Optics for Engineers

Chapter 3

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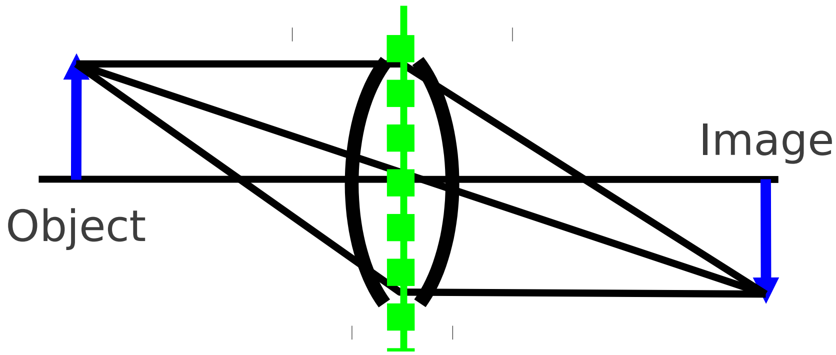
Chapter Overview

Thin Lens

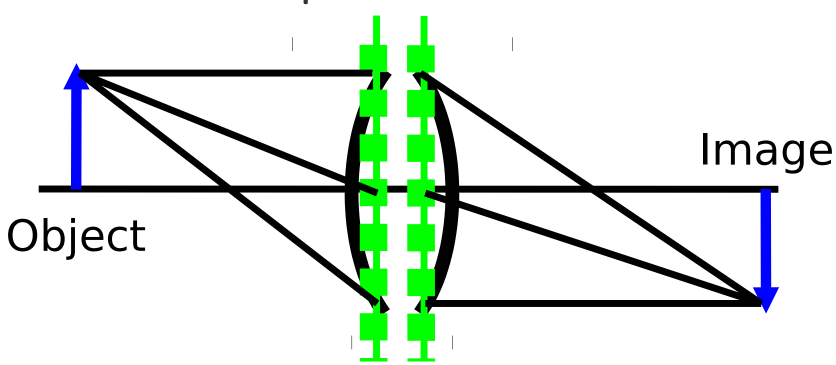


$$\frac{1}{s}+\frac{1}{s'}=\frac{1}{f}$$

Thick Lens

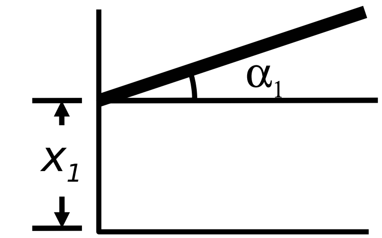
What are s, s', f?

Is this equation still valid?

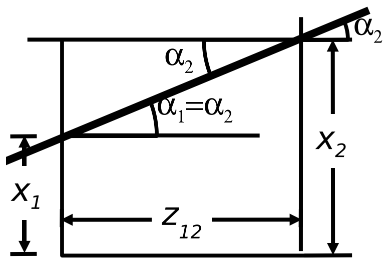


- Thin Lens (Ch. 2)
- Thick or Compound Lens
- Matrix Methods
- Abbe Invariant
  - $m_\alpha m = n/n'$
  - Fundamental Limit
- Principal Planes
- Imaging Equation
  - Thin Lens Equation for Thick Lens
- Exact Solution (Compound Lens)
- Approximation (Thick Lens)
  - "Rule of Thirds"

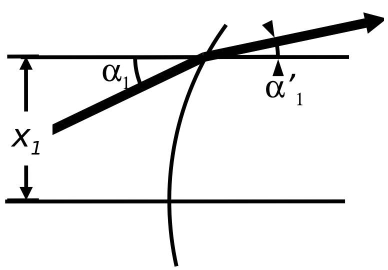
Compound Lens and Ray Definitions



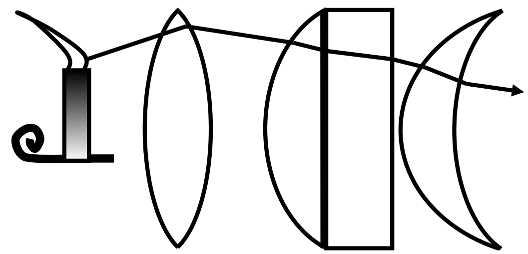
Ray Definition (Vector)



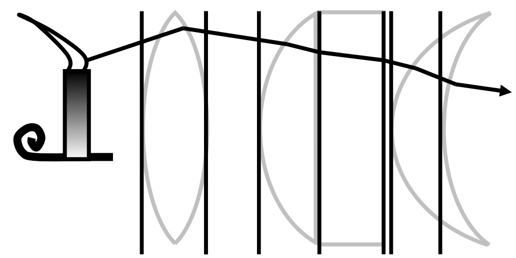
Translation (Matrix)



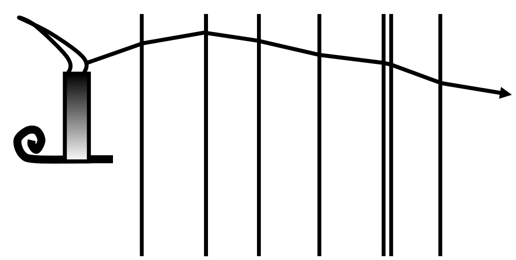
Refraction (Matrix)



Correct Ray



Vertex Planes



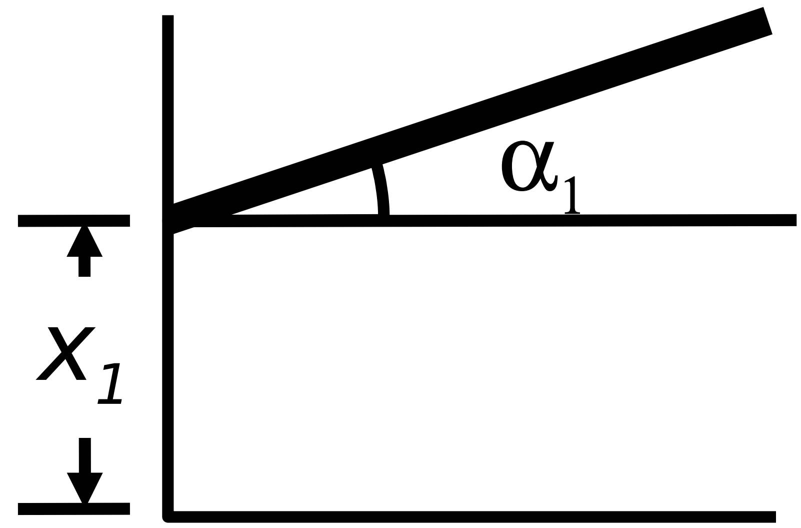
Matrix Optics Ray

Ray Definitions

- Ray Information
  - Straight Line
  - Two Dimensions (or 3)
  - Slope and Intercept
- Mathematical Formulation
  - Linear (Paraxial Approx.)
  - 2-Element Col. Vector
  - Intercept on Top
  - Reference to Local z
  - Angle on Bottom

$$\mathbf{V} = \begin{pmatrix} x \\ \alpha \end{pmatrix}$$

- Some Books Differ



- Arbitrary Operation (ABCD Matrix)

$$\mathcal{M} = \begin{pmatrix} A & B \\ C & D \end{pmatrix}$$

$$\mathbf{V}_{end} = \mathcal{M}_{start:end}\mathbf{V}_{start}$$

- Subscript for Vertex Number

Translation From One Surface to the Next

- Move Away from Source
- z_1 to z_2

$$\mathbf{V}_2 = \mathcal{T}_{12}\mathbf{V}_1$$

- Angle Stays Constant

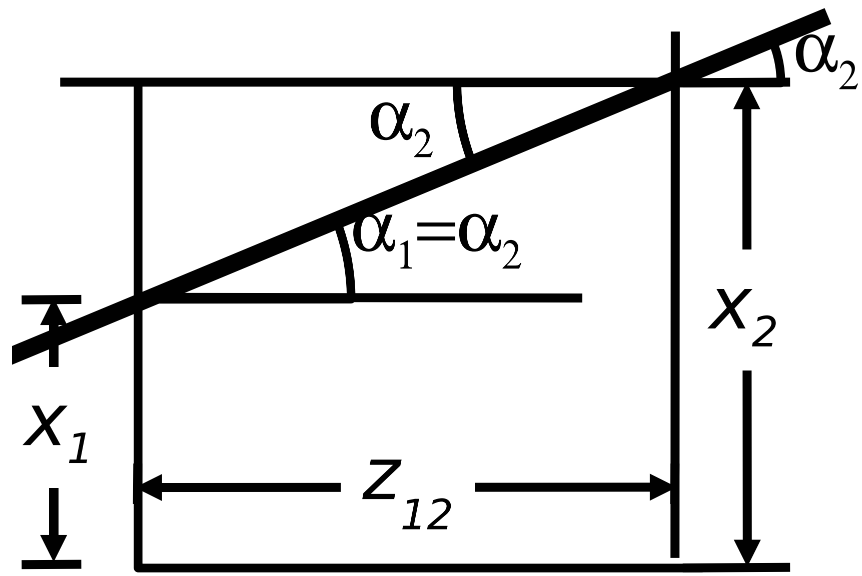
$$\alpha_2 = 1\alpha_1 + 0x_1$$

- Height Changes

$$x_2 = 1x_1 + z_{12}\alpha_1$$

- Matrix Form

$$\begin{pmatrix} x_2 \\ \alpha_2 \end{pmatrix} = \begin{pmatrix} 1 & z_{12} \\ 0 & 1 \end{pmatrix} \begin{pmatrix} x_1 \\ \alpha_1 \end{pmatrix}$$



$$\boxed{\mathcal{T}_{12} = \begin{pmatrix} 1 & z_{12} \\ 0 & 1 \end{pmatrix}}$$

Refraction at a Surface (1)

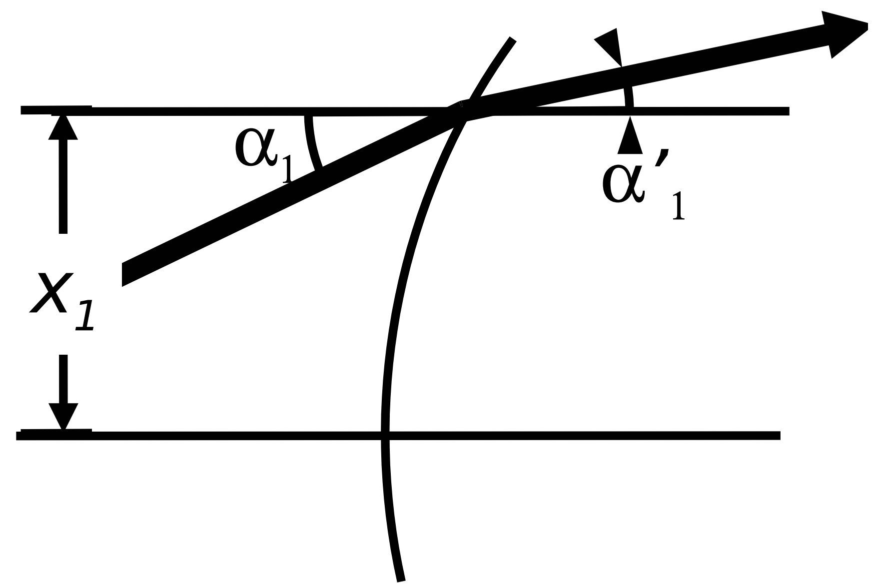
- Matrix Form

$$\mathbf{V}'_1 = \mathcal{R}_1 \mathbf{V}_1$$

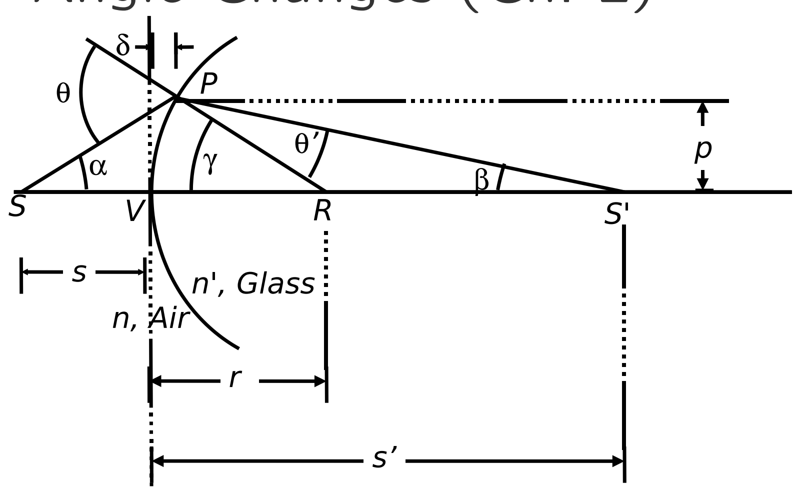
- Height Does Not Change

$$x'_1 = (1 \times x_1) + (0 \times \alpha_1)$$

$$\begin{pmatrix} x'_1 \\ \alpha'_1 \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ ? & ? \end{pmatrix} \begin{pmatrix} x_1 \\ \alpha_1 \end{pmatrix}$$



- Angle Changes (Ch. 2)



$$\theta = \gamma + \alpha \qquad \theta' = \gamma - \beta = \gamma + \alpha'$$

$$\tan\alpha = \frac{p}{s+\delta} \qquad \tan\beta = \frac{p}{s'-\delta}$$

$$\tan\gamma = \frac{p}{r-\delta}$$

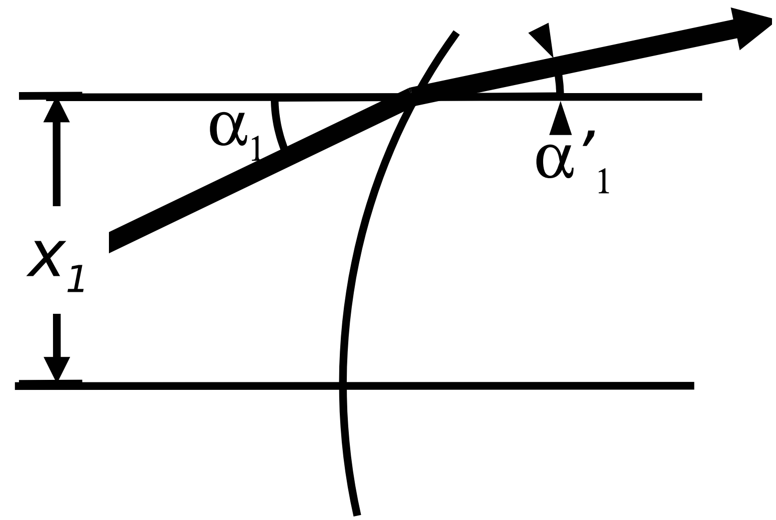
Refraction at a Surface (2)

- Height Does Not Change

$$\begin{pmatrix} x'_1 \\ \alpha'_1 \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ ? & ? \end{pmatrix} \begin{pmatrix} x_1 \\ \alpha_1 \end{pmatrix}$$

- Angle (See Prev. Page)

$$n\theta = n'\theta'$$



$$n\left(\gamma + \alpha\right) = n'\left(\gamma + \alpha'\right),$$

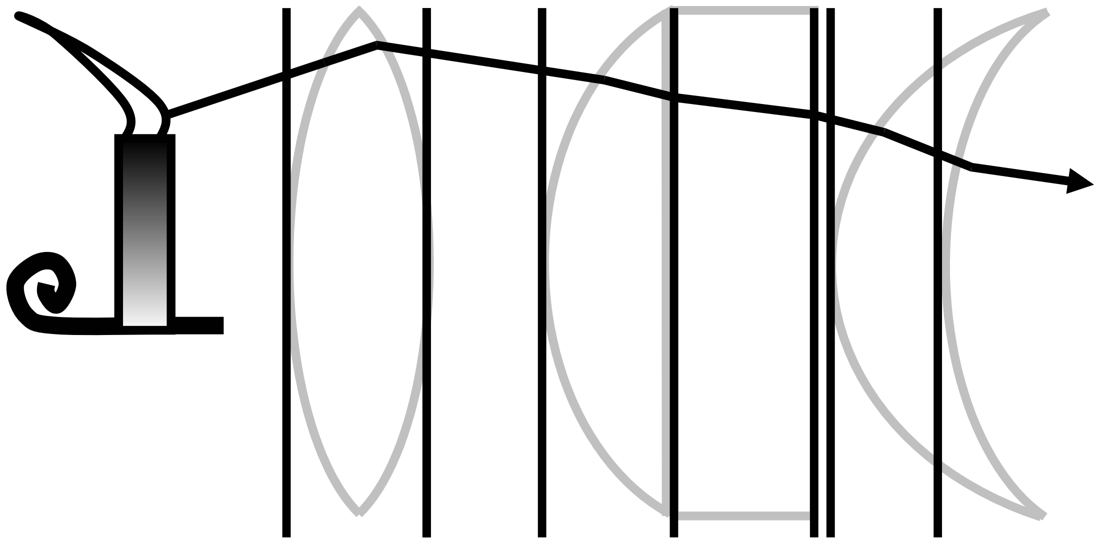
$$n\frac{x}{r} + n\alpha = n'\frac{x}{r} + n'\alpha'.$$

$$\alpha' = \frac{n - n'}{n'r}x + \frac{n}{n'}\alpha.$$

$$\boxed{\mathcal{R} = \begin{pmatrix} 1 & 0 \\ \frac{n-n'}{n'r} & \frac{n}{n'} \end{pmatrix}}$$

$$\mathcal{R} = \begin{pmatrix} 1 & 0 \\ -\frac{P}{n'} & \frac{n}{n'} \end{pmatrix}$$

Cascading Matrices



$\mathbf{V_1} = \mathcal{T}_{01}\mathbf{V_0} \qquad \mathbf{V_1}' = \mathcal{R}_1\mathbf{V_1} \qquad \mathbf{V_2} = \mathcal{T}_{12}\mathbf{V_1}' \qquad$ *etc.*

$$\mathbf{V_{end}} = \mathcal{M}_{0:end}\mathbf{V_0} \qquad \mathcal{M}_{0:end} = \mathcal{T}_{end-1:end}\cdots\mathcal{T}_{12}\mathcal{R}_1\mathcal{T}_{01}$$

Multiply from Right to Left as Light Moves from Left to Right.

The Simple Lens (1)

- First Surface

$$\mathbf{V}_1' = \mathcal{R}_1 \mathbf{V}_1$$

- Translation

$$\mathbf{V}_2 = \mathcal{T}_{12} \mathbf{V}_1'$$

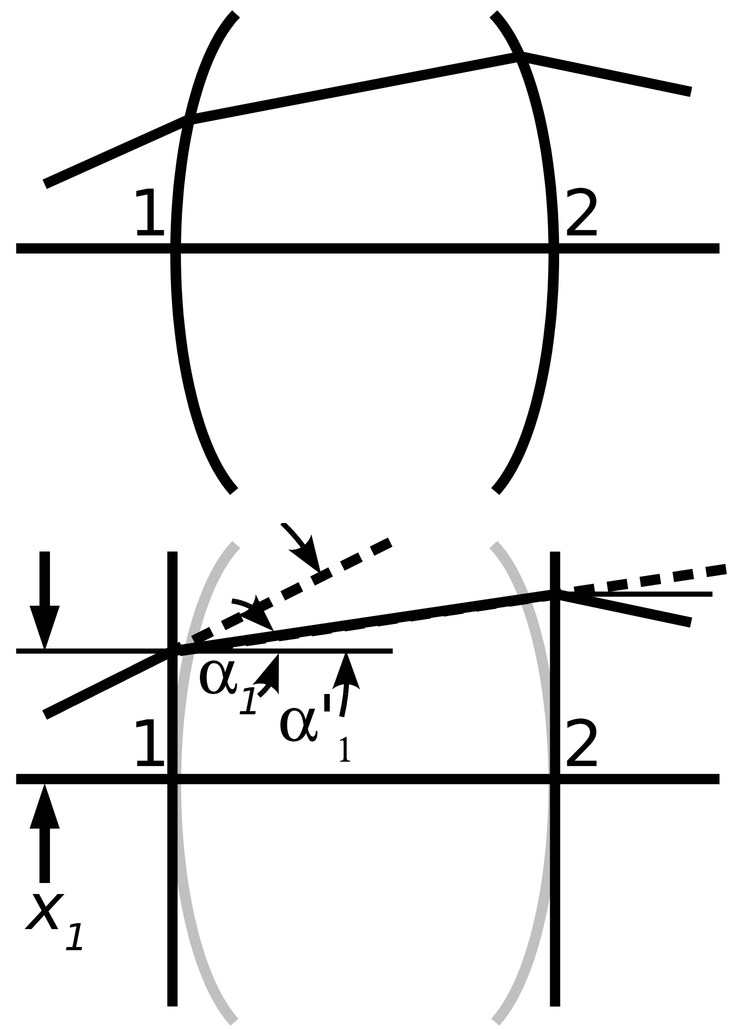
- Second Surface

$$\mathbf{V}_2' = \mathcal{R}_2 \mathbf{V}_2$$

- Result

$$\mathbf{V}_2' = \mathcal{L} \mathbf{V}_1$$

$$\boxed{\mathcal{L} = \mathcal{R}_2 \mathcal{T}_{12} \mathcal{R}_1}$$



The Simple Lens (2)

- From Previous Page $\mathcal{L} = \mathcal{R}_2\mathcal{T}_{12}\mathcal{R}_1$

$$\mathcal{L} = \begin{pmatrix} 1 & 0 \\ -\frac{P_2}{n_2'} & \frac{n_1'}{n_2'} \end{pmatrix} \begin{pmatrix} 1 & z_{12} \\ 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 \\ -\frac{P_1}{n_1'} & \frac{n_1}{n_1'} \end{pmatrix} \qquad \left(n_2 = n_1'\right)$$

- Strange but Useful Grouping

$$\mathcal{L} = \begin{pmatrix} 1 & 0 \\ -\frac{P_t}{n_2'} & \frac{n_1}{n_2'} \end{pmatrix} + \frac{z_{12}}{n_1'} \begin{pmatrix} -P_1 & n_1 \\ \frac{P_1 P_2}{n_2'} & -P_2\frac{n_1}{n_2'} \end{pmatrix} \qquad (P_t = P_1 + P_2)$$

- Initial: $n_1 = n$, Final: $n_2' = n'$, Lens: $n_1' = n_\ell$

$$\mathcal{L} = \begin{pmatrix} 1 & 0 \\ -\frac{P_t}{n'} & \frac{n}{n'} \end{pmatrix} + \frac{z_{12}}{n_\ell} \begin{pmatrix} -P_1 & n \\ \frac{P_1 P_2}{n'} & -P_2\frac{n}{n'} \end{pmatrix}$$

- n_ℓ implicit in P_1 and P_2, and thus P_t
  - User may not care about n_ℓ, r_1, r_2

The Thin Lens (1)

- The Simple Lens (Previous Page)

$$\mathcal{L} = \begin{pmatrix} 1 & 0 \\ -\frac{P_t}{n'} & \frac{n}{n'} \end{pmatrix} + \frac{z_{12}}{n_\ell} \begin{pmatrix} -P_1 & n \\ \frac{P_1 P_2}{n'} & -P_2 \frac{n}{n'} \end{pmatrix}$$

- Geometric Thickness, z_{12}/n_ℓ, Multiples Second Term

- Set $z_{12} \to 0$

$$\boxed{\mathcal{L} = \begin{pmatrix} 1 & 0 \\ -\frac{P}{n'} & \frac{n}{n'} \end{pmatrix} \qquad \text{(Thin Lens)}}$$

$P = P_t = P_1 + P_2$ Correction Term Vanishes

Fabrication Details (n_ℓ, r_1, r_2) Are Not Needed or Available

The Thin Lens (2)

- Thin Lens in terms of Focal Lengths

$$\mathcal{L} = \begin{pmatrix} 1 & 0 \\ -\frac{P}{n'} & \frac{n}{n'} \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ -\frac{1}{f'} & \frac{n}{n'} \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ -\frac{n}{n'f} & \frac{n}{n'} \end{pmatrix}$$

- Front Focal Length: $f = FFL$, Back: $f' = BFL$

- Special but Common Case: Thin Lens in Air

$$\boxed{\mathcal{L} = \begin{pmatrix} 1 & 0 \\ -P & 1 \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ -\frac{1}{f} & 1 \end{pmatrix} \qquad \text{(Thin Lens in Air)}}$$

$$f = f' = \frac{1}{P} \qquad FFL = BFL \qquad \text{Always True if } n' = n$$

Simple Lens Matrix Summary

Take–Away Messsge

- Matrix methods are valid in paraxial approximation
- A simple lens matrix is refraction, translation, refraction
- Result is thin lens plus a correction term
- Result reduces to the thin lens as thickness approaches zero

General Problems and the ABCD Matrix

- General Equation

$$\mathbf{V}_{end} = \mathcal{M}_{start:end}\mathbf{V}_{start} \qquad \begin{pmatrix} x_{end} \\ \alpha_{end} \end{pmatrix} = \begin{pmatrix} m_{11} & m_{12} \\ m_{21} & m_{22} \end{pmatrix} \begin{pmatrix} x_{start} \\ \alpha_{start} \end{pmatrix}$$

- Determinant Condition (Not Completely Obvious)

$$\boxed{\det \mathcal{M} = \frac{n}{n'} \qquad (\det \mathcal{M} = m_{11}m_{22} - m_{12}m_{21})}$$

- Abbe Sine Invariant (or Helmholz or Lagrange Invariant)

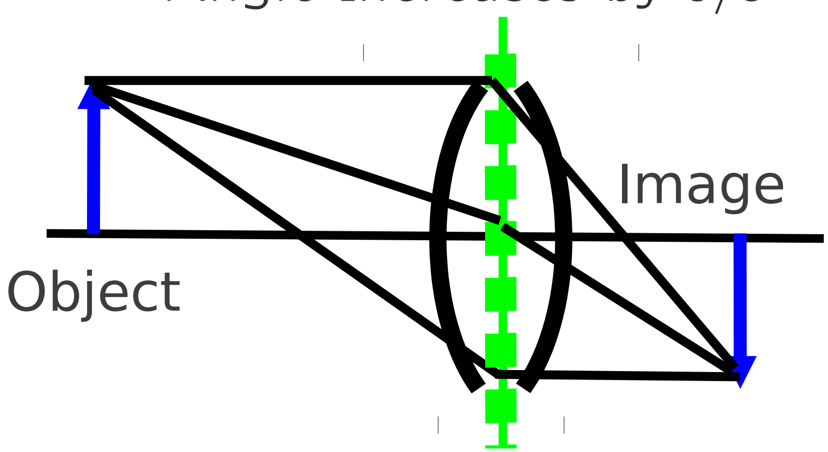
$$n'x'd\alpha' = nxd\alpha$$

Abbe Sine Invariant

- Equation

$$n'x'd\alpha' = nxd\alpha$$

- Alternative Derivations
  - Geometric Optics
  - Energy Conservation (C. 12)
- Lens Example
  - Height Decreases by s'/s
  - Angle Increases by s/s'



- Example: IR Detector
  - Diameter
    $D' = 100\mu\text{m}$
  - Collection Cone
    $FOV'_{1/2} = 30°$
- Telescope Front Lens
  - Diameter
    $D = 20\text{cm}$
  - Max. Field of View

$$FOV_{1/2} =$$

$$\frac{100 \times 10^{-6}\text{m} \times 30°}{20 \times 10^{-2}\text{m}} =$$

$$0.0150°$$

Principal Planes Concept (1)

- Thin Lens

$$\mathcal{L} = \begin{pmatrix} 1 & 0 \\ -\frac{P}{n'} & \frac{n}{n'} \end{pmatrix}$$

- Simple Equation
- Easy Visualization
  (“High–School Optics”)
- Good “First Try”

- Arbitrary Lens
- Vertex to Vertex

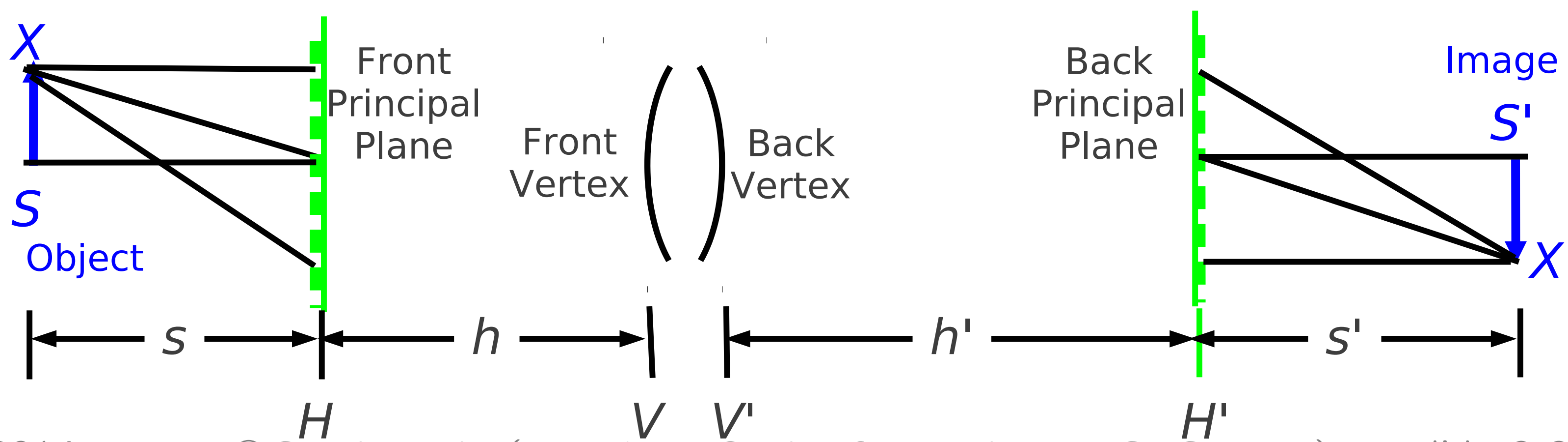
$$\mathcal{M}_{VV'} = \begin{pmatrix} m_{11} & m_{12} \\ m_{21} & m_{22} \end{pmatrix}$$

- Possible Simplification

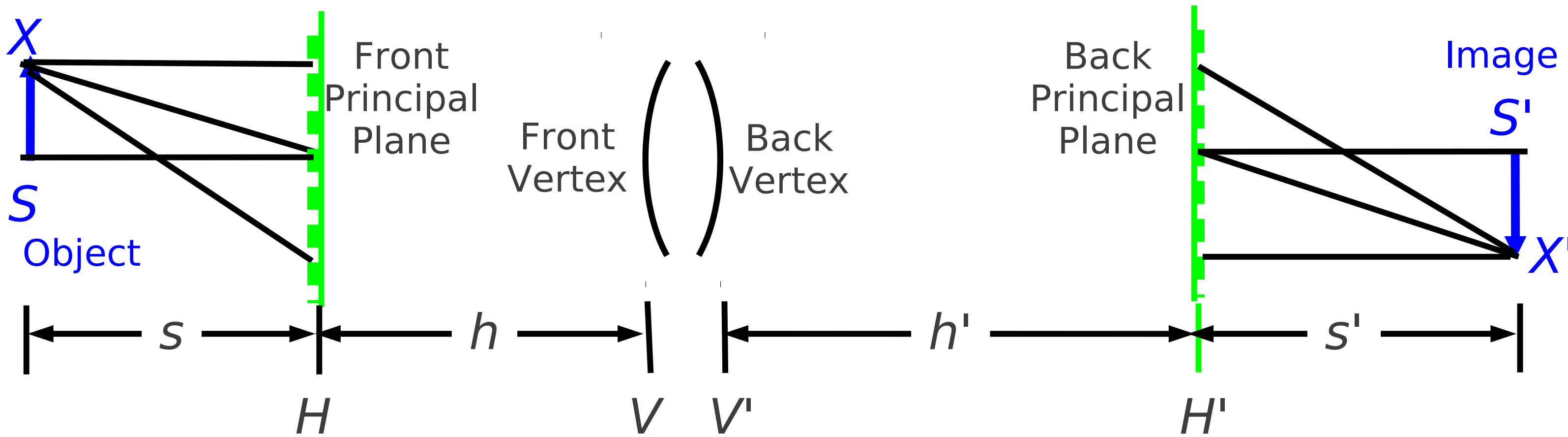
$$\mathcal{M}_{HH'} = \mathcal{T}_{V'H'}\mathcal{M}_{VV'}\mathcal{T}_{HV}$$

$$\mathcal{M}_{HH'} = \mathcal{L}$$

- Will It Work?

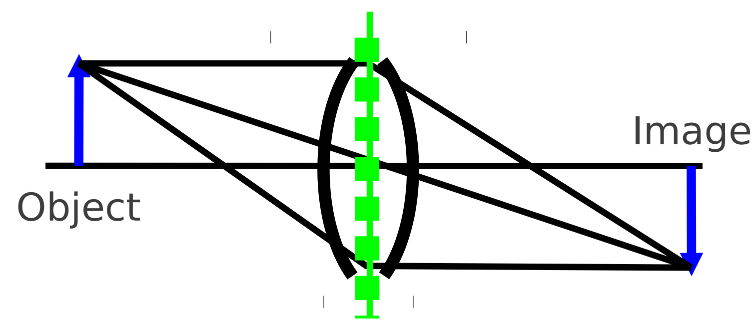


Principal Planes Concept (2)



Convert a Hard Problem to a Simple one

$$\mathcal{M}_{HH'} = \mathcal{T}_{V'H'}\mathcal{M}_{VV'}\mathcal{T}_{HV} \qquad \mathcal{M}_{HH'} = \mathcal{L}$$



Useful if a Solution Can Be Found
Very Useful if h and h' Are Not Too Large

Finding the Principal Planes

$$\mathcal{L} = \mathcal{M}_{HH'} = \mathcal{T}_{V'H'}\mathcal{M}_{VV'}\mathcal{T}_{HV}$$

$$\begin{pmatrix} 1 & 0 \\ -\frac{P}{n'} & \frac{n}{n'} \end{pmatrix} = \begin{pmatrix} 1 & h' \\ 0 & 1 \end{pmatrix} \begin{pmatrix} m_{11} & m_{12} \\ m_{21} & m_{22} \end{pmatrix} \begin{pmatrix} 1 & h \\ 0 & 1 \end{pmatrix}$$

$$\begin{pmatrix} 1 & 0 \\ -\frac{P}{n'} & \frac{n}{n'} \end{pmatrix} = \begin{pmatrix} m_{11} + m_{21}h' & m_{11}h + m_{12} + m_{21}hh' + m_{22}h' \\ m_{21} & m_{21}h + m_{22} \end{pmatrix}$$

$m_{11} + m_{21}h' = 1$	$m_{11}h + m_{12} + m_{21}hh' + m_{22}h' = 0$
$m_{21} = -\frac{P}{n'}$	$m_{21}h + m_{22} = \frac{n}{n'}$

Three Unknowns: Solution if Determinant Condition Satisfied

$h' = \frac{1-m_{11}}{m_{21}}$	Determinant Condition? Yes!
$P = -m_{21}n'$	$h = \frac{\frac{n}{n'}-m_{22}}{m_{21}}$

No Assumptions Were Made About $\mathcal{M}$: This Always Works.

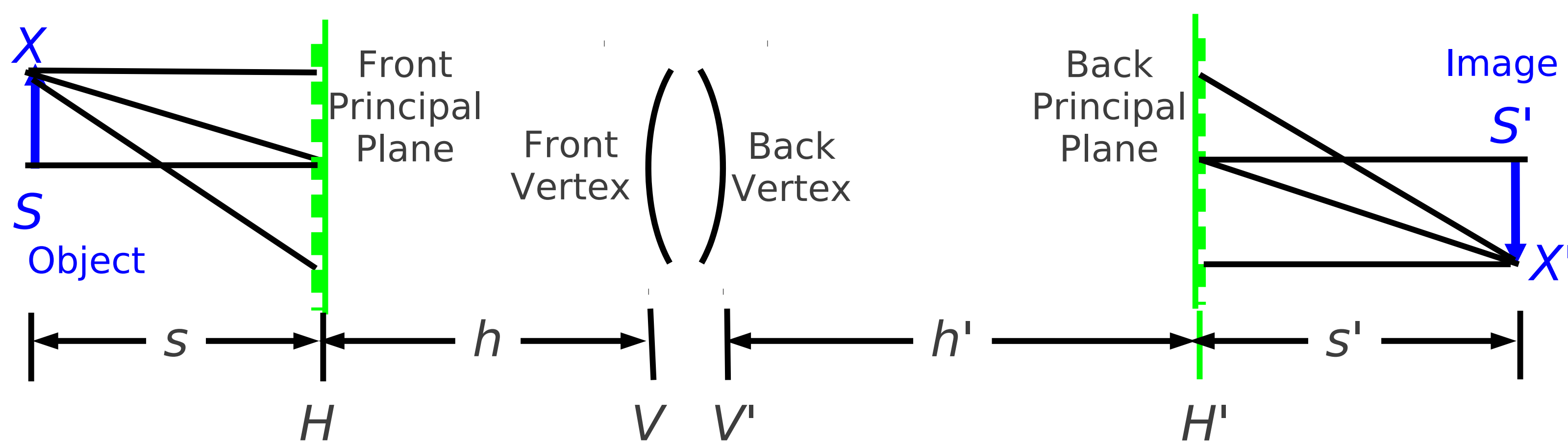
Principal Planes

- Principal Planes are Conjugates of Each Other ($m_{12} = 0$)

$$\begin{pmatrix} x_{H'} \\ \alpha_{H'} \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ -\frac{P}{n'} & \frac{n}{n'} \end{pmatrix} \begin{pmatrix} x_H \\ \alpha_H \end{pmatrix}$$

- Unit Magnification Between Them

$$x_{H'} = x_H$$



Note: Principal Planes May Not Be Accessible

Arbitrary Compound Lens

Take–Away Messsge

- No matter how many elements, we can find a lens matrix from the front principal plane to the back one . . .

$$\mathcal{M}_{HH'} = \begin{pmatrix} 1 & 0 \\ -\frac{P}{n'} & \frac{n}{n'} \end{pmatrix} \begin{pmatrix} x_H \\ \alpha_H \end{pmatrix}$$

- . . . and we can find the principal planes and optical power

$h' = \frac{1-m_{11}}{m_{21}}$	
$P = -m_{21}n'$	$h = \frac{\frac{n}{n'}-m_{22}}{m_{21}}$

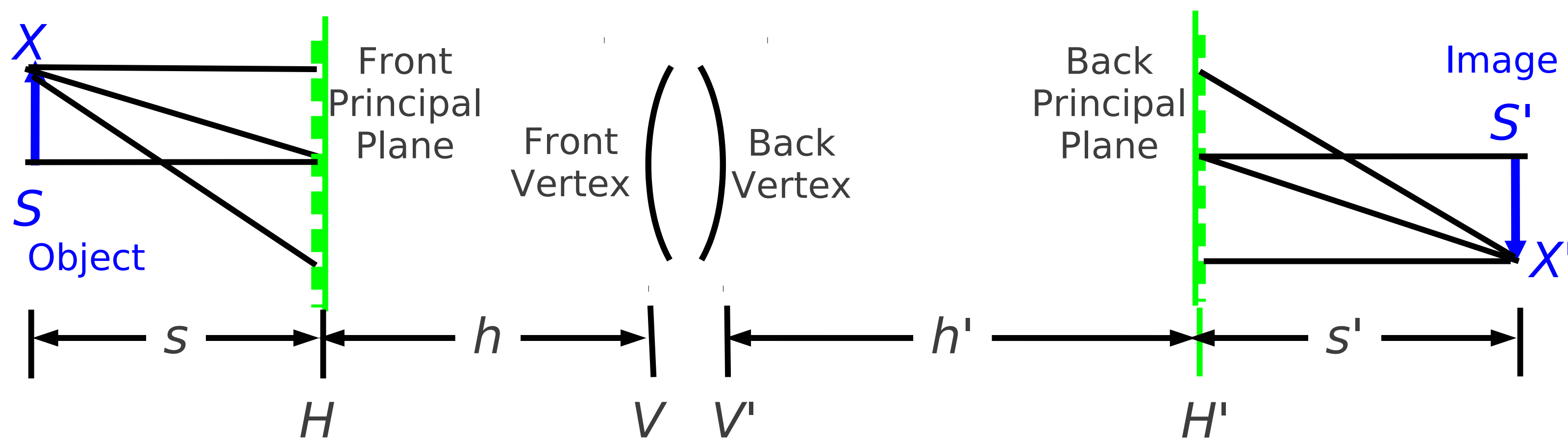
Imaging (We Know The Answer)

- Matrix from Object to Image

$$\mathcal{M}_{SS'} = \mathcal{M}_{H'S'}\mathcal{M}_{HH'}\mathcal{M}_{SH} = \mathcal{T}_{s'}\mathcal{M}_{HH'}\mathcal{T}_{s}$$

- Conjugate Planes

$$x' = (? \times x) + (0 \times \alpha) \qquad \mathcal{M}_{SS'} = \begin{pmatrix} m_{11} & 0 \\ m_{21} & m_{22} \end{pmatrix}$$



Imaging Equation for Compound Lens

$$\mathcal{M}_{SS'} = \begin{pmatrix} 1 & s' \\ 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 \\ -\frac{P}{n'} & \frac{n}{n'} \end{pmatrix} \begin{pmatrix} 1 & s \\ 0 & 1 \end{pmatrix} = \begin{pmatrix} 1 - \frac{s'P}{n'} & s - \frac{ss'P}{n'} + \frac{s'n}{n'} \\ -\frac{P}{n'} & -\frac{sP}{n'} + \frac{n}{n'} \end{pmatrix}$$

- Conjugate Plane Rule: $m_{12} = 0$

$$s - \frac{ss'P}{n'} + \frac{s'n}{n'} = 0$$

$$\boxed{\frac{n}{s} + \frac{n'}{s'} = P}$$

- Measure s and s' from H and H' respectively.

Compound Lens Matrix Results

Magnifications ($mm_\alpha = n'/n$)

$$m = 1 - \frac{s'P}{n'} = 1 - \frac{s'}{n'}\left(\frac{n}{s} + \frac{n'}{s'}\right) = -\frac{ns'}{n's}$$

$$m_\alpha = -\frac{s}{n'}(\frac{n}{s} + \frac{n'}{s'}) + \frac{n}{n'} = -\frac{s}{s'}$$

Imaging Matrix

$$\mathcal{M}_{SS'} = \begin{pmatrix} m & 0 \\ -\frac{P}{n'} & \frac{n'}{n}\frac{1}{m} \end{pmatrix}$$

Compound Lens

In–Practice

- For a compound lens, the thin lens equation still is valid
  - Measure f and f' from H and H' respectively.
  - Measure s and s' from H and H' respectively.

- The imaging matrix from s to s' gives the magnification, but the old equations are still right.

$$m = -\frac{s'}{s} \qquad m_\alpha = -\frac{n'}{n} \times \frac{1}{m}$$

Example of Matrix Application: Thick Lens

- Thick–Lens Equation (Vertex–to–Vertex)

$$\mathcal{L} = \begin{pmatrix} 1 & 0 \\ -\frac{P_t}{n'} & \frac{n}{n'} \end{pmatrix} + \frac{z_{12}}{n_\ell} \begin{pmatrix} -P_1 & n \\ \frac{P_1 P_2}{n'} & -P_2 \frac{n}{n'} \end{pmatrix}$$

- Power: $P = -m_{21} n'$

$$P = P_1 + P_2 - \frac{z_{12}}{n_\ell} P_1 P_2 \qquad f = \frac{n}{P} \qquad f' = \frac{n'}{P}$$

- Principal Planes

$$h = -\frac{n'}{n_\ell} \frac{P_2}{P} z_{12} \qquad h' = -\frac{n}{n_\ell} \frac{P_1}{P} z_{12}$$

Thick Lens in Air: The Thirds Rule for Principal Planes

- Principal Planes and Focal Length

$$f = f' = \frac{1}{P} \qquad h = -\frac{1}{n_\ell}\frac{P_2}{P}z_{12} \qquad h' = -\frac{1}{n_\ell}\frac{P_1}{P}z_{12}$$

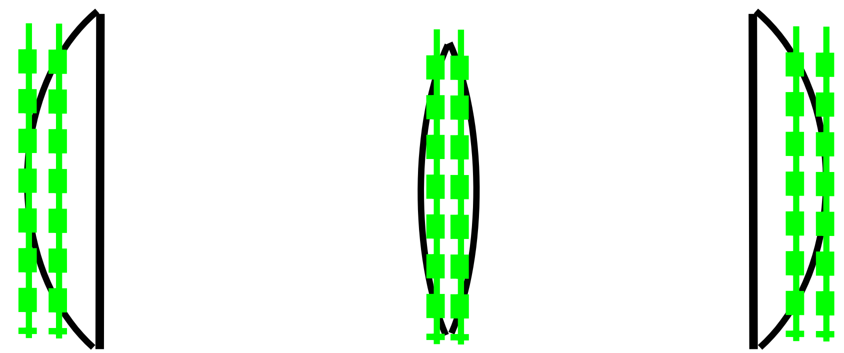
- Principal–Plane Spacing

$$z_{HH'} = z_{12} + h + h' = z_{12}\left(1 - \frac{P_2 + P_1}{n_\ell P}\right)$$

$$P \approx P_1 + P_2 \qquad z_{HH'} = z_{12} + h + h' \approx z_{12}\left(1 - \frac{1}{n_\ell}\right)$$

Glass $\qquad n_\ell \approx 1.5 \qquad z_{HH'} = \frac{z_{12}}{3}$

Special Cases



Convex–Plano Biconvex Plano–Convex

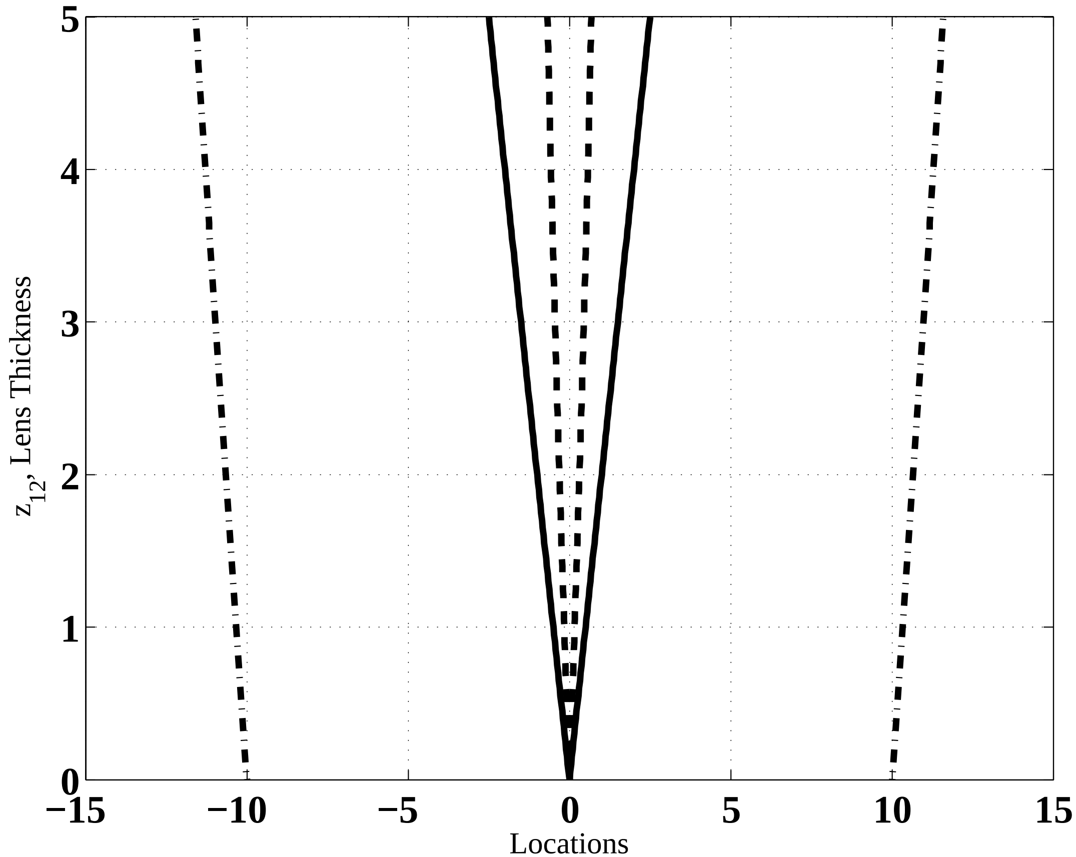
$$h = -\frac{n'}{n_\ell}\frac{P_2}{P}z_{12} \qquad h' = -\frac{n}{n_\ell}\frac{P_1}{P}z_{12}$$

h,h' Negative if P,P_1,P_2 Have Same Signs (Often True)

$h = 0$	if	$P_2 = 0$	Convex–Plano or Concave–Plano
$h' = 0$	if	$P_1 = 0$	Plano–Convex or Plano–Concave
$h' = h$	if	$P_2 = P_1$	Biconvex or Biconcave in Air
	and	$n' = n$	

Example: Biconvex Lens in Air

$P_1 + P_2 = 10$diopters, or $f = 10$cm (Biconvex: $P_1 = P_2$)
Solid=Vertices, Dashed=Principal Planes, Dash–Dot=Focal Planes

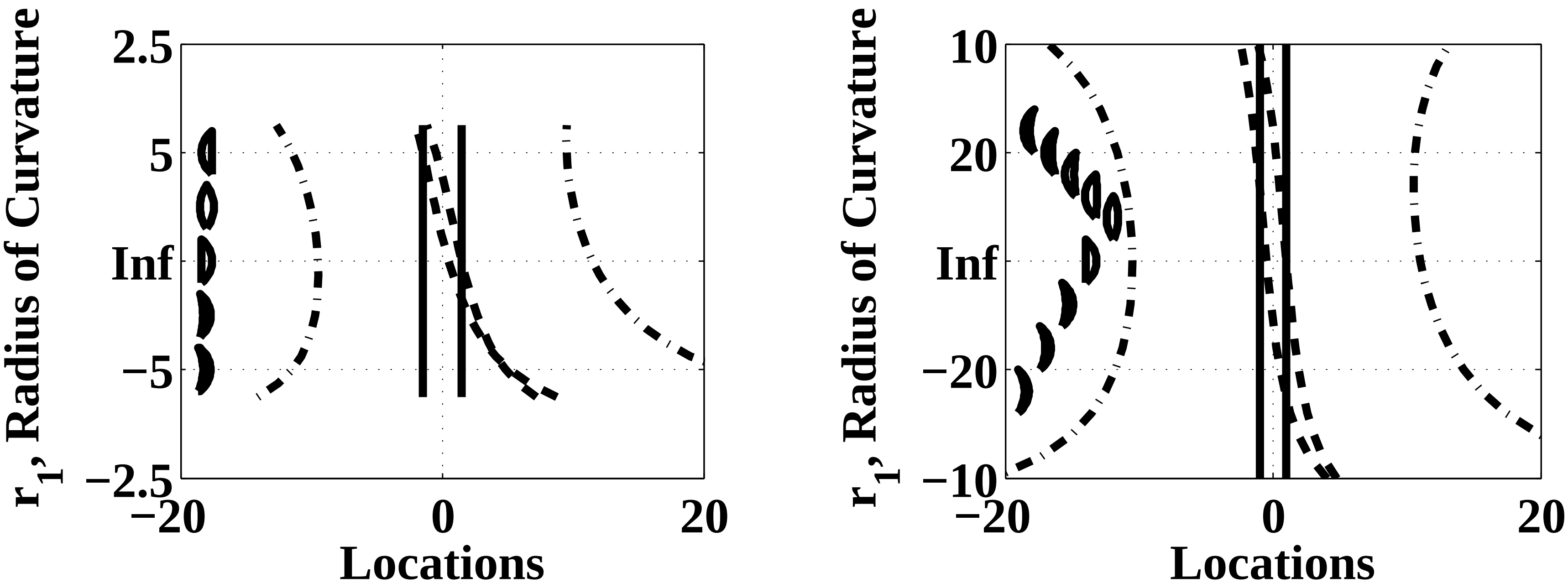


"Bending" the Lens (Including the Weird Cases)

$P_1 + P_2 = 10$diopters, or $f = 10$cm

Solid=Vertices, Dashed=Principal Planes, Dash–Dot=Focal Planes

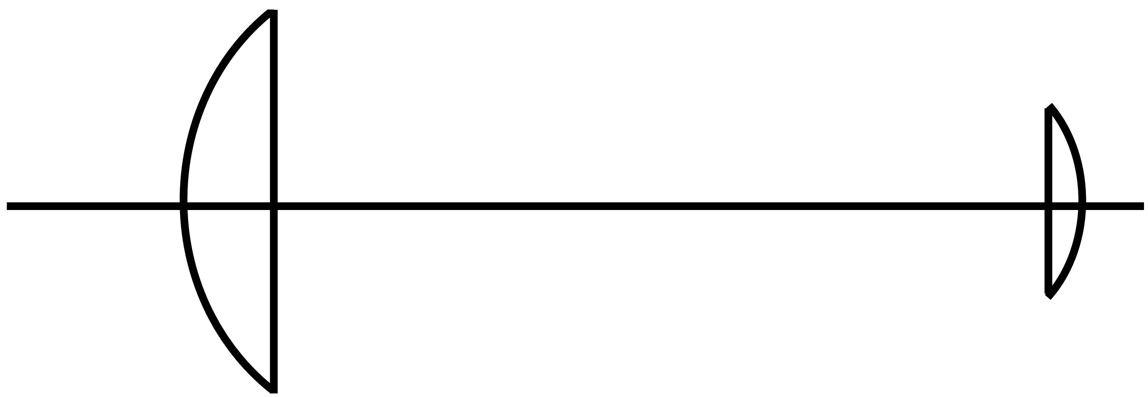
Note "Meniscus" Lenses in Germanium



A. Glass ($n = 1.5$)
$P : \pm 20$Diopters

B. Germanium ($n = 4$)
$P : \pm 30$Diopters

Example: Compound Lens Matrix (Two Thin Lenses)



- General Case

$$\mathcal{M}_{V_1,V_2'} = \mathcal{L}_{V_2,V_2'}\mathcal{T}_{V_1',V_2}\mathcal{L}_{V_1,V_1'}$$

- Both Lenses Thin

$$\mathcal{M}_{V_1,V_2'} = \begin{pmatrix} 1 & 0 \\ -\frac{1}{f_2} & 1 \end{pmatrix}\begin{pmatrix} 1 & z_{12} \\ 0 & 1 \end{pmatrix}\begin{pmatrix} 1 & 0 \\ -\frac{1}{f_1} & 1 \end{pmatrix}$$

$$\mathcal{M}_{V_1,V_2'} = \begin{pmatrix} 1-\frac{z_{12}}{f_1} & z_{12} \\ -\frac{1}{f_2}+\frac{z_{12}}{f_1 f_2}-\frac{1}{f_1} & 1-\frac{z_{12}}{f_2} \end{pmatrix}$$

600min 24 Jan 2014

Compound Lens Results

- Focal Length (Powers add for small separation)

$$\frac{1}{f} = \frac{1}{f_2} + \frac{1}{f_1} - \frac{z_{12}}{f_1 f_2}$$

- Principal Planes

$$h = \frac{\frac{z_{12}}{f_2}}{-\frac{1}{f_2} + \frac{z_{12}}{f_1 f_2} - \frac{1}{f_1}} = \frac{z_{12} f_1}{z_{12} - f_1 - f_2}$$

$$h' = \frac{\frac{z_{12}}{f_1}}{-\frac{1}{f_2} + \frac{z_{12}}{f_1 f_2} - \frac{1}{f_1}} = \frac{z_{12} f_2}{z_{12} - f_1 - f_2}$$

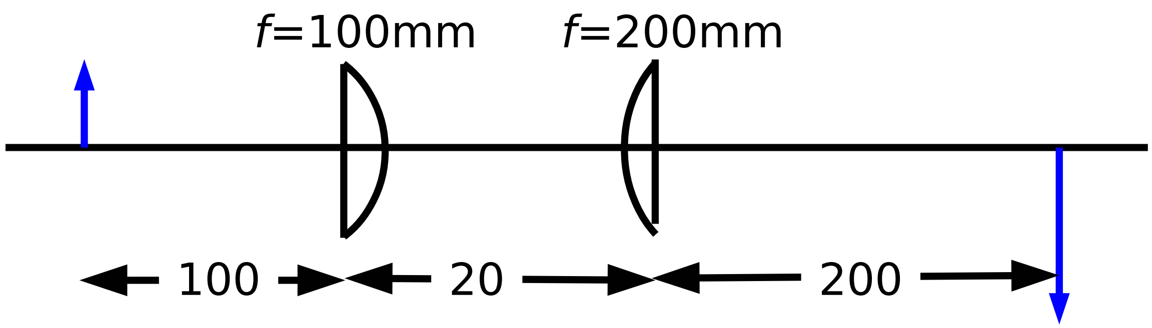
$h \rightarrow 0$ and $h' \rightarrow 0$ if $z_{12} \rightarrow 0$

Matrices and Principal Planes

In–Practice

- For a simple glass lens
  - The principal planes are separated by 1/3 the thickness.
  - For a lens with one plane surface, one principal plane is at the other vertex.
  - For a biconvex lens the principal planes are symmetrically located.
- For a compound lens the matrix calculation is needed
- In some cases, the principal planes can be in unusual (and inconvenient) places.

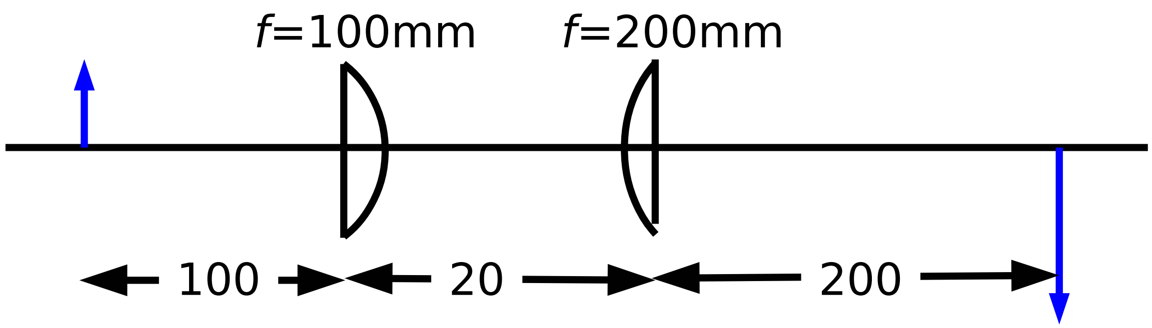
Example: 2X Magnifier (1)



- We Know How to Do This
  - Object at Front Focus of First Lens
  - Intermediate Image at Infinity
  - Final Image at Back Focus of Second Lens

- But Let's Use Matrix Optics for the Exercise

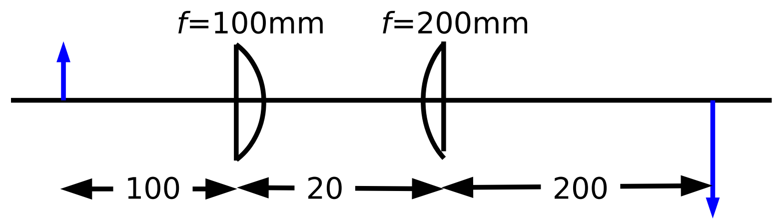
Example: 2X Magnifier (2)



Lens Vendor Data: Glass=BK7 ($n = 1.515$ at $\lambda = 633$nm

Parameter	Label	Value	
First Lens Focal Length	f_1	100	mm
First Lens Front Radius (LA1509 Reversed)	r_1	Infinite	
First Lens Thickness	$z_{v1,v1'}$	3.6	mm
First Lens Back Radius	r'_1	51.5	mm
First Lens "Back" Focal Length	$f_1 + h_1$	97.6	mm
Lens Spacing	$z_{v1',v2}$	20	mm
Second Lens Focal Length	f_2	200	mm
Second Lens Front Radius (LA1708)	r_2	103.0	mm
Second Lens Thickness	$z_{v2,v2'}$	2.8	mm
Second Lens Back Radius	r'_2	Infinite	
Second Lens Back Focal Length	$f'_2 + h'_2$	198.2	mm

Example: 2X Magnifier (Thin–Lens Approximation)



$$\frac{1}{f} = \frac{1}{100\text{mm}} + \frac{1}{200\text{mm}} - \frac{20\text{mm}}{100\text{mm} \times 200\text{mm}} \qquad f = 71.43\text{mm}$$

$$h = \frac{20\text{mm} \times 100\text{mm}}{20\text{mm} - 100\text{mm} - 200\text{mm}} = -7.14\text{mm}$$

$$h' = \frac{20\text{mm} \times 200\text{mm}}{20\text{mm} - 100\text{mm} - 200\text{mm}} = -14.28\text{mm}$$

$$m - -\frac{s'}{s} = -2 \qquad s' = 2s, \qquad \text{and} \qquad \frac{1}{f} = \frac{1}{s} + \frac{1}{s'} = \frac{1}{s} + \frac{1}{2s}$$

$$s = 3f/2 = 107.1\text{mm} \qquad s' = 3f = 214.3\text{mm}$$

Lens Thickness Effects

- Start with Equations for Thin Lenses
- Use Principal Planes in Place of Vertices

$$\mathcal{M}_{H_1,H_2'} = \mathcal{L}_{H_2,H_2'}\mathcal{T}_{H_1',H_2}\mathcal{L}_{H_1,H_1'}$$

- Same Equation as Thin Lens but Different Meaning

$$\mathcal{M}_{H_1,H_2'} = \begin{pmatrix} 1 & 0 \\ -\frac{1}{f_2} & 1 \end{pmatrix} \begin{pmatrix} 1 & z_{12} \\ 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 \\ -\frac{1}{f_1} & 1 \end{pmatrix}$$

  - f_1 from H_1 and f_1' from H_1'
  - f_2 from H_2 and f_2' from H_2'
  - z_{12} from H_1' to H_2

2X Magnifier, Revisited (1)

- Principal Planes

$$h = \frac{20\text{mm} \times 100\text{mm}}{20\text{mm} - 100\text{mm} - 200\text{mm}} = -7.14\text{mm} \qquad H_1 \text{ to } H$$

$$h' = \frac{20\text{mm} \times 200\text{mm}}{20\text{mm} - 100\text{mm} - 200\text{mm}} = -14.28\text{mm} \qquad H_2' \text{ to } H'$$

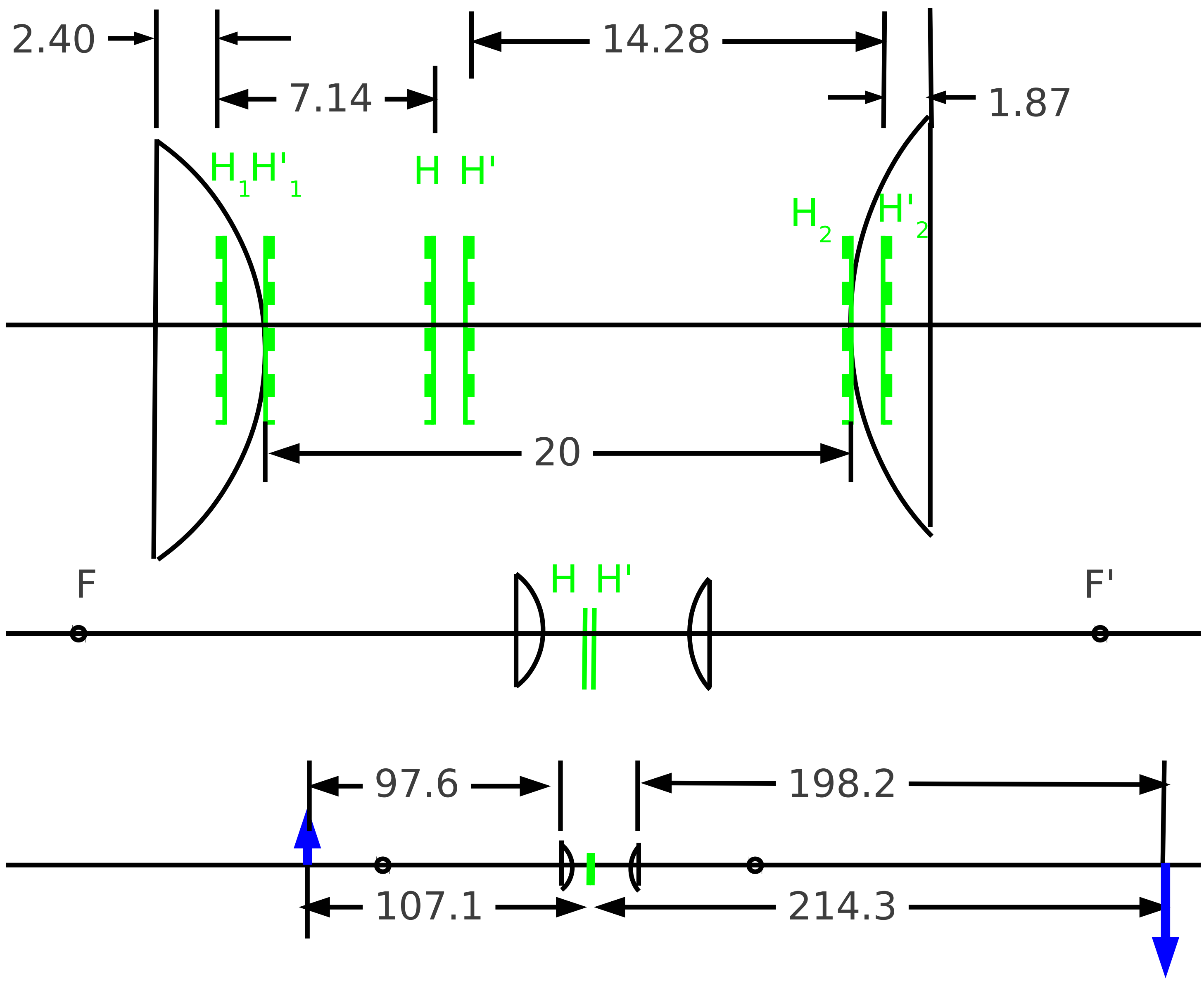
- Spacing (See Next Page)

$$0.713\text{mm}$$

- Object and Image Distances

$$s = \frac{3f}{2} = 107.1\text{mm} \qquad s' = 3f = 214.3\text{mm}$$

2X Magnifier, Revisited (2)



2X Magnifier Revisited (3)

In–Practice

- Assuming thin lenses in a compound lens is usually a good start.

- Locations of principal planes for the compound lens can be adjusted.

- Lens distances may change (although not in this example)

Q: How would you change the drawing if both lenses were reversed? Specifically, what would be the vertex–vertex distance between the lenses?

A Suggestion: Global Coordinates

- Notation: $zH1$
  - First Letter: z
  - The Remaining Characters: Plane Name (eg. $H1$)
- Need to Set One Plane as $z = 0$
- Example from the Magnifier
  - $z = 0$ at First Vertex
  - $zH1 = -h_1$
  - $zH = zH1 - h = -h_1 - h$
  - (Text Error: Not $z_H = zH1 - h = h_1 - h$)
  - *etc.*

Special Case: Afocal

$$z_{12} = f_1 + f_2$$

$$\frac{1}{f} = \frac{f_1}{f_1 f_2} + \frac{f_2}{f_1 f_2} - \frac{z_{12}}{f_1 f_2} = 0.$$

$$\boxed{m_{21} = 0, \qquad \frac{1}{f} = 0, \qquad \text{or} \qquad f \rightarrow \infty \qquad \text{(Afocal)}}$$

$$h \rightarrow \infty \qquad h' \rightarrow \infty$$

Principal Planes are not Very Useful Here.

Telescopes (1)

- Afocal Condition

$$\frac{1}{f} = \frac{1}{f_2} + \frac{1}{f_1} - \frac{z_{12}}{f_1 f_2} = 0 \qquad \text{if} \qquad z_{12} = f_1 + f_2$$

- Vertex Matrix

$$\mathcal{M}_{V_1,V_2'} = \begin{pmatrix} 1 - \frac{f_1+f_2}{f_1} & f_1 + f_2 \\ -\frac{1}{f_2} + \frac{f_1+f_2}{f_1 f_2} - \frac{1}{f_1} & 1 - \frac{f_1+f_2}{f_2} \end{pmatrix} = \begin{pmatrix} -\frac{f_2}{f_1} & f_1 + f_2 \\ 0 & -\frac{f_1}{f_2} \end{pmatrix}$$

- Imaging Matrix

$$\mathcal{M}_{SS'} = \begin{pmatrix} 1 & s' \\ 0 & 1 \end{pmatrix} \begin{pmatrix} -\frac{f_2}{f_1} & f_1 + f_2 \\ 0 & -\frac{f_1}{f_2} \end{pmatrix} \begin{pmatrix} 1 & s \\ 0 & 1 \end{pmatrix} = \begin{pmatrix} ? & 0 \\ ? & ? \end{pmatrix}$$

Telescopes (2)

$$\mathcal{M}_{SS'} = \begin{pmatrix} -\frac{f_2}{f_1} & -s\frac{f_2}{f_1} + f_1 + f_2 - s'\frac{f_1}{f_2} \\ 0 & -\frac{f_1}{f_2} \end{pmatrix} = \begin{pmatrix} ? & 0 \\ ? & ? \end{pmatrix}$$

$$-s\frac{f_2}{f_1} + f_1 + f_2 - s'\frac{f_1}{f_2} = 0$$

$$\boxed{m = -f_2/f_1, \qquad \text{(Afocal)}}$$

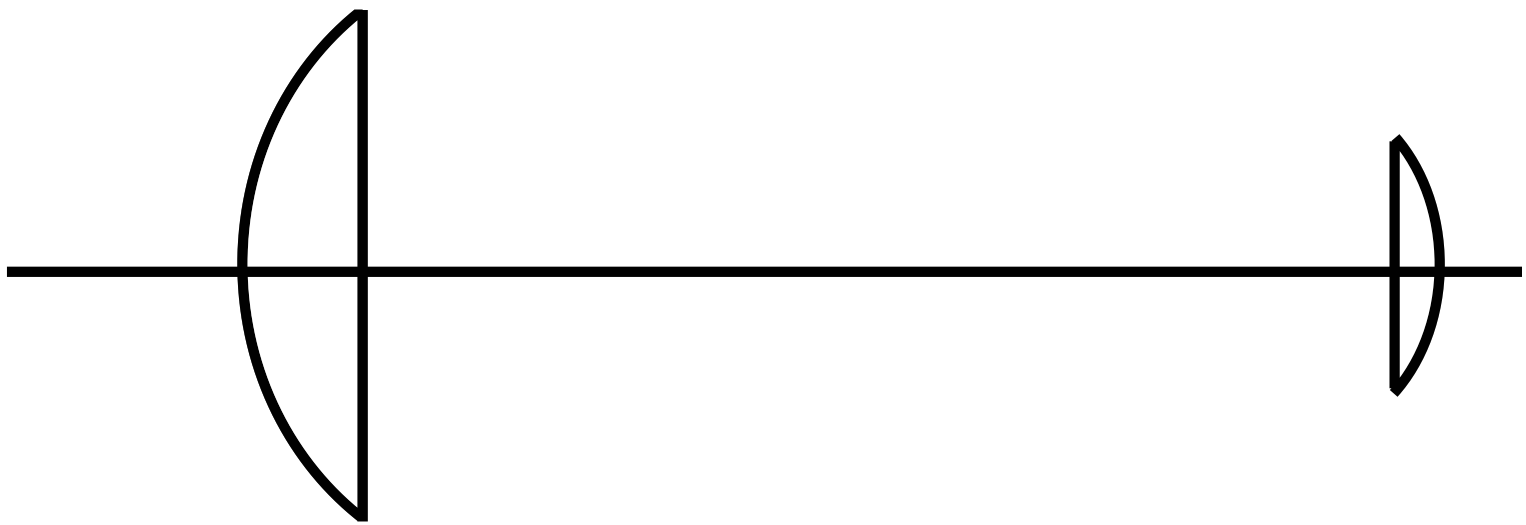
$$ms + f_1 + f_2 + s'/m = 0$$

$$s' = -m^2 s - f_1 m\,(1+m)$$

$$\mathcal{M}_{SS'} = \begin{pmatrix} m & 0 \\ 0 & \frac{1}{m} \end{pmatrix}$$

$$s' \approx -m^2 s \qquad s \to \infty$$

Astronomical Telescope



- Magnification: Image is smaller ($\ll 1$) $m = \frac{f_2}{f_1}$
- But a Lot Closer: ($m_z = -m^2$)
- Angular Magnification is Large ($m_\alpha = 1/m$)

700min 28 Jan 2014