Optics for Engineers
Chapter 4

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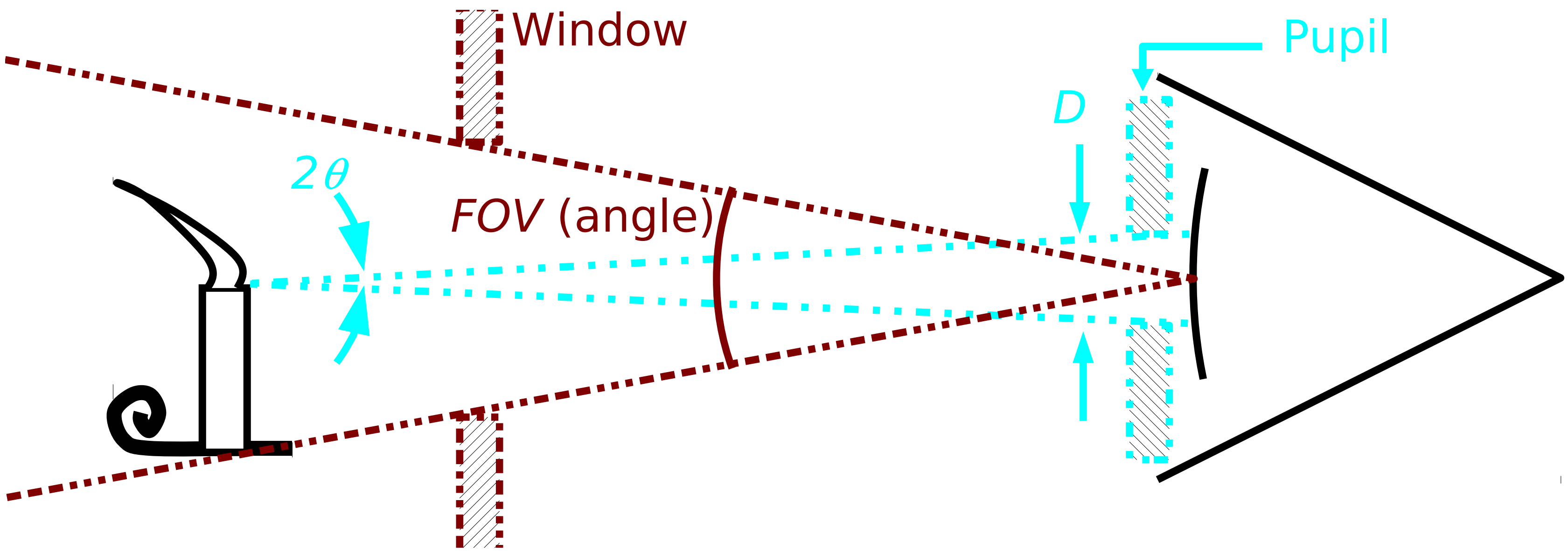
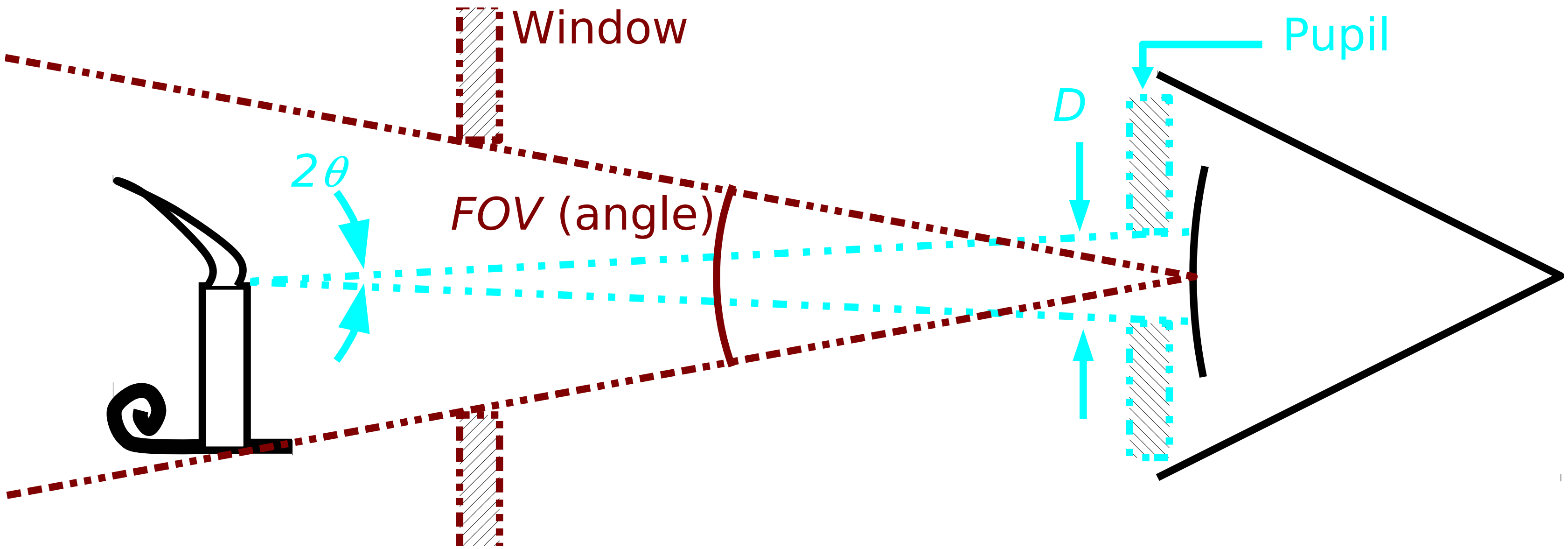
Stops



- Pupil Diameter, D, Limits Light Gathering Ability
  - Usually Defined by f–number or Numerical Aperture
- Window Limits Field of View
  - Usually Defined by Angle(s) or Linear Dimension(s)

Numerical Aperture



$$NA_{object} = n \sin\theta = n\frac{D/2}{\sqrt{s^2 + (D/2)^2}}$$

$$NA_{image} = n' \sin\theta' = n'\frac{D/2}{\sqrt{(s')^2 + (D/2)^2}}$$

F–Number and NA (1)

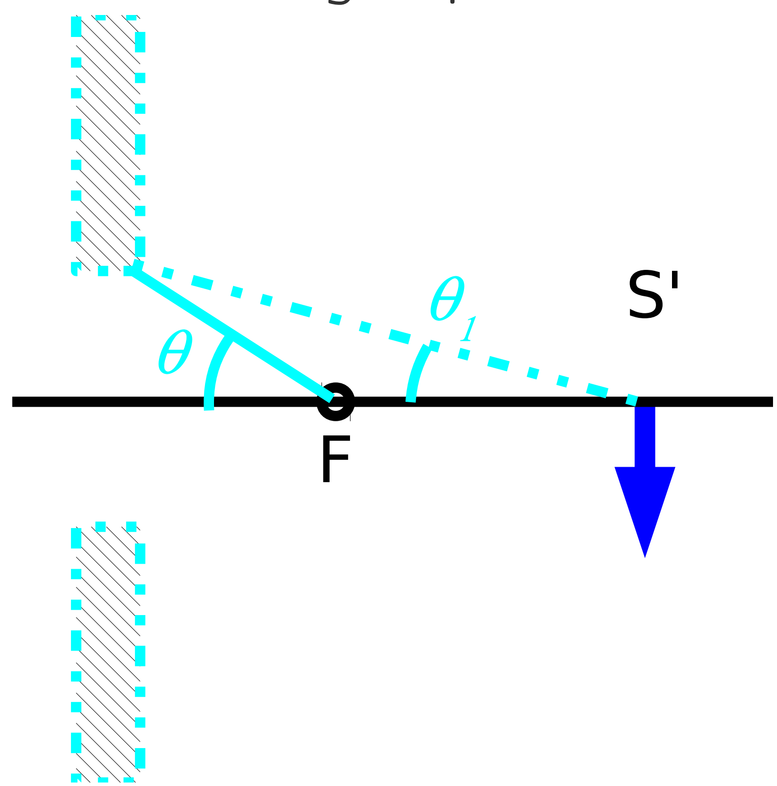
$$F = \frac{f}{D}$$

$$NA = n \sin\theta$$

Differences Summarized

	F	NA
Angle Vertex	Focal Point	Object or Image
Trig.	tan	sin
Dep.	Inv.	Lin.
"Fast" Lens	↓	↑
Aperture	Dia.	Rad.

Image Space



F–Number and NA (2)

$$F = \frac{f}{D} \qquad NA = n\sin\theta$$

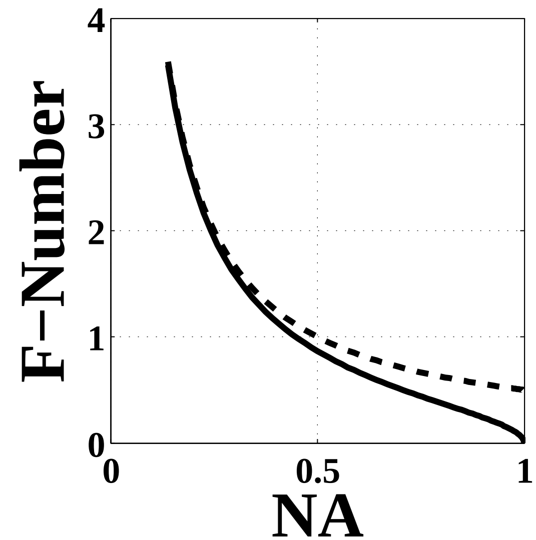
$$\frac{1}{f} = \frac{-m}{s'} + \frac{1}{s'} \qquad s' = (1-m)f$$

$$NA_{image} = n'\frac{1}{\sqrt{(|m-1|\times 2F)^2+1}}$$

$$NA_{object} = n\frac{1}{\sqrt{\left(\left|\frac{1}{m}-1\right|\times 2F\right)^2+1}}$$

Small NA, Large F

$$NA_{image} = n'\frac{1}{|m-1|\times 2F}$$



1:1 Relay ($m = -1$)

$$NA = \frac{1}{\sqrt{4F^2+1}}$$

Light–Gathering Ability

$$dP_{aperture} = I \times d\Omega$$

$$NA_{object} = n \sin\theta = n \frac{D/2}{\sqrt{s^2 + (D/2)^2}}.$$

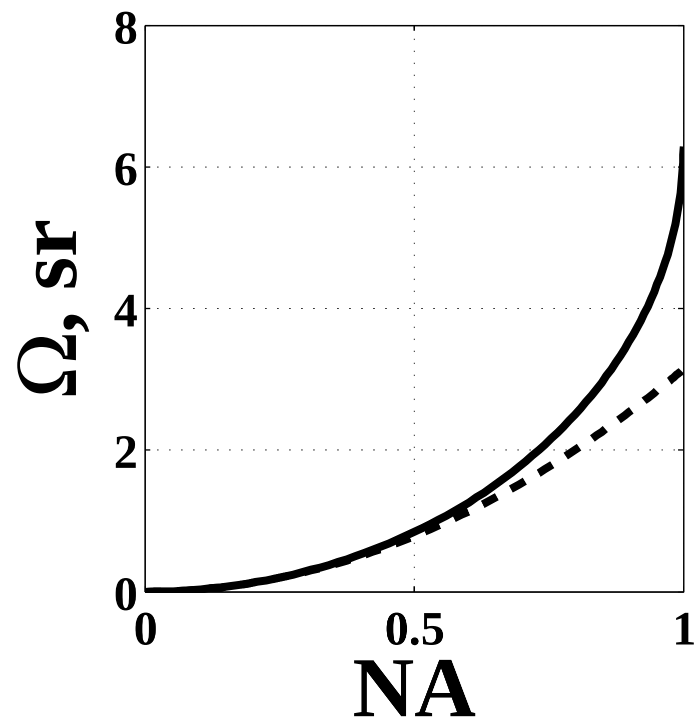
$$\Omega = 2\pi \left(1 - \sqrt{1 - \left(\frac{NA}{n}\right)^2}\right)$$

$$\Omega = \frac{\pi}{4}\left(\frac{D}{s}\right)^2 \qquad \text{(Small NA)}$$

For Constant I:

$$P = I\Omega$$

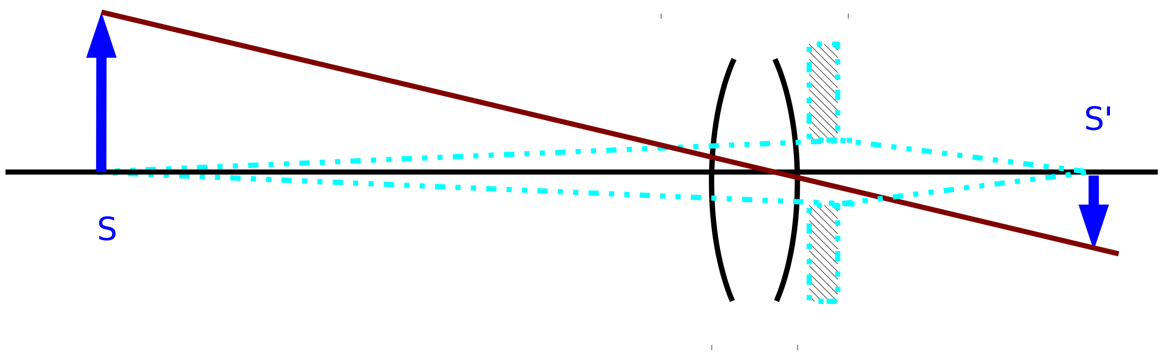
$$\Omega = \int_0^\theta \sin\theta' d\theta' d\phi'$$



Example: Camera (1)

Object: $dI = LdA$ (W/sr)
(Radiance, L in Ch. 12)
Change x_{pixel} to x'_{pixel} in Text

Object Distance	s	1000m
Camera Pixel	x'_{pixel}	7.4μm
Lens	f D	9mm $f/2$



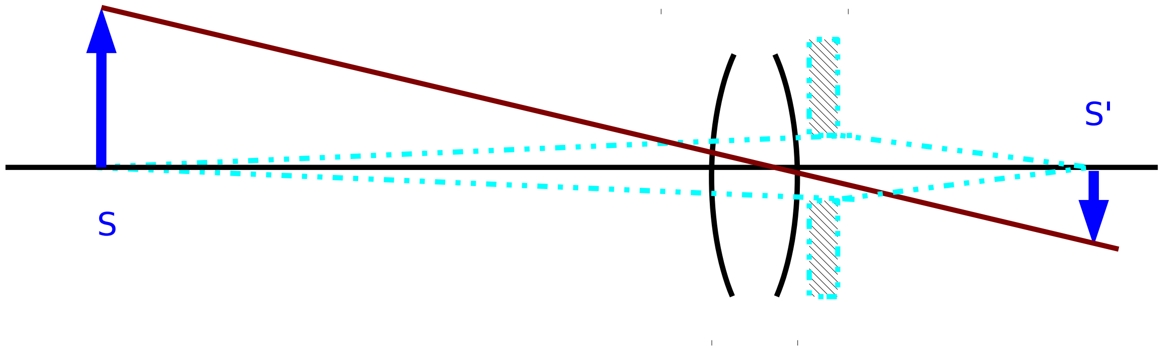
$$x_{pixel} = x'_{pixel}/m = \left(7.4 \times 10^{-6}\text{m}\right) / \left(9 \times 10^{-6}\right)$$

Area =0.68m^2

Intensity of Bright Scattered Sunlight
(1/4 in the Visible: Ch. 12)

$$\frac{1000\text{W/m}^2}{\pi} \times 0.25 \times 0.68\text{m}^2 \approx 50\text{W/sr}$$

Example: Camera (2)



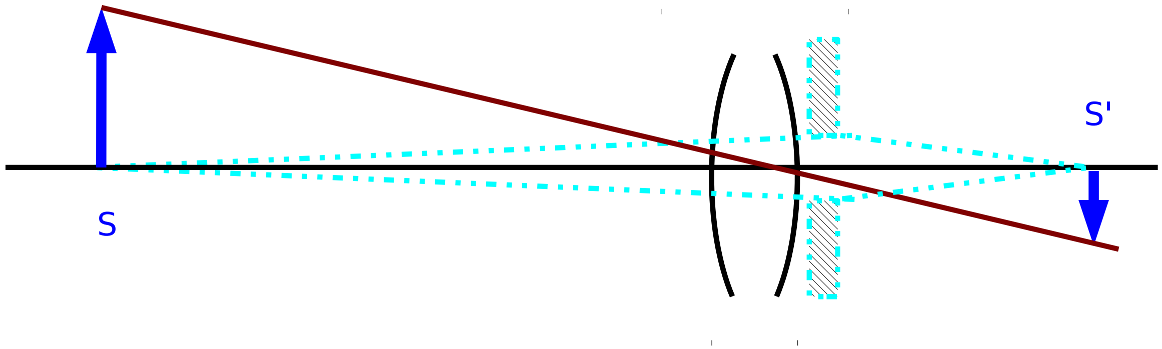
Intensity of Scattered Sunlight: 50W/sr

$$NA_{Object} \approx \frac{D}{2s} = \frac{f}{2Fs} = \frac{0.009\text{m}}{4 \times 1000\text{m}} = 2.25 \times 10^{-6}$$

$$\Omega \approx \pi NA^2 = 1.6 \times 10^{-11}\text{sr}$$

$$dP_{aperture} = dI\Omega = 50\text{W/sr} \times 1.6 \times 10^{-11}\text{sr} = 7.9 \times 10^{-10}\text{W}$$

Example: Camera (3)



$$dP_{aperture} = dI\Omega = 50\text{W/sr} \times 1.6 \times 10^{-11}\text{sr} = 7.9 \times 10^{-10}\text{W}$$

Photon Energy: $h\nu = hc/\lambda$

Photons (Lots of 'em!):

$$N = \frac{dP_{aperture}}{h\nu}\eta t$$

Wavelength (Green)	λ	500nm
Quantum Efficiency	η	0.4
Frame Time	t	1/30sec
Electrons	N	2.7×10^6

Example: Camera (4) Voltage

- Oxide Capacitance / Area
  - Typical MOS Wafer
  - See Any Electronics Text

$$C_A = 10\text{nF/cm}^2$$

- Number of Electrons
  - Previous Page
  - 2×10^6
- Voltage on Pixel: 79V
  - Unreasonable!
- Typical Full Well
  - 10^4electrons
  - 0.3V
- Decrease Aperture
  - At least f/2 to f/20
  - Typical for Bright Sun

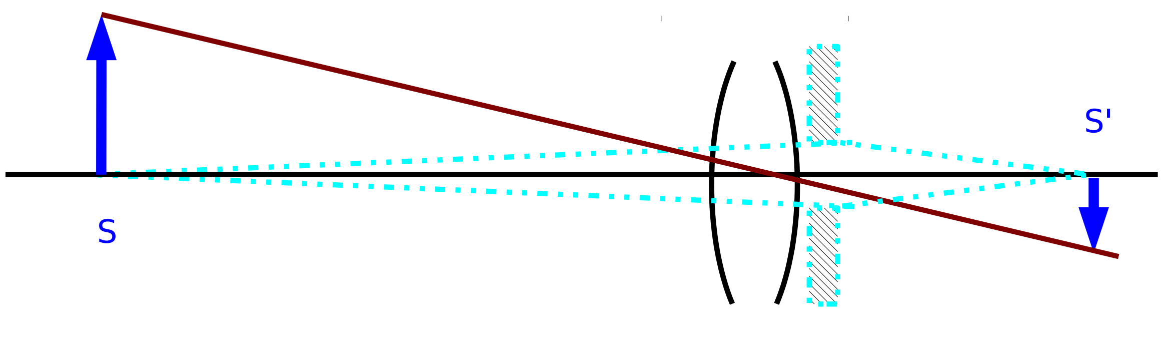
```
>> C_A=10*1e-9/1e-4
C_A =
   1.0000e-04
>> Area=(7.4e-6)^2
Area =
   5.4760e-11
>> capacitance=C_A*Area
capacitance =
   5.4760e-15
>> constant;
>> charge=2.7e6*q_electron
charge =
   4.3259e-13
>> voltage=charge/capacitance
voltage =
   78.9970
```

Camera Apertures

Abbe Invariant ($m_\alpha = 1/m$) Implies Constant Etendue (See Ch. 12)

$$A' = m^2 A \qquad \Omega' = \frac{1}{m^2}\Omega$$

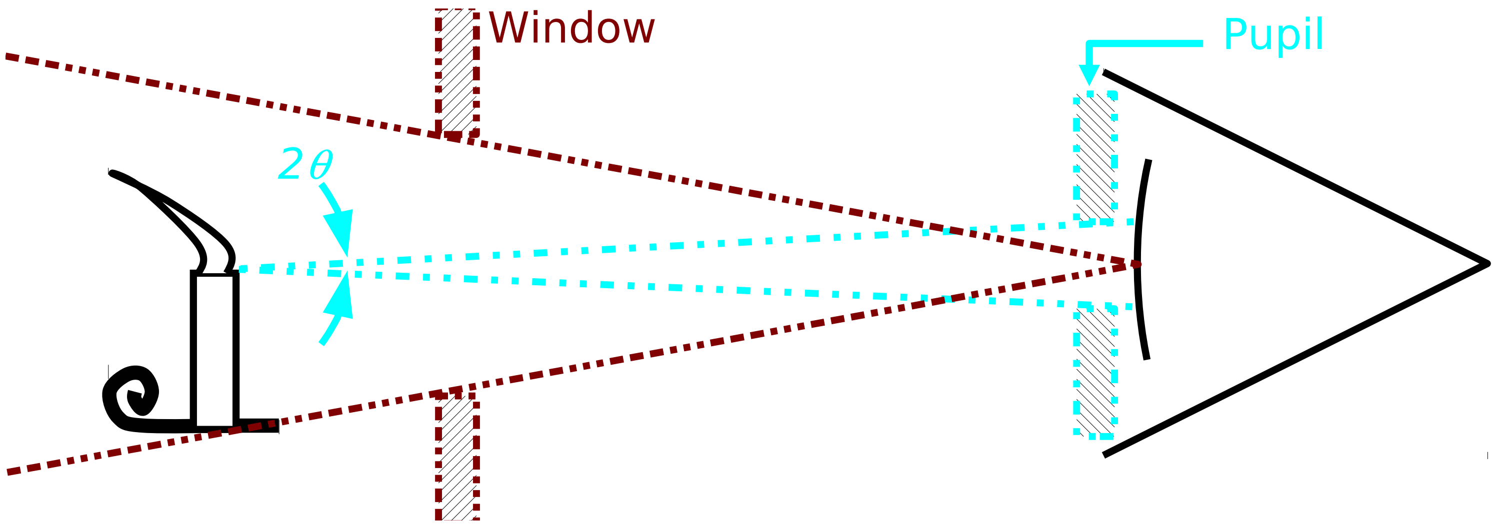
$$A\Omega = A'\Omega' \qquad LA\Omega = LA'\Omega'$$



Aperture Stops: Each Stop Is a Factor of 2

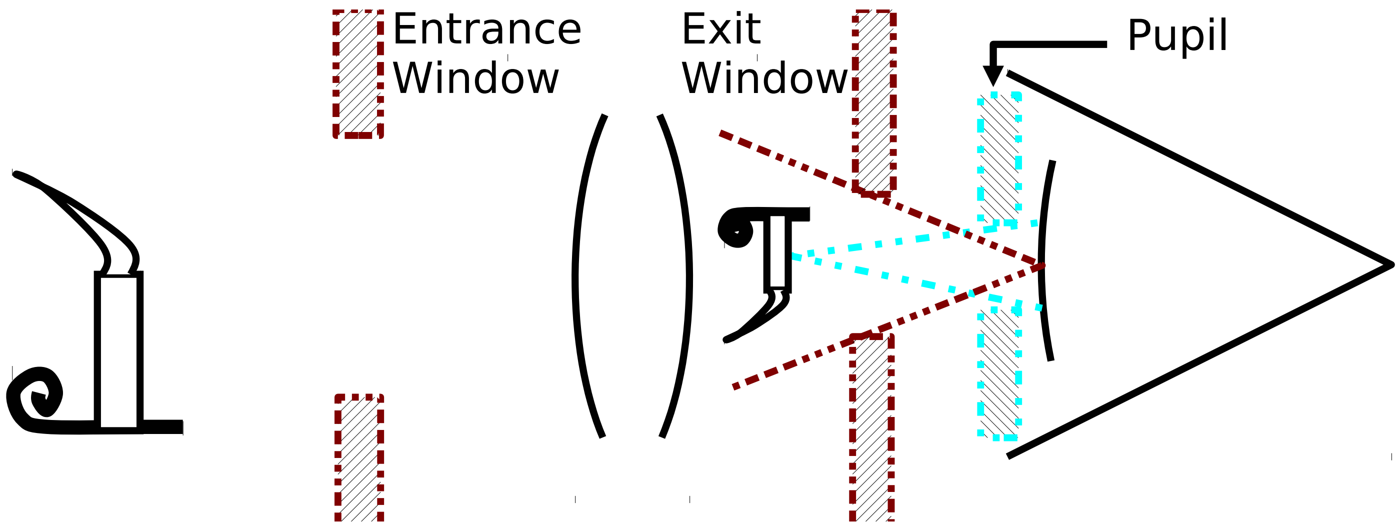
F, Indicated f–number	1.4	2	2.8	4	5.6	8	11
Actual f–number	$\sqrt{2}^1$	$\sqrt{2}^2$	$\sqrt{2}^3$	$\sqrt{2}^4$	$\sqrt{2}^5$	$\sqrt{2}^6$	$\sqrt{2}^7$
NA	0.3536	0.2500	0.1768	0.1250	0.0884	0.0625	0.0442
Ω, sr	0.3927	0.1963	0.0982	0.0491	0.0245	0.0123	0.0061

The Field Stop



- Simple Example as Shown
  - Window = Field Stop
  - Called the Entrance Window
- Field of View
  - Limits Size of Object (Diameter or Angle)...
  - ...or Size of Image

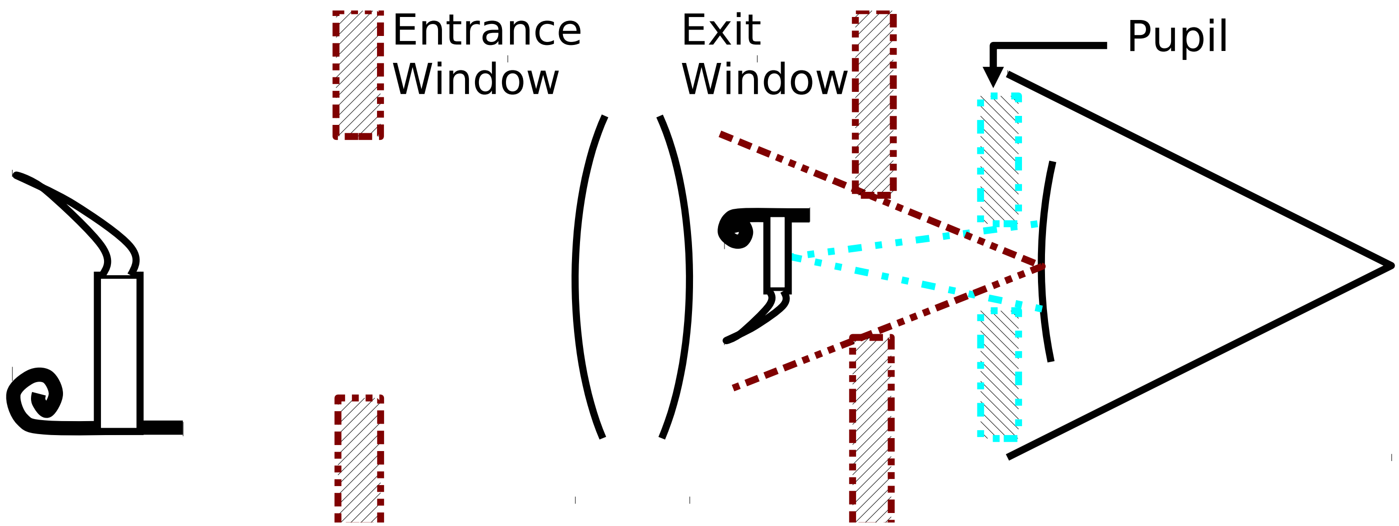
Exit Window: Example

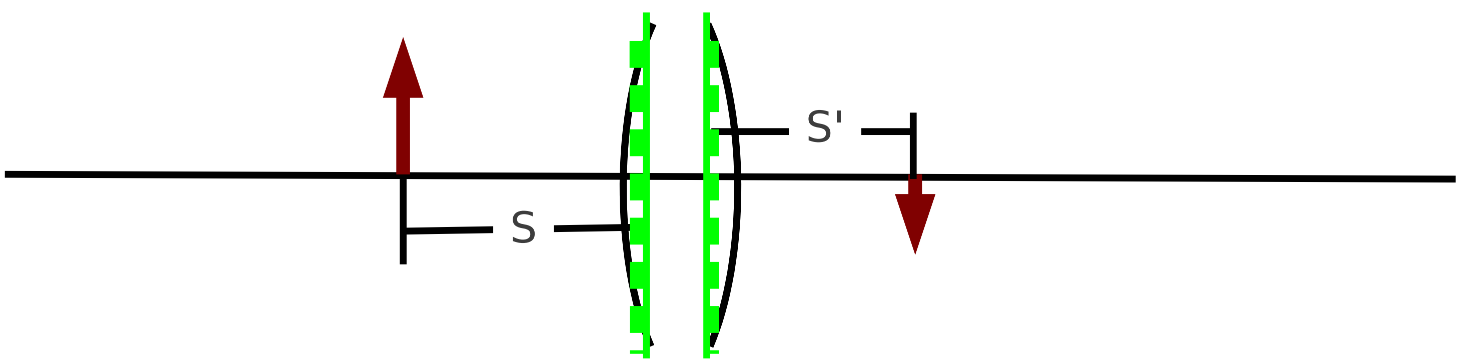


- Entrance Window Limits Size of Object
- Exit Window = Image of Field Stop
- Exit Window Limits Size of Image
- Location and Size from Imaging Equations
- Entrance and Exit Windows Real in this Example

800min 31 Jan 2014: Streaming video next week.

Finding the Exit Window





Use Imaging Equations: Location and Size

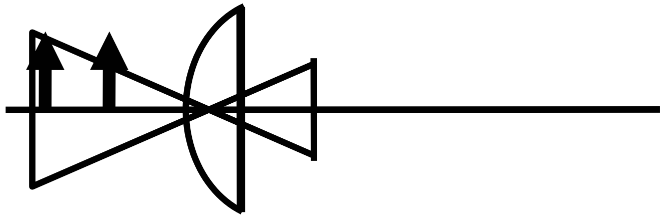
Camera FOV (1)

- Field of View Lmited in Image Rather than Object
  - Camera Chip is the Limit
  - 1/2.3in Compact Digital Camera
  - Diagonal Dimension = 11mm.
  - Image Field of View (Here Defined by Full Angle)

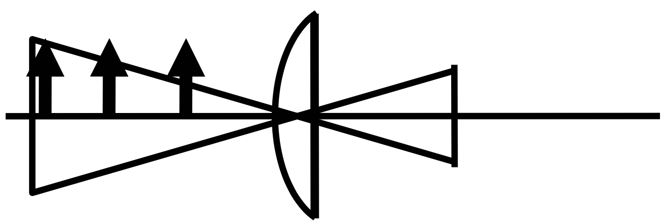
$$f = 10\text{mm} \qquad \text{(Normal Lens)} \qquad s \rightarrow \infty$$

$$FOV = 2 \arctan \frac{11\text{mm}/2}{10\text{mm}} = 58^\circ$$

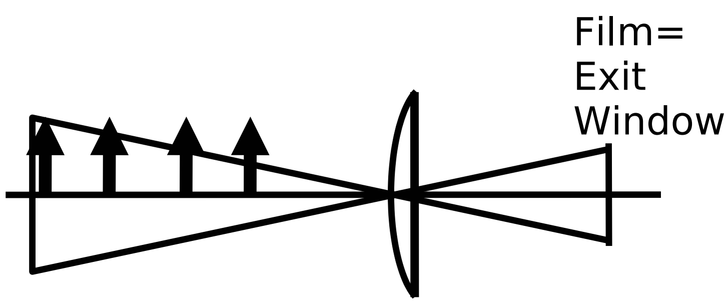
Camera FOV (2)

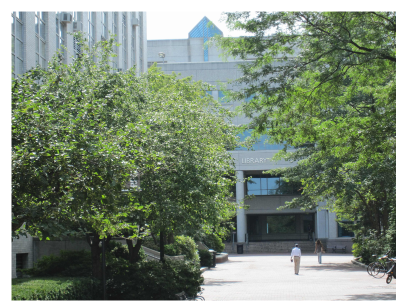


Wide–Angle Lens, $f = 5\text{mm}$



Normal Lens, $f = 10\text{mm}$





Telephoto Lens, $f = 20\text{mm}$

$$FOV = 2\arctan\frac{11\text{mm}/2}{f}$$

- Photographer Moved Away with Increasing f
- Same Linear FOV on the Building in Each Image
- Differences in Foreground Images

Typical Camera Lenses

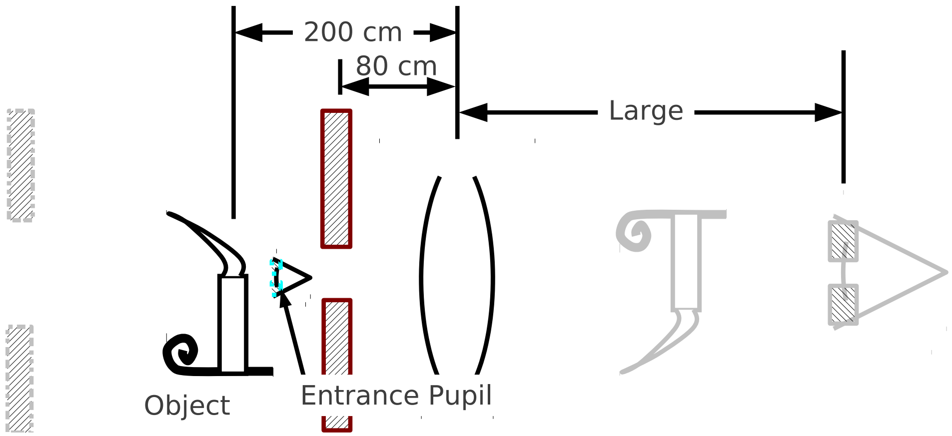
In–Practice

- Camera lens choices depend on exit window and application.
- Application determines field of view.
- Normal lens has 45 to 60 Degree FOV.

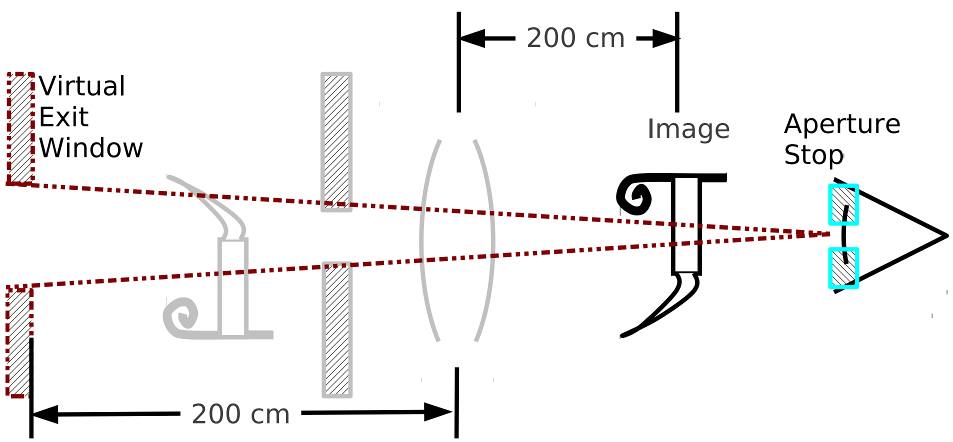
Application	35mm Camera	2/3–in Camera
Telephoto	>100mm	>20mm
Normal Lens	50mm	10mm
Wide–Angle	<30mm	<5mm

- These numbers are very rough guidelines

Another Example: Virtual Exit Window



$s = 200\text{cm},\ s_{fieldstop} = 80\text{cm}.$



$s' = 200\text{cm},\ s'_{fieldstop} = -400\text{cm}.$

$f = 100\text{cm}$

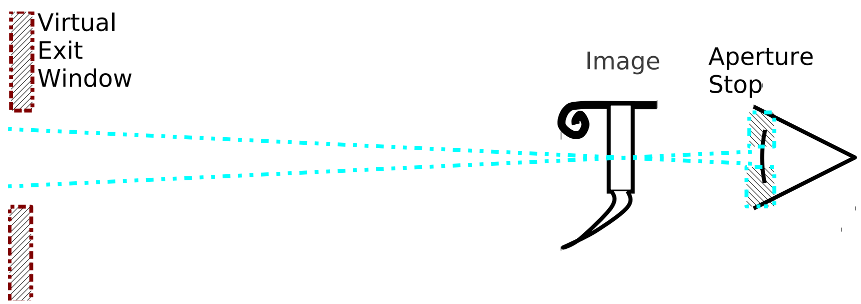
Field Stop Limits FOV (Top)

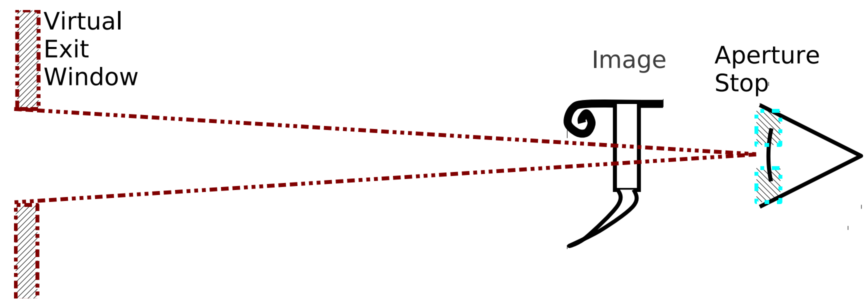
Virtual Exit Window Seen by Observer (Bottom)

Virtual Entrance Pupil in Object Space (Top)

Apertures Function Regardless of Apparent Order (Actual Sequence Matters).

Summary In Image Space

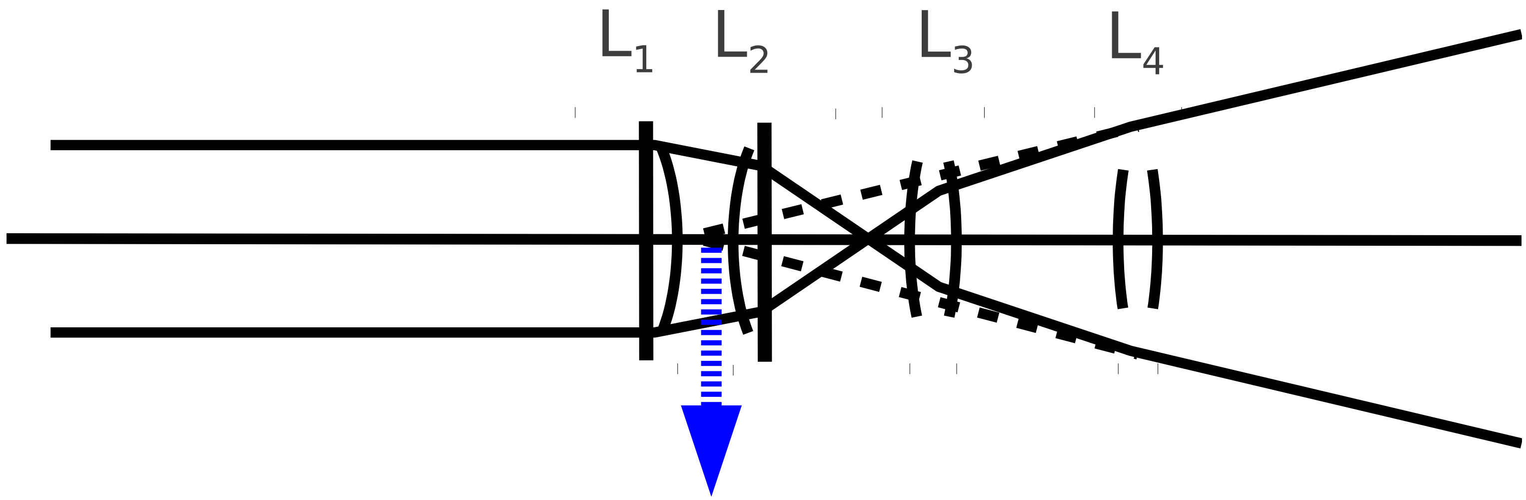




- Pupil Limits Light–Gathering Ability
- Cone of Rays From Image is Limited
- Solid Angle Determines Amount of Light Collected

- Window Limits Field of View
- Cone of Rays from Pupil is Limited
- FOV Defined by Angle or Linear Dimension

Where Are the Stops?



- Compound Lens
- Object to Left at Infinity
- Image as Shown
- Where Are the Stops Now?
  - Aperture Stop?
  - Field Stop?

- Important Concepts
  - Sequential Optics
  - Object and Image Space
  - Other Spaces (e.g. Infinity)

Object Space, Image Space, and Stop Definitions

Mapping from Object Space to Image Space through the Compound Lens

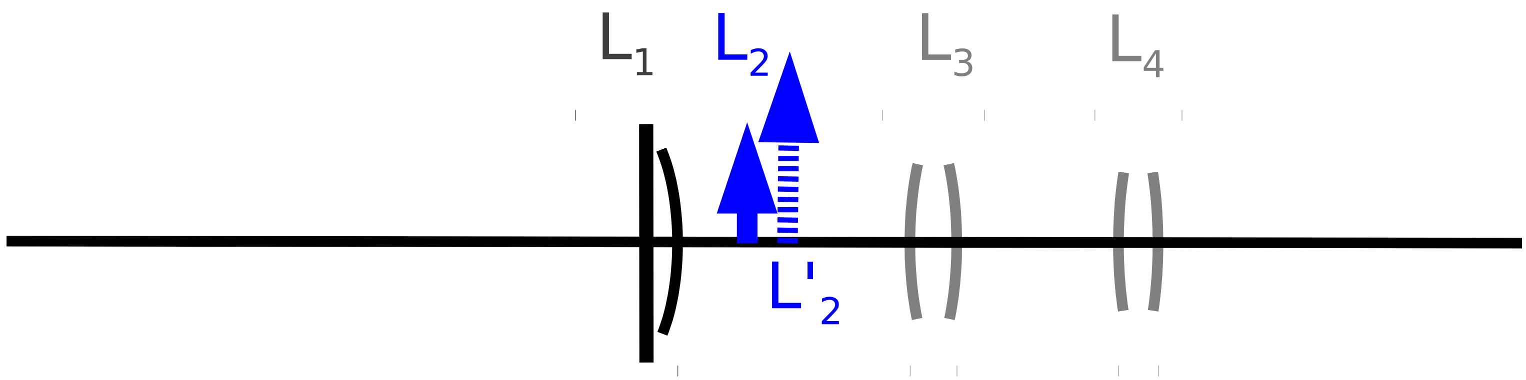
$$\frac{1}{s'} = \frac{1}{f} - \frac{1}{s} \qquad x' = -\frac{s'}{s}x$$

Pick one space and work in that.

Object Space	Physical Component	Image Space
Entrance Pupil: Image of Aperture Stop in Object Space. Limits Cone of Rays from Object	**Aperture Stop:** Limits Cone of Rays from Object which Can Pass Through the System.	**Exit Pupil:** Image of Aperture Stop in Image Space. Limits Cone of Rays from Image.
Entrance Window: Image of Field Stop in Object Space. Limits Cone of Rays From Entrance Pupil.	**Field Stop:** Limits Locations of Points in Object which Can Pass Through System	**Exit Window:** Image of Field Stop in Image Space. Limits Cone of Rays From Exit Pupil.

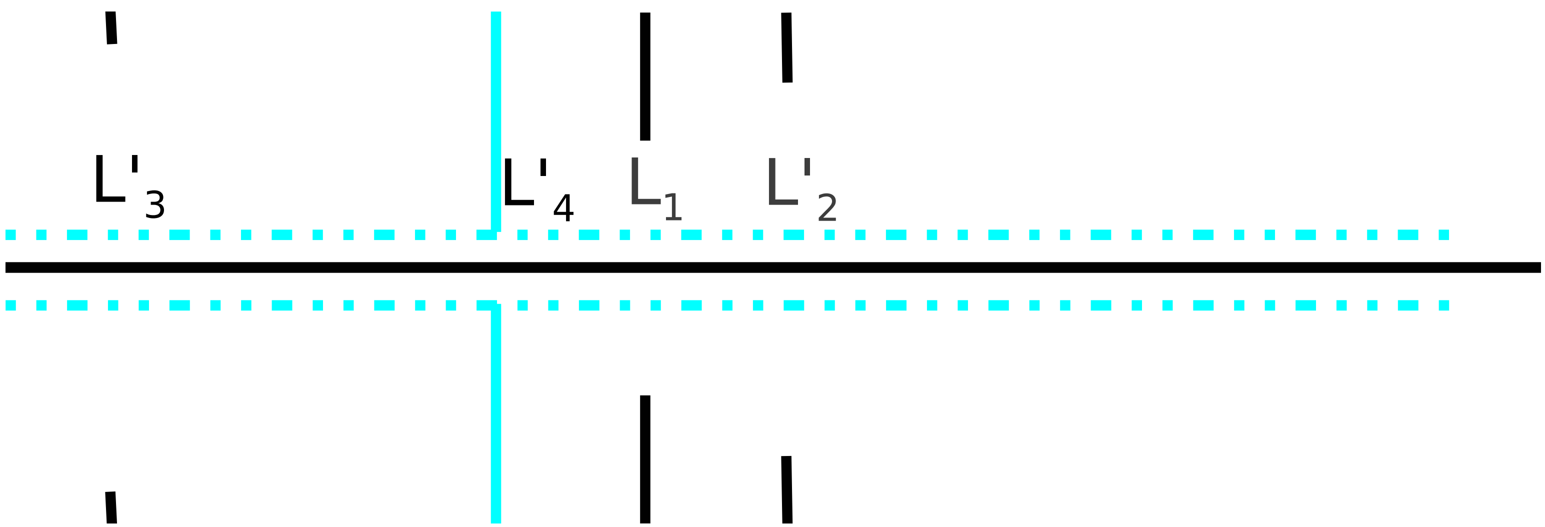
Finding the Stops in Object Space

- Find Each Lens as Seen in Object Space
  - Lens L_1
  - Lens L_2 as Seen Through L_1 $(=L_2')$
  - Lens L_3 as Seen Through L_2 and L_1 $(=L_3')$
  - Lens L_4 as Seen Through L_3, L_2 and L_1 $(=L_4')$



Finding the Entrance Pupil

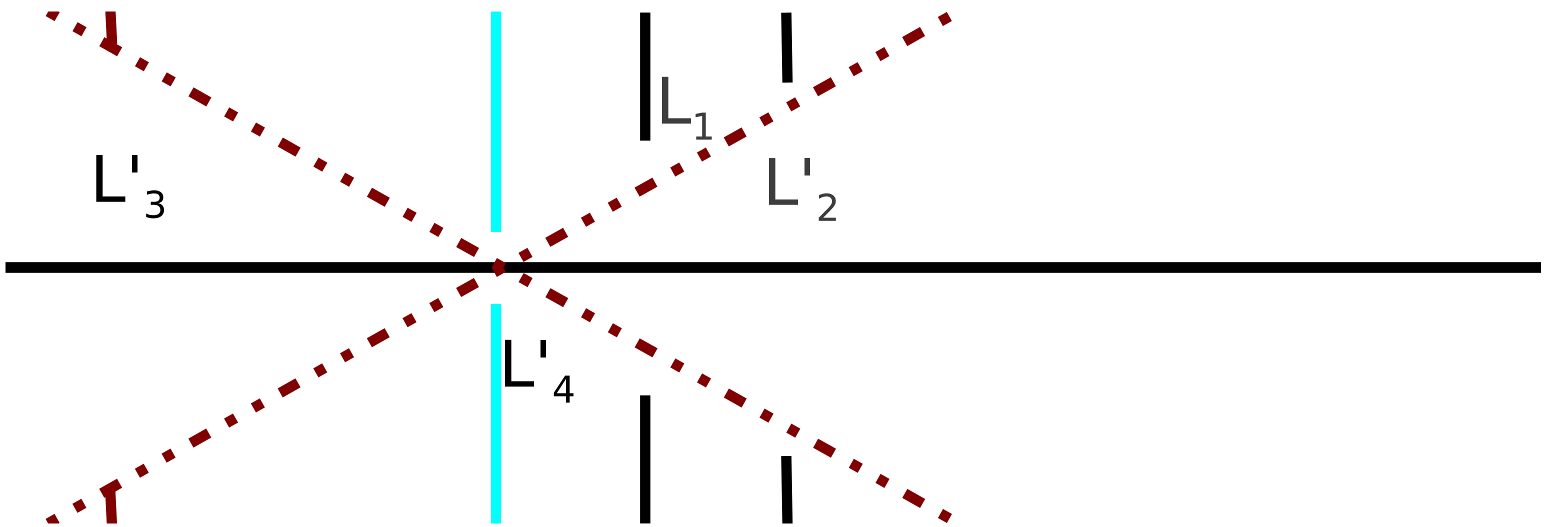
- Start at the Object (To the Left at Infinity Here)
- Find the Aperture that Limits Cone of Rays from Object
- Entrance Pupil is L'_4, Aperture Stop is L_4 (Smallest Angle)



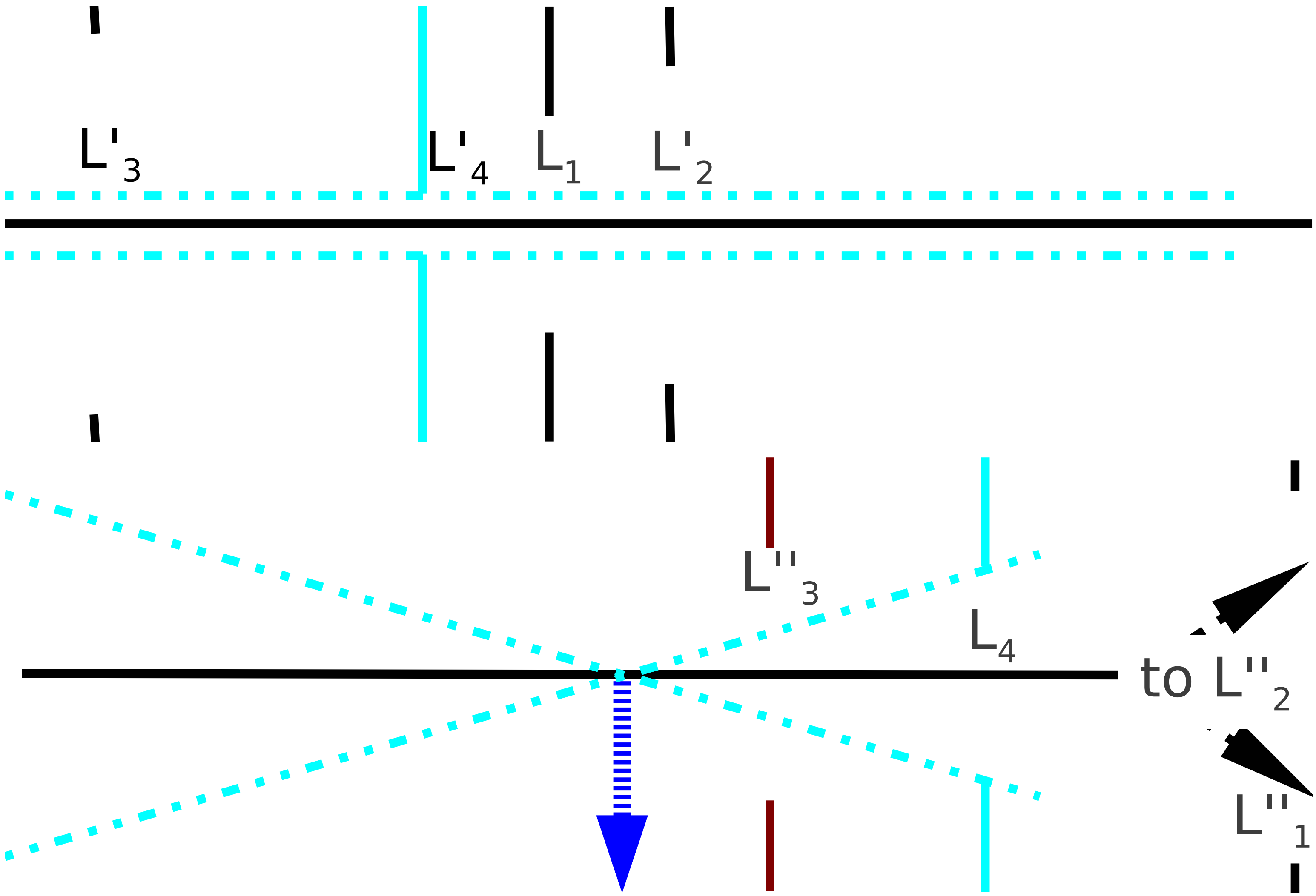
Finding the Entrance Window

- Start at the Entrance Pupil
- Find the Aperture that Limits Field of View from Pupil
- Entrance Window is L'_3, Field Stop is L_3
- Remember Entrance Pupil is L'_4, Aperture Stop is L_4
- The Remaining Apertures Don't Matter

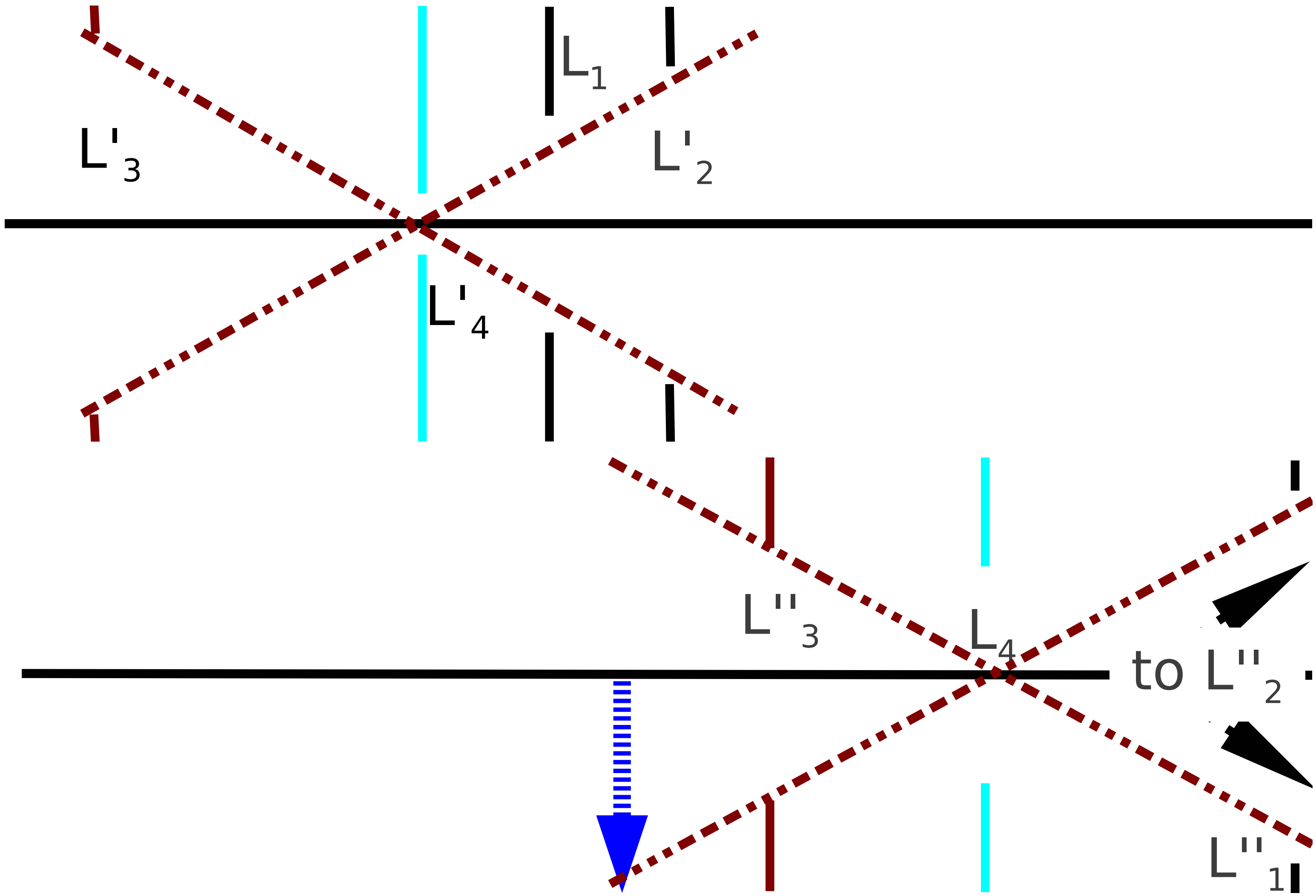
Fix figure in text (4-5-apexample.odg)



Object Space and Image Space: Entrance and Exit Pupils

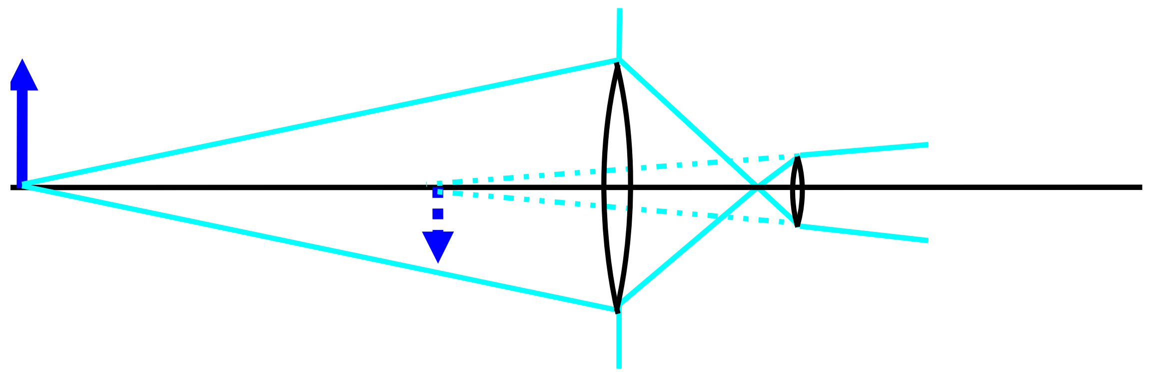


Object Space and Image Space: Entrance and Exit Windows

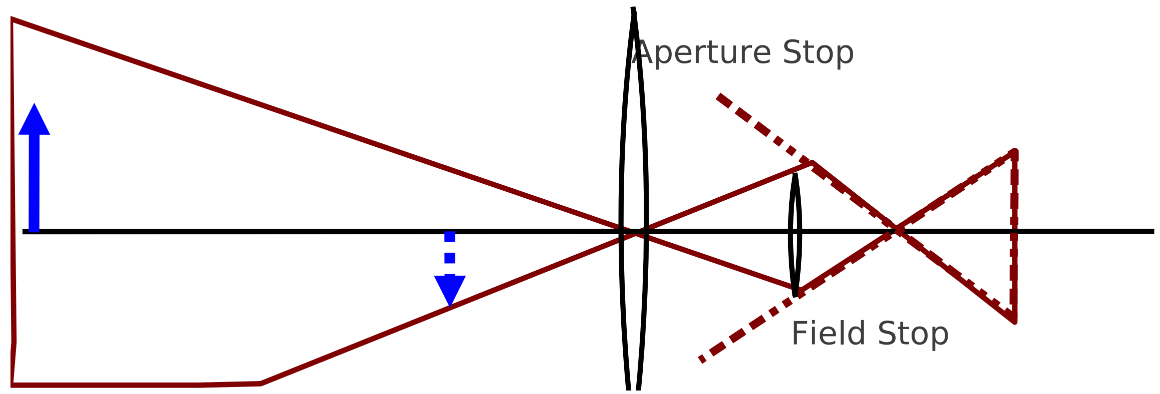


The Telescope

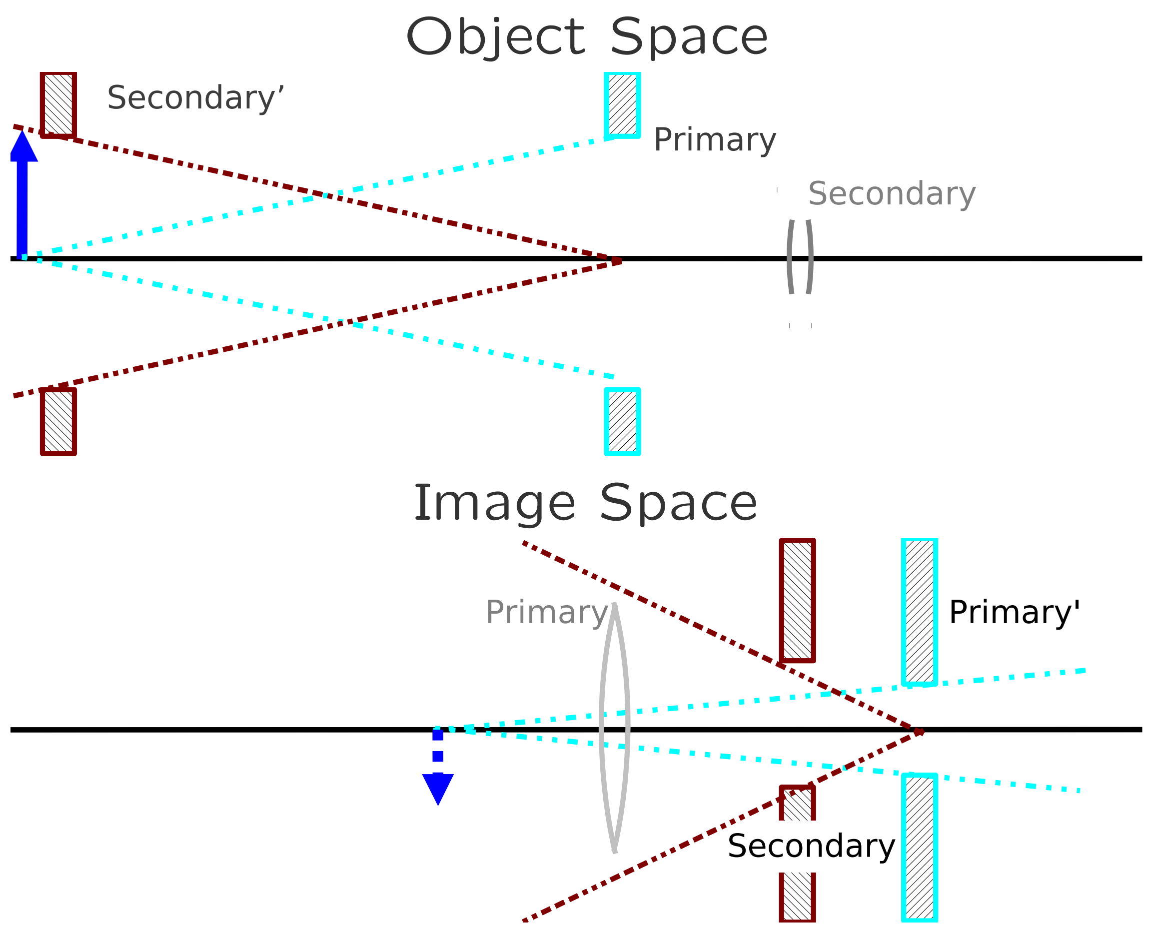
Primary as Aperture Stop



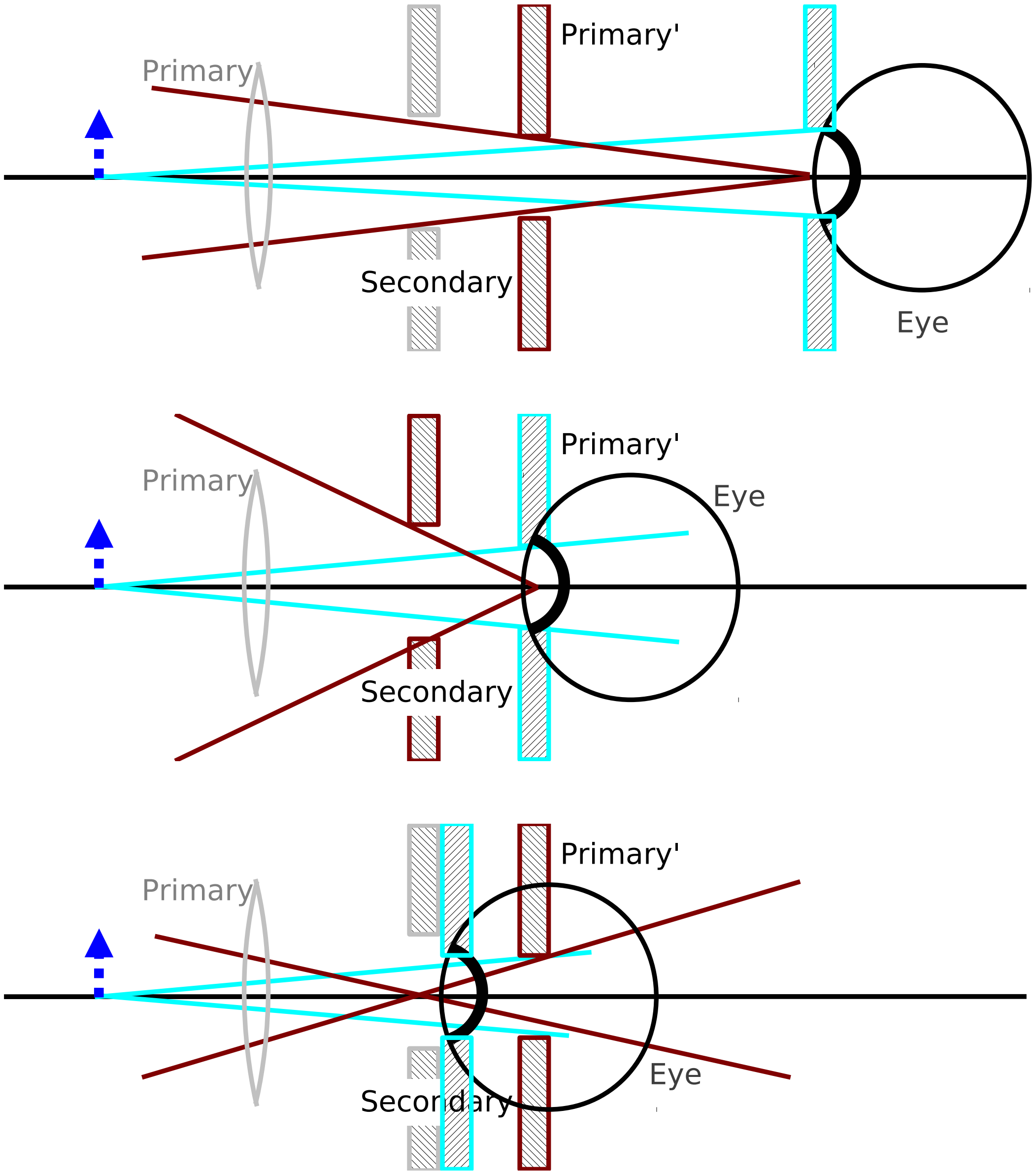
Secondary as Field Stop



The Telescope: Object and Image Space



Eye Relief: Matching Pupils

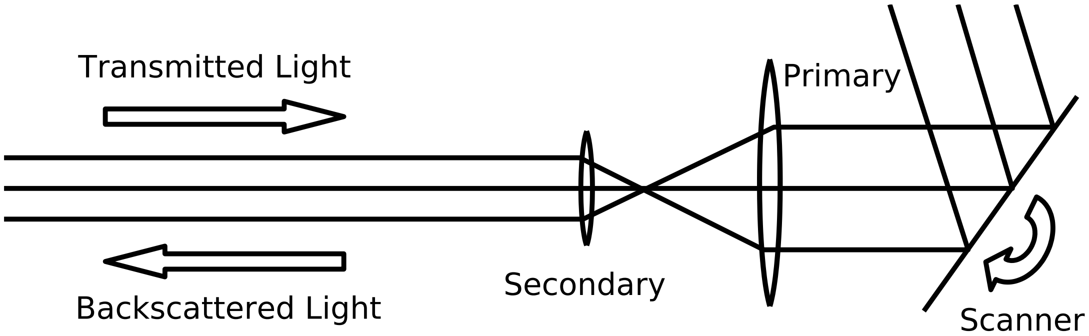


Eye Too Far Away: Telescope Exit Pupil Becomes Field Stop and Is Visible.

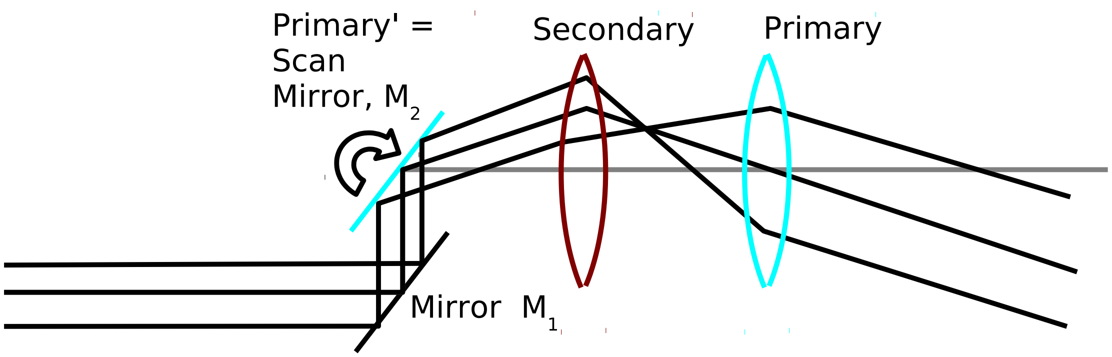
Pupils Matched: Actual Pupil is the Smaller of the Two.

Eye Too Close. Telescope Exit Pupil Becomes Field Stop but is Blurred (Distance is Negative).

Scanning and Pupils: Laser–Radar Example



Post–Expander Scanning



Pre–Expander Scanning

In–Practice

Pre–Expander Scanning can be useful, but requires careful design.

The tradoffs can be complicated; Smaller scan mirror, but larger secondary and larger scan angle.

Place the Scanning Mirrors in a Pupil Plane (or Close)

Small Mirrors Can Move Faster. . .

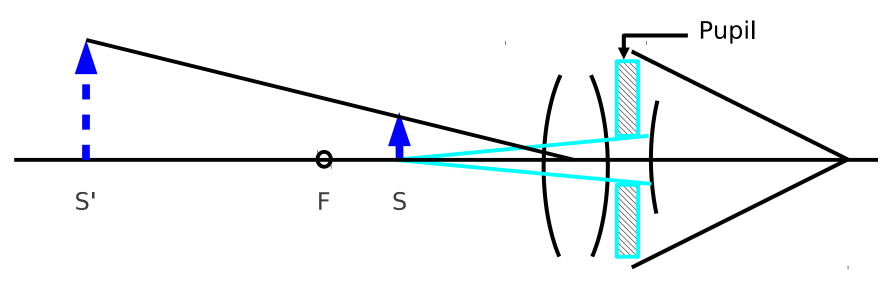
. . . but Remember the Angular Magnification

1000min 7 Feb 2014 by JH. Streaming video for 4 Feb.

The Simple Magnifier

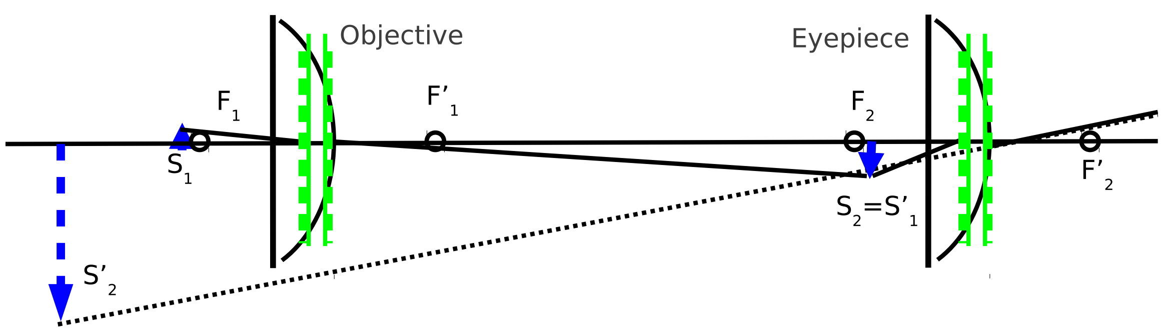
- The Simple Magnifier Has Practical Limitations
$$\frac{1}{s'} = \frac{1}{f} - \frac{1}{s}$$
- Large s' for Large m; $s \approx f$
$$s' = -\frac{fs}{f-s} \approx -\frac{f^2}{f-s}.$$
- s Slightly Smaller than f for Positive m and negative s'
$$m = \frac{-s'}{s} > 0$$
- No Limit on m ...
- But x'/s' is What Matters

- Define $M = m$ at $s' = 20\text{cm}$
$$M = \frac{20\text{cm}}{f}$$
- Hard to Make f/d Small
- $d \ll d_{eye}$ Costs Light
- Hard to Make $f \ll 1\text{cm}$ (but Leeuwenhoek did it)
- Better Solution: Compound Microscope



Compound Microscope

- The Solution: Use Two (or More) Lenses
- Now We Really Need to Understand Pupils and Windows
- Two–Lens Microscope
  - Objective (First Lens) Provides High Magnification and Real Image
  - Eyepiece Acts as a Simple Magnifier
  - Both Are Usually Compound Lenses (See Ch. 5)

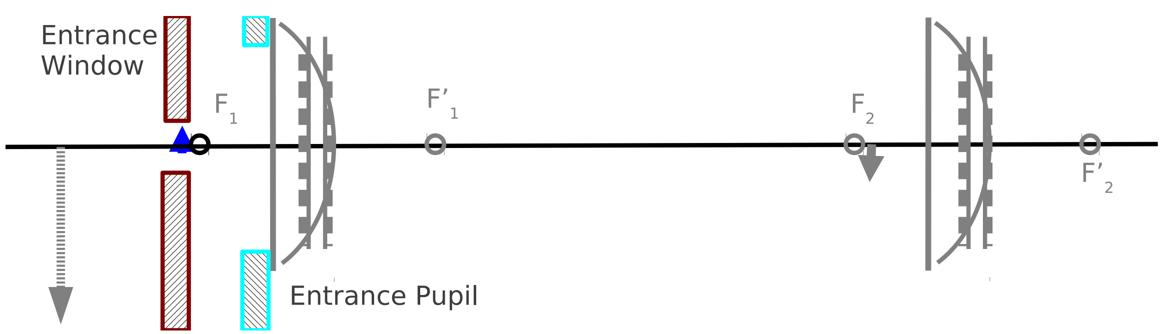


Compound Microscope: Object Space

- Objective Provides Magnification and Aperture Stop
  - Short Focal Length for High Magnification
  - High NA (Hopefully)
- Tube Length Provides Large Real Intermediate Image

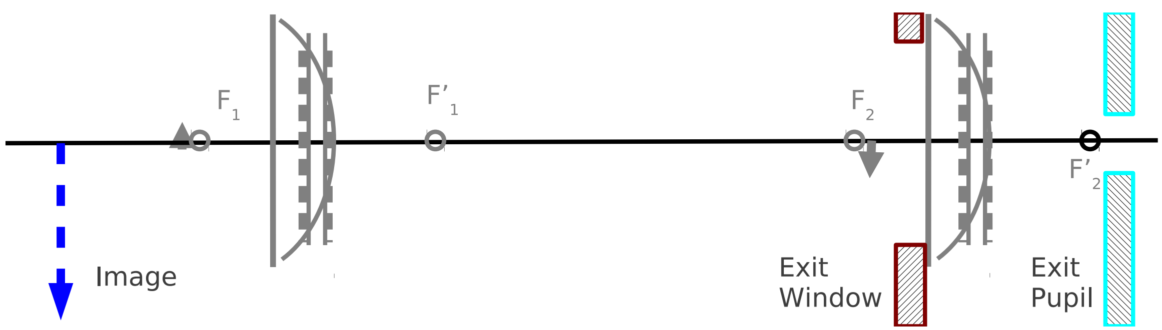
$$m_{objective} = -\frac{s'_{objective}}{f_{objective}} \approx -\frac{\ell_{tube}}{f_{objective}}$$

  - Standard Tube Length 160 mm
  - Many Variations
- Tube Length Makes Entrance Window Near Object



Compound Microscope: Image Space

- Eyepiece Acts as a Simple Magnifier and Field Stop
  - Moderate Focal Length for Moderate Magnification
  - Exact Magnification Not Critical (x'/s' Matters)
- Image Near Infinity (Virtual, Inverted)
- Tube Length Places Exit Pupil at Back Focus of Eyepiece
- Eye Relief for Pupil Matching

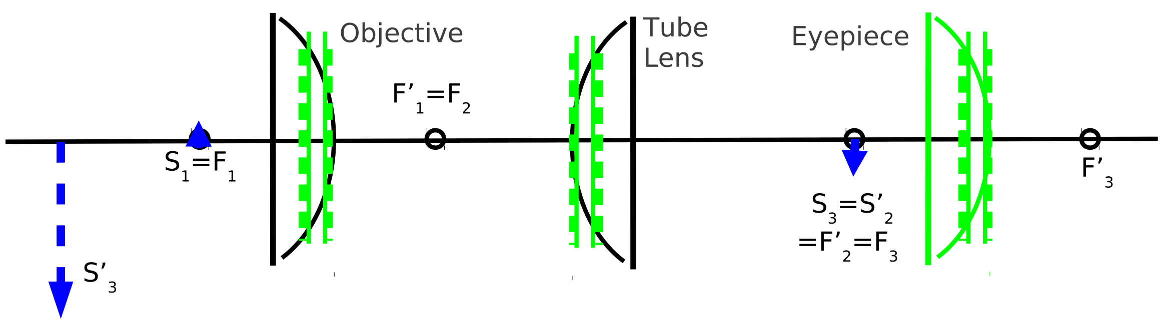


Infinity–Corrected Microscope

- Added Lens, Telecentric Configuration

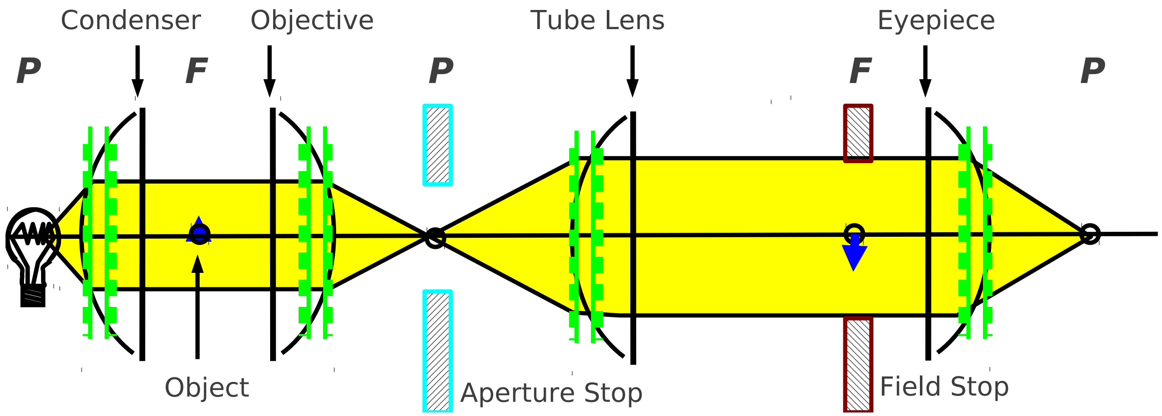
$$m_{objective} = -\frac{f_{tube}}{f_{objective}}$$

  – Improved Image Quality
- Infinity Space Between Objective and Tube Lens
  – Allows for Filters and Other Optics (Flat without Aberration: See Ch. 5)
  – Provides Real Pupil
- Camera Often Placed at Intermediate Image

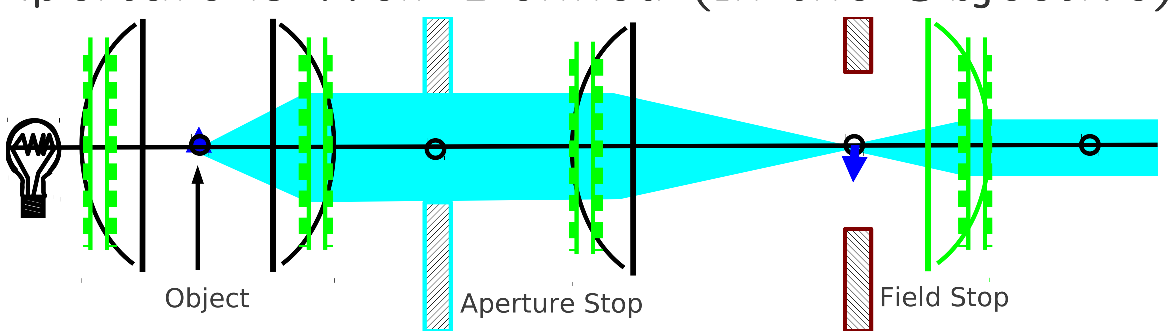


Infinity–Corrected Microscope

Illumination Should Match or Exceed FOV (& No Diffuser)

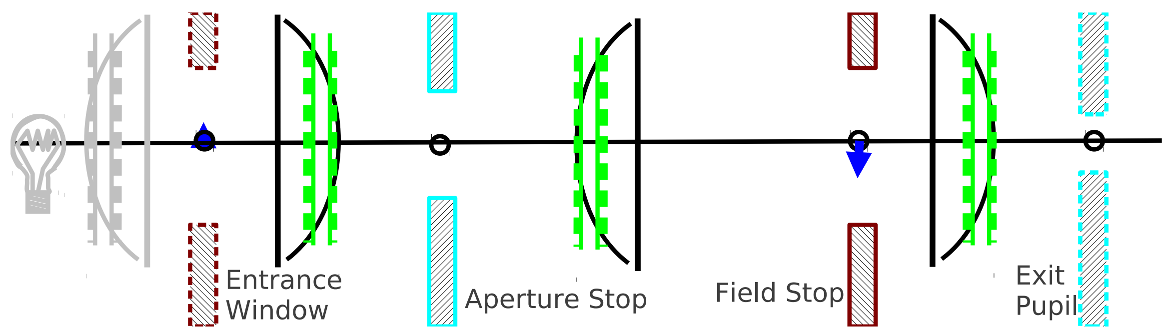


Aperture is Well–Defined (In the Objective)



Microscope Apertures

- Aperture Stop (< 10 to > 20mm)
  - In Back Focal Plane of Objective
  - Determines NA
  - Exit Pupil at Back Focal Plane of Eyepiece.
- Field Stop (> 20mm)
  - At Intermediate Image
  - Often Used for Camera or Detector
    - ∗ Camera Then Acts as Field Stop (Smaller FOV)
  - Entrance Pupil at Object (Front Focal Plane of Objective
- All in Focal Planes

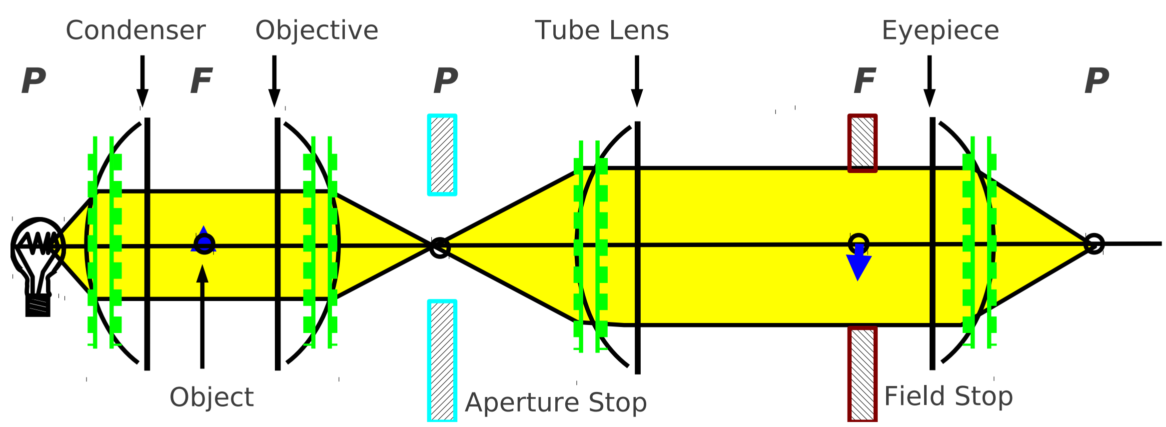


Köhler Illumination

- Light Source in a Pupil Plane
- Not Imaged (See Ch. 11 and Homework Problem There)
- Condenser Lens Determines FOV of Light Source

In–Practice

Köhler Illumination is normally used in a microscope.



1100min 11 Feb 2014