# Optics for Engineers
# Week 5

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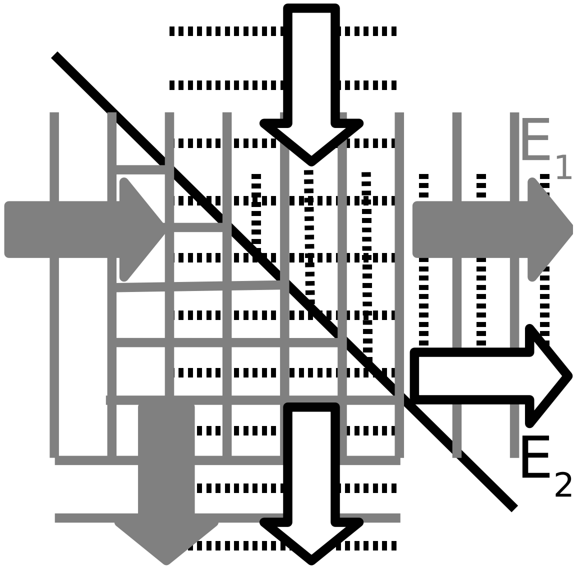
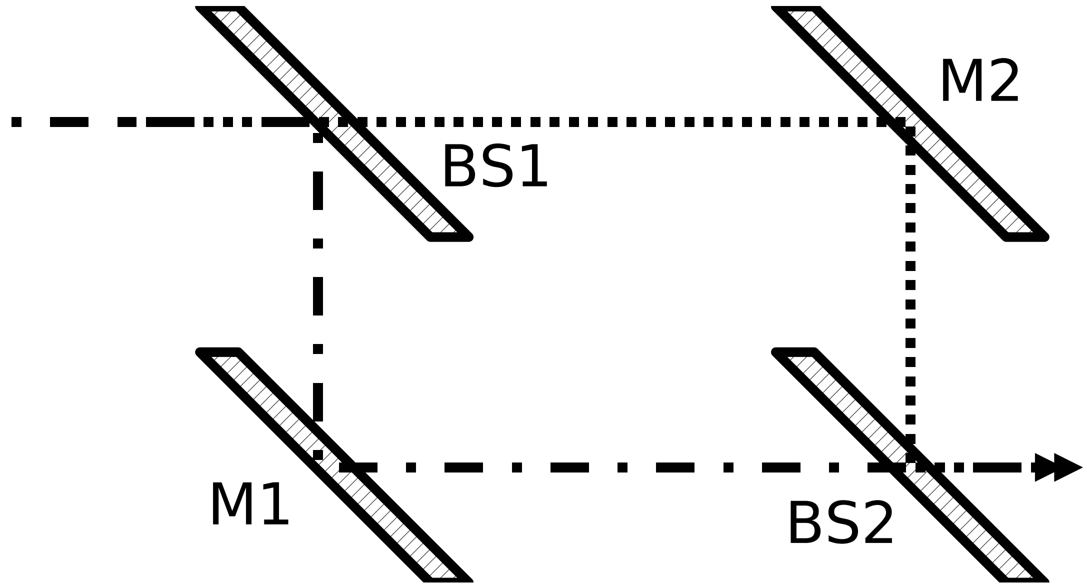
# Week 5 Agenda

- Coherent and Incoherent Light
- Mach–Zehnder Interferometer
- Michaelson Interferometer and Optical Testing
- Doppler Lidar
- Fabry–Perot Interfereometer
- Laser Cavities
- Dielectric Coatings

# Superposition




- Output Toward the Right (Similar for Downward)
- Coherent Addition

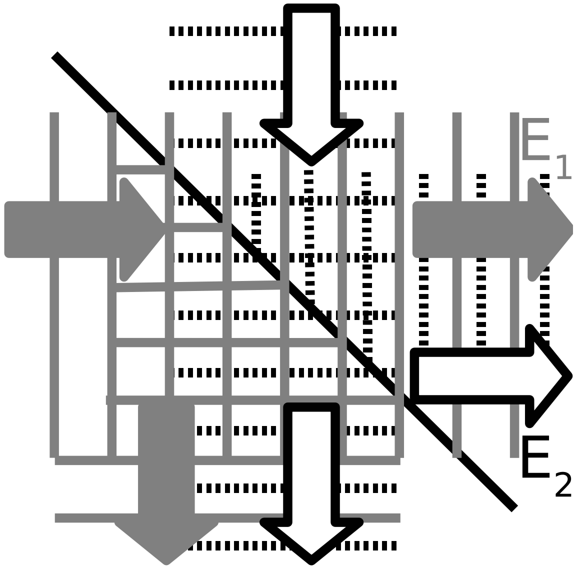
$$E = E_1 + E_2 \qquad I = |E|^2 = EE^*$$

- Incoherent Addition

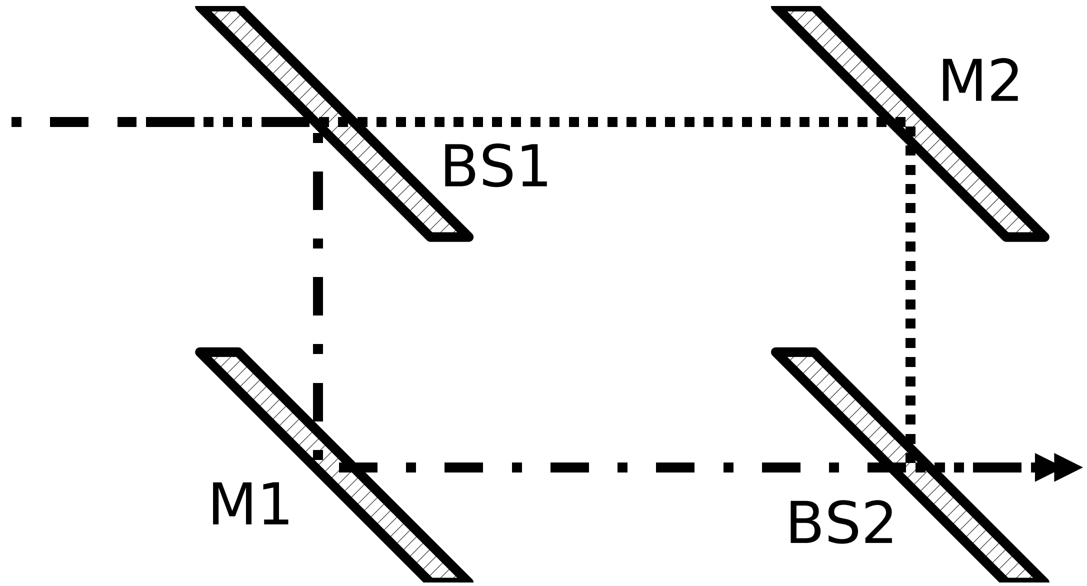
$$I = I_1 + I_2$$

# Coherent Superposition

Combining Waves



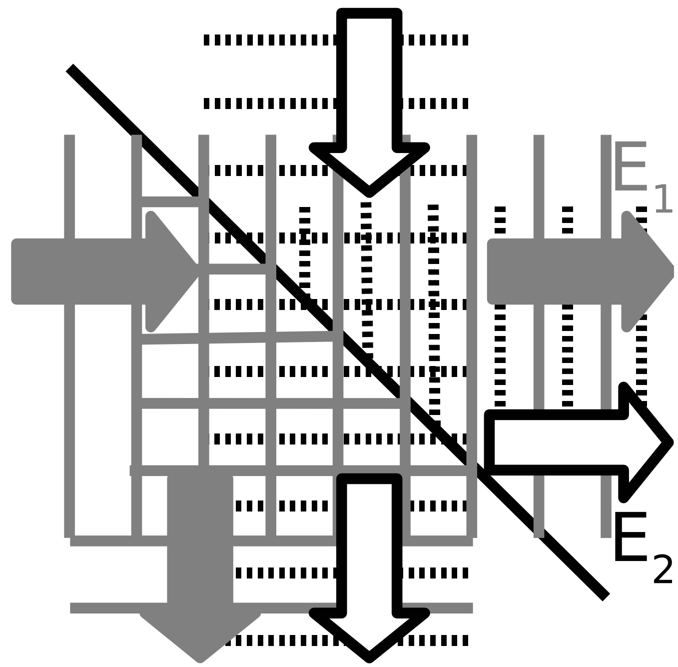

Interferometer




- Inputs Both Have $I = 1$
- Outputs Both Have $0 \le E_1 + E_2 \le 2$
- But $I_{right} + I_{down} = I_{from\ top} + I_{from\ left}$

# Measuring the Field Amplitude Is Hard

- Easy for Ocean Wave Height
- Easy for Acoustic Pressure
- Even Easy for Radio Waves
- No Direct Measurement for Light
  - Terahertz Frequencies
  - Sub–Micrometer Wavelengths
- Use Interferometry
  - Mix With Known Reference Wave
  - Measure Irradiance
  - Variations in Space or Time




$$I = \frac{|E|^2}{Z} = \frac{EE^*}{Z}$$

$$I = \frac{\left(E_1^* + E_2^*\right)(E_1 + E_2)}{Z}$$

# Interferometry Equations

- Irradiance

$$I = \frac{\left(E_1^* + E_2^*\right)(E_1 + E_2)}{Z}$$

- Expand

$$I = \frac{E_1^* E_1 + E_2^* E_2 + E_1^* E_2 + E_1 E_2^*}{Z}$$

  - First Two Terms are "DC" Terms
  - Third and Fourth are "Mixing" Terms
  - Complex Conjugate Pair (Real Sum)

$$I_{mix} = \frac{E_1 E_2^*}{Z} \qquad \text{and} \qquad I_{mix}^* = \frac{E_1^* E_2}{Z}$$

# Mixing Terms

- Complex Conjugates Add to Real Value

$$I_{mix} = \frac{E_1 E_2^*}{Z} \qquad \text{and} \qquad I_{mix}^* = \frac{E_1^* E_2}{Z}$$

- Magnitude

$$|I_{mix}| = |I_{mix}^*| = \sqrt{I_1 I_2}$$

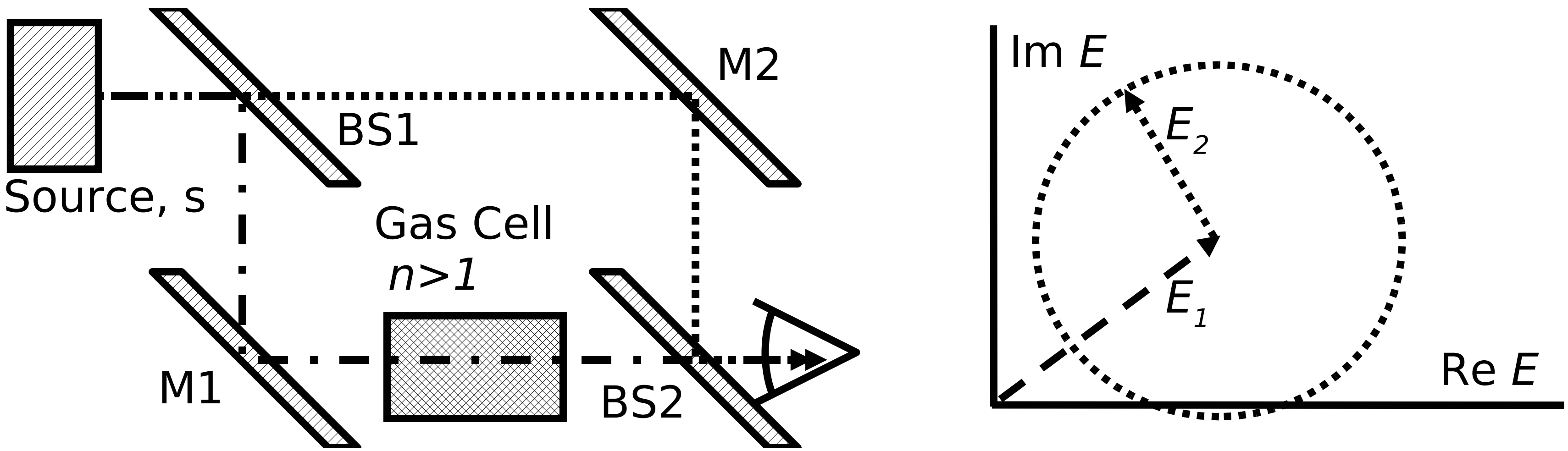
- Total Irradiance

$$I = I_1 + I_2 + 2\sqrt{I_1 I_2} \cos\left(\phi_2 - \phi_1\right)$$

- Random Fields: Incoherent Superposition

$$\bar{I}_{mix} = 0 \qquad \bar{I} = I_1 + I_2$$

# Mach–Zehnder Interferometer




$$I = I_1 + I_2 + 2\sqrt{I_1 I_2}\cos(\phi_2 - \phi_1)$$

- Add Gas Pressure to Cell; $n\uparrow$, $\Delta = \Delta OPL\uparrow$

$$\Delta = \delta(n\ell_c)$$

$$\delta\phi_1 = k\Delta = 2\pi\frac{\Delta}{\lambda} = 2\pi\frac{\ell_c}{\lambda}\delta n$$

# Fringe Amplitude and Contrast

- Total Signal

$$I = I_0 \left( R_1T_2 + T_1R_2 + 2\sqrt{R_1T_2T_1R_2}\cos\delta\phi \right)$$

- Fringe Amplitude

$$I_m = I_{max} - I_{min}$$

$$\sqrt{R_1T_1R_2T_2}I_0 = 0.25I_0 \qquad \text{for} R_1 = R_2 = 0.5$$

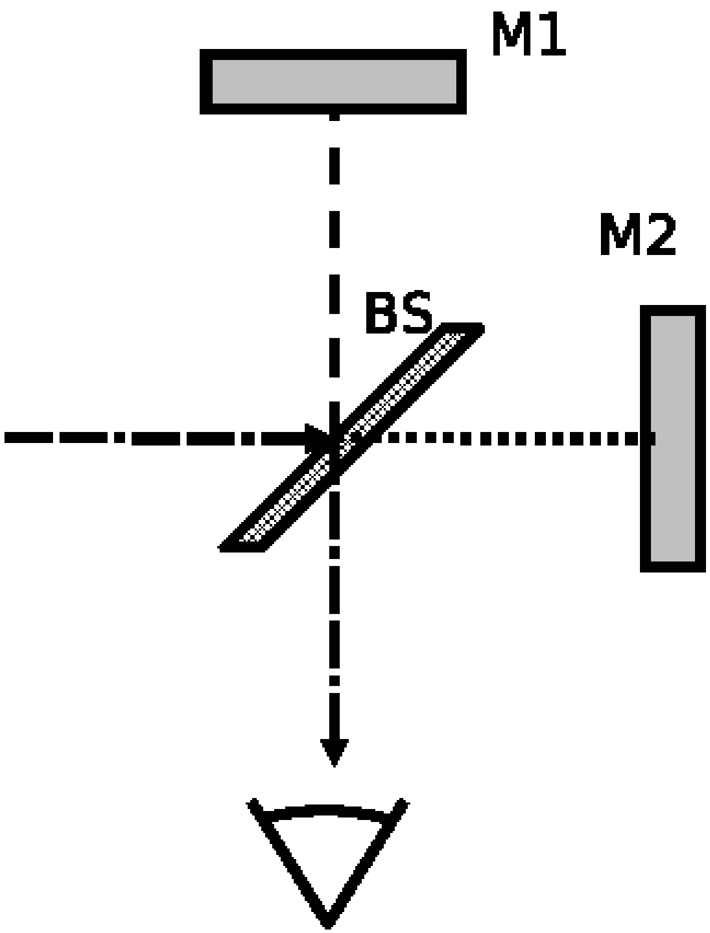
- Fringe Contrast Defined

$$V = \frac{I_{max} - I_{min}}{I_{max} + I_{min}} \qquad (0 \le V \le 1) \qquad V = 2\frac{\sqrt{R_1T_2T_1R_2}}{R_1T_2 + T_1R_2}$$

- For $R_1 = R_2 = R$ and $(T_1 = T_2 = T)$

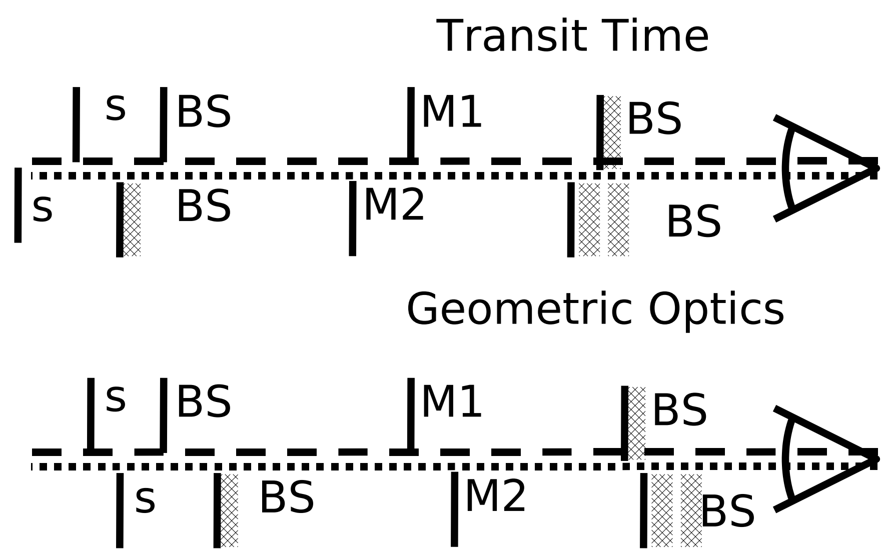
$$I = I_0 2RT\left(1 + \cos\delta\phi\right)$$

# Michelson Interferometer



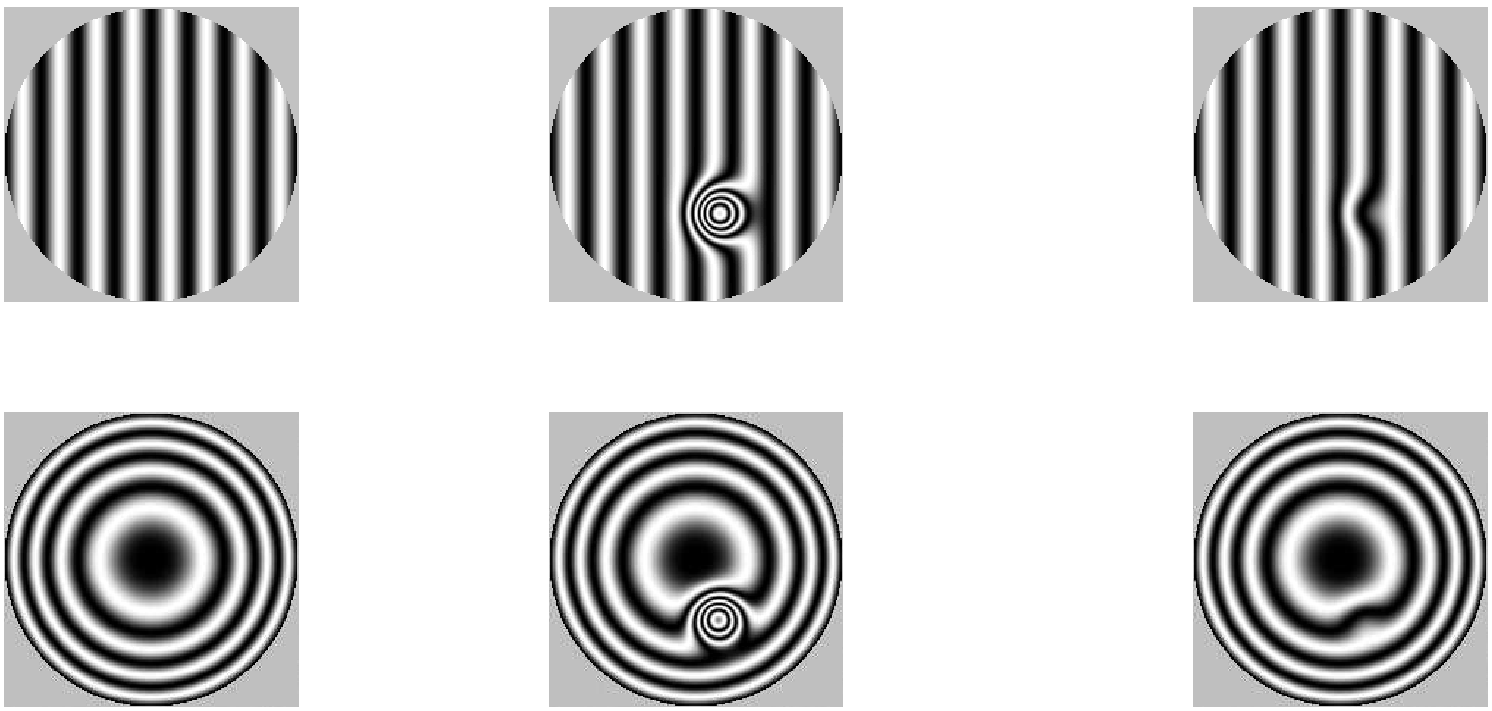

$$I = I_0 \left[R_{BS}R_{M1}T_{BS} + T_{BS}R_{M2}R_{BS} + \sqrt{R_{BS}^2 R_{M1} R_{M2} T_{BS}^2}\cos\left(\delta\phi\right)\right]$$

Straight–Line Layout



# Optical Testing

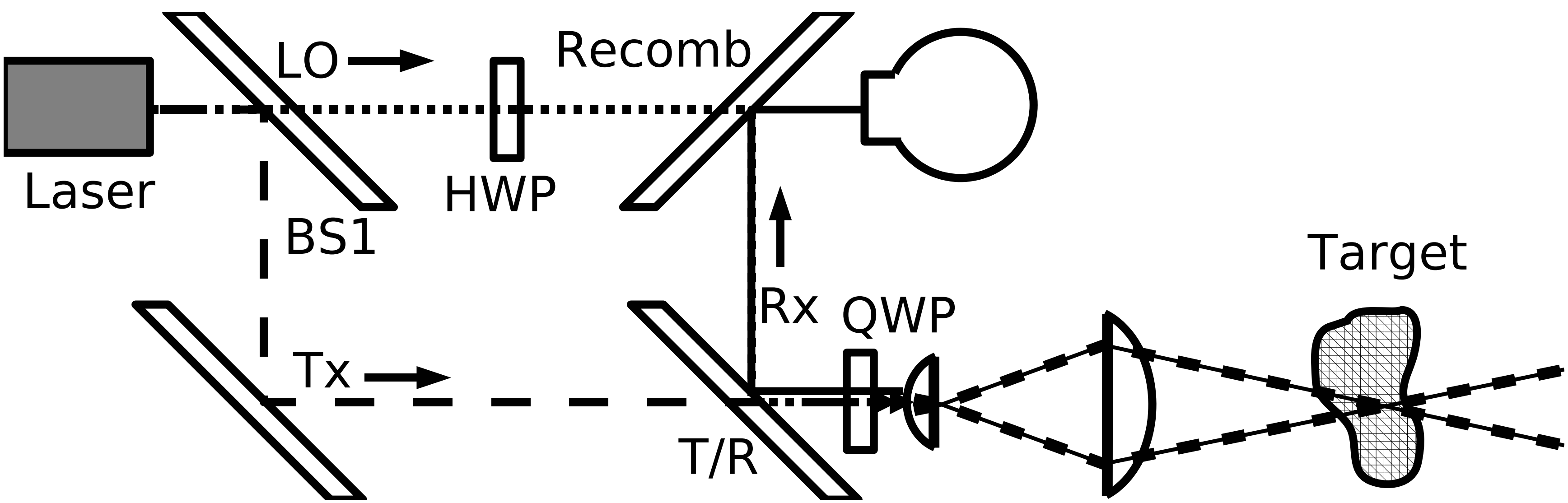
Synthetic Results Illustrating Fringe Patterns



Perfect Surface — 2–Wavelength Bump (4 Fringes) — 0.2–Wavelength Bump (0.4 Fringes)

# Coherent Laser Radar (Lidar)

- Similar to Mach–Zehnder (Modified Mach–Zehnder?)
- Common Transmit/Receive Aperture: Use T/R Switch (Ch. 6)
  - Transmitter Polarization: $P$ at Beamsplitters
  - Receiver Polarization: $S$: Need HWP in Reference (LO)
  - Ideally QWP Between Telescope and Target to Reduce Narcissus (Not Practical)
- BS1 and Recombining Beamsplitter High Reflectivity



# Doppler Velocity

- Target: Dust, Fo, Rain, Snow, Smoke, *etc.*
- Doppler Equation from Source to Target

$$2\pi f_d = \mathbf{k} \cdot \mathbf{v} \qquad (> 0 \text{ for Approaching Velocities})$$

$$f_d = \frac{v_{parallel}}{\lambda} \qquad \text{(Moving Source or Detector)}$$

- Doppler Lidar (or Radar) on Round Trip

$$f_{DR} = 2\frac{v_{parallel}}{\lambda}$$

($f_{DR} = 100$kHz for $v_{parallel} = 0.54$m/s at $\lambda = 10.59\mu$m)

# Pulsed Laser Radar

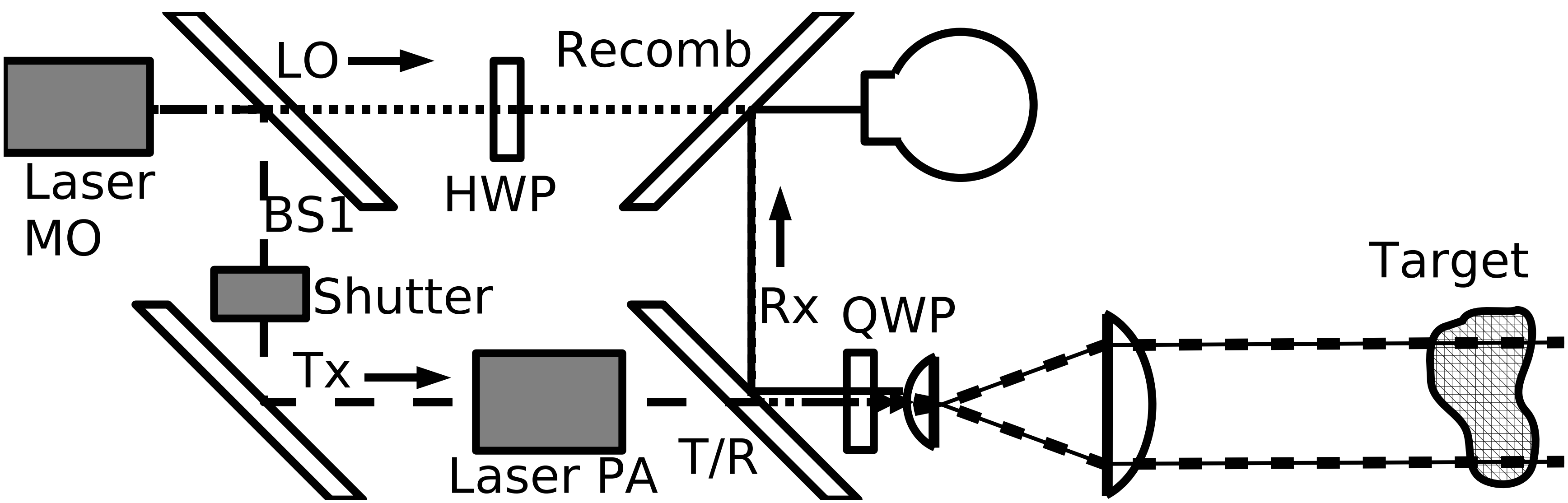
- Range Resolution and Velocity Resolution

$$\delta r = \frac{c\tau}{2} \qquad \delta f_{DR} \approx \frac{1}{\tau} \qquad \delta v_{parallel} \approx \frac{\lambda}{2\tau}$$

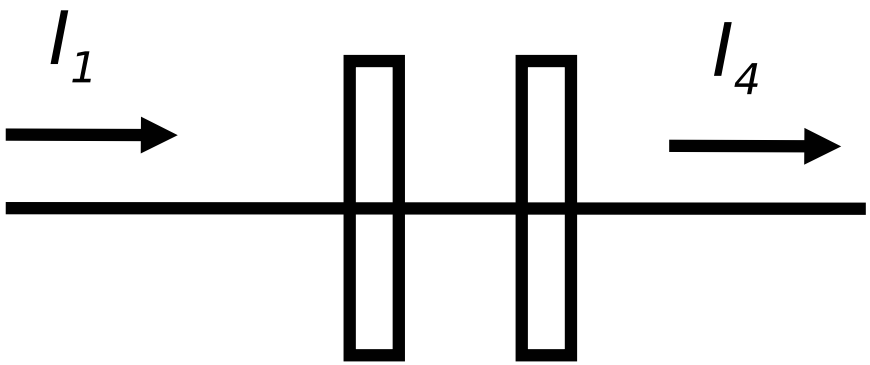
- Hundreds of Meters and $m/s$ Typical at 10.59$\mu$m
- Average Power

$$P_{avg} = P_{laser} \times \tau \times PRF$$

- Range Ambiguity May Limit PRF



# Fabry–Perot




- Resonant Frequencies and Free Spectral Range

$$f = N f_0$$

$$FSR = f_0 = \frac{c}{2\ell}$$

- Recirculating Power

$$P_{recirculating} = \frac{P_{out}}{T_2} =$$

$$\frac{P_{out}}{1 - R_2} = \frac{P_0}{1 - R_2}$$

- $N$ Round Trips

$$P_N = (R_1 R_2)^N$$

- 50% Probability

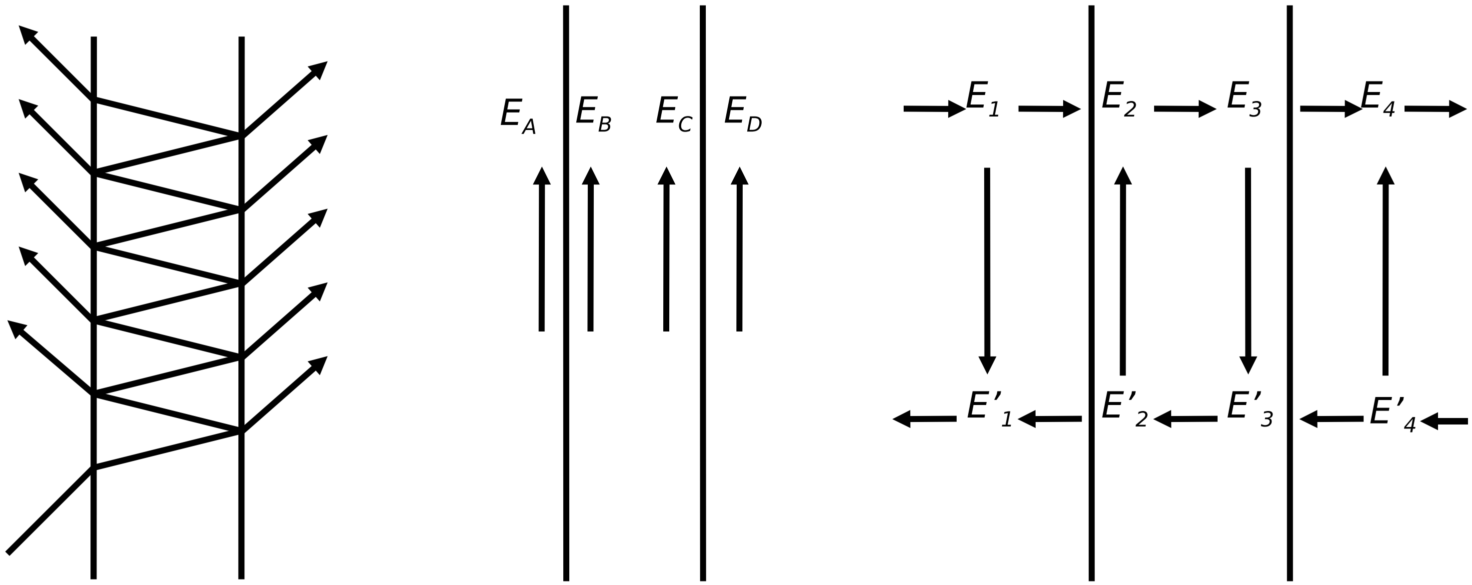
$$N = -\log 2 / \log (R_1 R_2)$$

*e.g.* $R_1 = R_2 = 0.999$: $N = 346$

- Resolution of a Longer Interferometer

# Fabry–Perot Equations

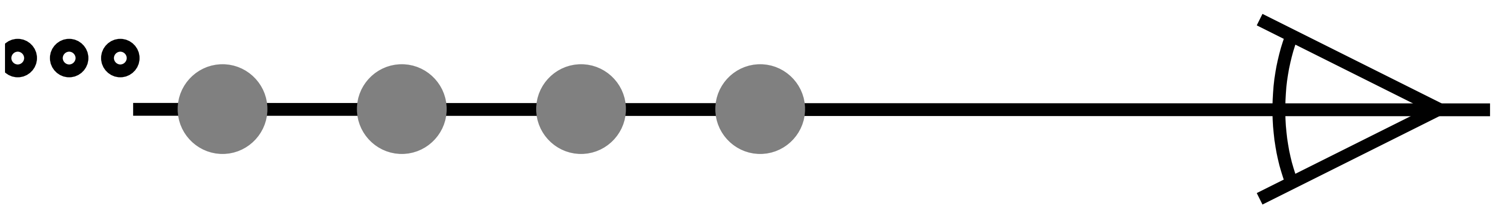



A. Infinite–Sum Solution

B. E/M Fields BC Solution

C. Network Approach with $\rho$ and $\tau$

The Infinite Sum, or “Barber’s Chair” Approach



# Computing the Sum

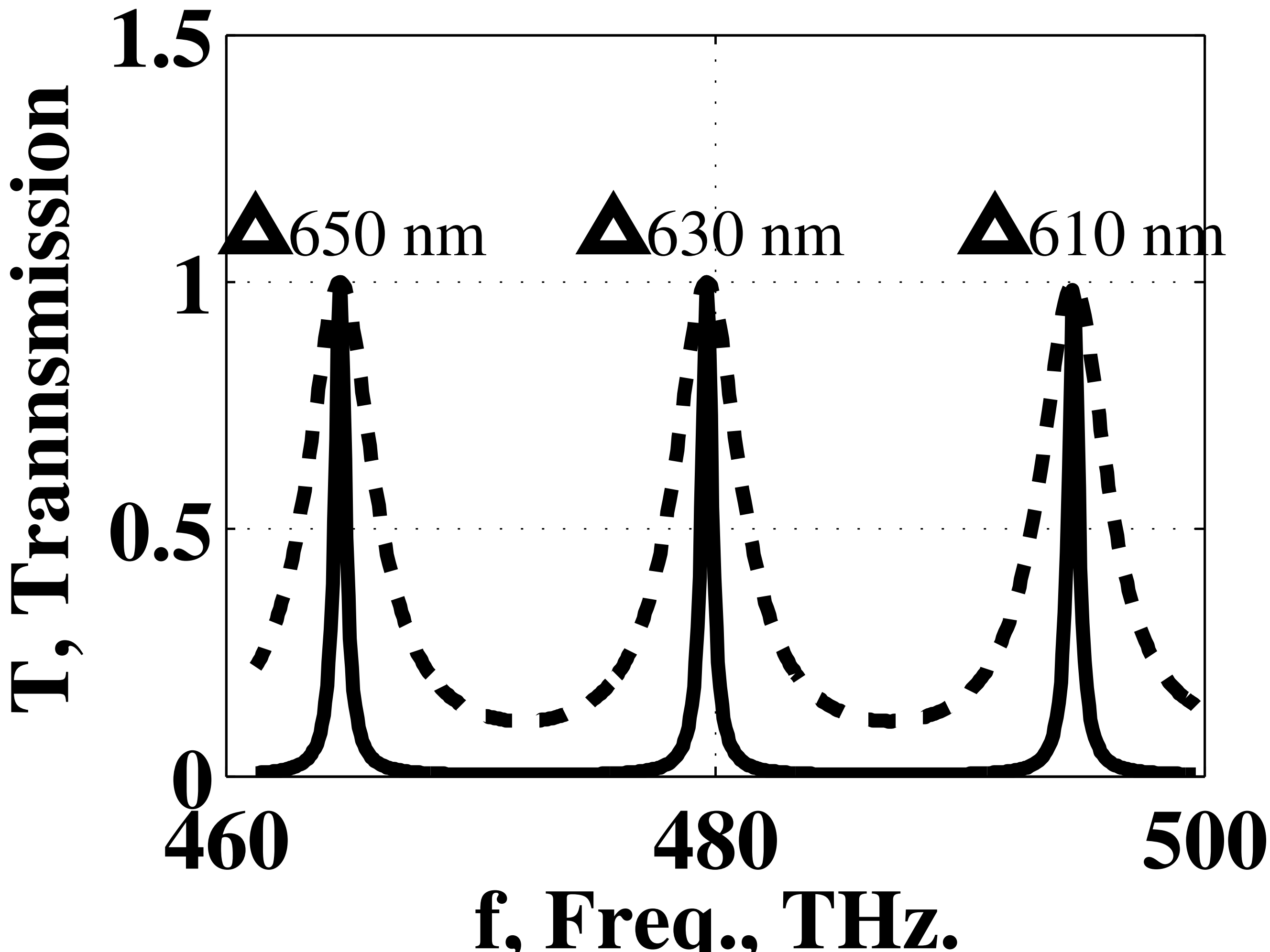
$$E_t = E_0 \tau_1 \tau_2 e^{jk\ell} \frac{1}{1 - \rho_1' \rho_2 e^{2jk\ell}}$$

$$E_r = E_0 \rho_1 + \tau_1 \tau_1' e^{2jk\ell} \frac{1}{1 - \rho_1' \rho_2 e^{2jk\ell}}$$

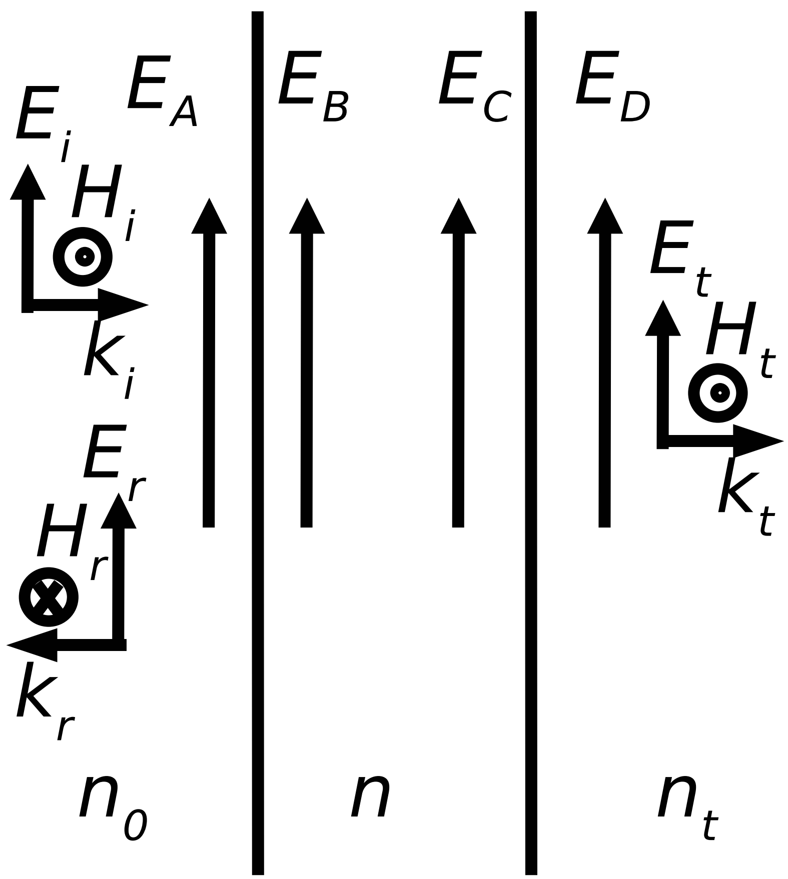
$$T = \left| \frac{E_t}{E_0} \right|^2 \qquad T = \tau_1 \tau_2 \tau_1^* \tau_2^* \frac{1}{1 - \rho_1' \rho_2 e^{2jk\ell}} \frac{1}{1 - (\rho')_1^* \rho_2^* e^{-2jk\ell}}$$

$$T = T_1 T_2 \frac{1}{1 - 2\sqrt{R_1 R_2} \cos(2k\ell) + R_1 R_2}$$

# Fabry–Perot Transmission



# Thin Films: Approach




- Normal Incidence
- Wave from Left

$$E_A = E_i + E_r \qquad E_D = E_t$$

$$H_A = H_i - H_r \qquad H_D = H_t$$

- Electric Field BC

$$E_B = E_A \qquad E_C = E_D$$

- Magnetic Field BC

$$H_B = H_A \qquad H_C = H_D$$

$$\frac{E_B}{nZ_0} = \frac{E_A}{n_0 Z_0} \qquad \frac{E_c}{nZ_0} = \frac{E_D}{n_t Z_0}$$

$$\frac{E_B}{n} = \frac{E_A}{n_0} \qquad \frac{E_C}{n} = \frac{E_D}{n_t}$$

# Thin Films: In the Medium

- Right–Propagating

$$E_{right}e^{inkz} \qquad \text{and} \qquad H_{right}e^{inkz} = \frac{1}{nZ_0}E_{right}e^{inkz}$$

- Left–Propagating

$$E_{left}e^{-inkz} \qquad \text{and} \qquad H_{left}e^{-inkz} = -\frac{1}{nZ_0}E_{left}e^{-inkz}$$

- Boundaries

$$E_B = E_{left} + E_{right} \qquad E_c = E_{left}e^{-jnk\ell} + E_{right}e^{jnk\ell}$$

- Result

$$E_i + E_r = E_t \cos(nk\ell) - E_t\frac{n_t}{n}\sin(nk\ell)$$

$$n_0E_i - n_0E_r = -inE_t \sin(nk\ell) + n_tE_t\cos(nk\ell)$$

# Thin Films: Matrix Equation

- Previous Result

$$E_i + E_r = E_t \cos(nk\ell) - E_t \frac{n_t}{n} \sin(nk\ell)$$

$$n_0 E_i - n_0 E_r = -inE_t \sin(nk\ell) + n_t E_t \cos(nk\ell)$$

- Matrix Equation

$$\begin{pmatrix} 1 \\ n_0 \end{pmatrix} E_i + \begin{pmatrix} 1 \\ -n_0 \end{pmatrix} E_r = \begin{pmatrix} \cos(nk\ell) & -\frac{i}{n}\sin(nk\ell) \\ -in\sin(nk\ell) & \cos(nk\ell) \end{pmatrix} \begin{pmatrix} 1 \\ n_t \end{pmatrix} E_t$$

- Characteristic Matrix

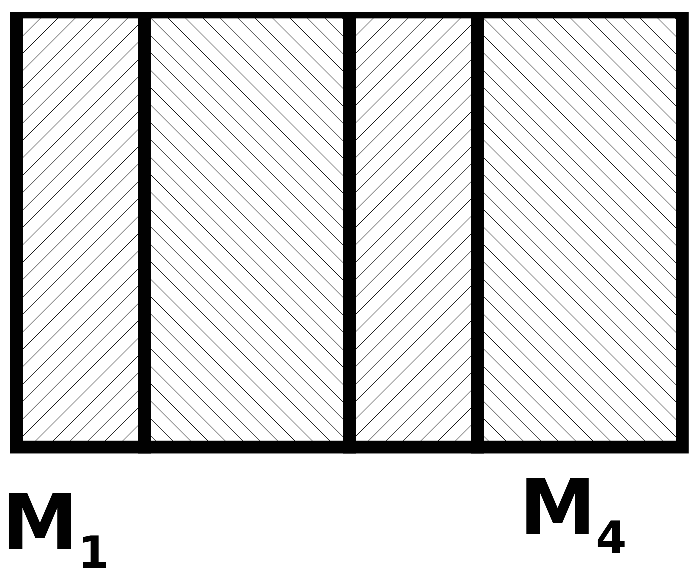
$$\mathcal{M} = \begin{pmatrix} \cos(nk\ell) & -\frac{i}{n}\sin(nk\ell) \\ -in\sin(nk\ell) & \cos(nk\ell) \end{pmatrix}$$

$$\begin{pmatrix} 1 \\ n_0 \end{pmatrix} + \begin{pmatrix} 1 \\ -n_0 \end{pmatrix} \rho = \mathcal{M} \begin{pmatrix} 1 \\ n_t \end{pmatrix} \tau$$

# Multiple Layers

$$\mathcal{M} = \begin{pmatrix} \cos(nk\ell) & -\frac{i}{n}\sin(nk\ell) \\ -in\sin(nk\ell) & \cos(nk\ell) \end{pmatrix}$$

$$\begin{pmatrix} 1 \\ n_0 \end{pmatrix} + \begin{pmatrix} 1 \\ -n_0 \end{pmatrix}\rho = \mathcal{M}\begin{pmatrix} 1 \\ n_t \end{pmatrix}\tau$$




$$\mathcal{M} = \begin{pmatrix} A & B \\ C & D \end{pmatrix}$$

$$\rho = \frac{An_0 + Bn_tn_0 - C - Dn_t}{An_0 + Bn_tn_0 + C + Dn_t}$$

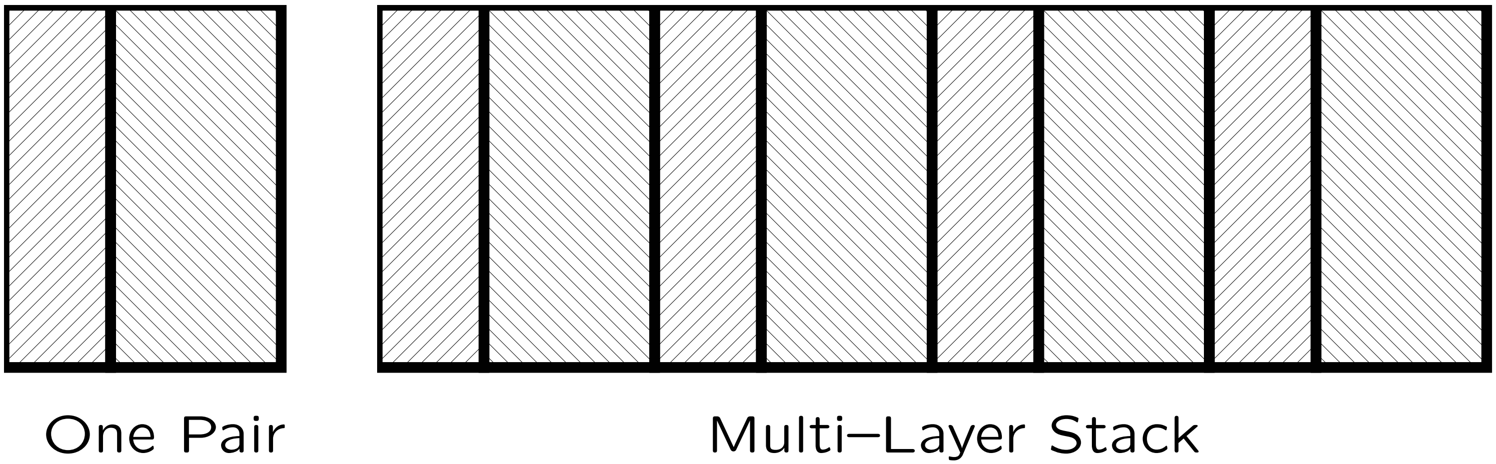
$$\tau = \frac{2n_0}{An_0 + Bn_tn_0 + C + Dn_t}$$

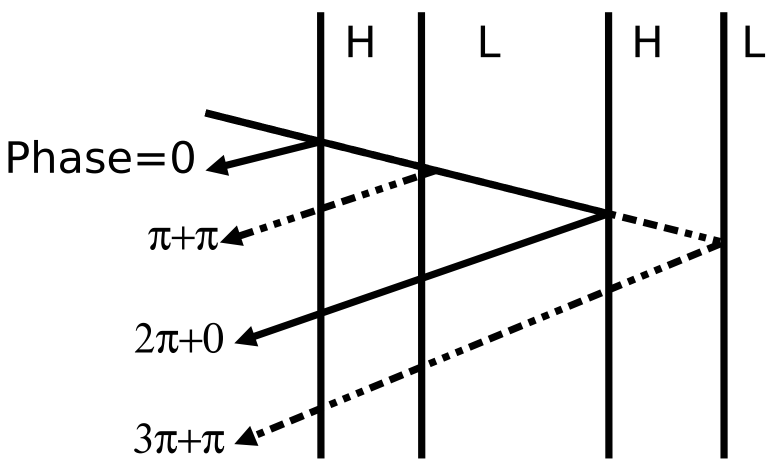
$$\mathcal{M} = \mathcal{M}_1\mathcal{M}_2\mathcal{M}_3\ldots$$

# Dielectric Stacks

- High Reflectivity (Often Better than Metal)
- Anti–Reflection Coatings or Stacks
- Narrow–Band Filters, Mirrors, *etc.*
- Bandpass Devices that Are Not Narrow–Band
- Hot Mirror or Cold Mirror
- Long–Pass Dichroic
- Short–Pass Dichroic
- Beamsplitters (Specific Reflectivity, Angle, Polarization, Wavelength Range, *etc.*)

# High–Reflectance Stack (1)




- One Layer ($\lambda/4$)

$$\mathcal{M}_i = \begin{pmatrix} 0 & -i/n_i \\ in_i & 0 \end{pmatrix}$$

- One Pair

$$\mathcal{M}_p = \begin{pmatrix} 0 & -i/n_h \\ in_h & 0 \end{pmatrix} \begin{pmatrix} 0 & -i/n_\ell \\ in_\ell & 0 \end{pmatrix} =$$

$$\begin{pmatrix} -n_\ell/n_h & 0 \\ 0 & -n_h/n_\ell \end{pmatrix}$$

# High–Reflectance Stack (2)

- One Pair of $\lambda/4$ Layers

$$\mathcal{M}_p = \begin{pmatrix} -n_\ell/n_h & 0 \\ 0 & -n_h/n_\ell \end{pmatrix}$$

- Multiple Pairs

$$\mathcal{M}_N = \begin{pmatrix} (-n_\ell/n_h)^N & 0 \\ 0 & (-n_h/n_\ell)^N \end{pmatrix}$$

- Reflectivity

$$R = \left( \frac{\left(\frac{n_\ell}{n_h}\right)^{2N} - \frac{n_t}{n_0}}{\left(\frac{n_\ell}{n_h}\right)^{2N} + \frac{n_t}{n_0}} \right)^2$$

  – Narrow Band ($\lambda/4$)
  – Almost Indep.of $n_t$, $n_0$

- Example
  – Zinc Sulfide, $n_h = 2.3$
  – Magnesium Fluoride, $n_\ell = 1.35$
  – 8 Layers

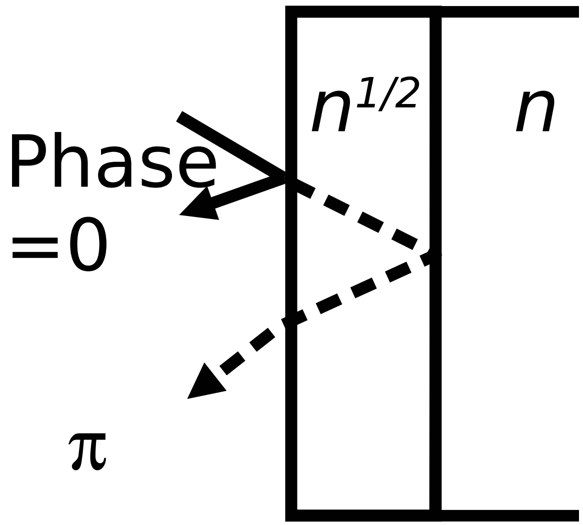
$$N = 4 \rightarrow R = 0.97$$

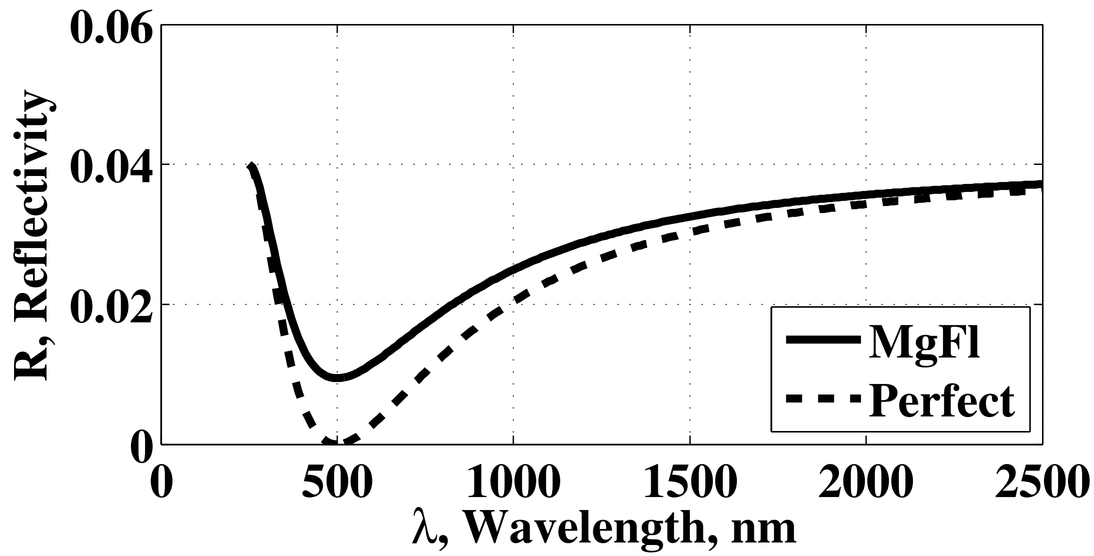
  – 30 Layers

$$N = 15 \rightarrow R = 0.999$$

  – For Lasers, *eg.* HeNe
- Compare Metal ($\approx 0.96$ Typical)
  – Near–Zero Heating

# Anti–Reflection Coating




- Ideal AR Coating
  - Quarter–Wave Coating

$$n_{layer} = \sqrt{n_t}$$

- Other Issues
  - Durability
  - Cost
  - Safety

- Example
  - One Layer
  - Magnsium Fluoride

$$n = 1.35$$

  - $R = 0.01$ at Design $\lambda$
  - $R = 0.04$ at Extremes
  - Looks Pink or Purple

# Multi–Layer AR Stacks