Optics for Engineers
Week 4

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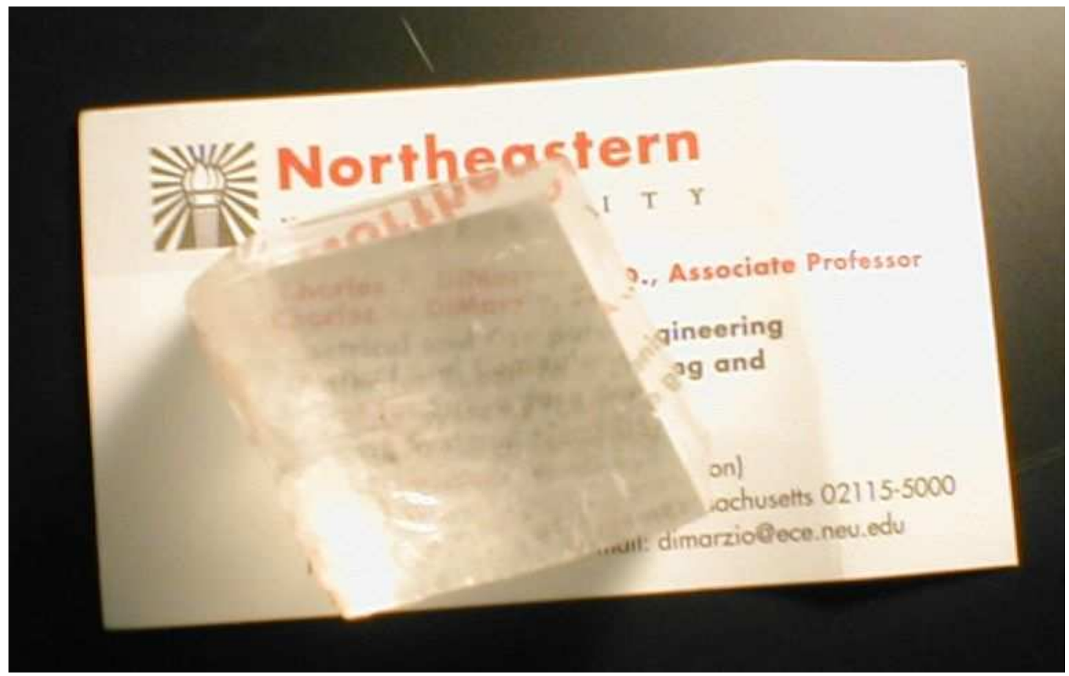
Week 4 Agenda

- Introduction and Some Definitions
- Linear Polarization
- Fresnel Coefficients
- Waveplates
- T/R Beamsplitter
- E/O Modulator
- Rotators

Overview of Polarized Light

- Fundamentals
- Devices
  (What They Do)
- Physics
  (How They Do It)
- Interfaces
- Jones Matrices
  (Bookkeeping)
- Coherency Matrices
  (Partial Polarization)
- Mueller Matrices
  (More Bookkeeping)



Linear Polarization

- Vertical and Horizontal Basis

$$\mathbf{E} = \left[E_v\hat{v} + E_h\hat{h}\right] e^{j(\omega t - kz)}$$

- x, y Basis

$$\mathbf{E} = \left[E_x\hat{x} + E_y\hat{y}\right] e^{j(\omega t - kz)}$$

$$\mathbf{H} = \left[-\frac{E_y}{Z}\hat{x} + \frac{E_x}{Z}\hat{y}\right] e^{j(\omega t - kz)}$$

Polarizing Devices

- Ideal Polarizers
  Pass or Block
- Others Transform

- Linear Polarizer
  - *e.g.* Pass x, Block y
  - Characterization
    * Direction
      (x,y, other)
    * Insertion Loss
      (Pass Direction)
    * Extinction
      (Block Direction)

- The Waveplate
  (Retarder)
  - Change Relative Phase
  - Characterization
    * Axis Direction
    * Phase Difference
    * Insertion Loss
- The Rotator
  (Circular Retarder)
  - Rotate Linear Pol.
  - Phase Change E_r *vs.* E_ℓ
  - Characterization
    * Rotation Angle
      or Phase Shift
    * Insertion Loss

Linear Polarizer

- Input Polarization Example (θ Direction)

$$\mathbf{E}_{in} = E_x\hat{x} + E_y\hat{y} = E_o\left[\cos(\theta)\,\hat{x} + \sin(\theta)\,\hat{y}\right]$$

- Perfect x Polarizer

$$\mathbf{E}_{out} = 1 \times E_x\hat{x} + 0 \times E_y\hat{y} = E_o\cos(\theta)\,\hat{x}$$

- Irradiance

$$|\mathbf{E}_{in}|^2 = E_o^2 \qquad\qquad |\mathbf{E}_{out}|^2 = E_o^2\cos^2\theta$$

- Transmission (Malus Law for This Case)

$$T = \frac{|\mathbf{E}_{out}|^2}{|\mathbf{E}_{in}|^2} \qquad\qquad T = \cos^2\theta$$

Polarizers in "Real Life"

- General Equation

$$\mathbf{E}_{out} = \tau_x \times E_x\hat{x} + \tau_y \times E_y\hat{y} \qquad \tau_x \approx 1 \qquad \tau_y \approx 0$$

- Insertion Loss

$$1 - |\tau_x|^2 \qquad \text{or in dB,} \quad 10\log_{10}|\tau_x|^2$$

- Extinction

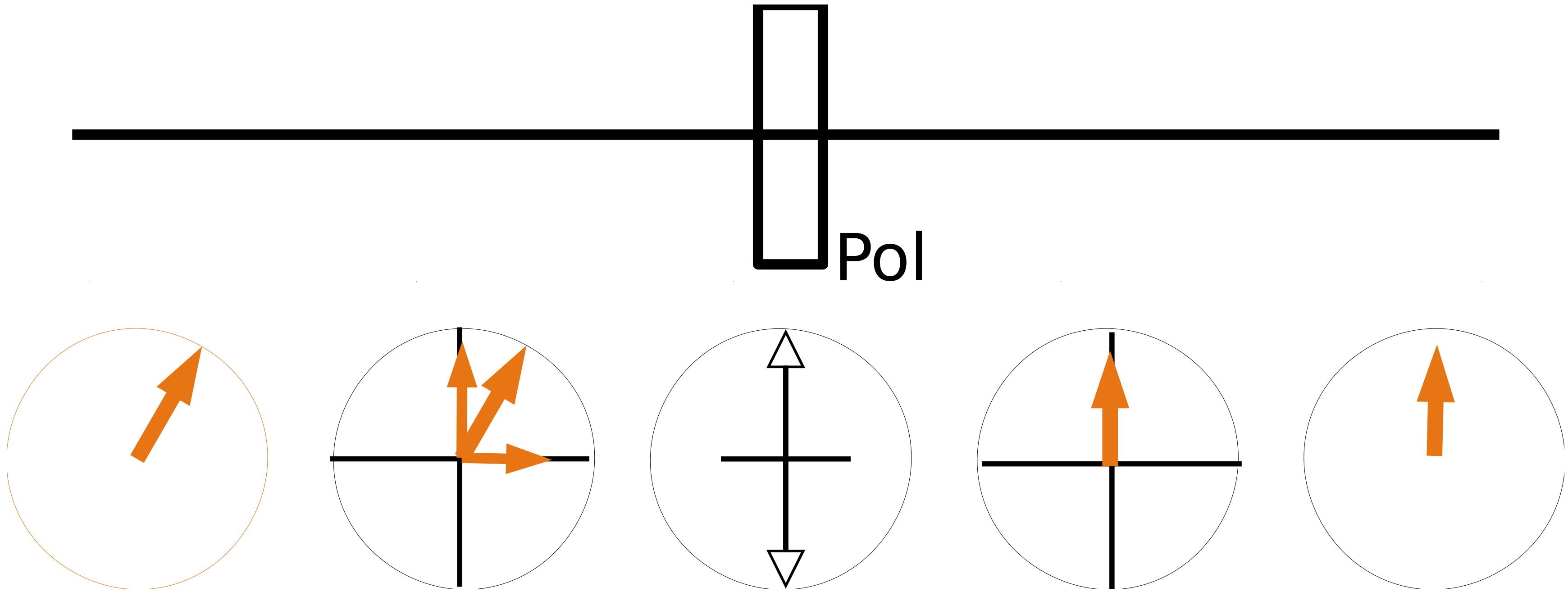
$$|\tau_y|^2 \qquad \text{or in dB,} \quad 10\log_{10}|\tau_y|^2$$

- Extinction Ratio

$$|\tau_x|^2 / |\tau_y|^2$$

  – Good Extinction $\approx 10^5$ or 45dB

Linear Polarizer Analysis



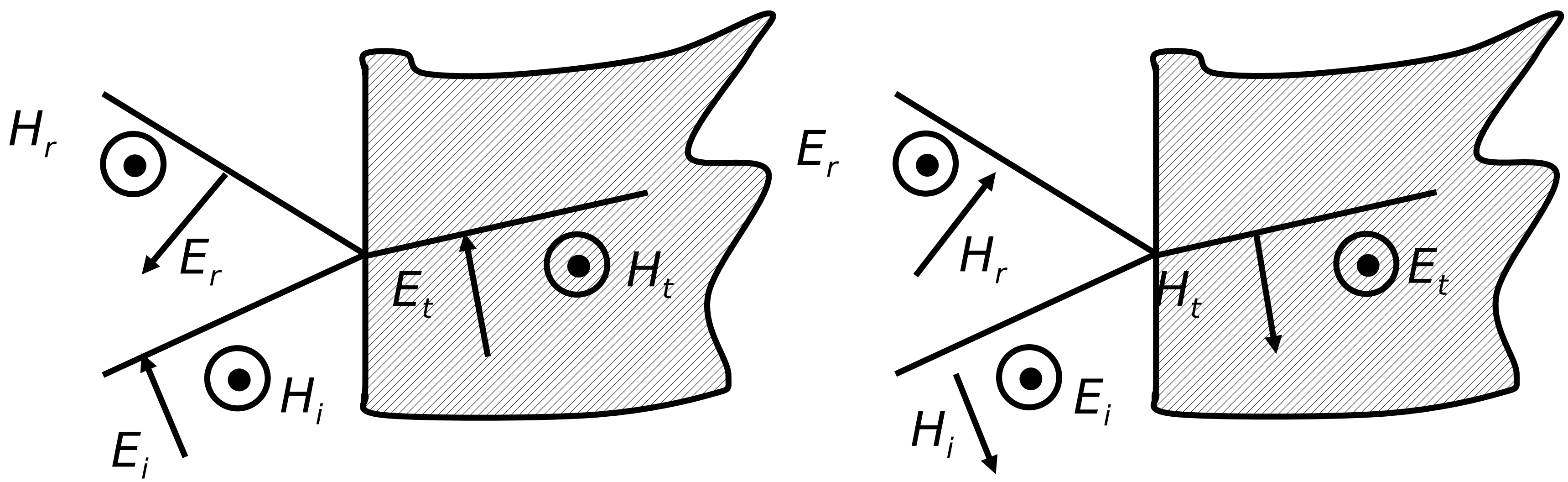
$E_V \times 1$
$E_H \times 0$

Derive the Cosine–Squared Law

S,P Basis at an Interface

- P Means $\mathbf{E}$ Parallel to Plane of Incidence
- S Means $\mathbf{E}$ Perpendicular (Senkrecht) to Plane of Incidence

$$\mathbf{E} = \left[E_s\hat{s} + E_p\hat{p}\right] e^{j(\omega t - kz)}$$



P Polarization (TM) S Polarization (TE)

Fresnel Coefficents

- S Polarization

$$\rho_s = \frac{E_r}{E_i} = \frac{\cos\theta_i - \sqrt{\left(\frac{n_2}{n_1}\right)^2 - \sin^2\theta_i}}{\cos\theta_i + \sqrt{\left(\frac{n_2}{n_1}\right)^2 - \sin^2\theta_i}} \qquad \tau_s = 1 + \rho_s$$

- P Polarization ($|\rho_P| \leq |\rho_S|$)

$$\rho_p = \frac{\sqrt{\left(\frac{n_2}{n_1}\right)^2 - \sin^2\theta_i} - \left(\frac{n_2}{n_1}\right)^2 \cos\theta_i}{\sqrt{\left(\frac{n_2}{n_1}\right)^2 - \sin^2\theta_i} - \left(\frac{n_2}{n_1}\right)^2 \cos\theta_i} \qquad \tau_p = (1 + \rho_p)\frac{n_1}{n_2}$$

Air To Glass

$$|\rho_p| = |\rho_s|$$

at $\theta = 0^\circ$

Q: Why Abs?

$$|\rho_p| = |\rho_s| = 1$$

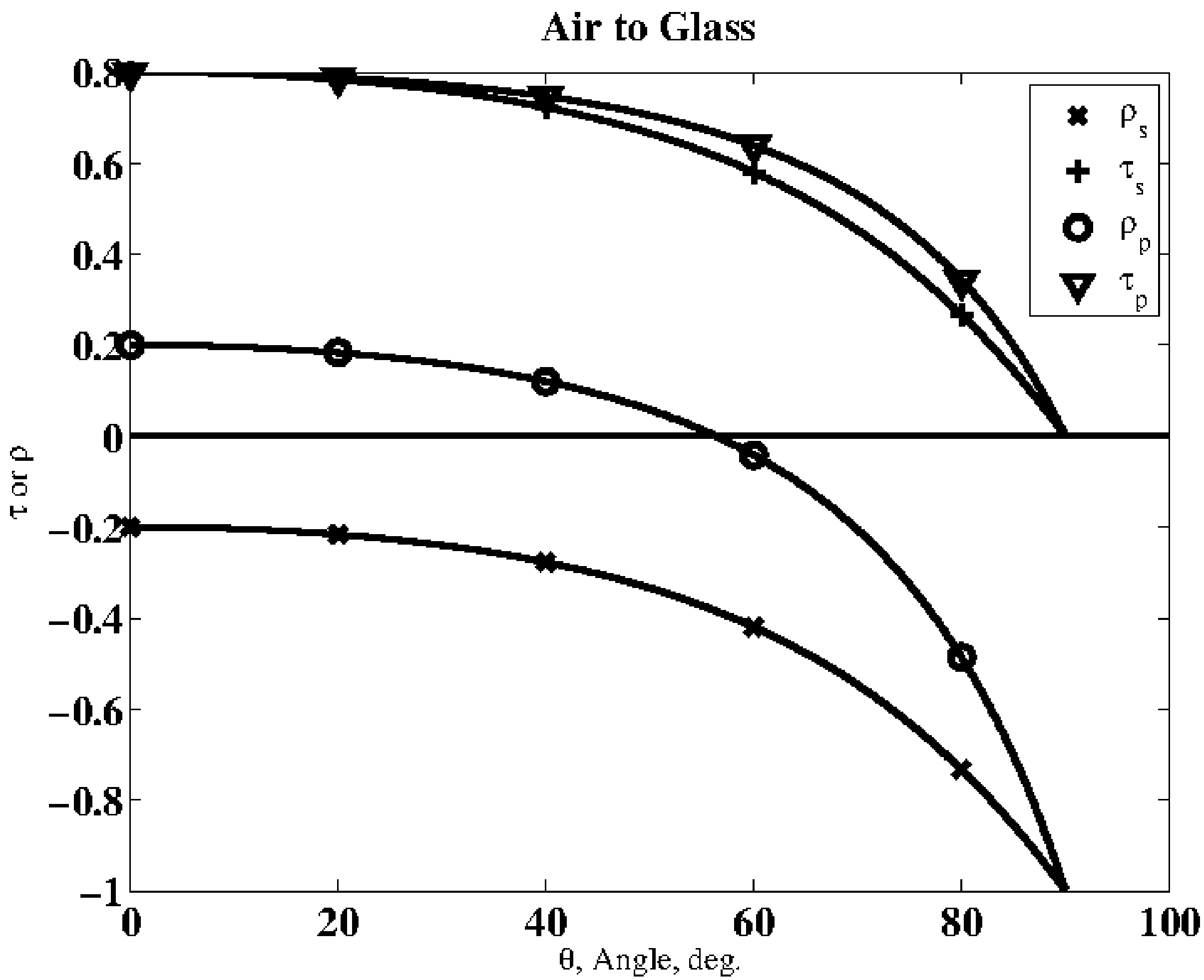
at $\theta = 90^\circ$

and

$$\rho_p = 0$$

at $\theta \approx 56^\circ$

(Brewster's Angle)

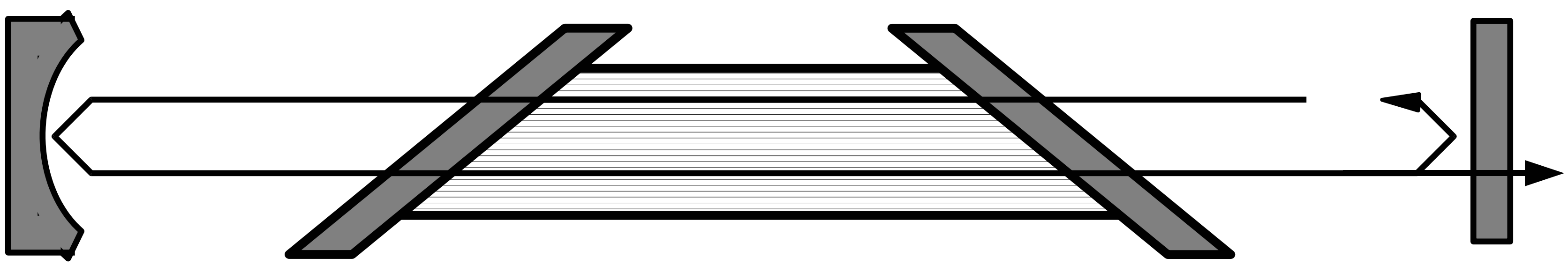


Brewster's Angle

- $\rho_p = 0$ Means No Reflection
- 100% Transmission (Different from $\tau_p = 1$) Q: Why?

$$\tan\theta_B = \frac{n_2}{n_1}$$

- Application: Windows in Laser (Polarized Laser)



- Q: What is the Direction of Polarization?

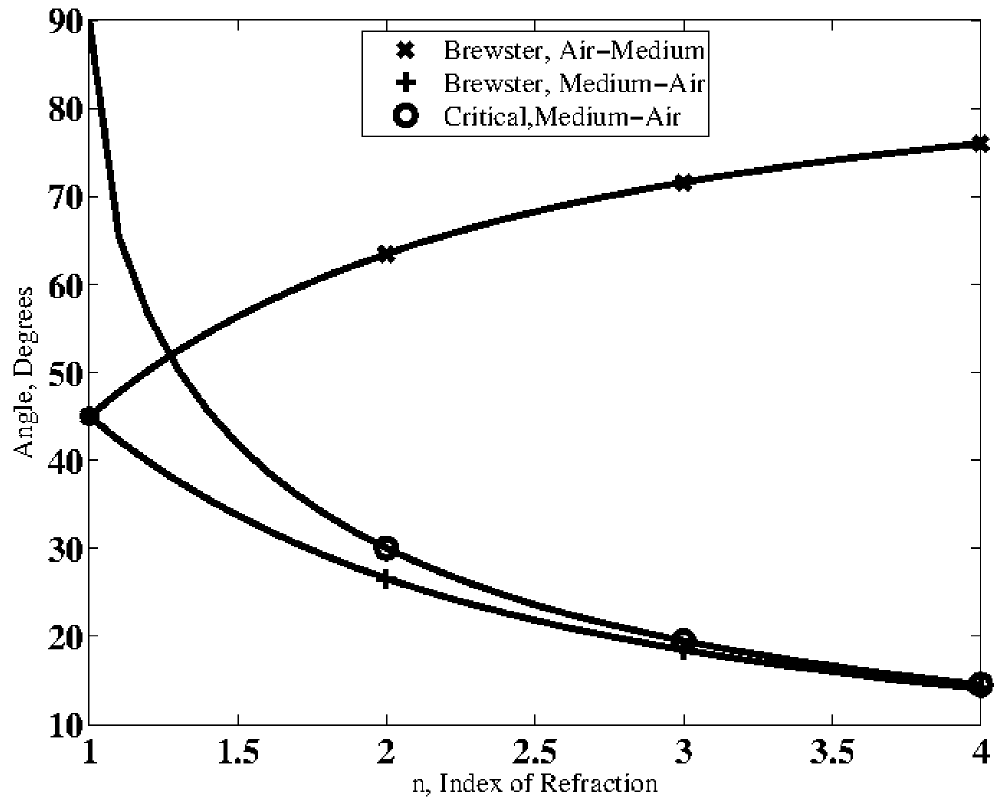
Critical Angle

- Critical Angle ($n_1 > n_2$)

$$\sin\theta_C = \frac{n_2}{n_1}$$

- Brewster's Angle

$$\tan\theta_B = \frac{n_2}{n_1}$$



Irradiance and Power

- Irradiance

$$I = \frac{|\mathbf{E}|^2}{Z}, \qquad I = \frac{dP}{dA'} = \frac{dP}{\cos\theta dA}$$

- Reflection

$$\frac{I_r}{I_i} = R = \rho\rho^*$$

- Transmission

$$\frac{I_t}{I_i} = T = \tau\tau^* \frac{Z_1}{Z_2}\frac{\cos\theta_t}{\cos\theta_i} = \tau\tau^* \frac{n_2}{n_1}\frac{\sqrt{\left(\frac{n_2}{n_1}\right)^2 - \sin^2\theta_i}}{\cos\theta_i}$$

- Conservation

$$T + R = 1$$

Fresnel Reflection at Normal Incidence

- Reflection

$$R(0) = \left| \frac{(n_2/n_1) - 1}{(n_2/n_1) + 1} \right|^2$$

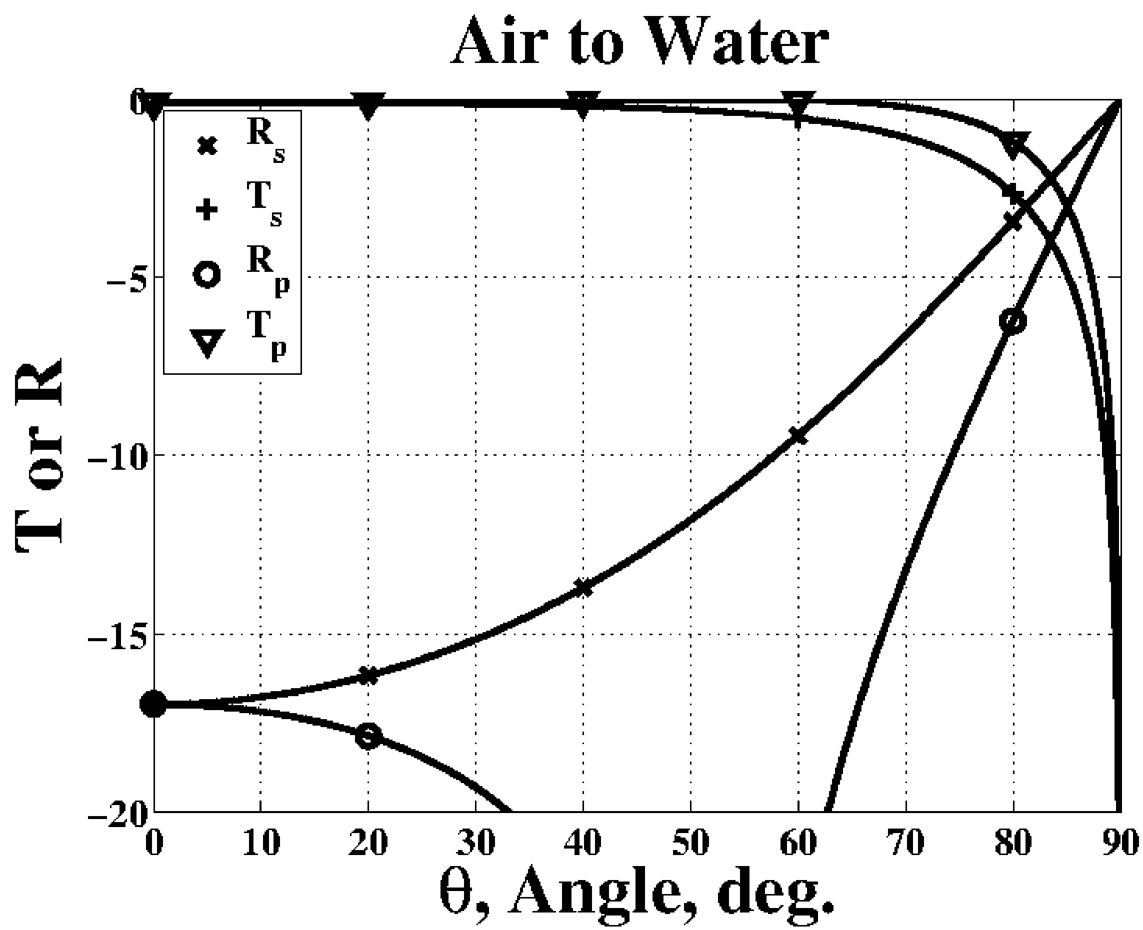
- Special Case (Air to Medium)

$$R(0) = \left| \frac{n - 1}{n + 1} \right|^2$$

- Examples

Air–Water:	$n = 1.33$	$R(0) = 0.02$
Air–Glass:	$n = 1.5$	$R(0) = 0.04$
Air–Germanium (IR):	$n = 4$	$R(0) = 0.36$

Air to Water (dB)



Air–Water:

$$R(0) = 0.04$$

Generally:

$$R_s(0) = R_p(0)$$

$$R\left(90^{\circ}\right) = 1$$

Elsewhere

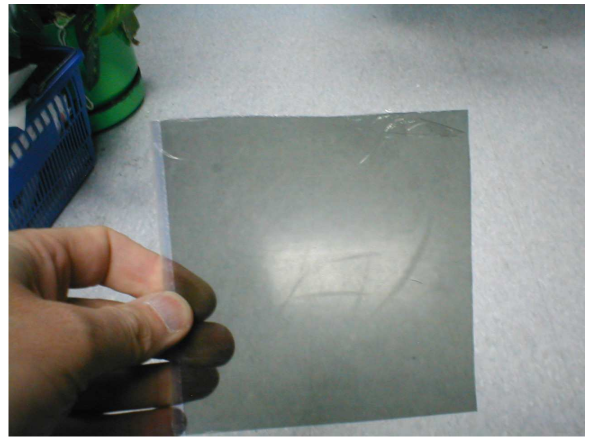
$$R_s(0) > R_p(0)$$

$$R_p\left(\theta_b\right) = 0$$

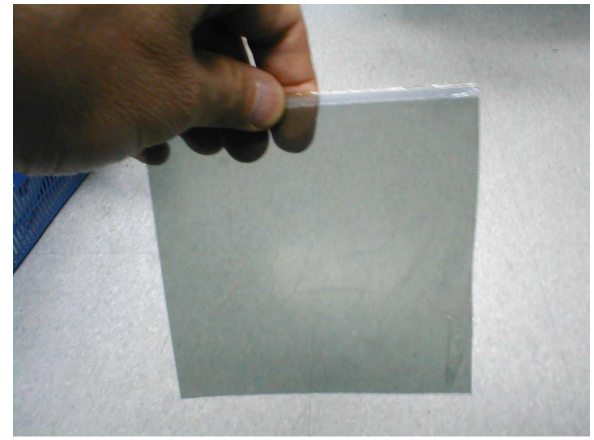
$R\left(\theta\right)$ for n to $n' = R\left(\theta'\right)$ for n' to n

Polished–Floor Reflection

No Polarizer



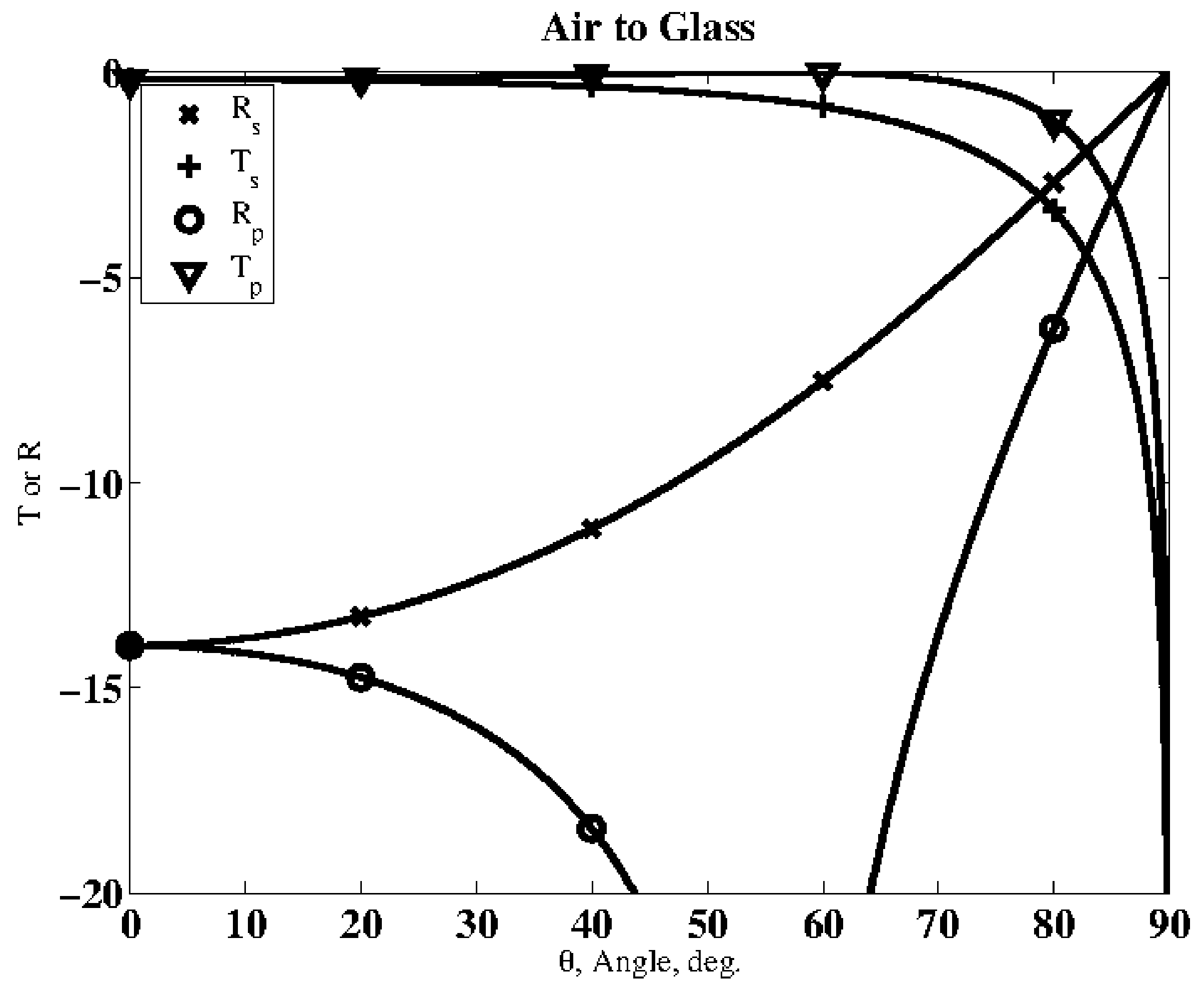
Horizontal Polarizer



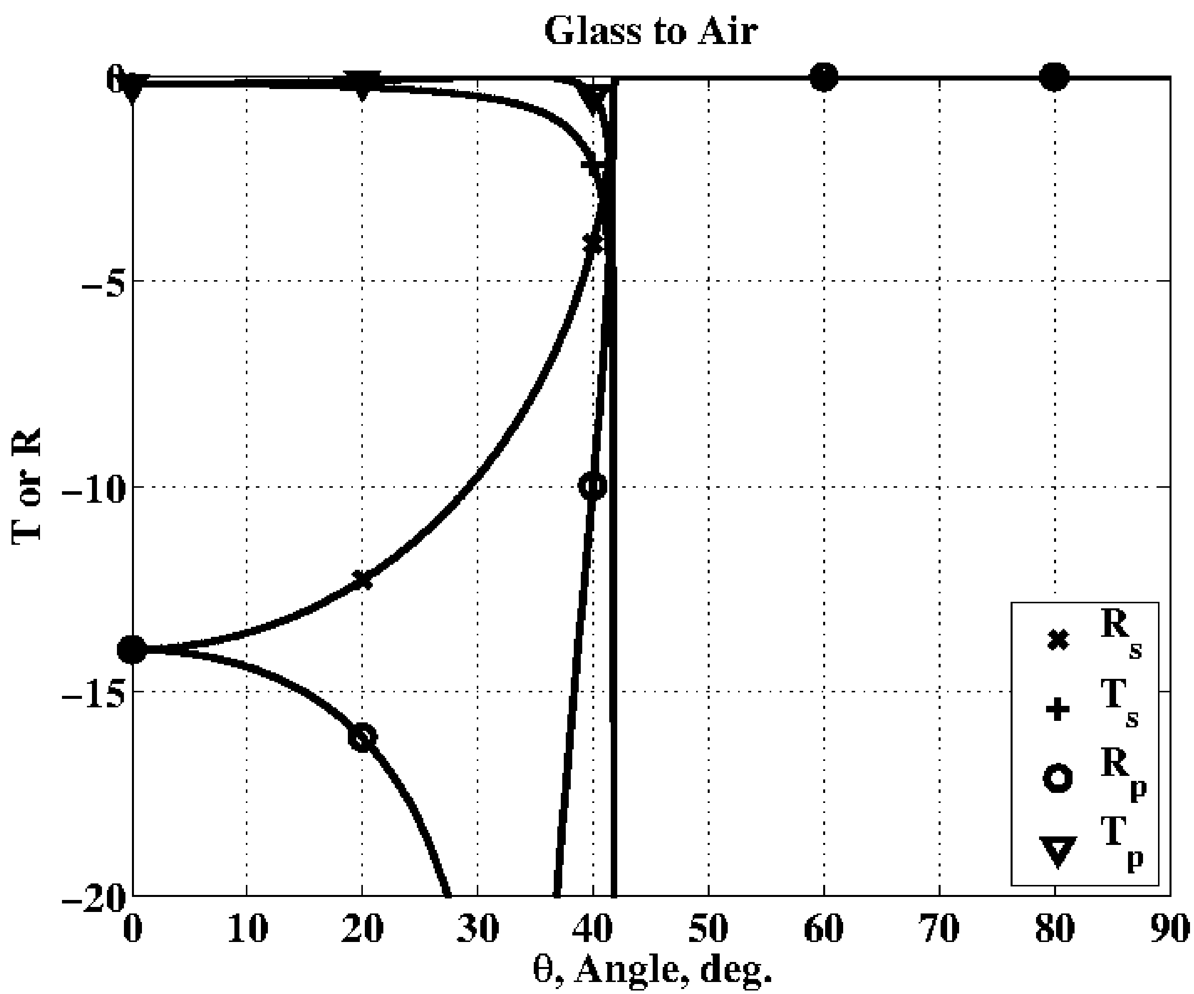
Vertical Polarizer

Q: Which is Which?

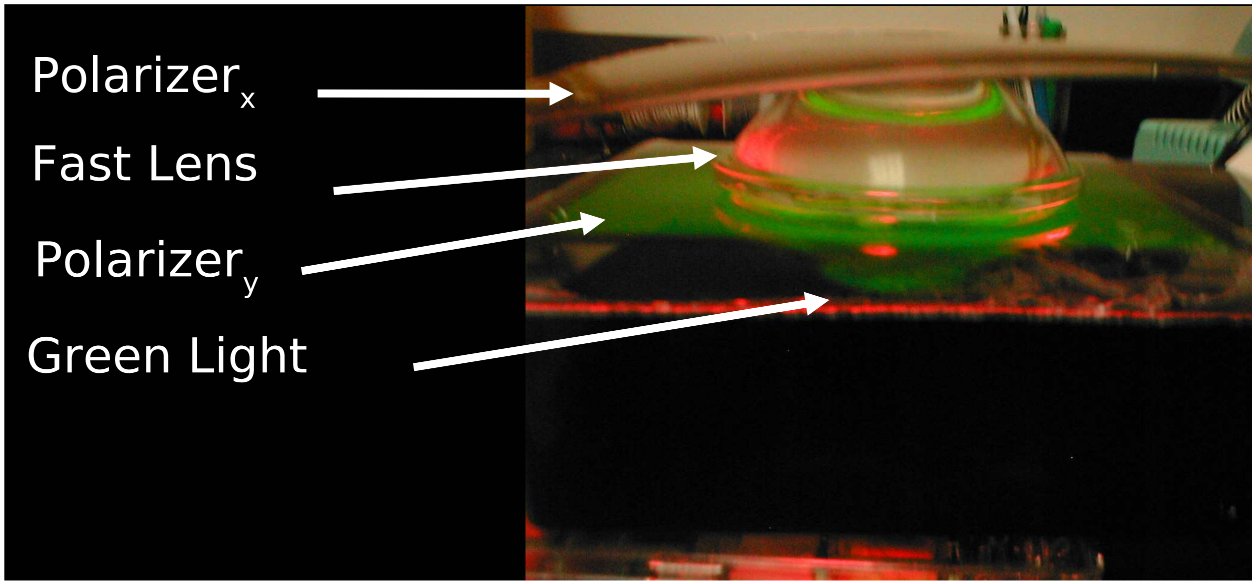
Air to Glass (dB)

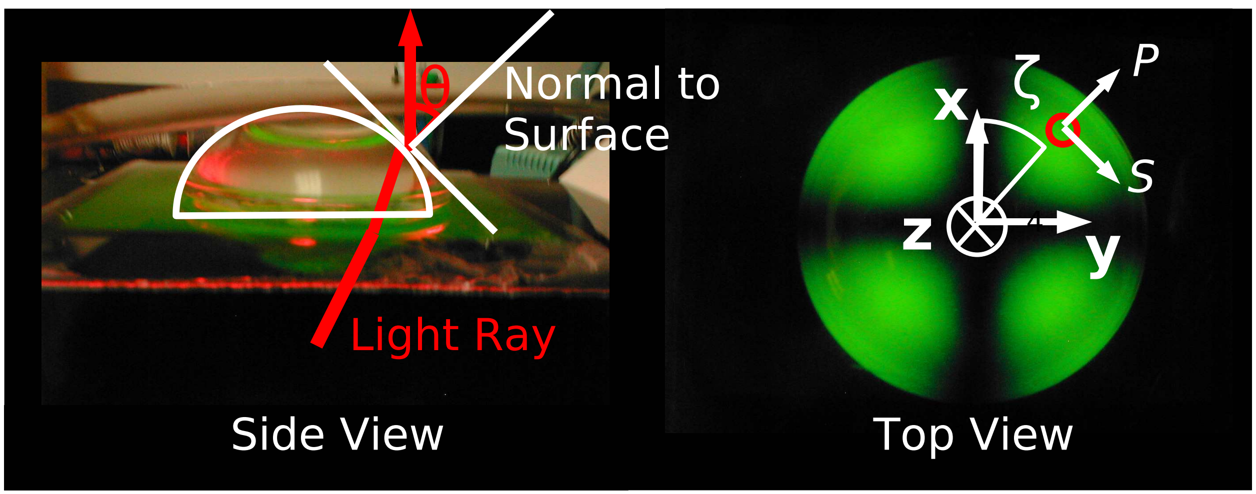


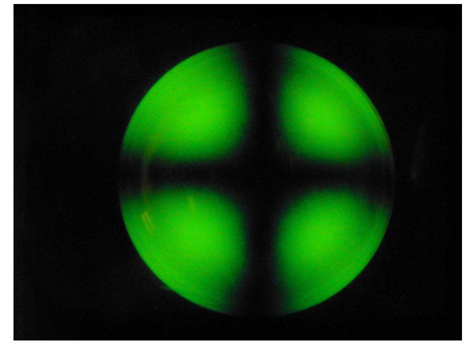
Glass to Air (dB)

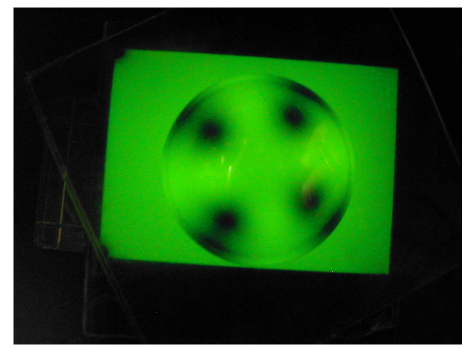


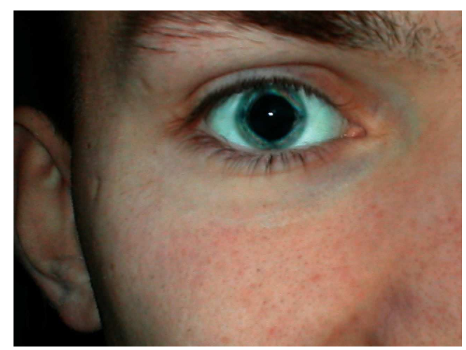
Maltese Cross









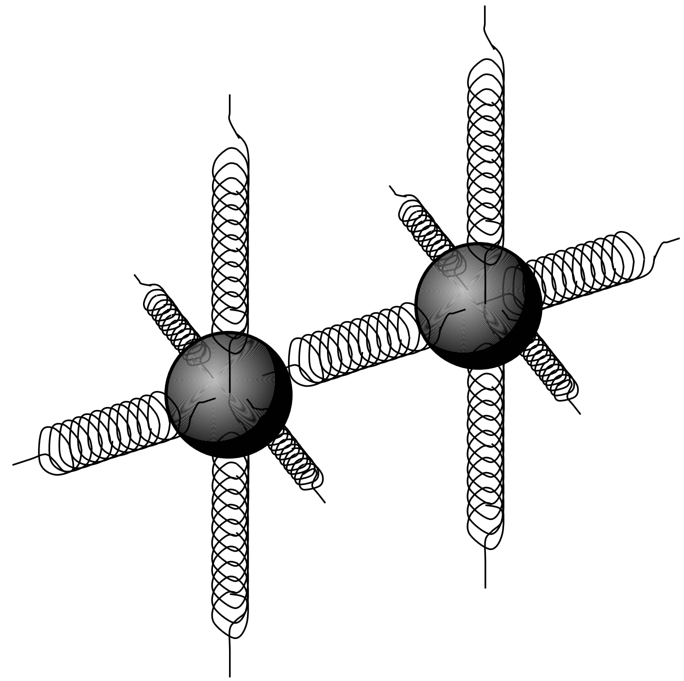


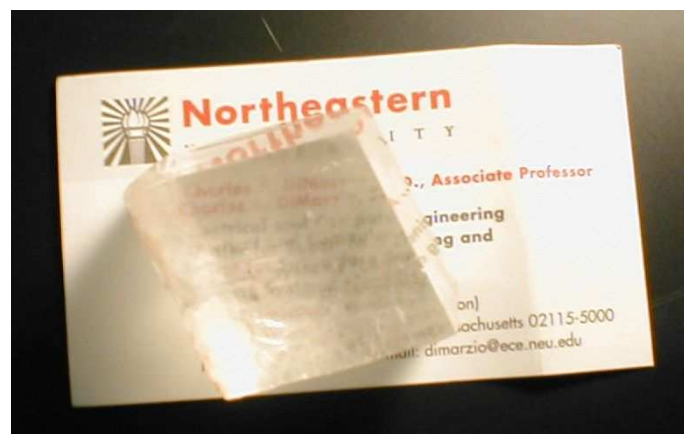
Birefringence

- Two Indices of Refraction
  - Different Ray Bending (Double Image)
  - Different Speeds
- Epsilon Tensor
  - 3–D Matrix
  - Can be Diagonalized
  - Two or Three Eigenvalues
    * Uniaxial

$$\mathcal{E} = \begin{pmatrix} \epsilon_{xx} & 0 & 0 \\ 0 & \epsilon_{yy} & 0 \\ 0 & 0 & \epsilon_{yy} \end{pmatrix}$$

      · Ordinary Ray (y Polarized)
      · Extraordinary Ray (x)
    * Biaxial (All 3 Different)





The Wave Plate

- Input Polarization Example (θ Direction Again)

$$\mathbf{E}_{in} = E_x\hat{x} + E_y\hat{y} = E_o\left[\cos\left(\theta\right)\hat{x} + \sin\left(\theta\right)\hat{y}\right]$$

- Half–Wave Plate

$$\tau_x = 1 \qquad \tau_y = -1$$

$$\mathbf{E}_{hwp} = E_o\left[\cos\left(\theta\right)\hat{x} - \sin\left(\theta\right)\hat{y}\right] \qquad \angle\mathbf{E}_{out} = -\theta$$

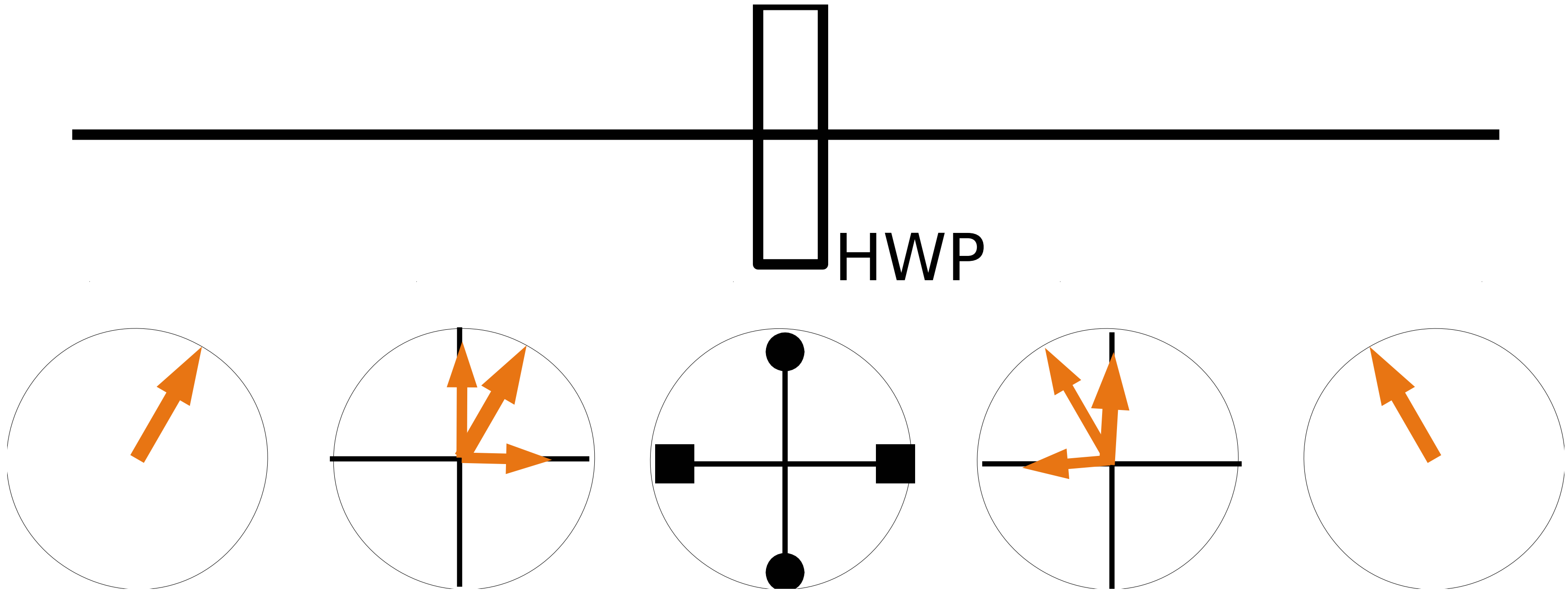
- Quarter–Wave Plate

$$\tau_x = 1 \qquad \tau_y = j,$$

$$\mathbf{E}_{qwp} = E_o\left[\cos\left(\theta\right)\hat{x} + j\sin\left(\theta\right)\hat{y}\right]$$

- Circular Polarization at $\theta = 45^{\circ}$ (Q: Left or Right?)
- Other Waveplates Sometimes Used

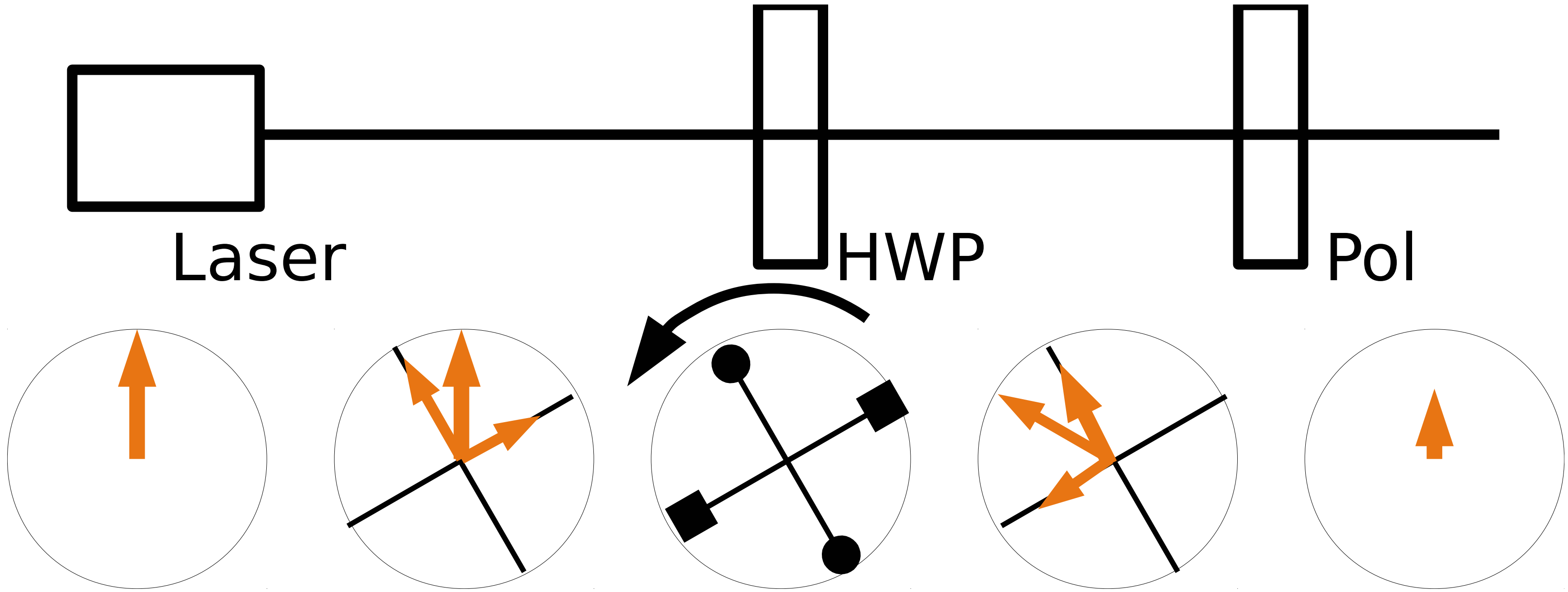
Waveplate Analysis



$$E_V \times 1$$
$$E_H \times (-1)$$

HWP Flips Polarization

Variable Attenuator



4 Peaks, 4 Nulls

Not Linear

T/R Switch (Optical Circulator)

- Common Aperture
  - $T + R = 1$
  - Round–Trip

$$(1 - R)\, F_{target} R$$

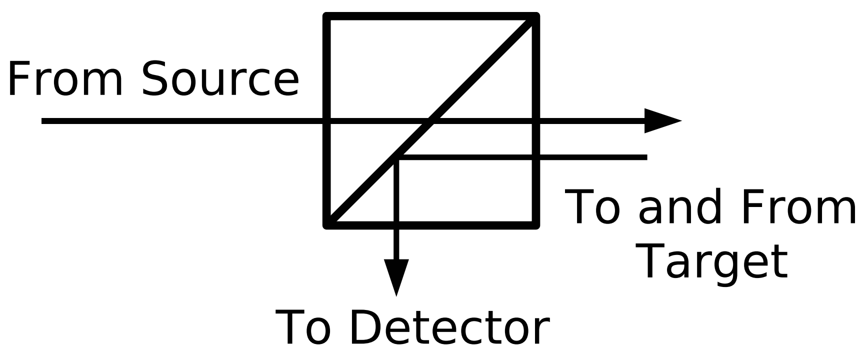
  - Optimize (Not Great)

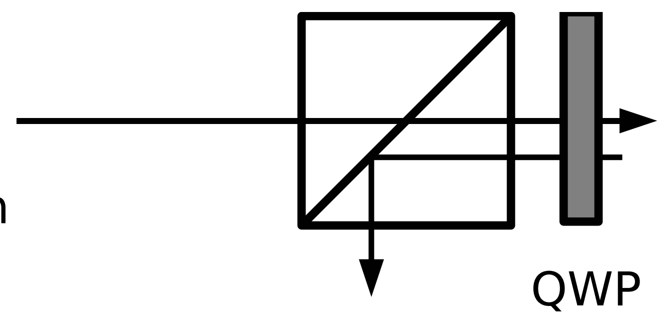
$$d\left[(1 - R)\, R\right]/dR = 0$$

$$R = \frac{1}{2} \qquad R(1 - R) = \frac{1}{4}$$

Polarization Analysis

- $\hat{p}$–Polarized Source: High Transmission
- QWP Makes Circular Pol.
- Target Keeps Polarization (RHC to LHC)
- QWP Makes $\hat{s}$ Polarization: High Reflection
- $T_P + R_S \neq 1$





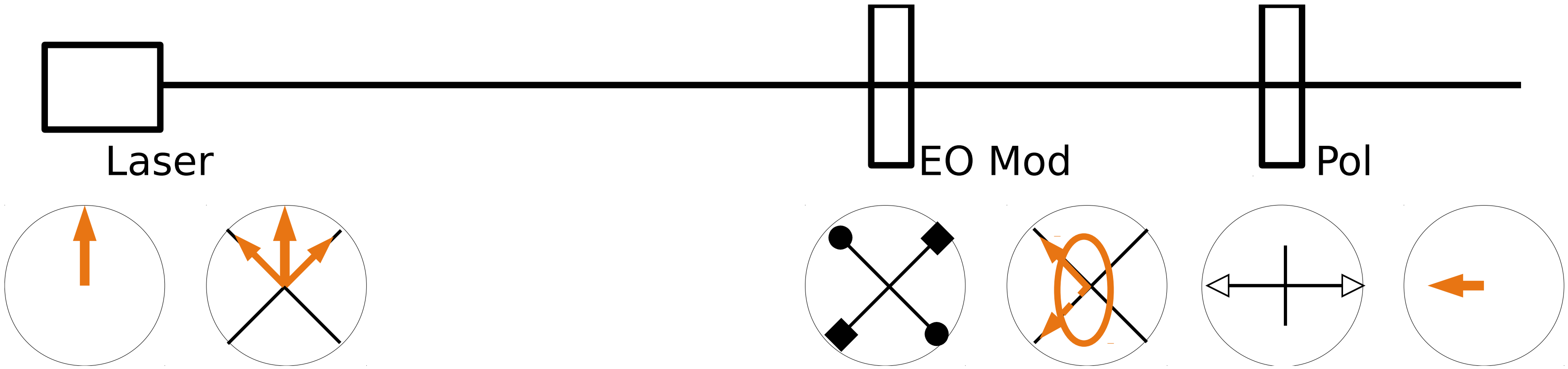
Electrically–Induced Birefringence

- Eletric Field Alters Symmetry
- Birefringence Proportional to DC Voltage

$$\delta\phi = \pi \frac{V}{V_\pi}$$

- Applications
  - Phase Modulation (Field Paralel to One Axis)
  - Frequency Modulation
    (Phase Modulation in Laser Cavity)
  - Amplitude Modulation
    (Field at 45° with Crossed Polarizer at Output)

E/O Modulator



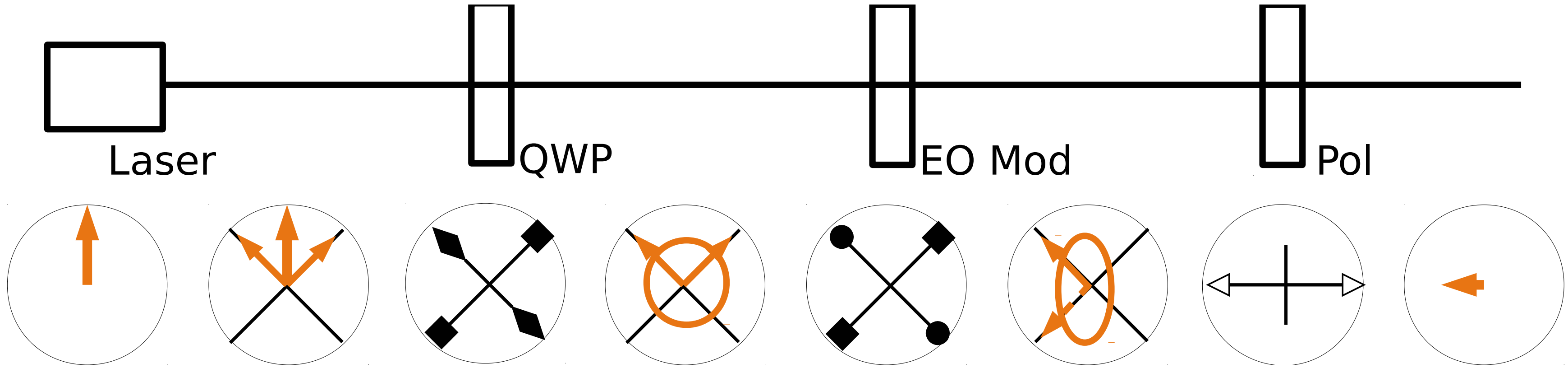
- Voltage Controlled Waveplate

$$\delta\phi = \pi \frac{V}{V_\pi}$$

- $T = 1$ at $V = V_\pi$ and $T = 0$ at $V = 0$,

- Linear Transmission Near Quarter–Wave $V \approx V_\pi/2$

Modulator with Bias

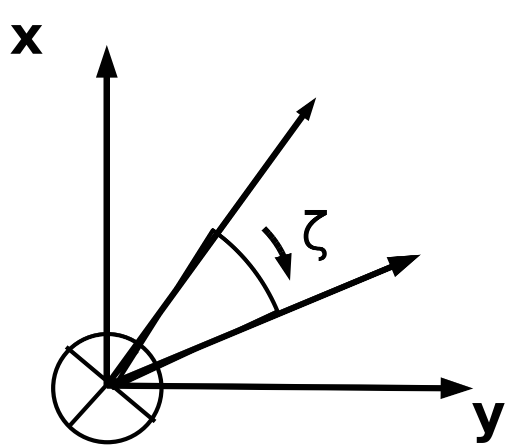


Quarter–Wave Phase Difference at $V = 0$

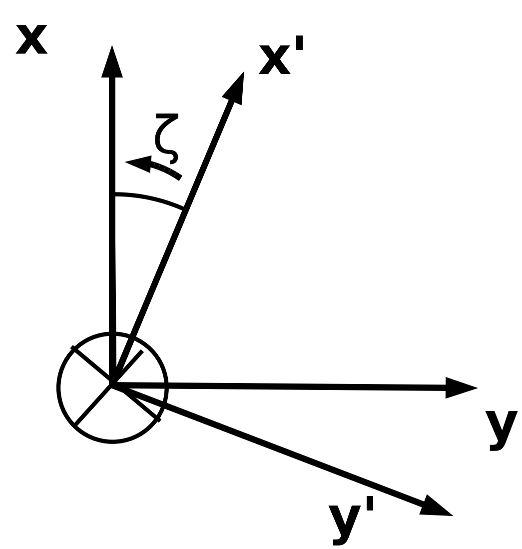
Rotator

- General Equation

$$\begin{pmatrix} E_{x:out} \\ E_{y:out} \end{pmatrix} = \begin{pmatrix} \cos\zeta_r & -\sin\zeta_r \\ \sin\zeta_r & \cos\zeta_r \end{pmatrix} \begin{pmatrix} E_{x:in} \\ E_{y:in} \end{pmatrix}$$



Polarization Rotator



Rotation of Coordinates

Polarization Rotator

- Reciprocal Rotator (*e.g.* Sugar in Water)

$$\delta\zeta = \kappa C \ell$$

  - $\kappa =$ Specific Rotary Power
  - $C =$ Concentration
  - $\ell =$ Length
- Rotation in Either Direction
  - Left (Levulose) $C > 0$
  - Right (Dextrose) $C < 0$
- Same Sign for Reverse Propagation (*e.g.* Reflection)
  - Round–Trip Restores Original Polarization
- Non–Reciprocal Rotator (*e.g.* Fraday Rotator)
  - Underlying Physics (DC Magnetic Field)

$$\mathbf{a} = -\frac{e}{m}\mathbf{v} \times \mathbf{B}$$

  - Result: ($v =$ Verdet Constant)

$$\delta\zeta = v\mathbf{B} \cdot \hat{z}\ell$$

- Reverse Propagation

$$\delta\zeta = v\mathbf{B} \cdot (-\hat{z})\ell$$

  - Round–Trip Doubles Rotation
  - Application: Faraday Isolator