser Cavity
  - Kogelnik and Li, 1966
  - Spherical Gaussian Waves
  - "Gaussians Are Forever"

- Imaginary Part of Field
  - Gaussian Profile
  - Spherical Wavefront



- Focusing and Propagation
  - Simple Equations

Physical Meaning of Parameters



$$E \approx \sqrt{\frac{2P}{\pi w^2}} e^{jkz} e^{jk\frac{x^2+y^2}{2\rho}} e^{-\frac{x^2+y^2}{w^2}} e^{-\jmath\psi}$$

- Distance from Waist, z
- Rayleigh Range, b

$$b = \frac{\pi w_0^2}{\lambda} = \frac{\pi d_0^2}{4\lambda}$$

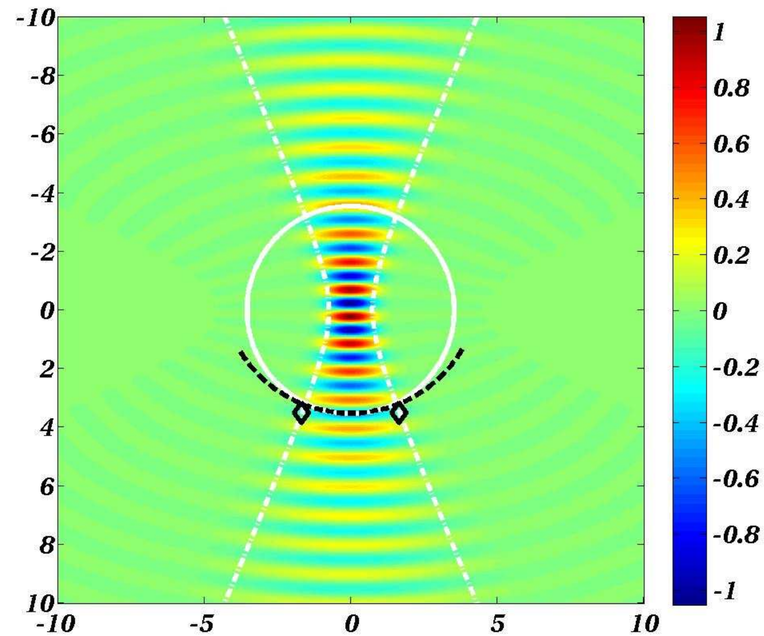
- Radius of Curvature, ρ
  Dashed Black Line
- Beam Diameter, d
  Black Diamonds

Gouy Phase

- Phase Term

$$\psi = \arctan\frac{z}{b}$$

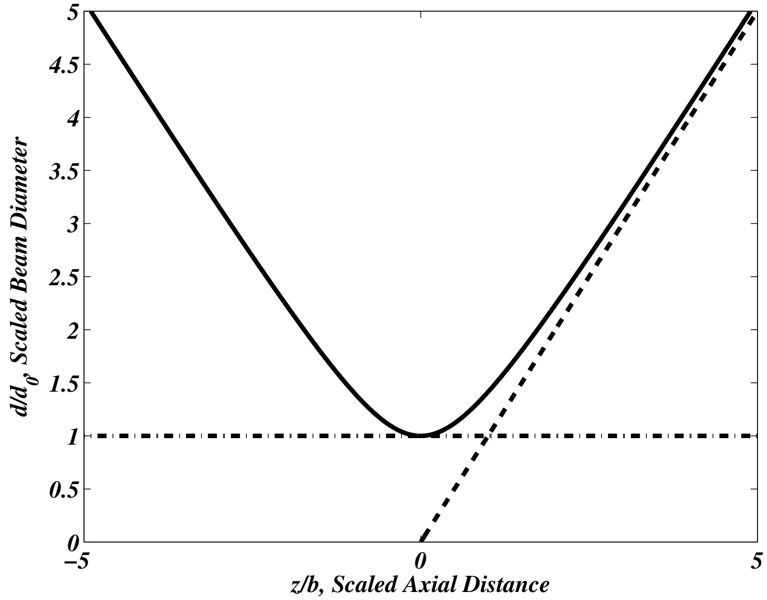
- See White Circle
- Plot is $\Im E$



The Really Useful Equations

- Beam Diameter, d

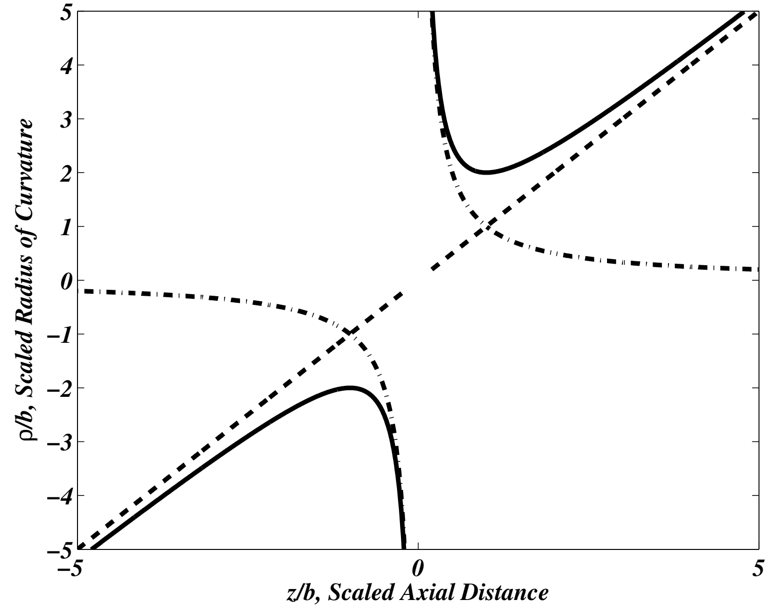
$$d = d_0\sqrt{1+\frac{z^2}{b^2}}$$



- Near Field $d_g \approx d_0$
- Far Field $d_d \approx \frac{4}{\pi}\frac{\lambda}{d_0}z$

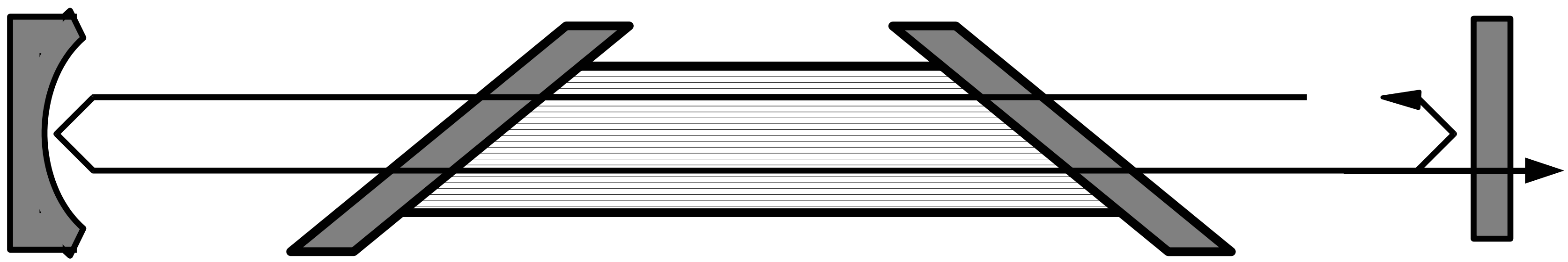
- Radius of Curvature, ρ

$$\rho = z + \frac{b^2}{z}$$

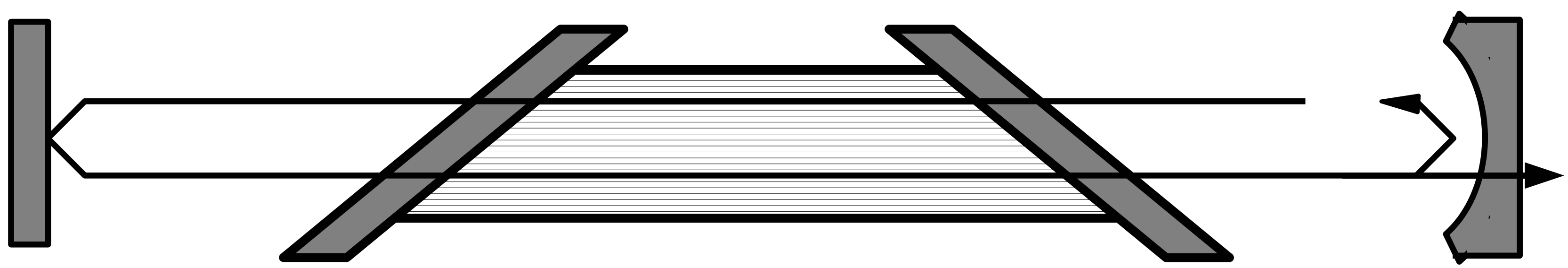


- Near Field $\rho \approx b^2/z \to \infty$
- Far Field $\rho \approx z \to \infty$

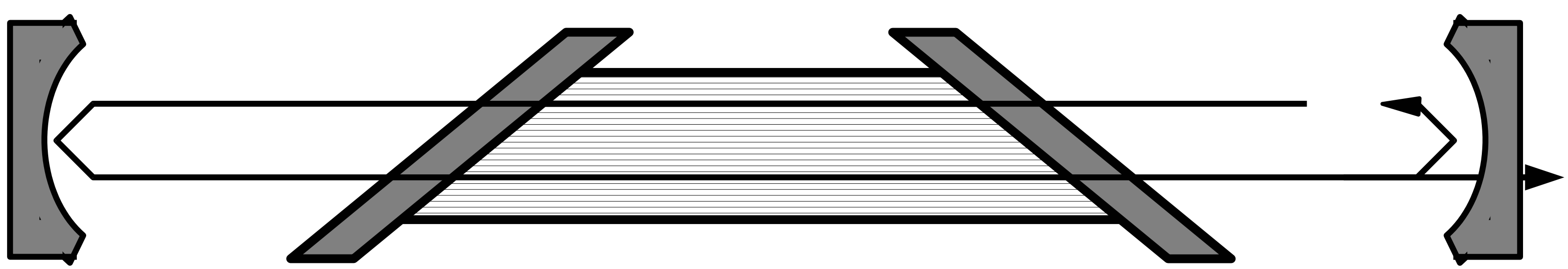
Stable Laser Cavity Design



A. Flat Output Coupler



B. Flat Rear Mirror



C. Confocal Resonator

Steady State in Laser Cavity

- The amplitude after a round trip is unchanged. This means that any loss (including power released as output) must be offset by corresponding gain. (Gain Saturation)

- The phase after a round trip must be unchanged. We discussed, in our study of interference, how this requirement on the axial phase change, e^{jkz}, affects the laser frequency.

- The beam shape must be unchanged, so that the phase and amplitude is the same for all x and y. This is the subject to be considered in this section.

Design Problem

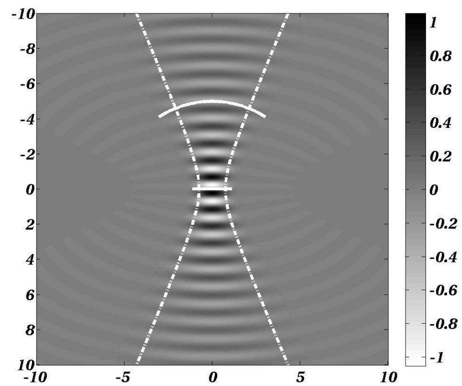
- Carbon Dioxide Laser: P(20), $\lambda = 10.59\mu$m
- Beam Output: Collimated, 5mm Diameter
- Cavity Length: 1m (Probably because of Gain)
- Solution
  - Collimated Output: Flat Output Coupler
  - Rear Mirror to Match Curvature at $z = -1$m

$$b = 1.85\text{m} \qquad \rho = -4.44\text{m} \qquad d = 5.7\text{mm}$$

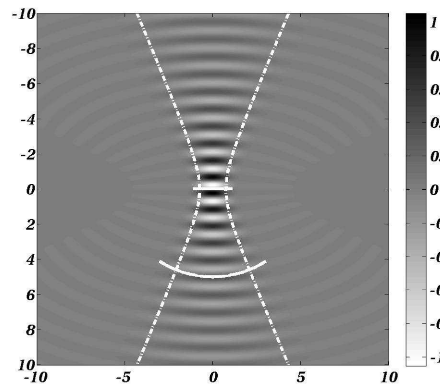
  - Rear Mirror Concave, $\rho = -4.44$m
  - Diameter Larger than $d = 5.7$mm (Typically 1.5X)

Stable Cavity Examples

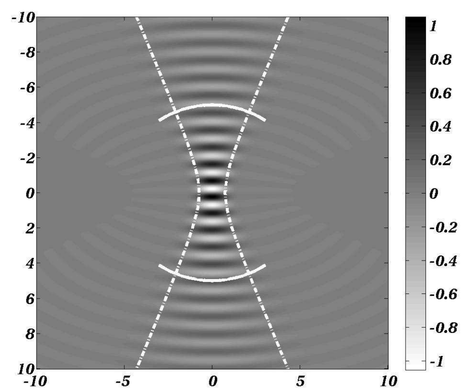
Output toward Bottom



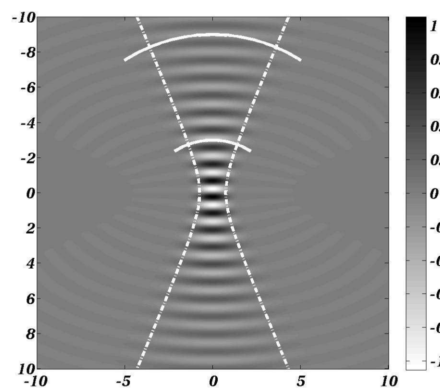
A. Flat Output Coupler



B. Flat Rear Mirror

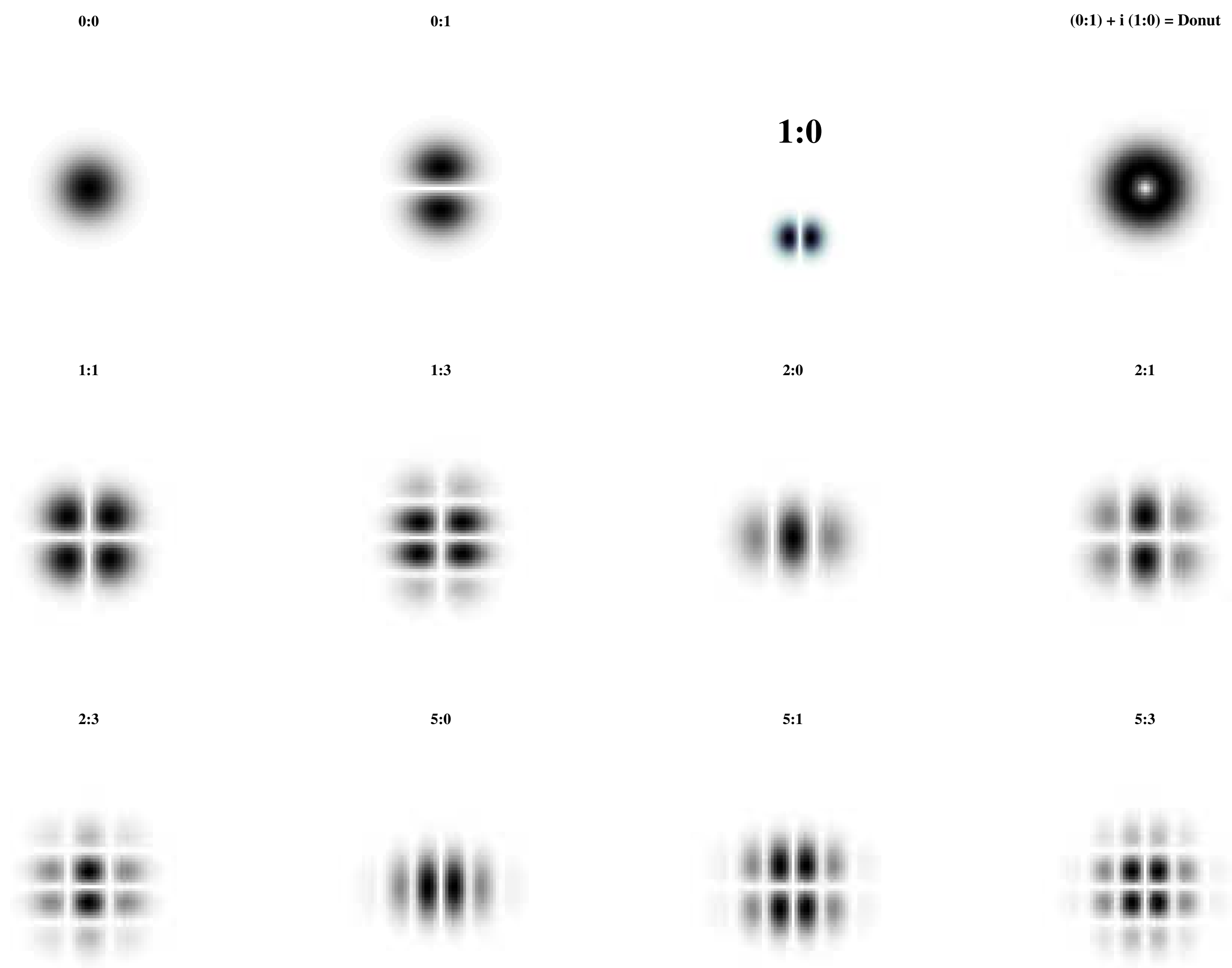


C. Confocal Resonator

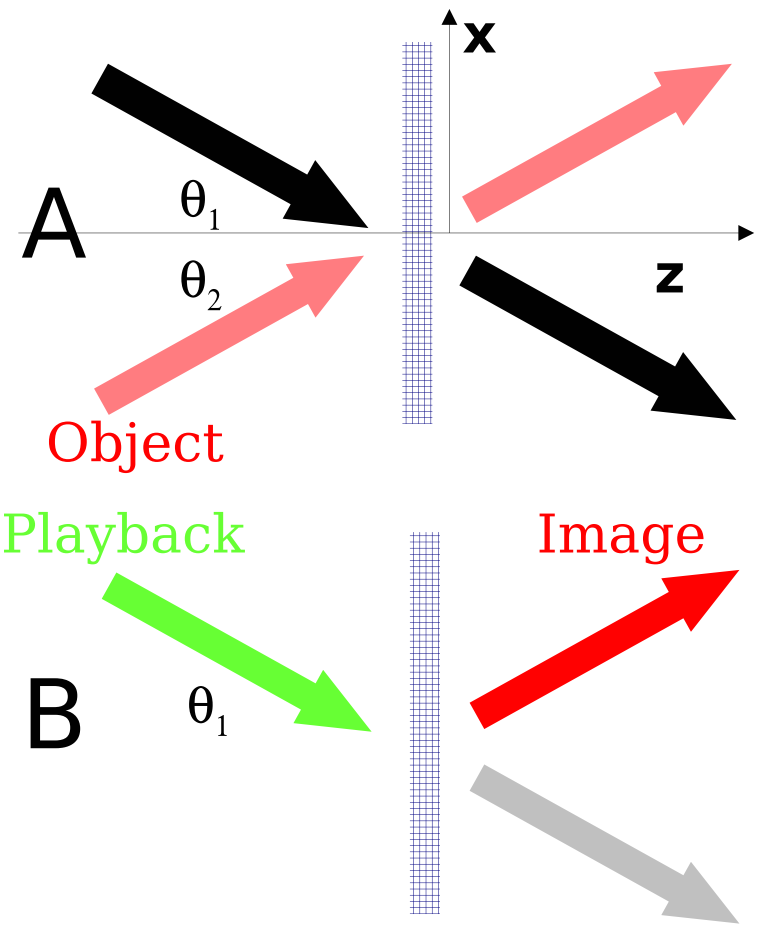


D. Focusing Cavity

Some Hermite–Gaussian Modes



Making a Hologram



Adapted from Lei Sui

R = Reference Wave
O = Object Wave

Expose Film: (Or use PRC for memory.) Irradiance at Film Plane:

$$I = (R + O)(R^* + O^*)$$

Develop Film:

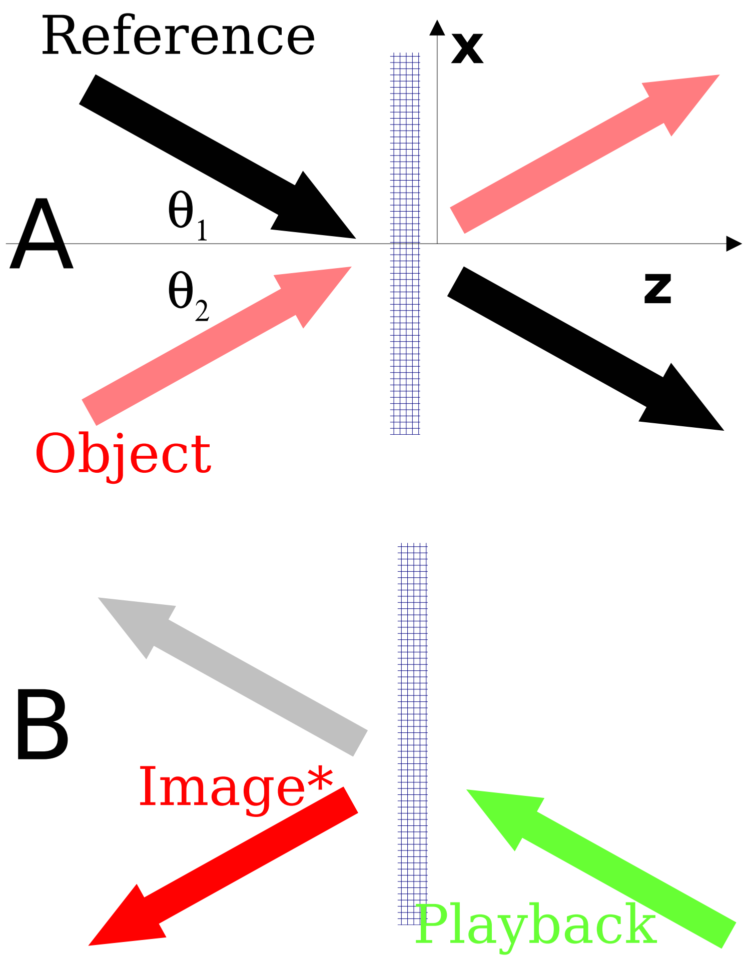
$$H = PR^*O + PRO^* + \ldots$$

Match Playback to Reference:

$$I = RR^*O$$

$$Ghost = RRO^*$$

Conjugate Hologram



R = Reference Wave

O = Object Wave

Expose Film: Irradiance at Film Plane:

$$I = (R + O)(R^* + O^*)$$

Develop Film:

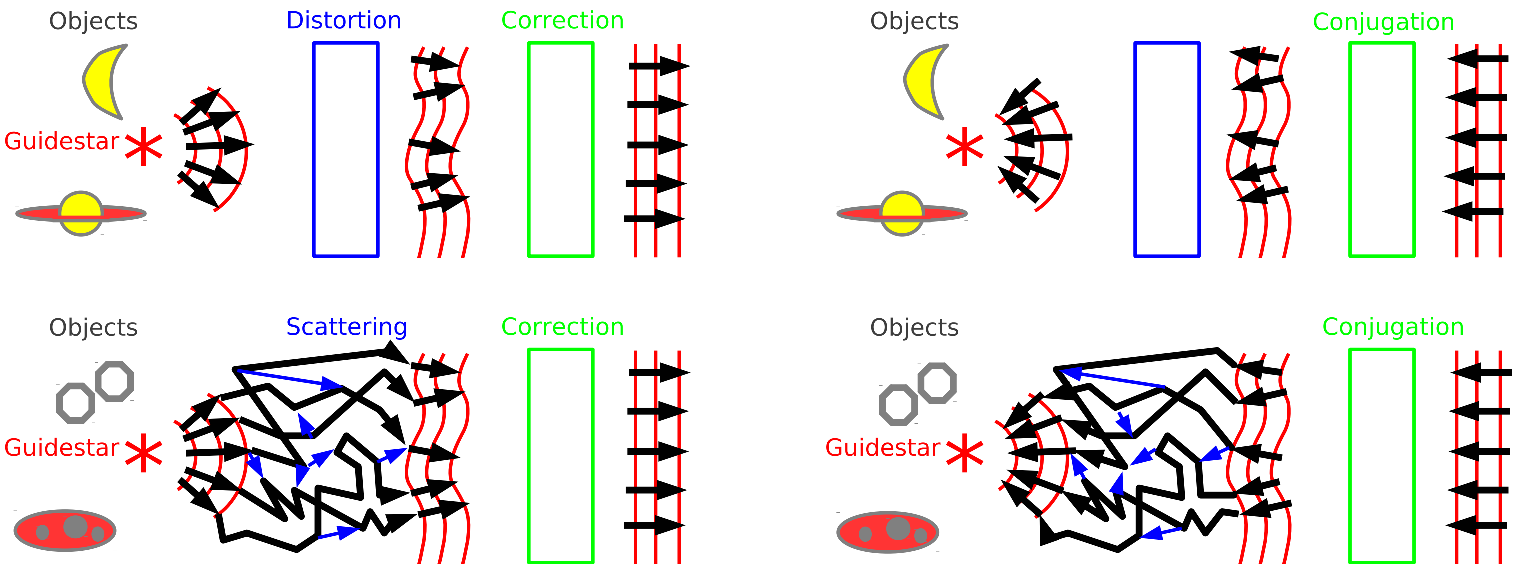
$$H = PR^*O + PRO^* + \ldots$$

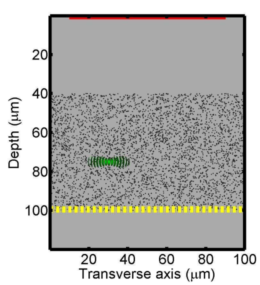
Match Playback to Conjugate of Reference:

$$Ghost^* = R^*R^*O$$

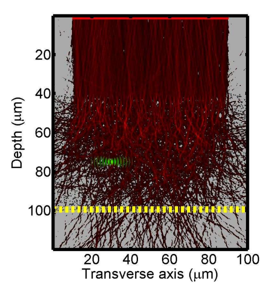
$$I^* = R^*RO^*$$

Guidestar

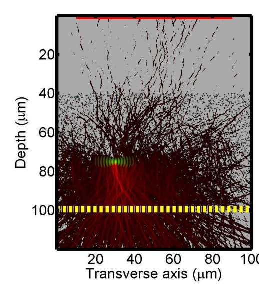




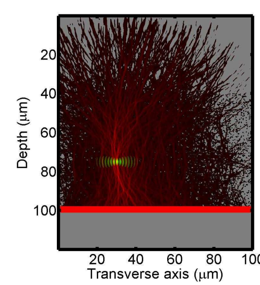
A. Layout



B. Probe



C. Sideband



D. PCS