Optics for Engineers

Chapter 5

Charles A. DiMarzio
Northeastern University

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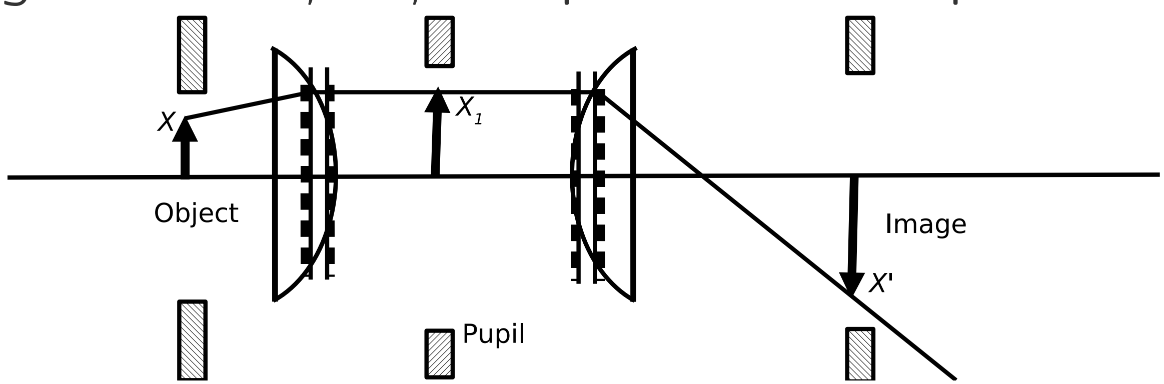
The Story So Far

- Small–Angle Approximation

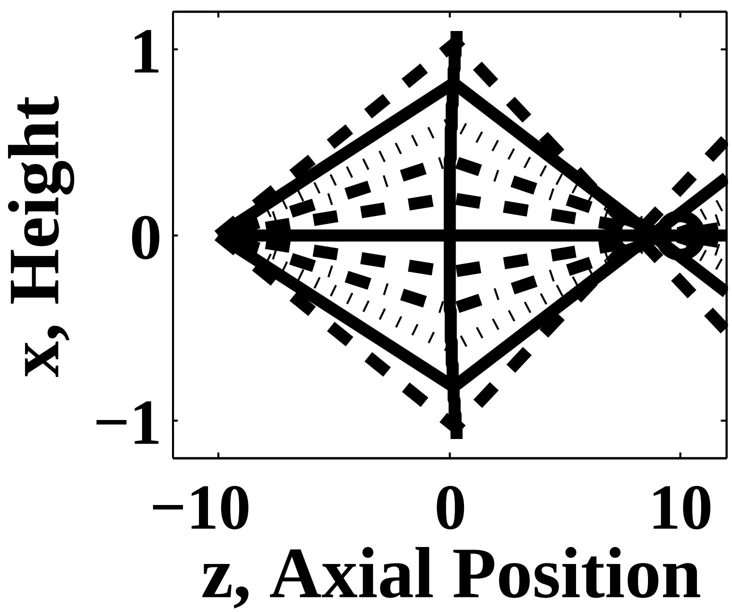
$$\sin\theta = \theta = \tan\theta \qquad \text{and} \qquad \cos\theta = 1$$

- Perfect Imaging
  - Lens Equation (Image Location)
  - Magnification (Image Size)
  - Matrix Optics and Other Bookkeeping Tricks
  - Point Images as Point, or . . .
  - Image Position, X', Independent of Pupil Position, X_1

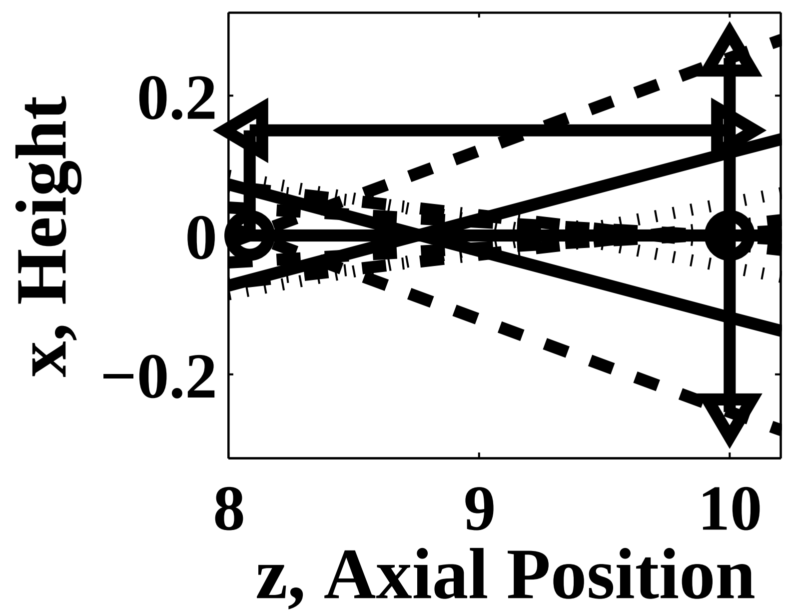


Using Snell's Law Exactly

- Example: Single Convex Air–to–Glass Interface
- Paraxial Rays Follow Small–Angle Approximation
- Edge Rays May Focus Quite Differently
- Rays Do Not Intersect at a Single Point (or at all in 3D)
- Large "Shot Pattern" at "Paraxial" Focus
- "Best" Focus Translated and Depth of Focus Increased



Complete Ray Trace



Expanded View Near Image

Looking Ahead: Diffraction

- Diffraction Theory (Ch. 8) Predicts a Minimum Spot Size
  - Rooted in Fundamental Physics
  - $\approx \frac{\lambda}{D_{pupil}} z$
- Ray Tracing Result Below this Limit is "Good Enough"
  - Characterized as "Diffraction–Limited"
- Larger Ray–Tracing Result Indicates Degraded Imaging
  - Can Characterize Roughly by "XDL"

Ray Tracing: Overview

- Setup: Launch a Fan of Rays (*eg.* Fill FOV and Pupil)
- Loop On Rays
  - Loop On Elements (Like Matrix But No Approximations)
    - Translation (Straight–Line Propagation)
    - Refraction or Reflection (Interfaces)
  - Close (End the Ray Calculation)
- Report (*eg.* Spot Size *vs.* Field Position)

Ray Tracing: Translation (1)

- Parametric Eq. for Ray

$$\mathbf{x} = \mathbf{x}_0 + \ell \hat{\mathbf{v}} \qquad \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} x_0 \\ y_0 \\ z_0 \end{pmatrix} + \ell \begin{pmatrix} u \\ v \\ w \end{pmatrix}$$

- Surface Eq. (Eg. Sphere, Centered on Axis)

$$(\mathbf{x} - \mathbf{x}_c) \cdot (\mathbf{x} - \mathbf{x}_c) = r^2 \qquad x^2 + y^2 + (z - z_c)^2 = r^2$$

- Combine to Find Intersection

$$\hat{\mathbf{v}} \cdot \hat{\mathbf{v}} \ell^2 + 2 (\mathbf{x}_0 - \mathbf{x}_c) \cdot \hat{\mathbf{v}} \ell + (\mathbf{x}_0 - \mathbf{x}_c) \cdot (\mathbf{x}_0 - \mathbf{x}_c) - r^2 = 0$$

$$u^2 \ell^2 + v^2 \ell^2 + w^2 \ell^2 + 2 x_0 u \ell + 2 y_0 v \ell + 2 (z_0 - z_c) w \ell +$$

$$x_0^2 + y_0^2 + (z_0 - z_c)^2 - r^2 = 0$$

Ray Tracing: Translation (2)

- Solution (Quadratic in ℓ)

$$a_q \ell^2 + b_q \ell + c_q = 0,$$

$$a_q = \hat{\mathbf{v}} \cdot \hat{\mathbf{v}} = 1 \qquad b_q = 2\left(\mathbf{x}_0 - \mathbf{x}_c\right) \cdot \hat{\mathbf{v}}$$

$$c_q = \left(\mathbf{x}_0 - \mathbf{x}_c\right) \cdot \left(\mathbf{x}_0 - \mathbf{x}_c\right) - r^2$$

- Zero to Two Real Solutions

$$\ell = \frac{-b_q \pm \sqrt{b_q^2 - 4a_q c_q}}{2a_q}$$

- Pick the “Right” One and Find Intersection

$$\mathbf{x}_A = \mathbf{x}_0 + \ell \hat{\mathbf{v}}$$

Ray Tracing: Refraction

- Find the Normal in Order to Apply Snell's Law

$$\hat{\mathbf{n}} = \frac{\mathbf{x} - \mathbf{x}_c}{\sqrt{(\mathbf{x} - \mathbf{x}_c) \cdot (\mathbf{x} - \mathbf{x}_c)}}$$

- At the New Origin from Translation Calculation

$$\mathbf{x}_A$$

- Compute the New Direction (Snell's Law in Vector Notation)

$$\hat{\mathbf{v}}' = \frac{n}{n'}\hat{\mathbf{v}} + \left[\sqrt{1 - \left(\frac{n}{n'}\right)^2 \left[1 - (\hat{\mathbf{v}} \cdot \hat{\mathbf{n}})^2\right]} - \frac{n}{n'}\hat{\mathbf{v}} \cdot \hat{\mathbf{n}}\right]\hat{\mathbf{n}}$$

Ray Tracing: Close

- After Iterating Translation and Refraction as Needed...

- Answer Some Question

  – "Where is Intersection with Paraxial Focal Plane?" ...

$$\mathbf{x}_B \cdot \hat{z} = z_{close}$$

$$(\mathbf{x}_A + \ell \hat{\mathbf{v}}_1) \cdot \hat{z} = z_{close}$$

  – or Any of a Collection of More Complicated Questions

Ray Tracing: Report (and More)

- Spot–Diagram (*eg. vs.* Field Position or Depth)
- Through–Focus Spot–Diagrams
- RMS or Maximum Spot Size (*eg.* XDL)
- Optical Path Length (*eg. vs.* Field Position)
- Many More
- Advanced Ideas (Many Commercial Programs)
  - Optimization (*eg.* Vary Radii of Curvature and Distances)
  - Use Vendor's Stock Lenses
  - Use Vendor's Existing Tools
- Commercial Optical Designers

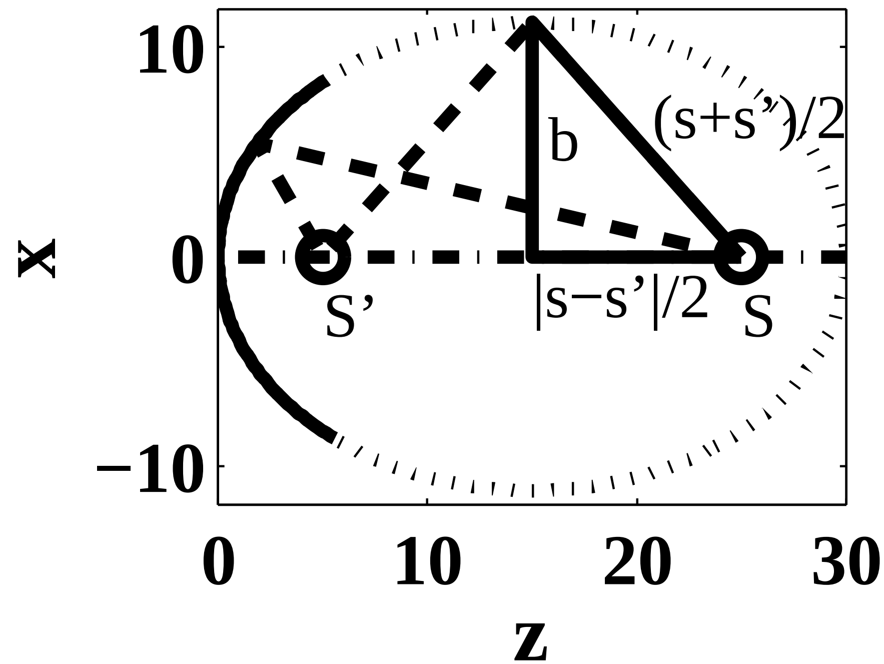
Ray Tracing

Take–Away Message

- Ray Tracing Gives Exact Answers (Except for Diffraction)
- Paraxial Rays Obey the Small–Angle Equations of Ch. 3.
- Diffraction–Limited System
  - Spot Less than $z\lambda/D$
  - Such a Design is "Good Enough."
- Commercial Programs Exist

Ellipsoidal Mirror (1)

- Path:
  - S to Surface to S'
- Fermat's Principle:
  - Minimal Time
- Imaging:
  - All Paths Minimal



$$\sqrt{(z-s)^2+x^2+y^2}+\sqrt{\left(z-s'\right)^2+x^2+y^2}=s+s'$$

Ellipsoidal Mirror (2)

- Vertex at Origin

$$\left(\frac{z-a}{a}\right)^2+\left(\frac{x}{b}\right)^2+\left(\frac{y}{b}\right)^2=1$$

$$a=\frac{s+s'}{2}$$

$$b^2=\left(\frac{s+s'}{2}\right)^2-\left(\frac{s-s'}{2}\right)^2=ss'$$

$$\frac{1}{f}=\frac{1}{s}+\frac{1}{s'}=\frac{s'+s}{s's}=\frac{2a}{b^2}$$

$$f=\frac{b^2}{2a}$$

- Spherical Mirror ($s=s'$)

$$z^2+x^2+y^2=r^2$$

$$\boxed{r=s=2f}$$

- Parabolic Mirror

$$z=\frac{x^2}{4s'}+\frac{y^2}{4s'}\qquad(s\rightarrow\infty)$$

$$\boxed{z=\frac{x^2}{4f}+\frac{y^2}{4f}}$$

Ellipsoidal Mirror (3)

- Ellipse

$$\left(\frac{z-a}{a}\right)^2+\left(\frac{x}{b}\right)^2+\left(\frac{y}{b}\right)^2=1$$

$$f=\frac{b^2}{2a}$$

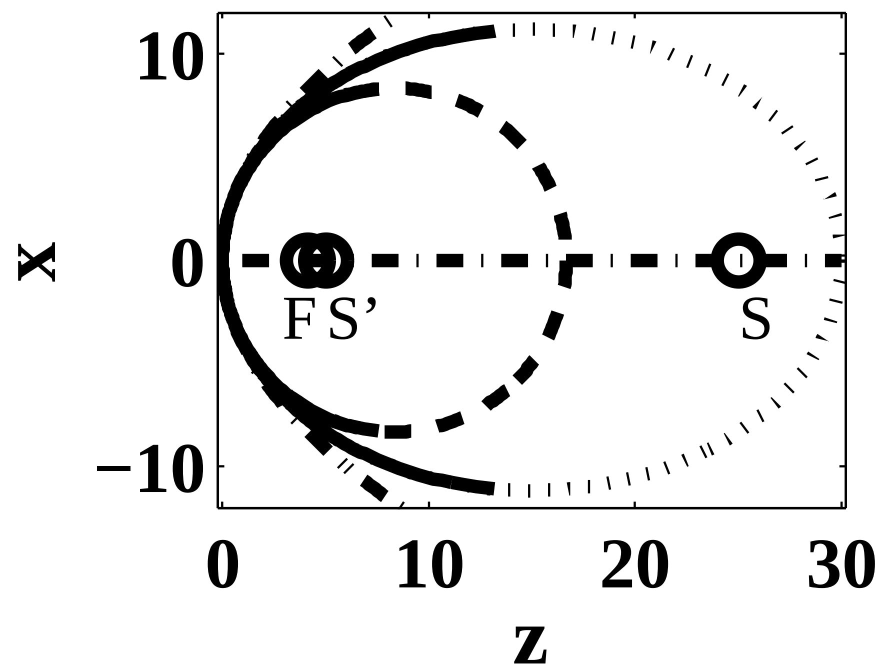
- Best–Fit Sphere

$$\boxed{r=2f}$$

- Best–Fit Parabola

$$\boxed{z=\frac{x^2}{4f}+\frac{y^2}{4f}}$$

- Ellipse Perfect for One Point Object
- Sphere or Parabola May be Best Overall



Mirror Aberrations: Definitions

- Match Second Derivatives at Origin (or See Previous Slide)

- Perfect Ellipsoid Defined by z, and Δ Represents OPL Error

$$z_{sphere} = z + \Delta_{sphere}/2 \qquad z_{para} = z + \Delta_{para}/2$$

$$(z-a)^2 = a^2 - \left(\frac{a}{b}\right)^2 \left(x^2+y^2\right) \qquad \text{(Ellipsoid)}$$

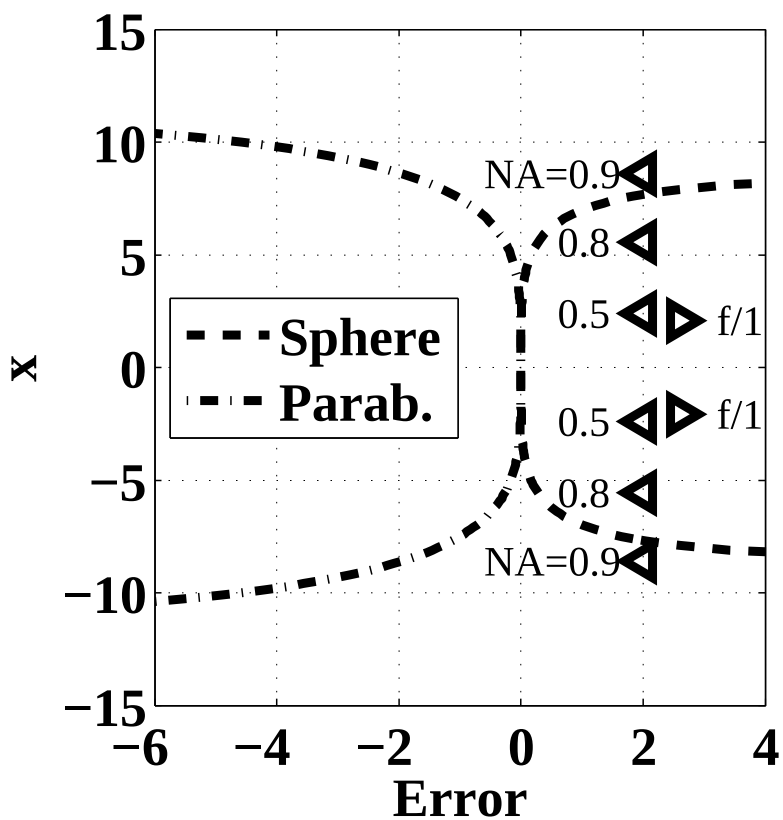
$$\left(z + \frac{\Delta_{sphere}}{2} - r\right)^2 = r^2 - x^2 - y^2 \qquad \text{(Sphere, } r = 2f\text{)}$$

$$z + \frac{\Delta_{para}}{2} = \frac{x}{4f} \qquad \text{(Paraboloid)}$$

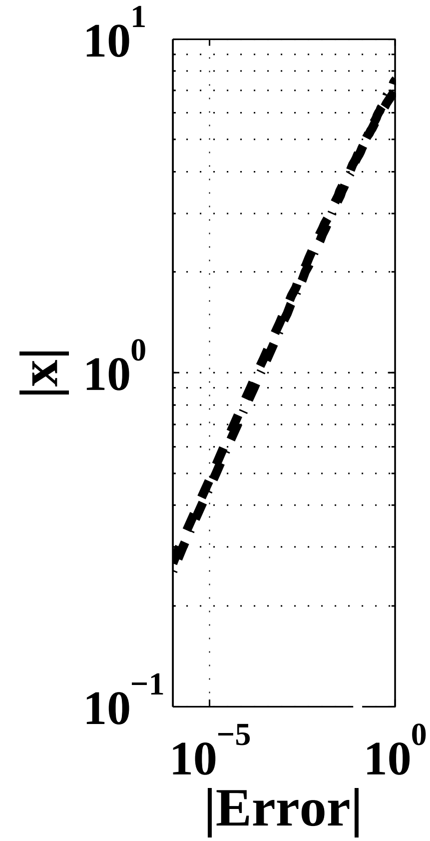
- Solve for Δ for Sphere or Parabola

Mirror Aberrations: OPL Errors

- Sphere or Parabola ok if $\Delta \ll \lambda$



Errors for $s = 25$ and $s' = 5$
Relative to Ellipsoid



Log Scale
Errors vary with $|x|^4$

Summary of Mirror Aberrations

- An ellipsoidal mirror is ideal to image one point to another.
- For any other pair of points, aberrations will exist.
- A paraboloidal mirror is ideal to image a point at its focus to infinity.
- Spherical and paraboloidal mirrors may be good approximations to ellipsoidal ones for certain situations and the aberrations can be computed as surface errors.
- Aberrations are fundamental to optical systems. Although it is possible to correct them perfectly for one object point, in an image with non–zero pupil and field of view, there will always be some aberration.
- Aberrations generally increase with increasing field of view and increasing numerical aperture.

Seidel Aberrations and OPL

- Small–Angle Approximation

$$\sin\theta \approx \theta$$

- Next–Best Approximation (Third Order)

$$\sin\theta \approx \theta + \frac{\theta^3}{3!}$$

- Wavefront Aberrations: $\Delta =$ Error in OPL (*eg.* Tilt)

$$\frac{d\Delta_{tilt}}{dx_1} = a_1 \approx \delta\theta$$

- Approach: Δ *vs.* Field Position and Pupil Position

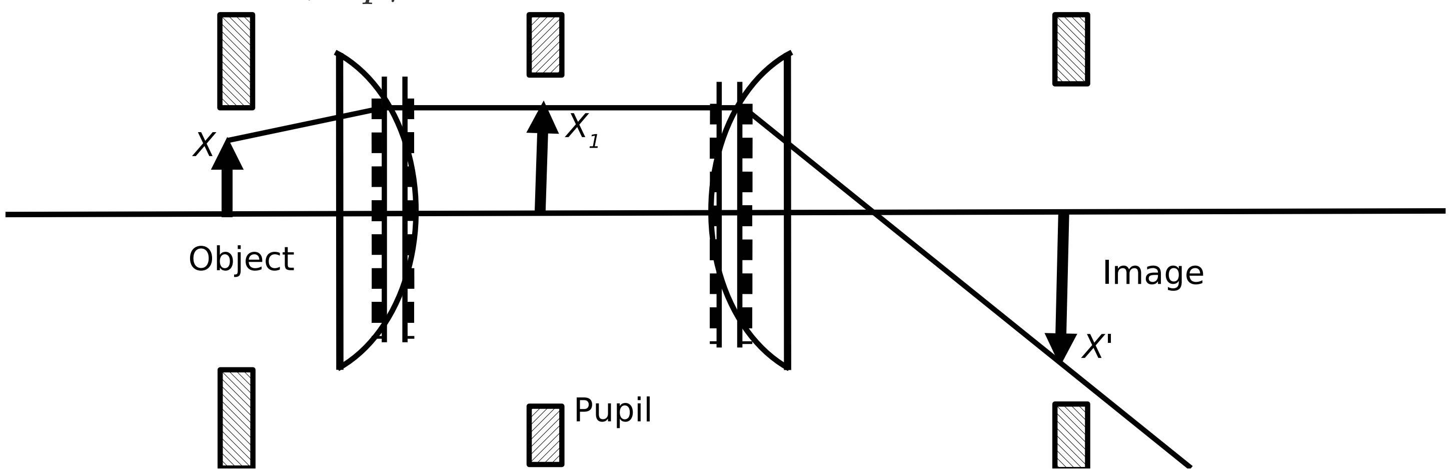
General Expression for Error

- Pupil Definitions, ρ, ϕ

$$x_1 = \rho d_p/2 \cos\phi$$

$$y_1 = \rho d_p/2 \sin\phi$$

- Field Point: $(x, 0)$
  - Defines Coordinates
- Expand Δ in ρ, ϕ, x
  - Even Orders to 4th



$$\begin{aligned}\Delta = \quad & a_0 + \\ & b_0 x^2 + b_1\rho^2 + b_2\rho x\cos\phi + \\ & c_0 x^4 + c_1\rho^4 + c_2 x^2\rho^2\cos^2\phi + \\ & c_3 x^2\rho^2 + c_4 x^3\rho\cos\phi + \\ & c_5 x\rho^3\cos^3\phi + \ldots\end{aligned}$$

The Easy Terms

- Constant Term (The Very Easy One)
  - "Piston" (Just a Phase Change)

$$a_0$$

- Tilt (Linear Terms Displace the Image)
- Quadratic Term
  - Defocus

$$b_1\rho^2$$

    * Can be Corrected
    * Does not Affect Image Quality
- That's the End of the Easy Ones

Spherical Aberration

- Previously Analyzed for Mirrors
- First Quartic Term

$$\Delta_{sa} = c_1 \rho^4 \qquad \text{(Spherical Aberration)}$$

- Analysis
  - ρ^2 is Focus
  - $c_1 \rho^2 \rho^2$ Means Focus Varies With ρ^2
  - Different Focus for Different Parts of Pupil: Blur
  - Blur Occurs Even for $x = 0$

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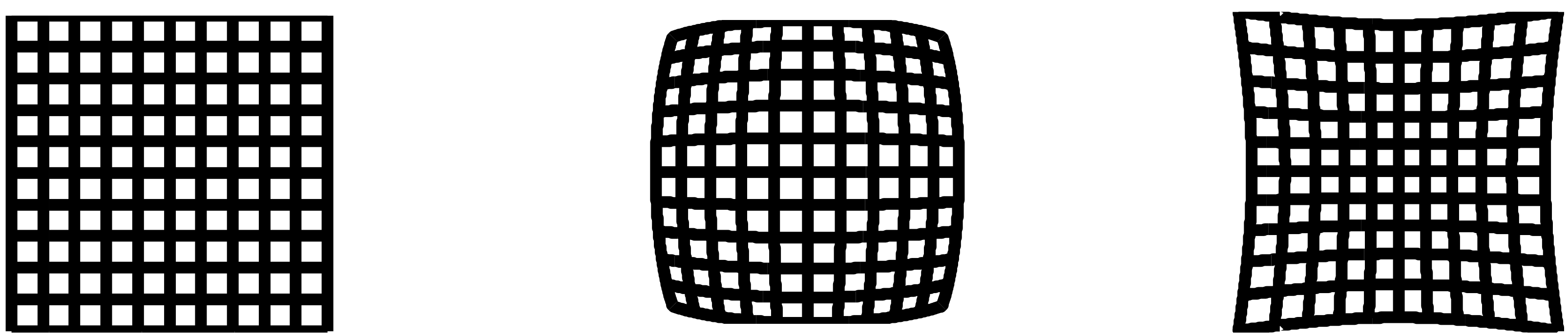
Distortion

- Quartic Terms

$$\Delta_d = c_4 x^3 \rho \cos\phi \qquad \text{(Distortion)}$$

- Analysis
  - $\rho\cos\phi$ Is Wavefront Tilt
  - $c_4 x^3 \rho\cos\phi$ Means Tilt Varies with x^3
  - Tilt Increases ($c_4 > 0$) or Decreases ($c_4 < 0$) as x^3
  - No Error at $x = 0$



Object Barrel Pincushion

Coma

- Quartic Terms

$$\Delta_c = c_5 x \rho^3 \cos^3 \phi \qquad \text{(Coma)}$$

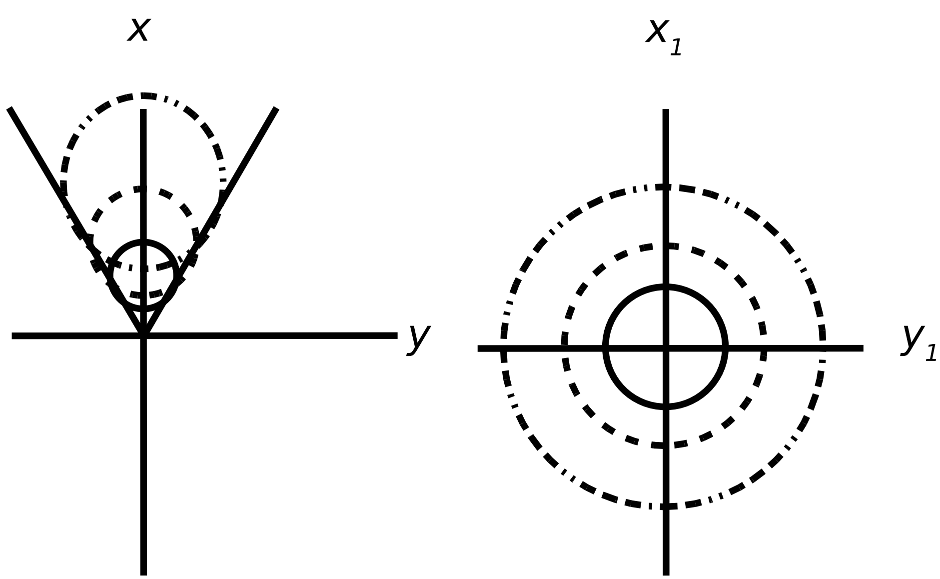


Image Rel. to Field Point (x)

Pupil

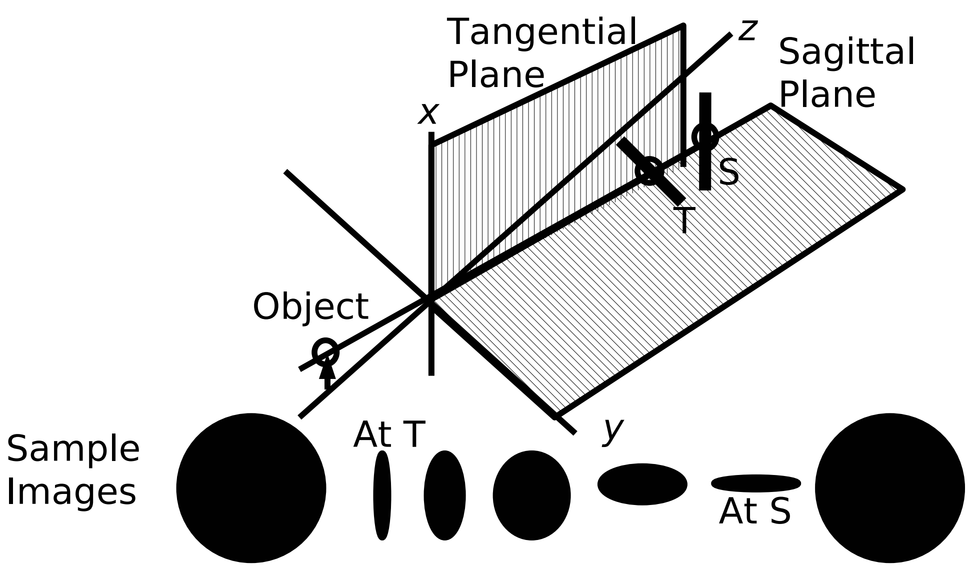
- Analysis
  - $\rho \cos \phi$ Is Wavefront Tilt
  - $c_5 x \rho^2 \cos^2 \phi \rho \cos \phi$ Means Tilt Varies
    - Linearly with x (No Error at $x = 0$)
    - Quadratically with $\rho \cos \phi$ (Symmetric in Pupil)
  - Comet–Like Image of a Point

Field Curvature and Astigmatism

- Quartic Terms

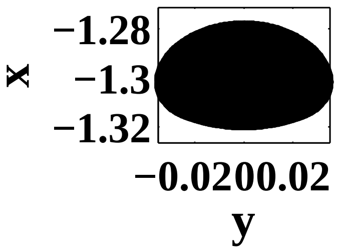
$$\Delta_{fca} = c_2 x^2 \rho^2 \cos^2\phi + c_3 x^2 \rho^2$$

(Field Curvature and Astigmatism)

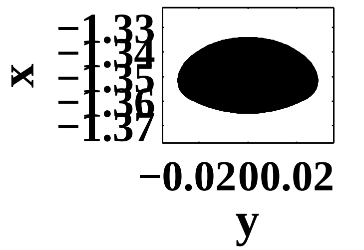


- Analysis
  - ρ^2 is Focus
  - $c_3 x^2 \rho^2$ Means Focus Varies with $c_3 x^2$ (Field Curvature)
  - $c_2 x^2 \cos^2\phi \rho^2$ Means Focus Varies with $c_2 x^2 \cos^2\phi$ (Astigmatism in $\cos^2\phi$)
  - No Effect at $x = 0$
  - Astigmatism Increases with x^2

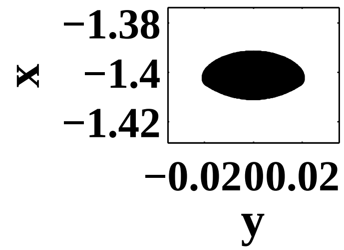
Astigmatism Examples (Ray Tracing)



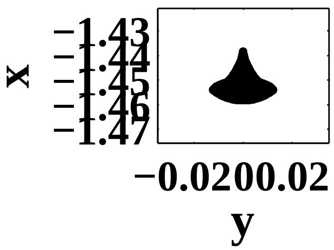
$z = 8.0$



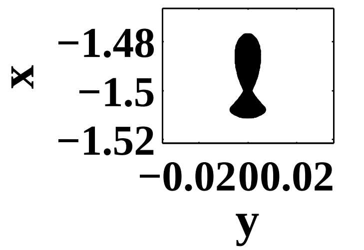
$z = 8.3$



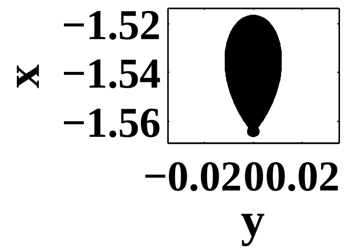
$z = 8.6$ (sag image)



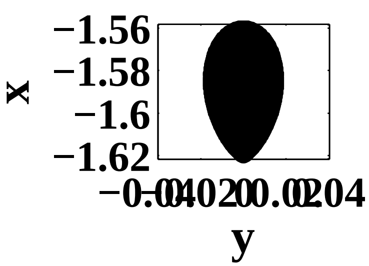
$z = 9.2$



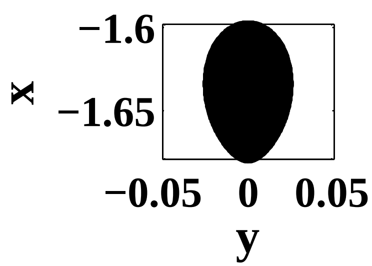
$z = 9.5$ (tan image)



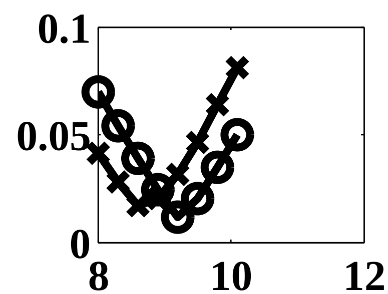
$z = 9.8$



$z = 10.1$



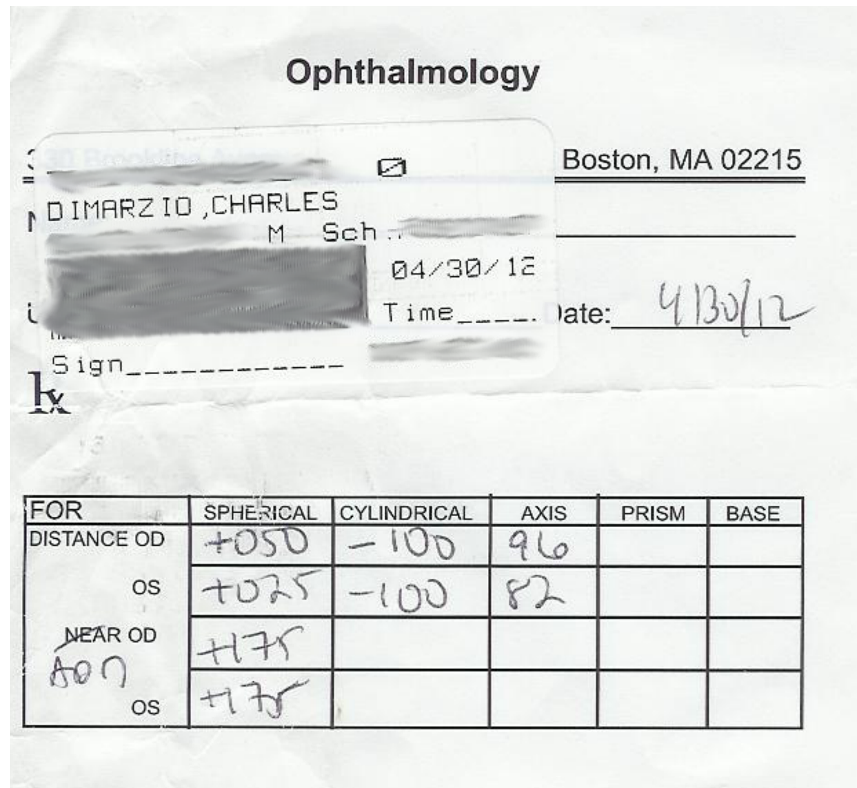
$z = 10.3$



Summary

"Deliberate Astigmatism"

- Setup
  - Cylindrical Lens
  - (With Spherical?)
  - Ellipsoidal Lens
- Result
  - Astigmatism On Axis
  - Different Paraxial Foci
- Some Applications
  - Eyes and Eyeglasses
  - CD Player Focusing
    - Quadrant Detector



Ophthalmology

[redacted] Boston, MA 02215

DIMARZIO ,CHARLES

[redacted] M Sch [redacted]

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Time____ Date: 4/30/12

Sign___________ [redacted]

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FOR	SPHERICAL	CYLINDRICAL	AXIS	PRISM	BASE
DISTANCE OD	+050	-100	96		
OS	+025	-100	82		
~~NEAR~~ ADD OD	+175				
OS	+175				

Seidel Aberrations Summary

	x^0	x^1	x^2	x^3
ρ^1	Tilt			Distortion
ρ^2	Focus		F. C. & Astig.	
ρ^3		Coma		
ρ^4	Spherical			

Expressions for aberrations. Aberrations are characterized according to their dependence on x and ρ.

On Axis the Only Aberration is Spherical

Aberrations

Take–Away Message

- Seidel Aberrations Cause Distortion and Blurring
- Only Spherical for On–Axis Object
- Aberrations Increase With Field of View and NA

Spherical Aberration in a Thin Lens: Coddington Factors

- Given s and s' What is the Best Thin Lens?

$$\frac{1}{f} = \frac{1}{s} + \frac{1}{s'} \qquad \frac{1}{f} = (n-1)\left(\frac{1}{r_1} - \frac{1}{r_2}\right)$$

- Two Unknowns, r_1 and r_2

- Definition: Coddington Position Factor (f, p: Lens Use)

$$p = \frac{s' - s}{s' + s} = \frac{1+m}{1-m}$$

- Coddington Shape Factor (f, q: Lens Manufacture)

$$q = \frac{r_2 - r_1}{r_2 + r_1}$$

Spherical Aberration in a Thin Lens: Computing Aberration

- Equations for Surface Radii

$$\frac{1}{f} = (n-1)\left(\frac{1}{r_1} - \frac{1}{r_2}\right)$$

$$r_1 = 2f\frac{q}{q+1}(n-1) \qquad r_2 = -2f\frac{q}{q-1}(n-1)$$

- Longitudinal Aberration a Function of Height in Pupil

$$\text{Diopters} \qquad L_s = \frac{1}{s'(x_1)} - \frac{1}{s'(0)} =$$

$$\boxed{\frac{x_1^2}{8f^3}\frac{1}{n(n-1)}\left(\frac{n+2}{n-1}q^2 + 4(n+1)pq + (3n+2)(n-1)p^2 + \frac{n^3}{n-1}\right)}$$

Spherical Aberration in a Thin Lens: Aberration Distances

- Focal Change in Diopters

$$\frac{1}{s'(x_1)} - \frac{1}{s'(0)} = \frac{s'(0) - s'(x_1)}{s'(x_1)s'(0)} \approx \frac{s'(0) - s'(x_1)}{s'(0)^2}$$

- Displacement of Focal Position

$$\Delta s'\left(x_1\right) \approx \left[s'\left(0\right)\right]^2 L_s\left(x_1\right)$$

- Transverse Displacement

$$\Delta x'\left(x_1\right) = x_1 \frac{\Delta s'\left(x_1\right)}{s'(0)}$$

Spherical Aberration in a Thin Lens: Minimizing Aberration

- Set Derivative to Zero

$$\frac{dL_s}{dq} = 0$$

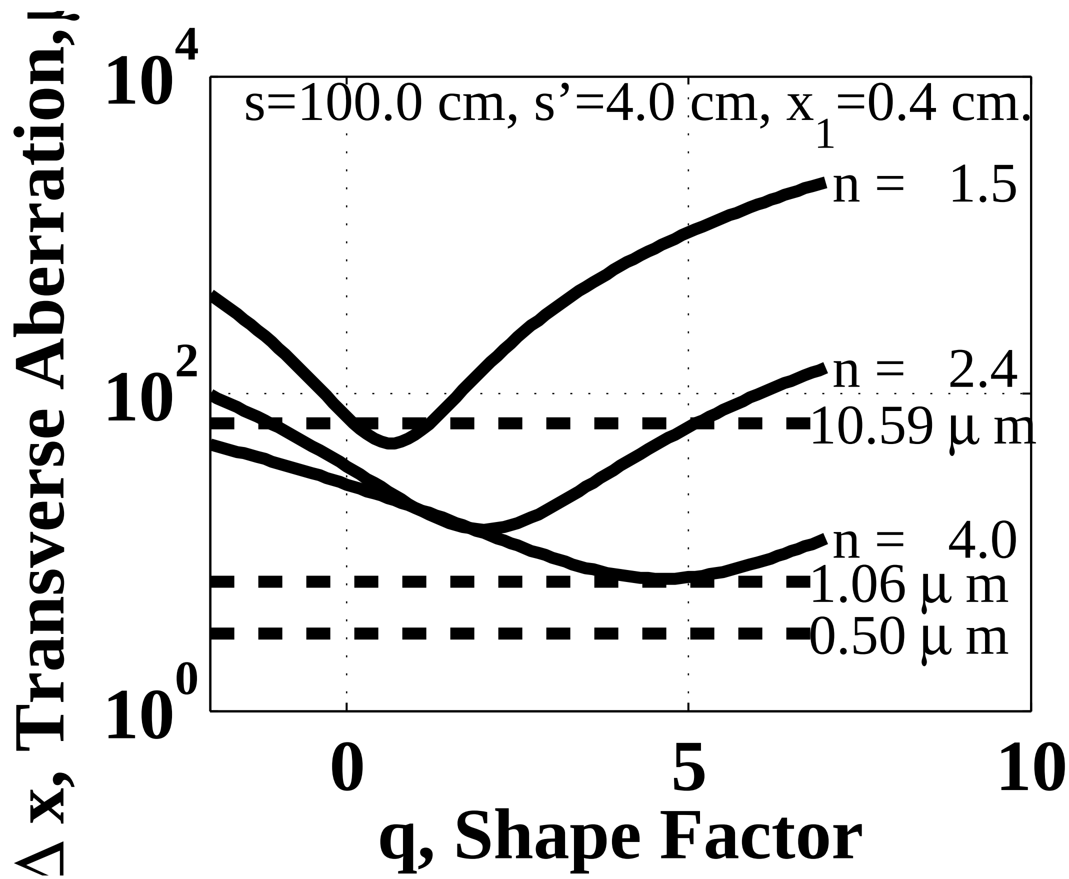
- Solve for Best q

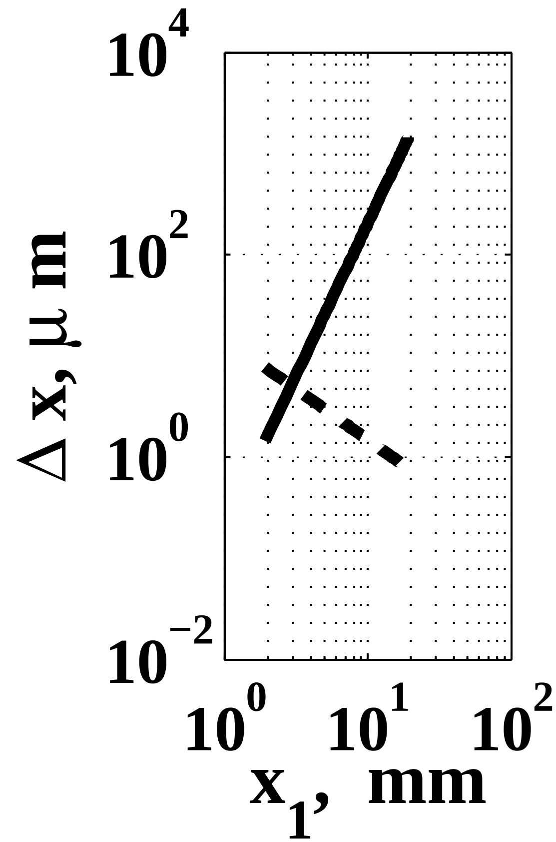
$$\boxed{q_{opt} = -\frac{2\left(n^2-1\right)p}{n+2}}$$

- Transverse Aberration: Varies with NA^3

$$\Delta x\left(x_1\right) = \frac{x_1^3 s'(0)}{8f^3}\left[\frac{-np^2}{n+2} + \left(\frac{n}{n-1}\right)\right]$$

Spherical Aberration Examples





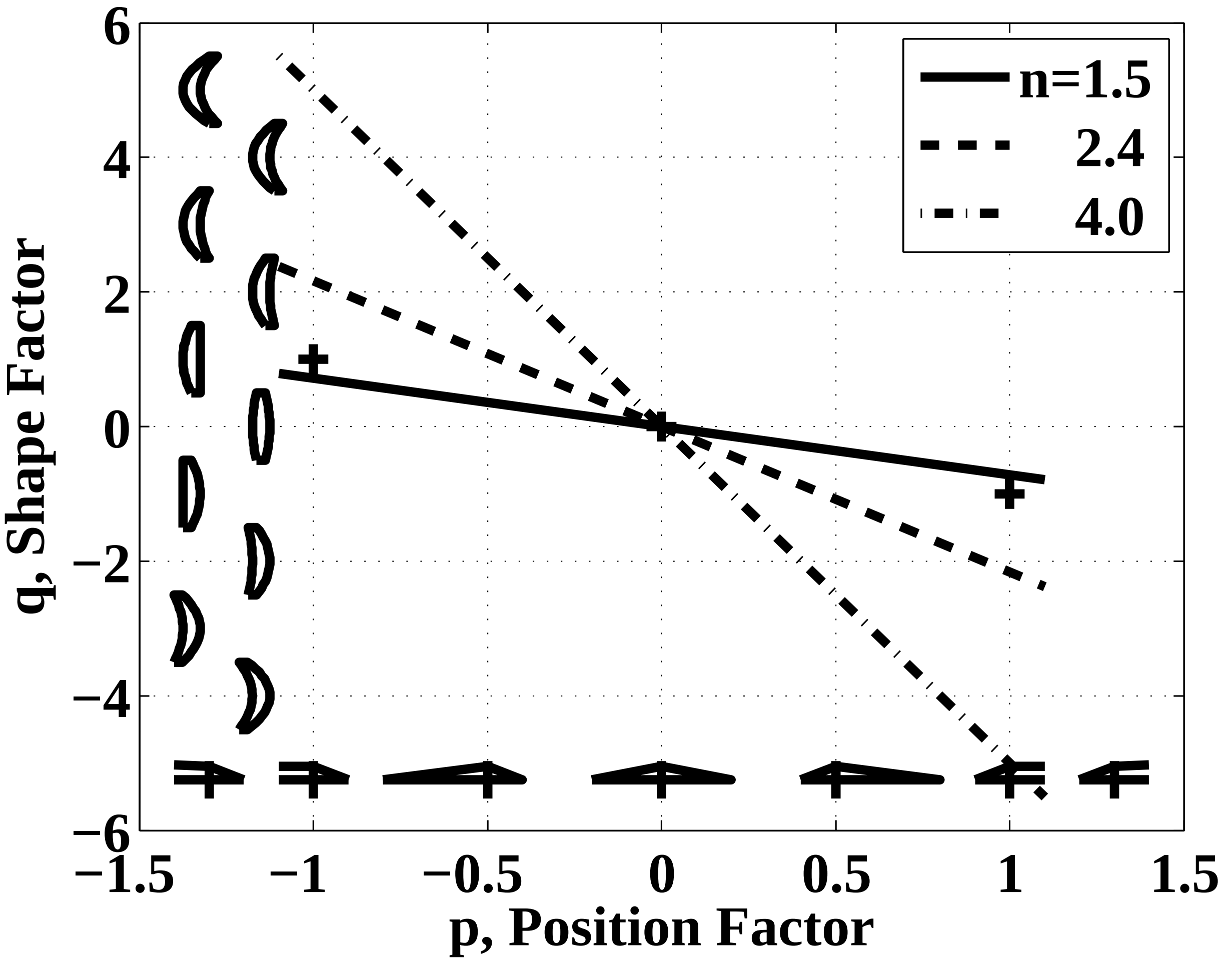
Minimum May be Good Enough
(or not)

$\Delta X \propto x_1^3$
$D.L. \propto 1/x_1$

$\approx f/5$ Lens. $\approx$Plano–Convex Best for Glass, Meniscus Best for Ge

Spherical Aberration in a Thin Lens: Designing the Lens

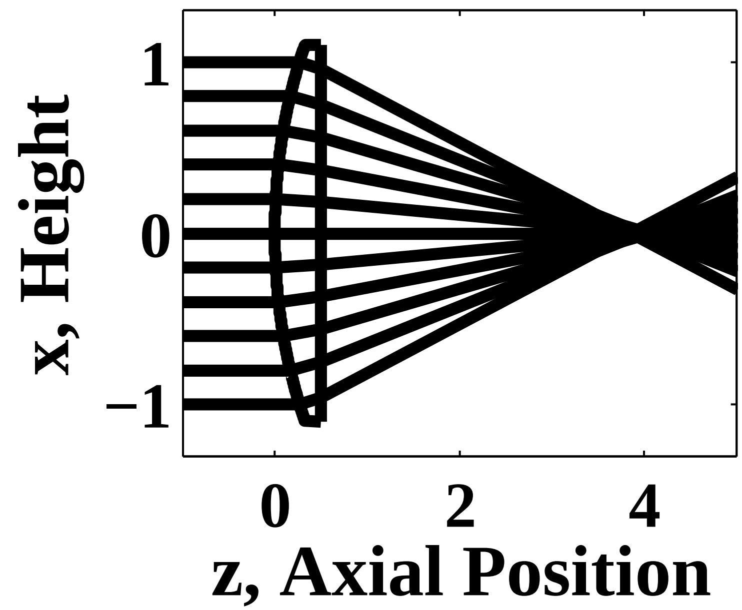
+ signs show Plano–Convex, Biconvex, Convex–Plano



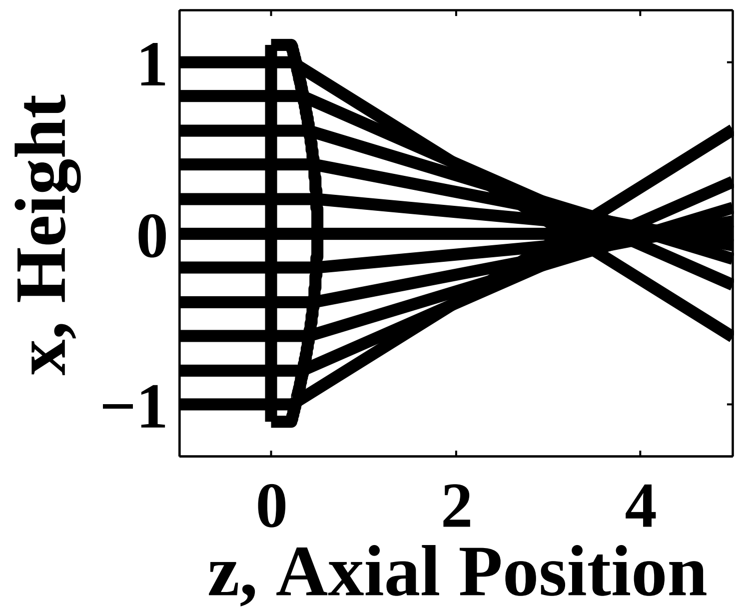
Guideline: Share the Bending

Take–Away Message

- Share the Bending for Best Aberration
- Watch Principal Planes
  - IR Detector Lens Example



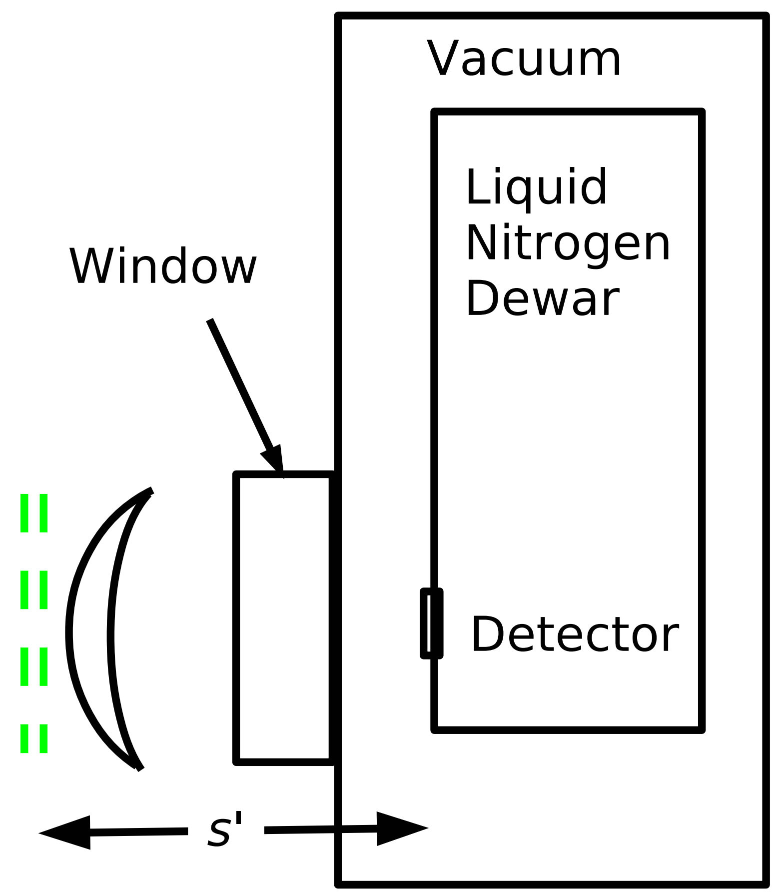
A. Correct Orientation



B. Reversed

IR Detector Lens Problem

- Small Detector
- Inside Dewar
- Germanium Lens
- Meniscus
- Short Focal Length
- High NA (Small Spot)
- Principal Planes
  - Outside Lens
  - Badly Positioned
- Bad Solution
  - Reverse the Lens
  - Focal Point on Detector
  - Bad Aberrations



Chromatic Aberration

- Focal Length Depends on Index of Refraction
- Index of Refraction Depends on Wavelength
  - See Glass Map in Ch. 1
- Different Colors Have Different Focal Lengths
  - Important for White Light Spectrum or a Portion of It
  - Important for Multi–Wavelength Systems (*eg.* Fluorescence, $\lambda_{excitation} \neq \lambda_{emission}$)
  - Important for Short Pulses ($\delta f = 1/\delta t$): Remember

$$\frac{\delta\lambda}{\lambda} = \frac{\delta f}{f} = \frac{1}{f\delta t} = \frac{1}{\text{Cycles per Pulse}}$$

- Correction is Possible
- Reflective Optics Eliminate Chromatic Aberration

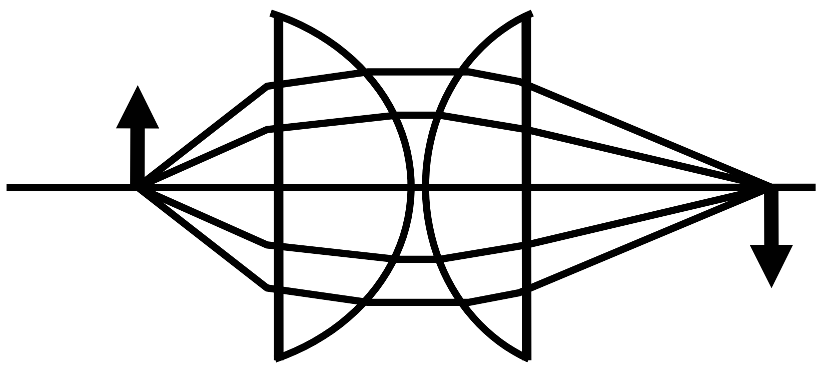
Lens Design Ideas

When One Doesn't Work . . .

Use Two (or more)

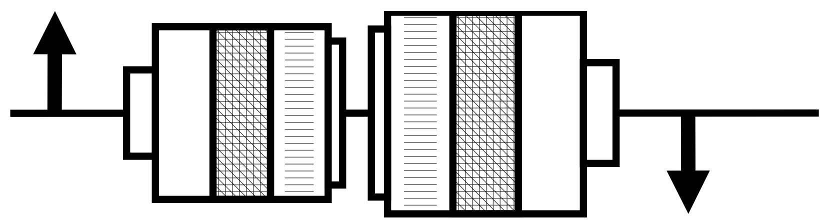
and Share the Bending

(The More the Better)

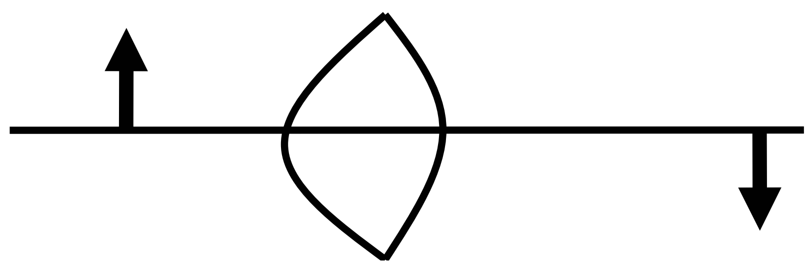


or "Let George Do It"

(Use Commercial Lenses)



or Try an Aspheric



1300min 18 Feb 2014