Optics for Engineers
Week 1

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EECE–4646
Northeastern University

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Week 1 Agenda

- Administrivia
- Introductions
- History of Optics
- Overview: Spectrum, Maxwell's Equations, wave equation, index of refraction, Fermat's principle.
- Geometric Optics: Reflection and refraction, total internal reflection, retroreflectors, optical fibers.

Me

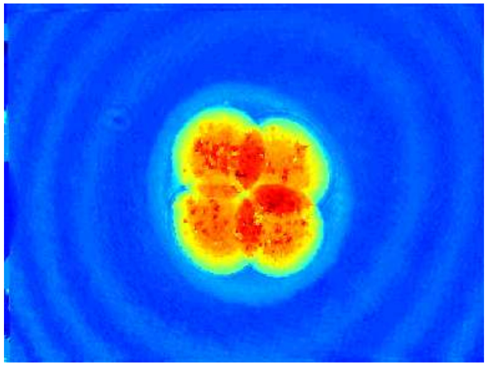
- Education
  - 1969: BS in Engineering Physics, University of Maine
  - 1973: MS in Physics, WPI
  - 1996: Ph.D. in Electrical Engineering, Northeastern
- Employment
  - 1973 — 1987: Raytheon Company (Laser Radar)
  - 1983 — 1987: Northeastern (Part–Time Lecturer)
  - 1987 — 2000: Northeastern (Research Scientist)
  - 2000 — Present: Northeastern ECE Faculty (MIE/BioE)
  - 2014 — 2020: Topical Editor for *Optics Letters*
  - 2014 — 2016: Associate Chair of ECE
- Home: Cambridge, with my Wife, Sheila
- Family: 2 Children, 3 Grandchildren
- Home Ski Area: Killington, Vermont

Personal History

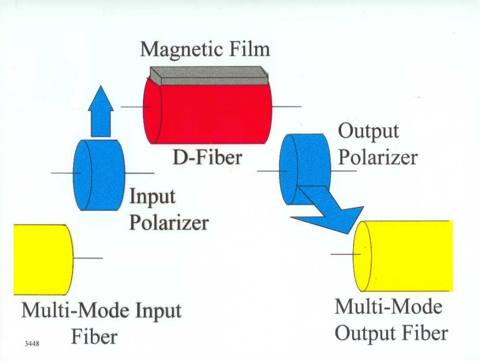
- Raytheon (Jelalian)
  - Aircraft Wake LIDAR
  - Airborne LIDAR
- Northeastern University
  - LIDAR
  - MOKE Sensors
  - Landmine Detection
  - Hyperspectral Imaging (Biomed)
  - Light and Sound
  - Optical Quadrature
  - Multi-Modal Microscopy

Severe Storms

Coal–Dust Lidar



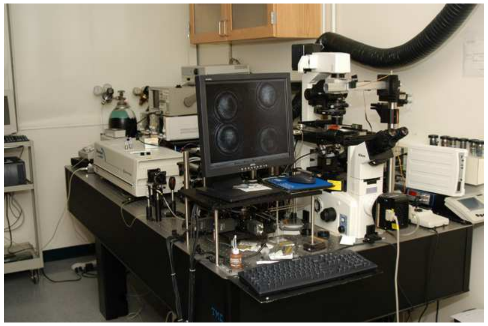
Cell Counting



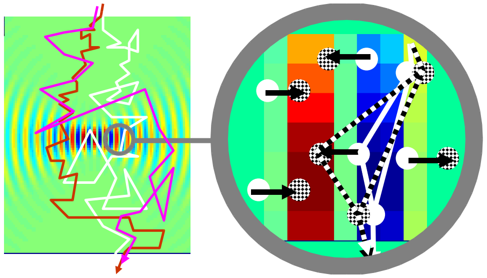
Magnetic Sensor

Our Current Research

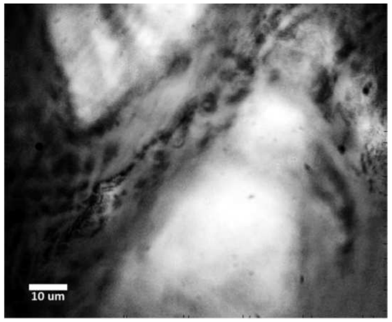
- Multi–Modal Microscopy
- Light and Sound
- Structured Illumination
- Collagen Orientation
- Stepwise 3–Photon Fluorescence in Melanin
- Lidar (Laser Radar)



Multi–Modal



Light and Sound



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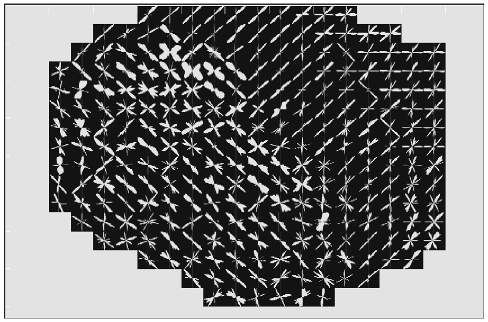
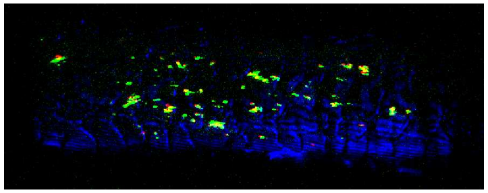


Collagen



Melanin

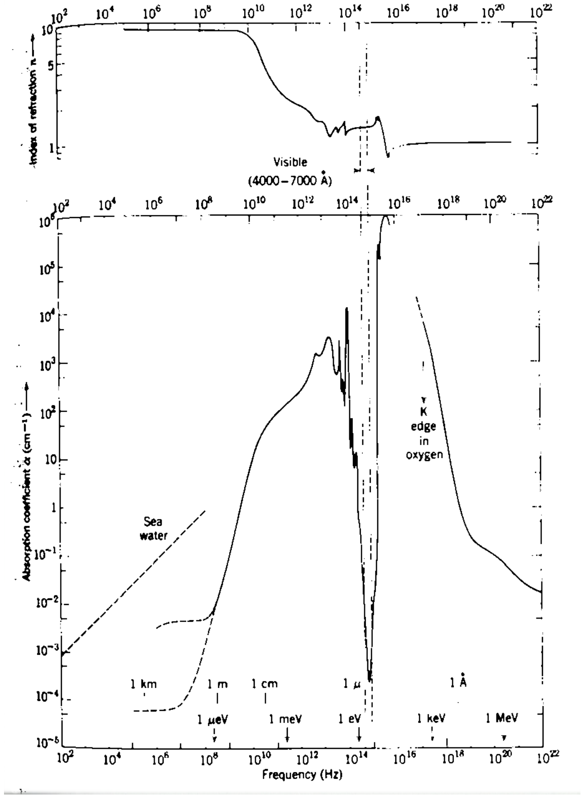
Lidar

Engineers at Play

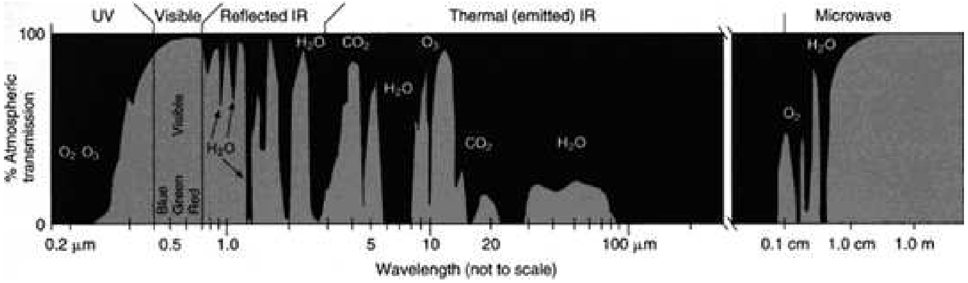
Course Components

- Lectures (Synchronously and Recorded)
- Slides (Available on the Website)
- Homework
- Quizzes
- Exams
- Projects
- Participation
- Office Hours on Zoom or in person

Why Optics?



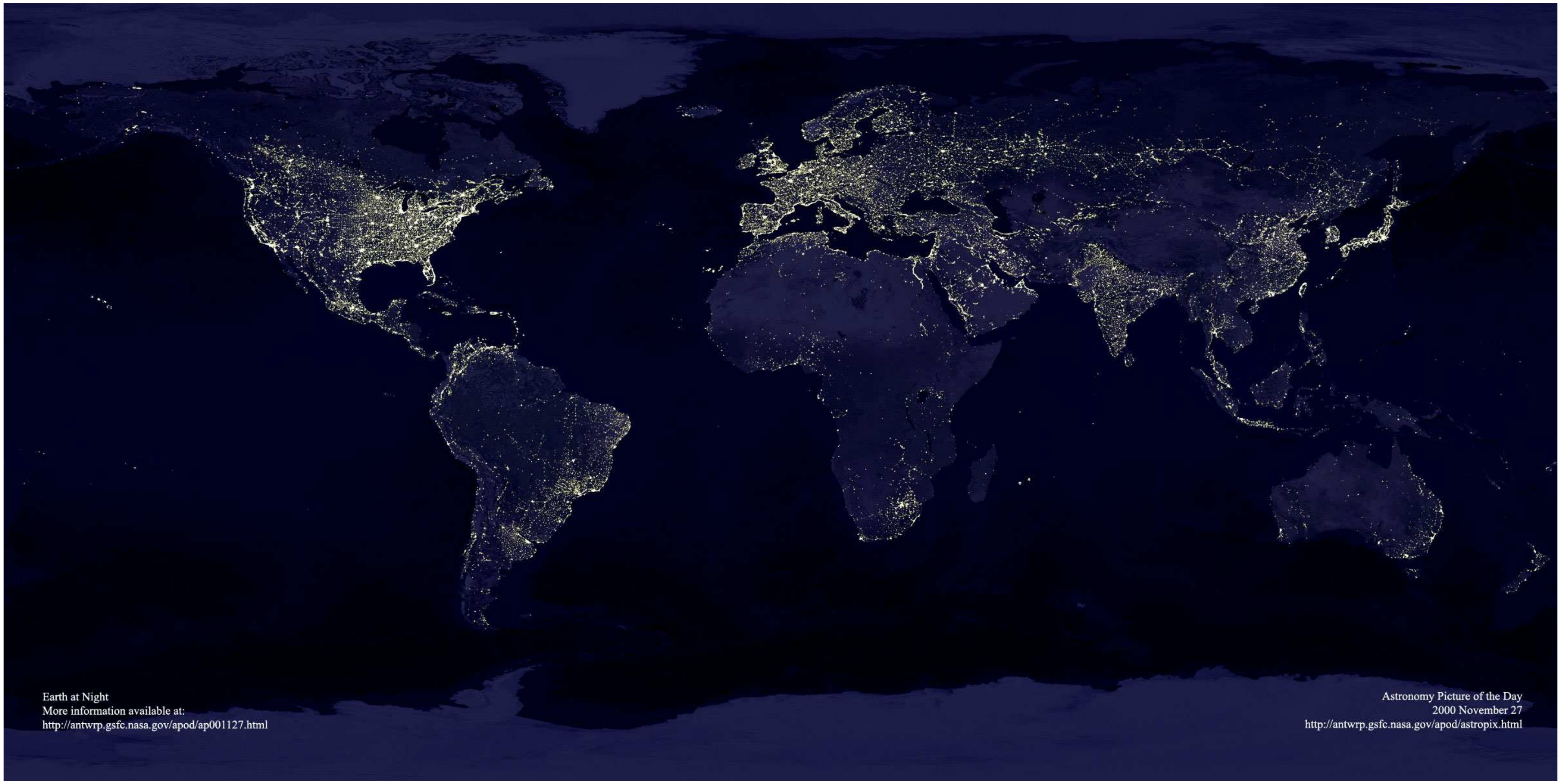
A. Liquid Water



B. Water Vapor

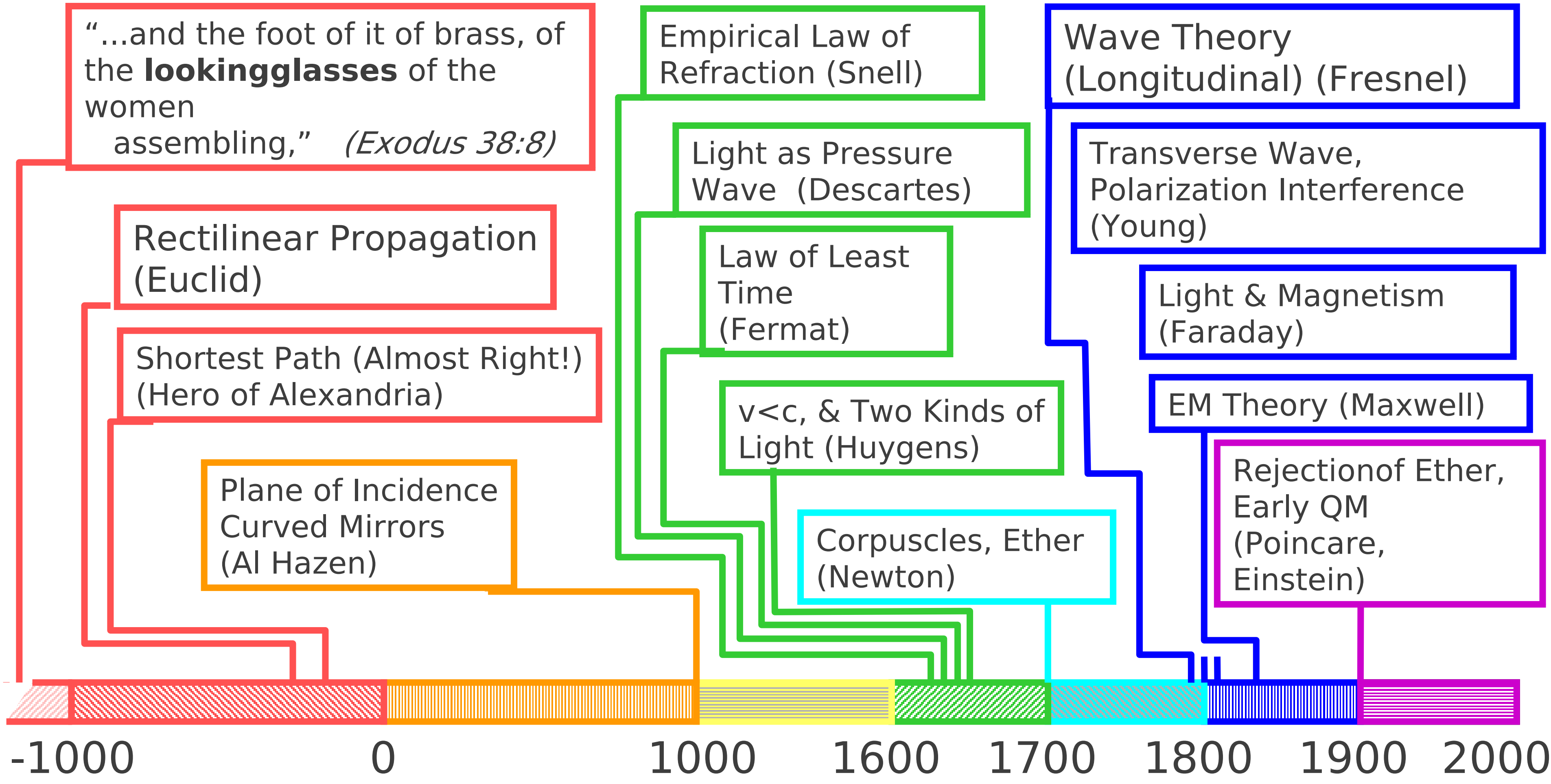
ELECTROMAGNETIC TRANSMISSION. Water strongly absorbs most electromagnetic waves, with the exception of wavelengths near the visible spectrum (A, From Jackson *Classical Electrodynamics*, ©1975). The atmosphere also absorbs most wavelengths, except for very long wavelengths and a few transmission bands (B, NASA's Earth Observatory).

Why Optics?



C. Mayhew & R. Simmon (NASA/GSFC), NOAA/ NGDC, DMSP Digital Archive).

History of Optics (1)



“...and the foot of it of brass, of the **lookingglasses** of the women
assembling,” *(Exodus 38:8)*

Rectilinear Propagation (Euclid)

Shortest Path (Almost Right!) (Hero of Alexandria)

Plane of Incidence
Curved Mirrors
(Al Hazen)

Empirical Law of Refraction (Snell)

Light as Pressure Wave (Descartes)

Law of Least Time (Fermat)

v<c, & Two Kinds of Light (Huygens)

Corpuscles, Ether (Newton)

Wave Theory (Longitudinal) (Fresnel)

Transverse Wave, Polarization Interference (Young)

Light & Magnetism (Faraday)

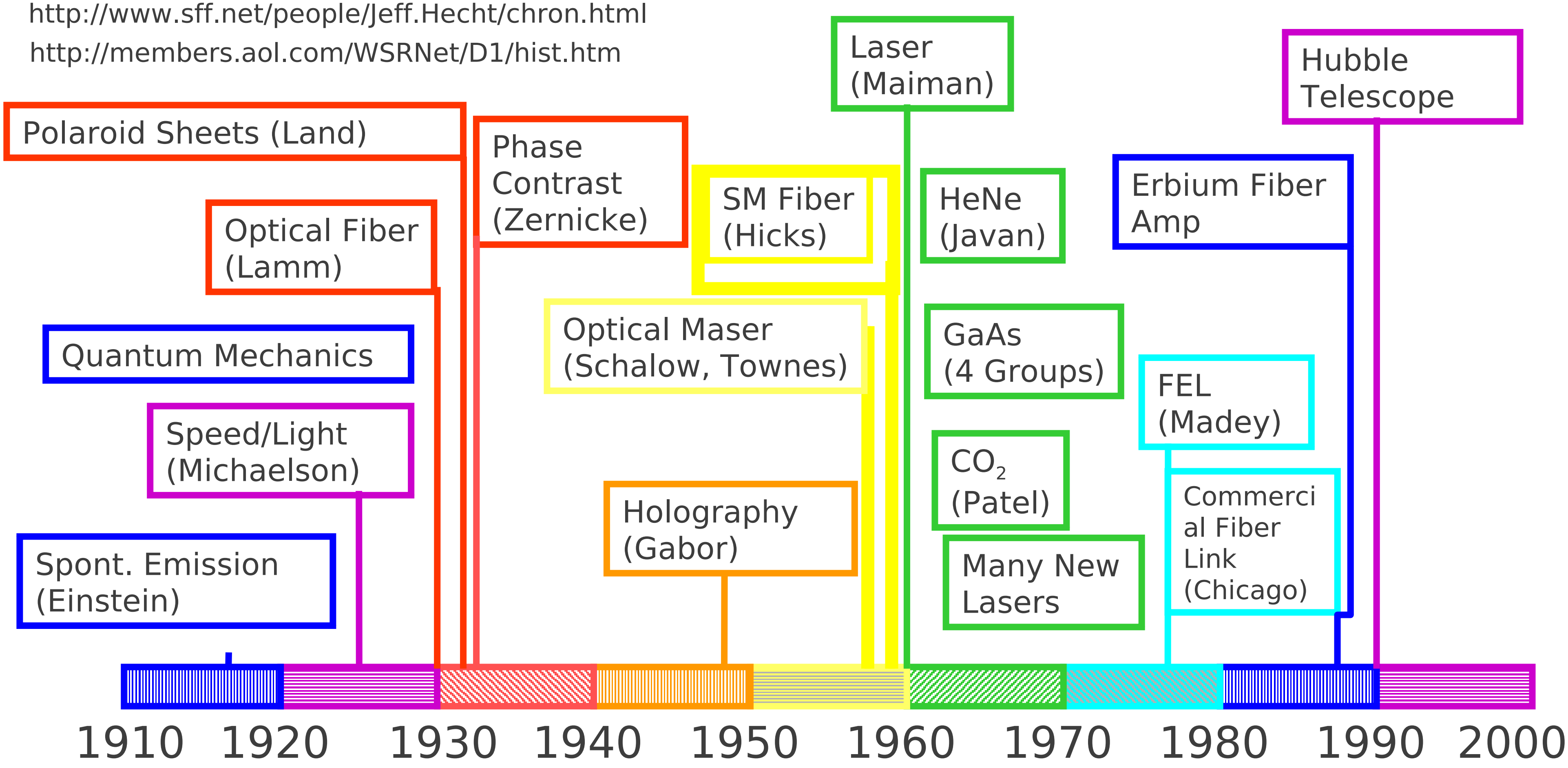
EM Theory (Maxwell)

Rejectionof Ether, Early QM (Poincare, Einstein)

History of Optics (2)

http://www.sff.net/people/Jeff.Hecht/chron.html

http://members.aol.com/WSRNet/D1/hist.htm



Laser (Maiman)

Hubble Telescope

Polaroid Sheets (Land)

Phase Contrast (Zernicke)

SM Fiber (Hicks)

HeNe (Javan)

Erbium Fiber Amp

Optical Fiber (Lamm)

Quantum Mechanics

Optical Maser (Schalow, Townes)

GaAs (4 Groups)

FEL (Madey)

Speed/Light (Michaelson)

CO$_2$ (Patel)

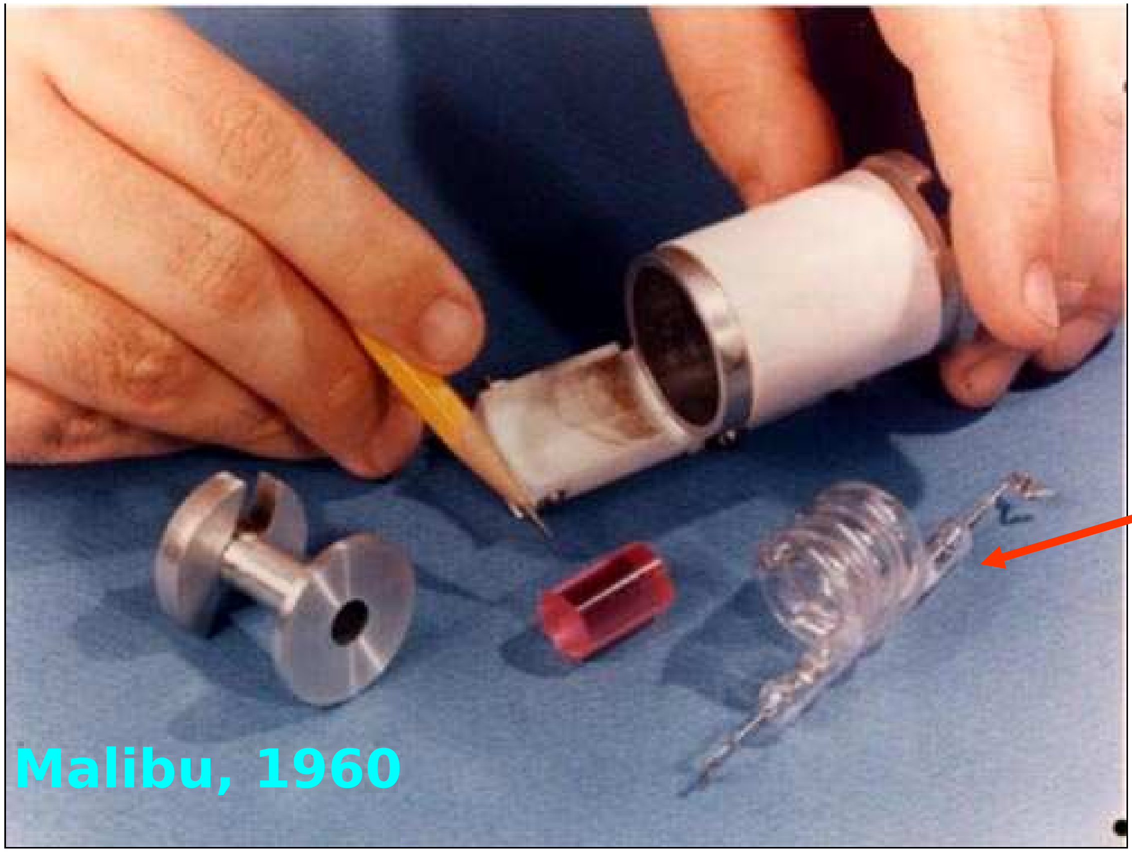
Commerci al Fiber Link (Chicago)

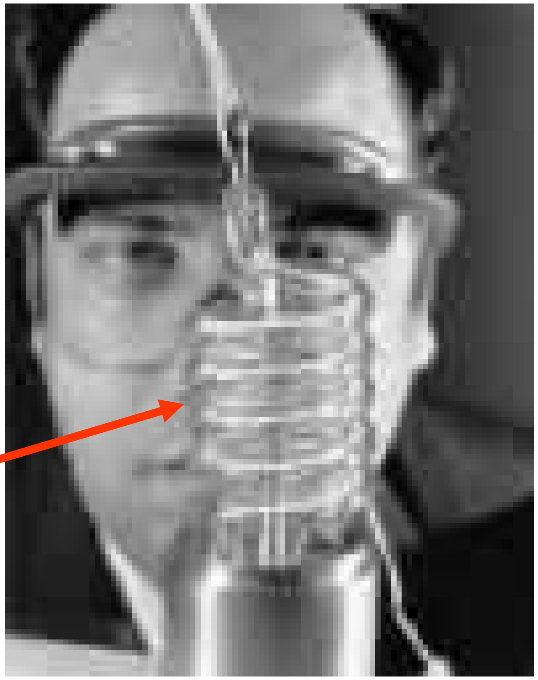
Holography (Gabor)

Spont. Emission (Einstein)

Many New Lasers

The First Laser?





News Photo of the First Laser

"Laser, inter eximia naturae dona numeratum plurimis compositionibus inseritur*"

* "The laser is numbered among the most miraculous gifts of nature and lends itself to a variety of applications." Pliny, *Natural History XXII*, 49.

The Math of Optics

- Maxwell's Equations
- Wave Equation for $\mathbf{E}$ or other
- Ray Propagation
- Other Approximations

Maxwell's Equations

Faraday's Equation

$$\boxed{\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t}}$$

Ampere's Equation
(No Currents)

$$\boxed{\nabla \times \mathbf{H} = \frac{\partial \mathbf{D}}{\partial t}}$$

Gauss' Equations

$$\boxed{\nabla \cdot \mathbf{D} = \rho = 0}$$

$$\boxed{\nabla \cdot \mathbf{B} = 0}$$

Constitutive Parameters

$$\mathbf{D} = \varepsilon \mathbf{E}$$

$$\mathbf{B} = \mu \mathbf{H}$$

Getting to the Wave Equation

$$\begin{tikzcd}
& \downarrow \text{Material Properties} \arrow[d] & \\
& \mathbf{D}=\varepsilon\mathbf{E} \arrow[dl] \arrow[dr] & \\
\mathbf{D} & & \mathbf{E} \\
\nabla\times\mathbf{H}=\frac{\partial\mathbf{D}}{\partial t} \arrow[u] \arrow[d] & & \nabla\times\mathbf{E}=-\frac{\partial\mathbf{B}}{\partial t} \arrow[u] \arrow[d] \\
\mathbf{H} & & \mathbf{B} \\
& \mathbf{B}=\mu_0\mathbf{H} \arrow[ul] \arrow[ur] &
\end{tikzcd}$$

In the case that $\mathcal{E}$ is a scalar constant, ϵ,

$$\boxed{\nabla^2\mathbf{E}=\mu\epsilon\frac{\partial^2\mathbf{E}}{\partial t^2}}$$

The Wave Equation

$$\epsilon = \epsilon_0 n^2 \qquad \mu = \mu_0$$

$$\boxed{\nabla^2 \mathbf{E} = \left(\frac{n}{c}\right)^2 \frac{\partial^2 \mathbf{E}}{\partial t^2}}$$

Second order differential equation in space and time
Solutions: Sinusiods
One Important Solution: A Plane Wave

$$\mathbf{E} = \hat{x} E_0 e^{j(\omega t - nkz)},$$

Electric field vector in $\hat{x}$ direction.
Wave propagation in $\hat{z}$ direction.

Waves in Space and Time

- Plane Wave

  $n = 1$

  $$E = E_0 e^{j(\omega t - kz)}$$

- Wavelength

  $\lambda = 2\pi/k$

  Vertical Line

- Period

  $T = 2\pi/\omega$

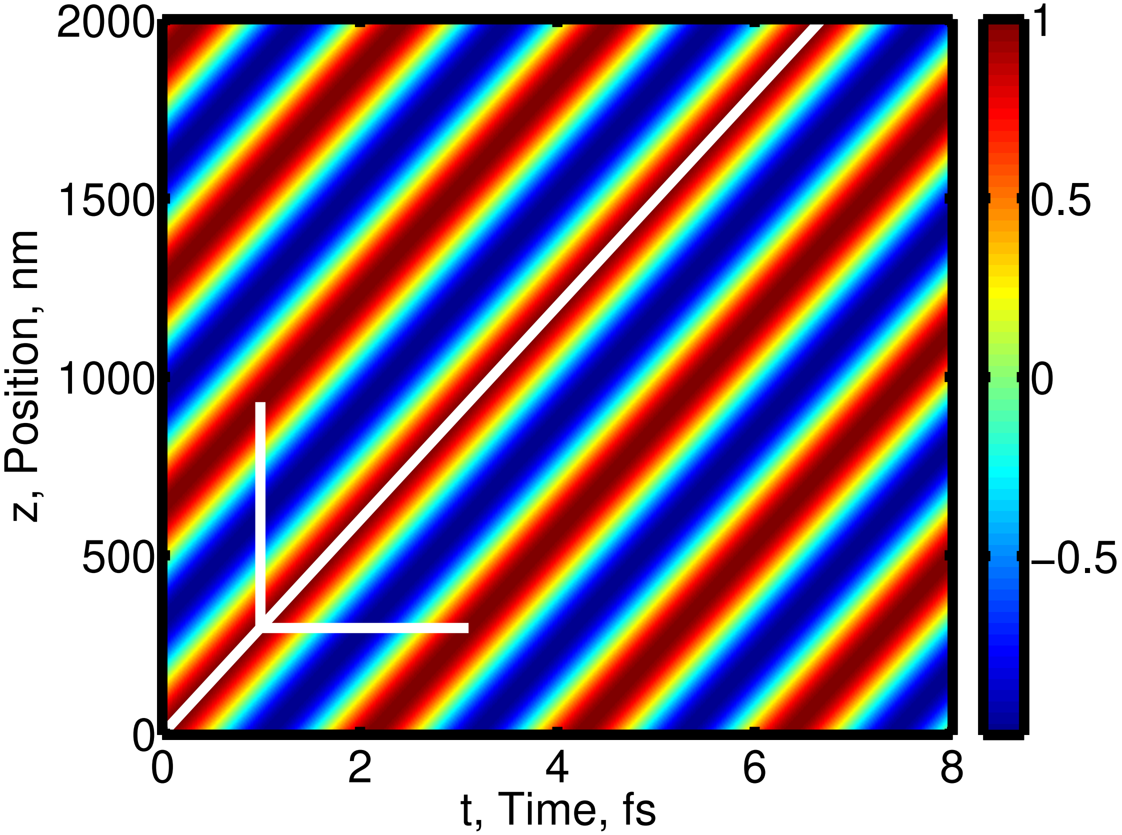
  Horizontal Line

- Speed

  v=c=299,792,458 m/s

  Constant Phase

  Diagonal Line



In general, $v = dz/dt = c/n$

Wavelength $= \lambda/n$

Index of Refraction

Material	**Approximate Index**
Vacuum	1.00
Air	1.0003
Water (visible to NIR)	1.33
Glass (Visible to NIR)	1.5
ZnSe (Infrared)	2.4
Germanium (Infrared)	4.0

Index of Refraction Summary

Take–Away Message

- n Depends on Material
- Small Values, Limited Choices, Slight Variations
  - $n \approx 1$ for Air
  - $n \approx 1.33$ for Water
  - $n \approx 1.5$ for Glass

Getting to the Wave Equation

$$\begin{tikzcd}
 & & \text{Material Properties} \arrow[d] & & \\
 & \mathbf{D} & \mathbf{D} = \varepsilon \mathbf{E} \arrow[l] \arrow[r] & \mathbf{E} & \\
\nabla \times \mathbf{H} = \frac{\partial \mathbf{D}}{\partial t} \arrow[ur] \arrow[dr] & & & & \nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t} \arrow[ul] \arrow[dl] \\
 & \mathbf{H} & \mathbf{B} = \mu_0 \mathbf{H} \arrow[l] \arrow[r] & \mathbf{B} &
\end{tikzcd}$$

In the case that $\mathcal{E}$ is a scalar constant, ϵ,

$$\boxed{\nabla^2 \mathbf{E} = \mu \epsilon \frac{\partial^2 \mathbf{E}}{\partial t^2}}$$

Circuits Analogy

$V = IR$	$\lvert\mathbf{E}\rvert = \lvert\mathbf{H}\rvert Z$	$\lvert\mathbf{E}\rvert = \lvert\mathbf{H}\rvert \eta$
$P = IV$	$\mathbf{S} = \mathbf{E} \times \mathbf{H}$	
	$\lvert\mathbf{S}\rvert = \lvert\mathbf{E}\rvert \lvert\mathbf{H}\rvert$	For Plane Wave ($\mathbf{E} \perp \mathbf{H}$)
$P = \frac{V^2}{R}$	$\lvert\mathbf{S}\rvert = \frac{\lvert\mathbf{E}\rvert^2}{Z}$	
$P = I^2R$	$\lvert\mathbf{S}\rvert = \lvert\mathbf{H}\rvert^2 Z$	Not Often Used in Optics
V in Volts	$\mathbf{E}$ in $\frac{\text{Volts}}{\text{meter}}$	
I in Amperes	$\mathbf{H}$ in $\frac{\text{Amperes}}{\text{meter}}$	
R in Ohms	Z or η in Ohms	
P in Watts	$\mathbf{S}$ in $\frac{\text{Watts}}{\text{meter}^2}$	$\lvert\mathbf{S}\rvert$ is Irradiance

Irradiance

Equation: $I = |\mathbf{S}| = \frac{|\mathbf{E}|^2}{Z}$

Often Used: $I = |\mathbf{E}|^2$

Assumes Z is Constant
Is OK for Ratios under that Assumption
Provides Incorrect Relationship Between I and $\mathbf{E}$, but . . .
Fields can Never be Directly Measured
And Fields are Seldom of Interest Anyway
So Strange Units for Fields Seldom Cause Problems . . .

But, When in Doubt, Do it the Right Way.

Wavelength, Frequency, Photons

Quantity	Equation	Typical Units	Example
Vacuum Wavelength	λ	nm or μm	Green: 500nm
Frequency	$f = \nu = \frac{c}{\lambda}$	THz.	600THz
Wave vector	$\lvert k\rvert = \frac{2\pi}{\lambda}$		
Wavenumber*	$\tilde{\nu} = \frac{1}{\lambda}$	cm^{-1}	$20,000\mathrm{cm}^{-1}$
Photon Energy	$h\nu$	J	4×10^{-19}J
**	$\frac{h\nu}{e}$	eV	2.5eV
Photon Momentum	$\mathbf{p} = \frac{h\mathbf{k}}{2\pi}$		$1.3 \times 10^{-27}\frac{kg\ m}{s}$

* Used in Spectroscopy (proportional to energy)

** Electron Volts; Energy Units

Spectral Regions

Band	**Low λ**	**High λ**	**Characteristics**
Vacuum Ultraviolet	100nm	300nm	Requires vacuum for propagation.
Ultraviolet C (UVC)	100nm	280nm	
Oxygen Absorption		280nm	
Ultraviolet B (UVB)	280nm	320nm	Causes sunburn. Is partially blocked by glass.
Glass Transmission	350nm	2.5μm	
Ultraviolet A (UVA)	320nm	400nm	Is used in a "black light." Transmits through glass.
Visible Light	400nm	710nm	Is visible to humans, transmitted through glass, detected by silicon.
Near–Infrared (NIR)	750nm	1.1μm	Is transmitted through glass and biological tissue, is detected by silicon.
Si Band Edge	1.2μm		Is not a sharp edge.
Water Absorption	1.4μm		Is not a sharp edge.
Mid–Infrared	3μm	5μm	Is used for thermal imaging. Is transmitted by Zinc Selenide and Germanium.
Far–Infrared (FIR)	8μm	$\approx 14\mu$m	Is used for thermal imaging. Is transmitted through ZnSe Ge, detected by HgCdTe *etc*.

Spectral Regions

In Practice

- Glass is Transparent in UVA, Visible, NIR
  - $n \approx 1.5$
  - Useful for Windows, Lenses, Prisms, *etc.*
- Silicon Absorbs at these Wavelengths
  - Useful for Detectors
- Different Materials Requred in UVC, UVB, Mid– and Far–IR
  - Normally Expensive Ones!

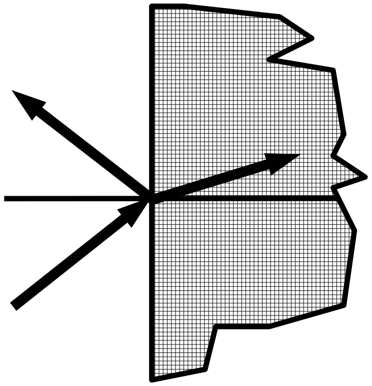
Photon Example

- Pulsed Laser
  - Average Power: $P_{av} = 1$W
  - Pulse Repetition Frequency: $PRF = 80$MHz
  - Pulse Width: $\tau = 100$fs
  - Wavelength: 800nm
- Questions
  - Pulse Energy?
  - Photons per Pulse?
  - Peak Power?

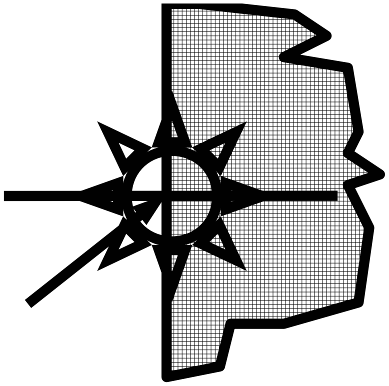
Photon Example

- Pulsed Laser
  - Average Power: $P_{av} = 1\text{W}$
  - Pulse Repetition Frequency: $PRF = 80\text{MHz}$
  - Pulse Width: $\tau = 100\text{fs}$
  - Wavelength: 800nm
- Answers
  - Pulse Energy: $Q = \frac{P_{av}}{PRF} = 12.5\text{nJ}$
  - Photons per Pulse: $N = \frac{Q}{h\nu} = \frac{Q\lambda}{hc} = 5 \times 10^{10}$
  - Peak Power: $P_{pk} = \frac{Q}{\tau} = \frac{P_{av}}{\tau\ PRF} = 125\text{kW}$

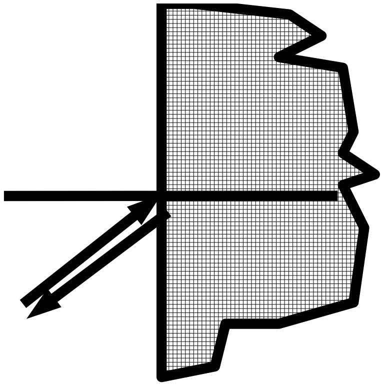
Light at an Interface



Specular Behavior



Scattering and Diffuse Behavior



Retroreflection

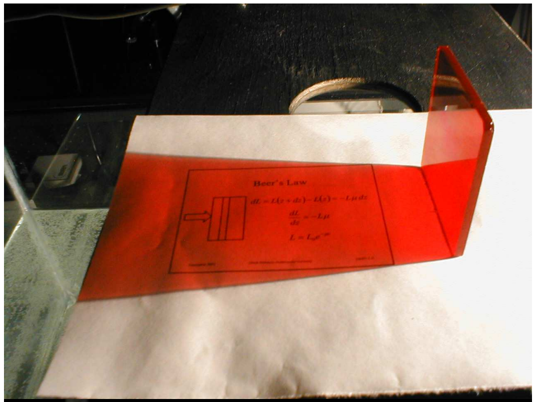
In these interactions, energy of the photon is conserved.

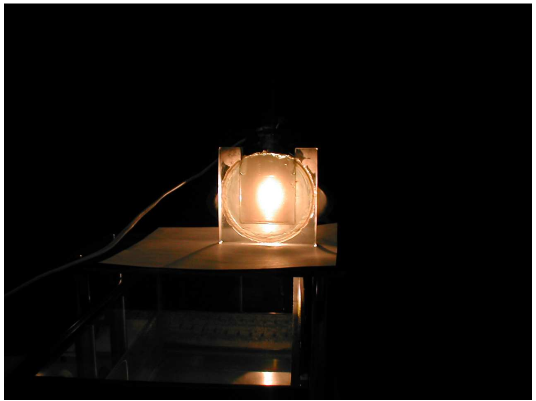
Take–Away Message

Light at an interface has specular, diffuse, and retro–reflective behavior. We will mostly consider specular behavior until much later.

Interface Examples

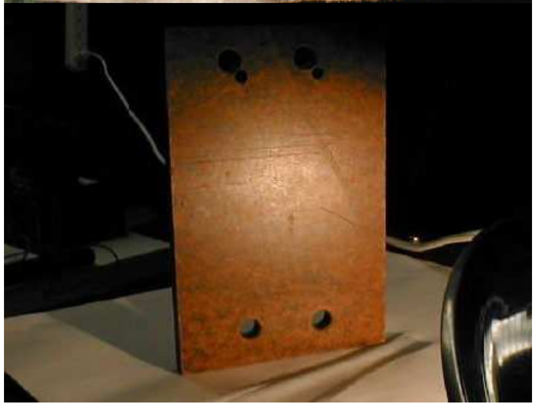
Transmission
& Absorption
(Colored
Glass)





Transmission
and Scattering
(Milk and
Water)

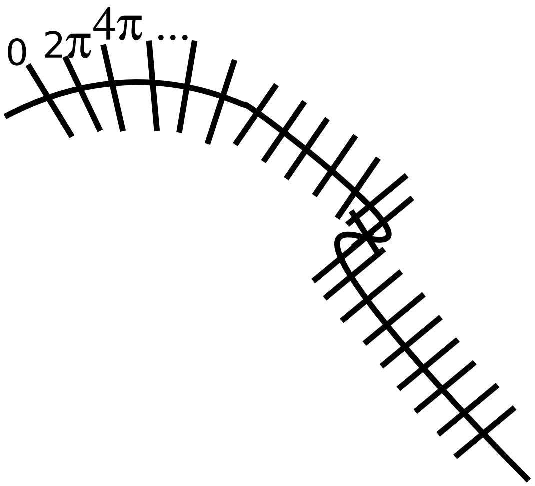
Diffuse
Reflection
(Rusty Iron)



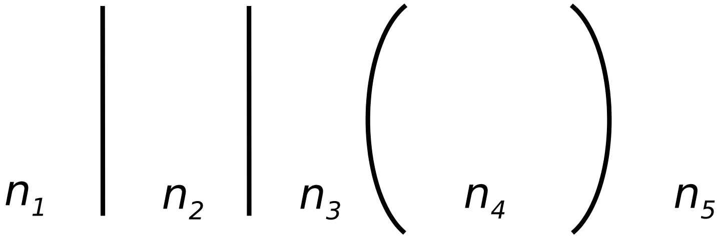
Diffuse and
Specular
Reflection
(Floor)

Photon energy is still conserved. In absorption, a fraction of the photons is absorbed, but the photon energy is still unchanged.

Optical Path Length

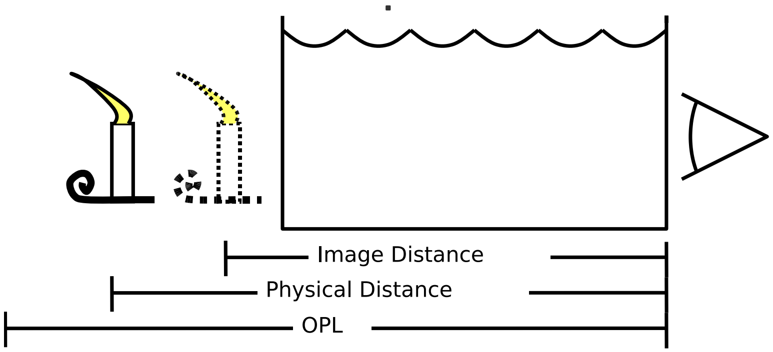


Optical Path Length. The OPL between any two points is obtained by integrating along a path between those two points. The eikonal equation ensures that the integral does not depend on the path chosen. The phase difference between the two points can be computed from the OPL.



Optical Path in Discrete Materials:

$$OPL = \ell = \sum_m n_m \ell_m$$



Optical Path in Water. The optical path is longer than the physical path, but the geometric path is shorter.

Optical Path Length

Take–Away Message

The Eikonel Equation is very general and powerful. It can be used for complicated problems like gradient–index fibers and mirages and others beyond our scope here.

Most important here is the concept of Optical Path Length in discrete materials.

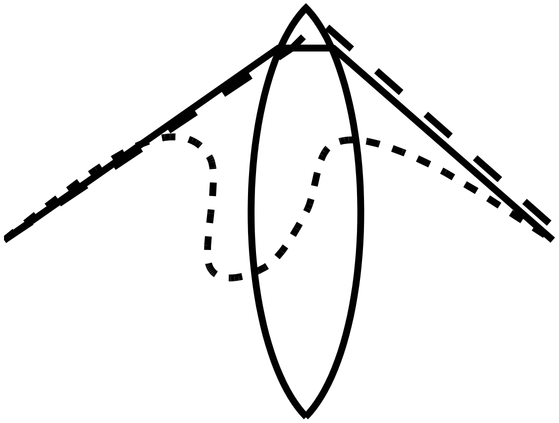
$$OPL = \ell = \sum_m n_m \ell_m .$$

We will use it to understand refraction and thus prisms and lenses, and later we will use it in interferometry (Ch. 10).

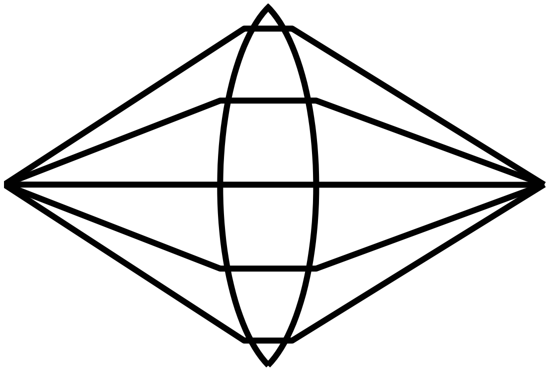
Fermat's Principle in Action

$$\sum_{m=1}^{3} n_m \ell_m = \ell_1 + n_{glass}\ell_2 + \ell_3,$$

- Center of Lens
  - Thick Glass
    $OPL > PPL$
  - Less Air, Low Index
- Edge of Lens
  - Less Glass
  - More Air
- All Paths Equal?
  - Rays Arrive in Phase
  - Point Is Imaged
  - Object and Image are Said to be Conjugates



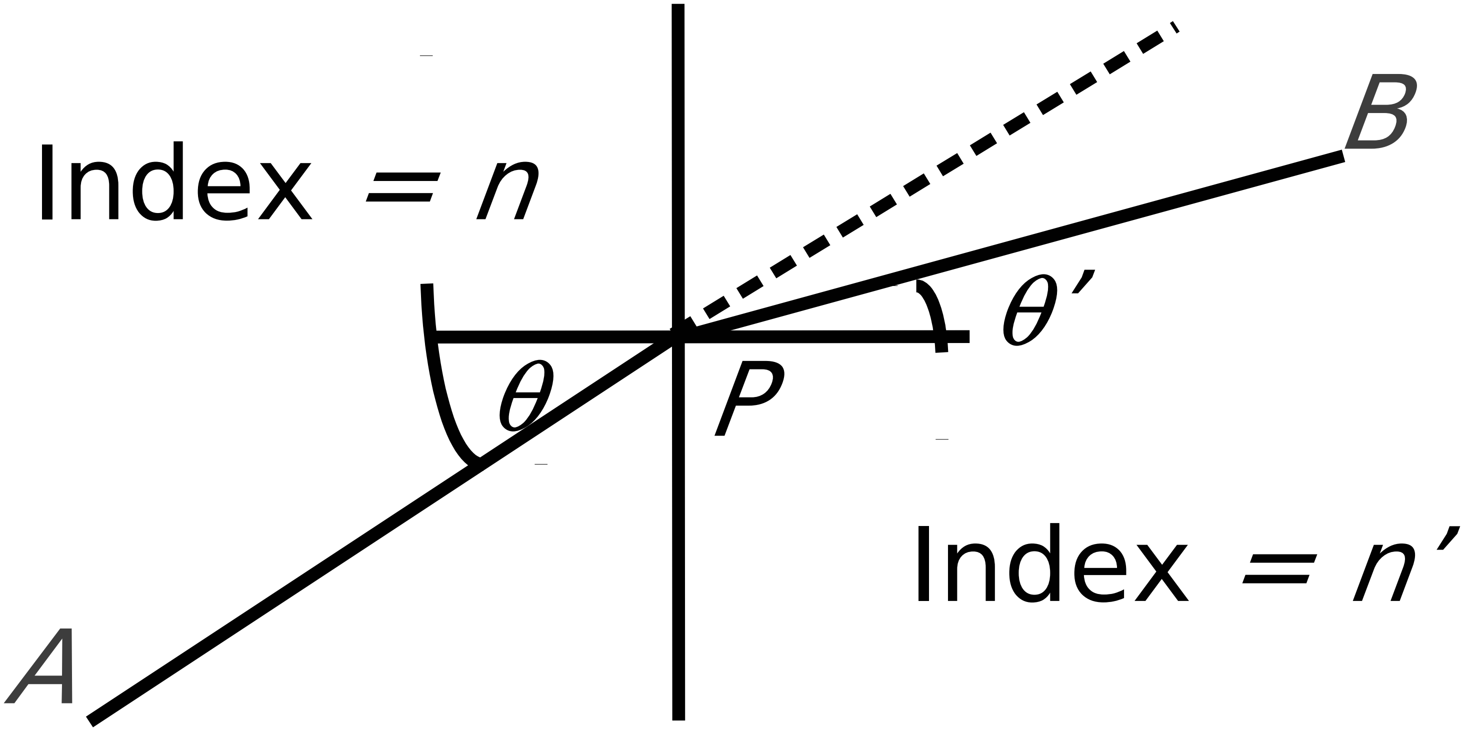
Fermat's Principle. Light travels the shortest optical path.



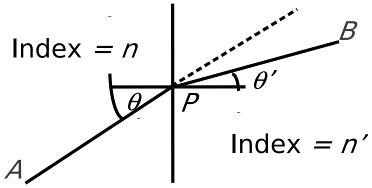
Imaging. All paths are Minimal, Points are conjugate; One is the Image of the Other.

Concepts for Refraction

- Plane of Incidence Contains Incident (and Exiting) Ray and Normal (and is the plane of the 2–D drawing)
- Angle of Incidence Is Defined Relative to Normal

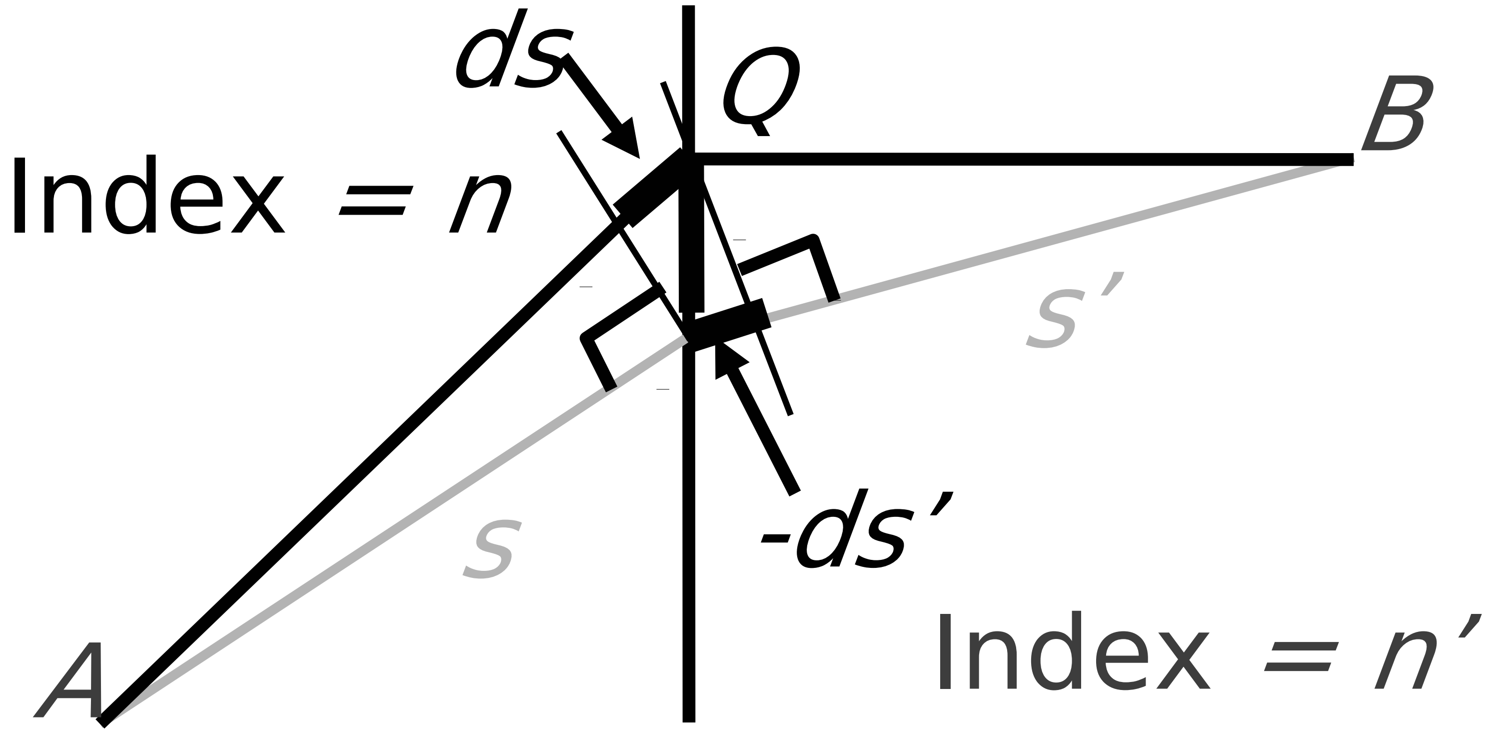


Snell’s Law

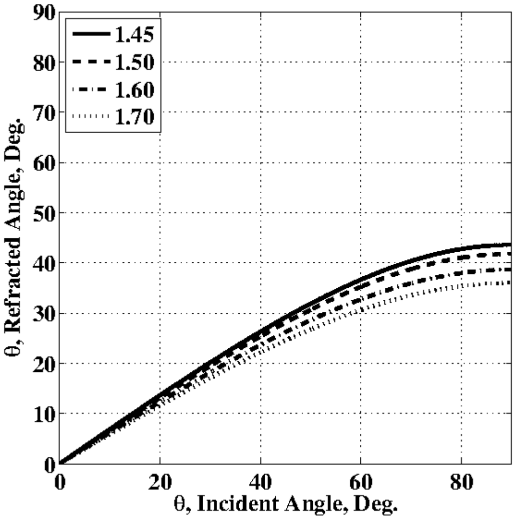


Variational Approach from Minimal Path, AB (Fermat) $nds = n'ds'$

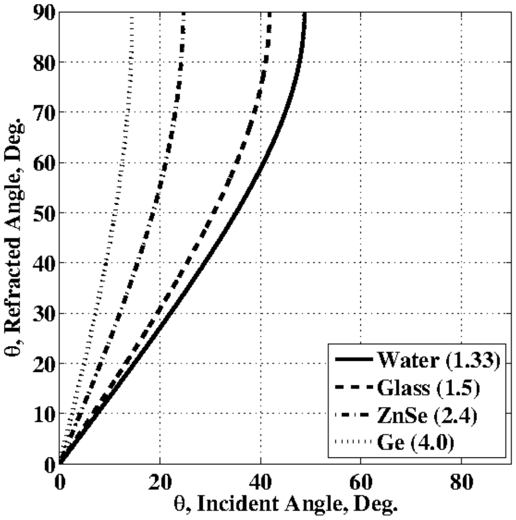
$$\boxed{n \sin\theta = n' \sin\theta'.}$$



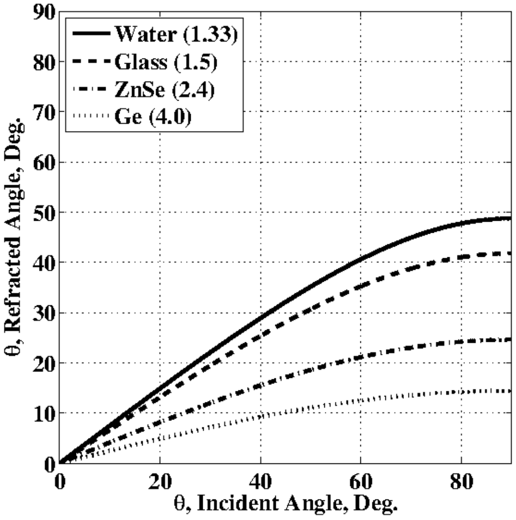
Snell's Law: Examples



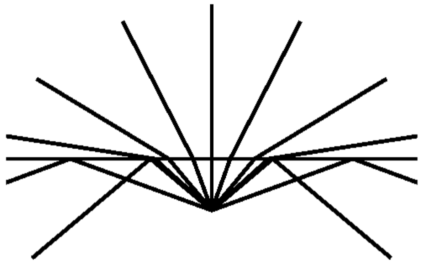
Air to Glass



Material to Air



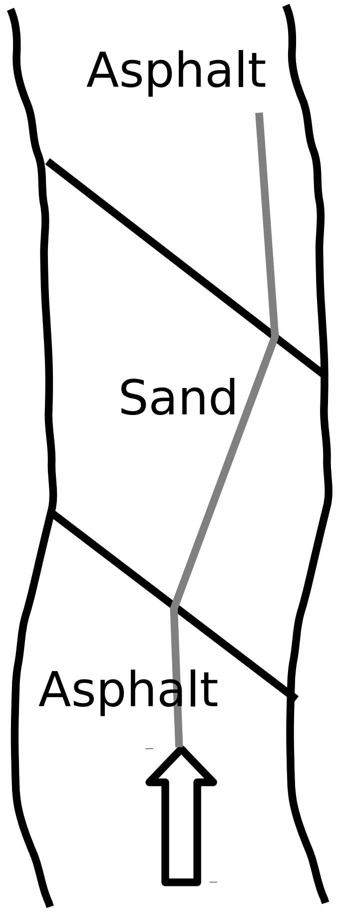
Air to Material



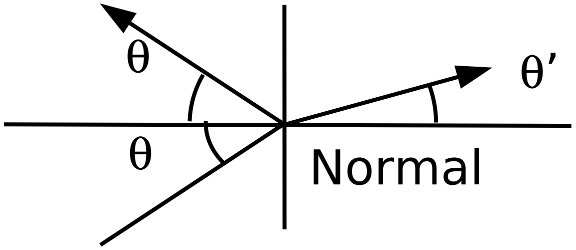
Total Internal Reflection

Snell's Law: Analogy

- Driving a car from an asphalt road onto a dirt road
  - Asphalt–to–dirt line is diagonal.
  - Dirt slows speed.
- Car tries to turn to the right.
- Dirt–to–Asphalt: Car turns back to the left.
- This is just an analogy to remember the direction.



Reflection and Refraction



Reflection:

$$\theta_r = \theta.$$

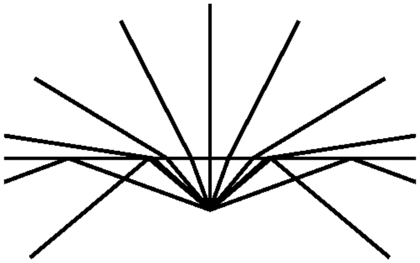
Refraction

Total Internal Reflection

- Critical Angle (No Solution for θ')

$$\boxed{n \sin \theta_c = 1}$$

- For $\theta < \theta_c$ Reflection and Refraction
- For $\theta > \theta_c$ 100% Reflection



Snell's Window

Carol Grant

Q: Why is it black outside the "window?"

Reflection at a Plane Mirror

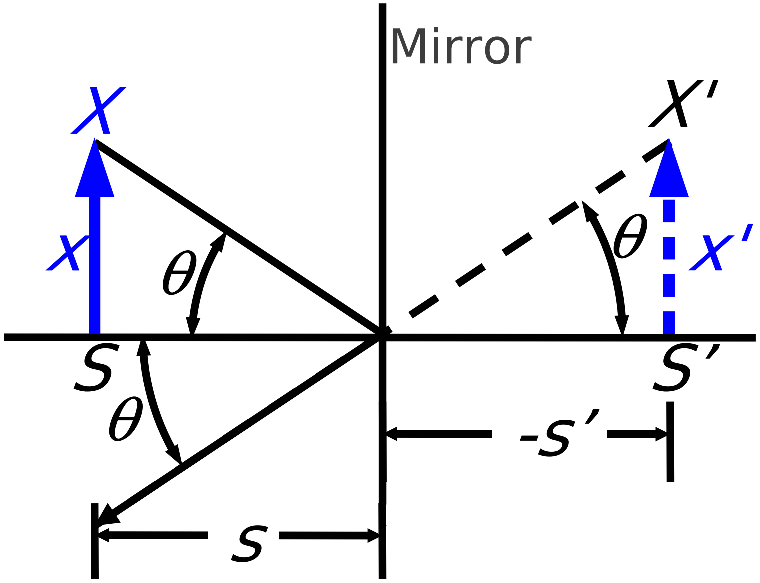
- **Image Location** Similar Triangles

$$s' = -s$$

- **Magnification (Transverse)**
  - More Similar Triangles
  - Result: $x' = x \quad m = 1$

$$m = \frac{x'}{x} = \frac{-s'}{s} = 1$$

(Planar reflector)



Upright ($m > 0$) & Virtual (Dotted Lines)

The Retroreflector