Optics for Engineers

Chapter 2

Charles A. DiMarzio
Northeastern University

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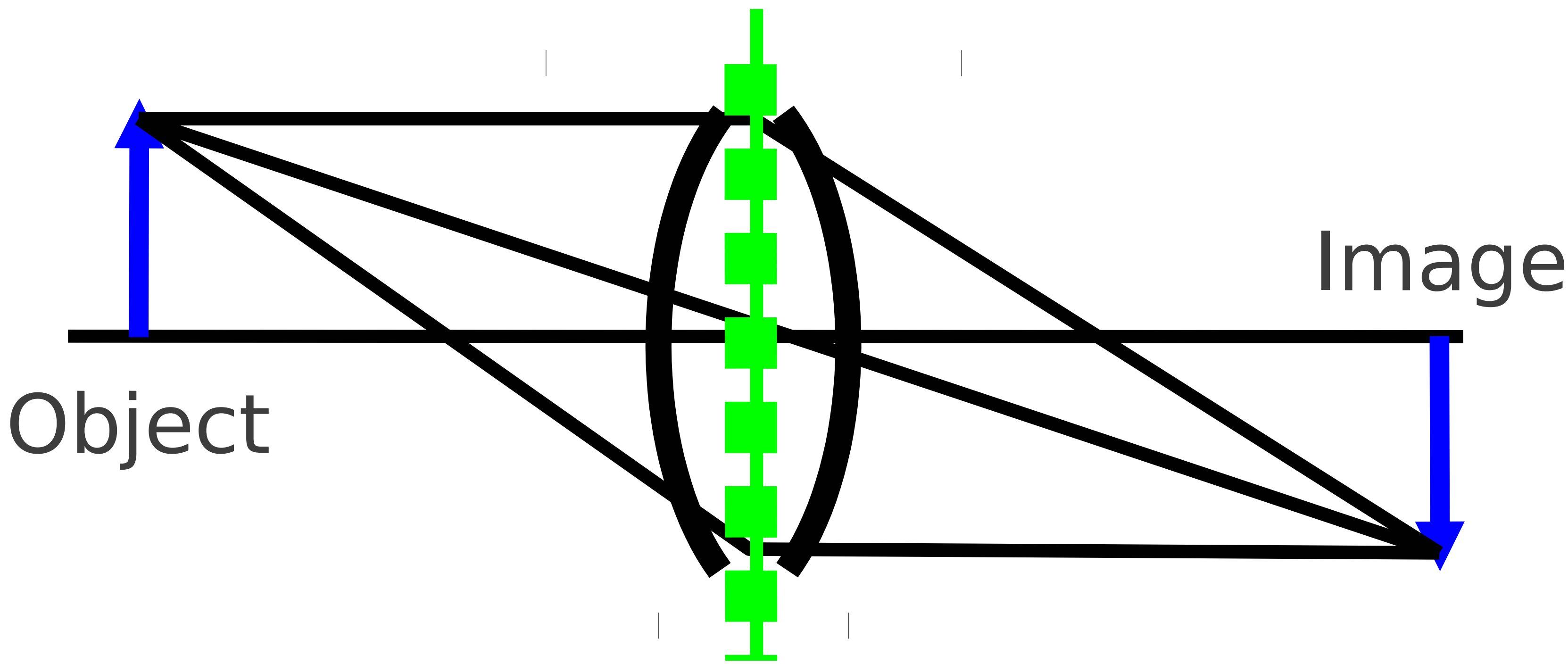
Outline of Geometric Optics

- Chapter 2
  - Snell's Law from Fermat's Principle
  - Mirrors and Refractive Surfaces
  - Multiple Surfaces: Simple Lenses: The Thin Lens
  - Image Location, Orientation, Magnification
- Chapter 3: Matrix Optics: Principal Planes
- Chapter 4: Stops Limit Light Gathering and FOV
- Chapter 5: Aberrations Limit Resolution
- Later: Wave Optics: Diffraction–Limited Resolution in Ch. 8

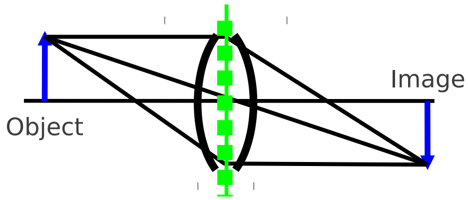
“High–School Optics”



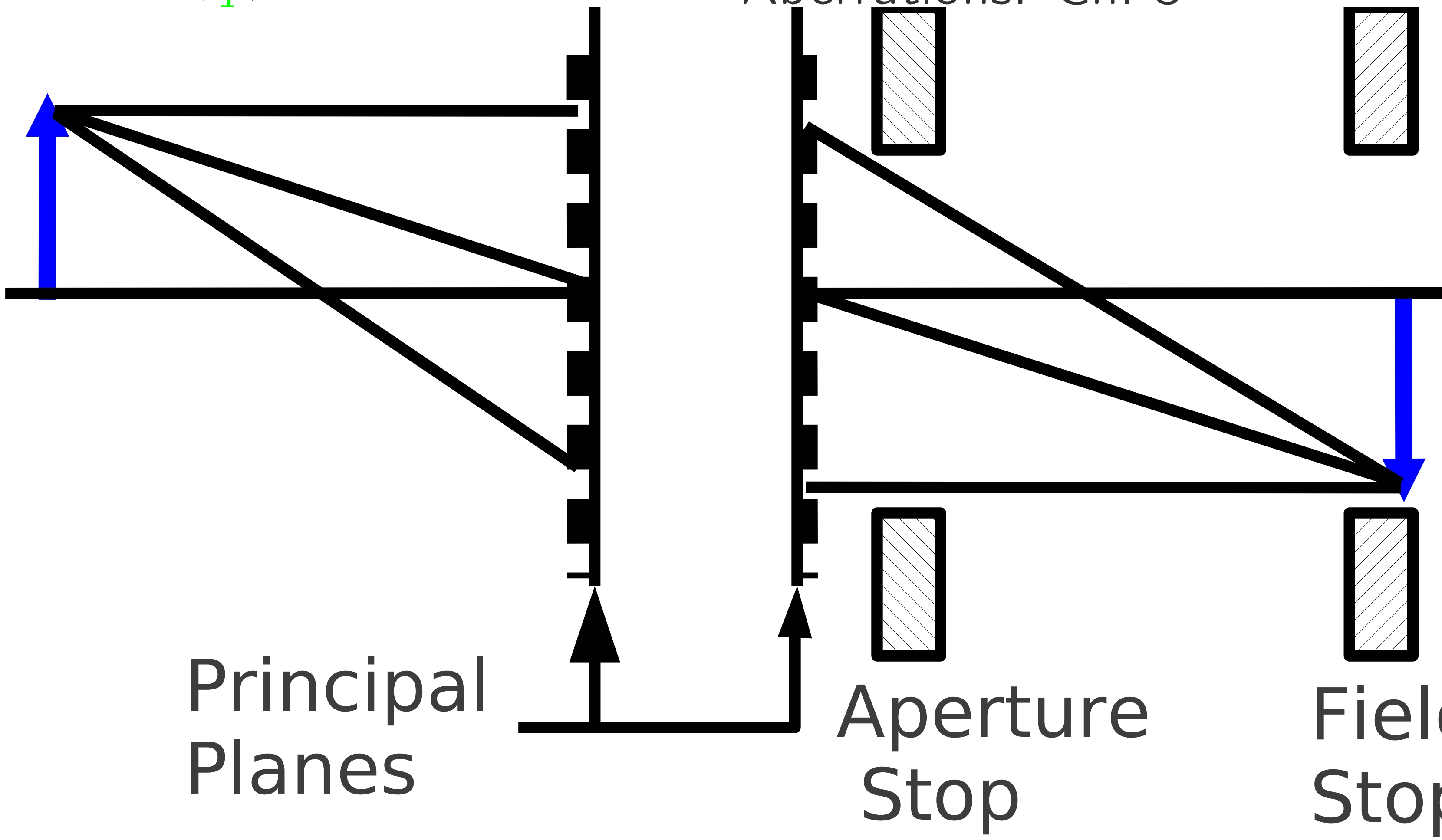
“High–School Optics Rules”

- Find front and back focal points, F and F', located f in front of, and in back of, the lens.
- Trace the ray from the object arrow parallel to the axis, refracting out through the back focal point.
- Trace the ray from the object arrow through the front focal point, out parallel to the axis.
- They intersect at the image.
- Check by tracing the ray through the center of the lens which does not refract.

“The AP Version”

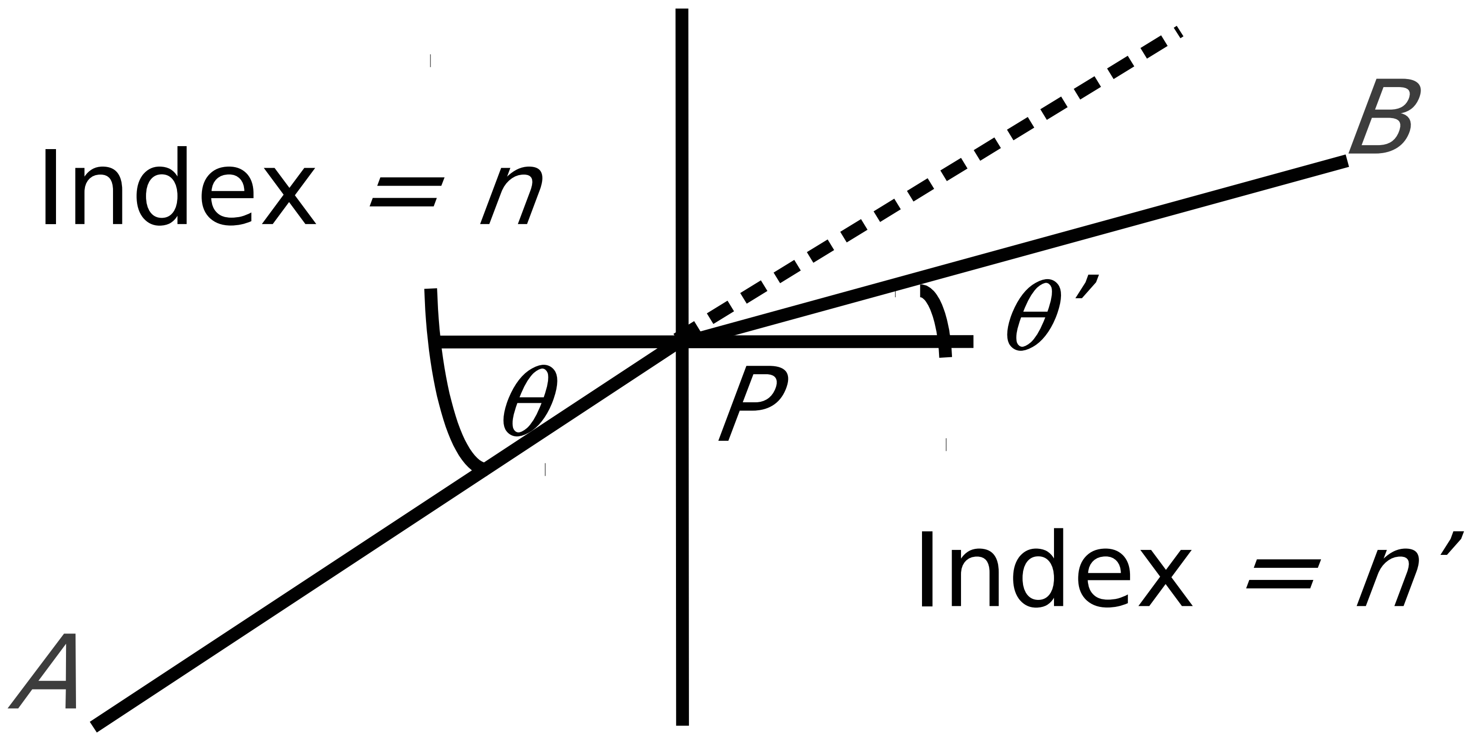


Principal Planes: Ch. 3
Stops: Ch. 4
Aberrations: Ch. 5

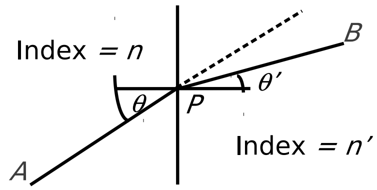


Concepts for Refraction

- Plane of Incidence Contains Incident (and Exiting) Ray and Normal (and is the plane of the 2–D drawing)
- Angle of Incidence Is Defined Relative to Normal

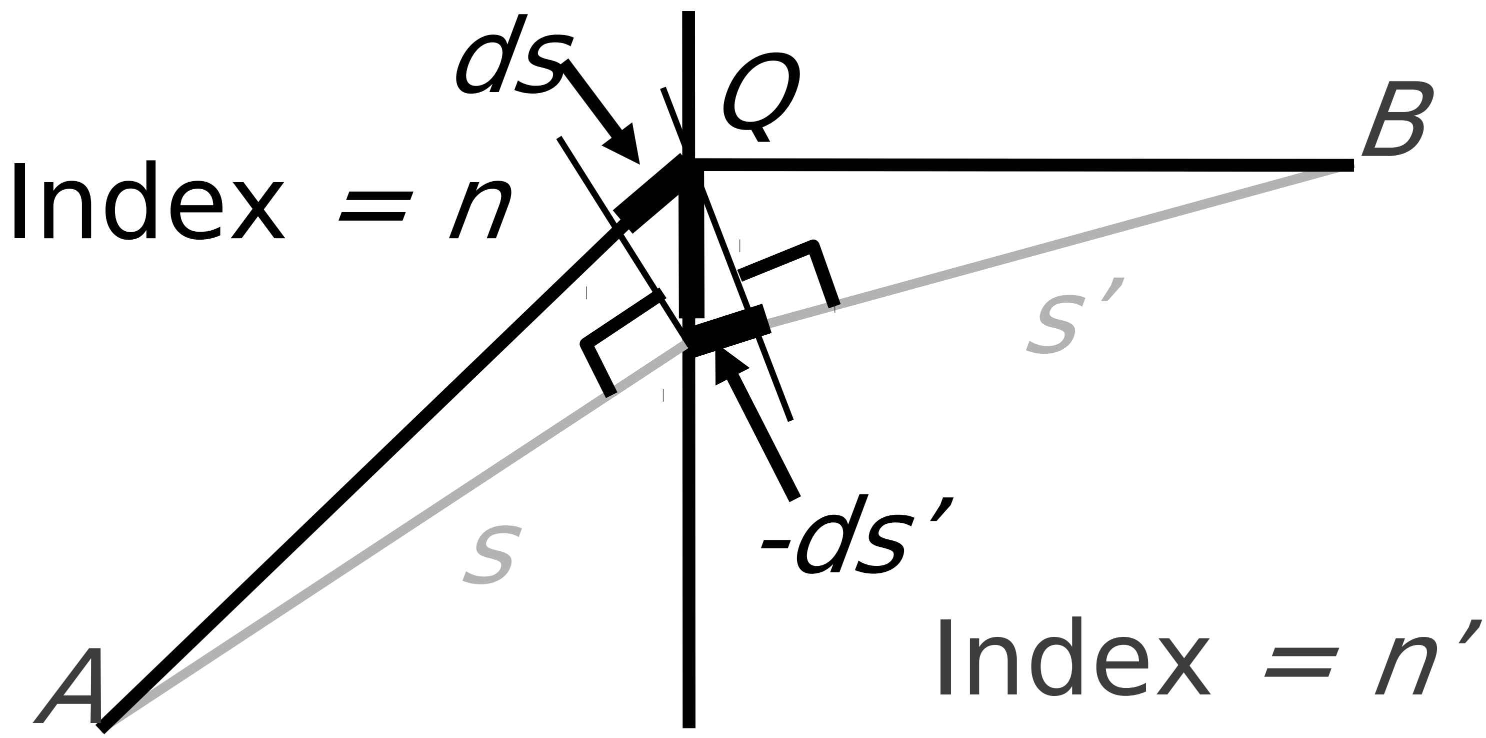


Snell's Law

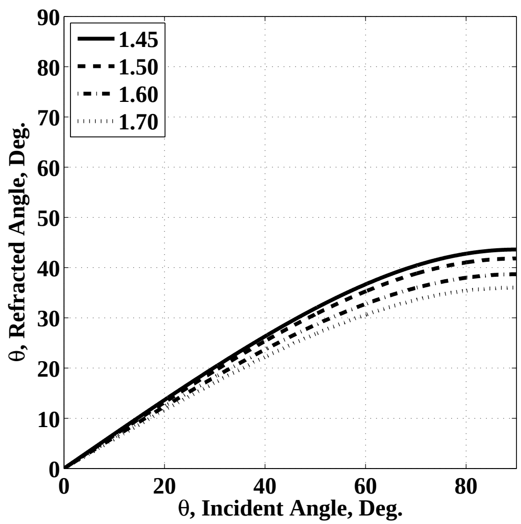


Variational Approach from
Minimal Path, AB (Fermat) $nds = n'ds'$

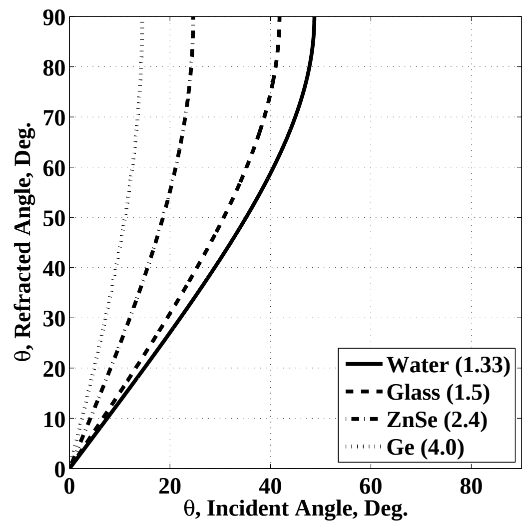
$$n \sin\theta = n' \sin\theta'.$$



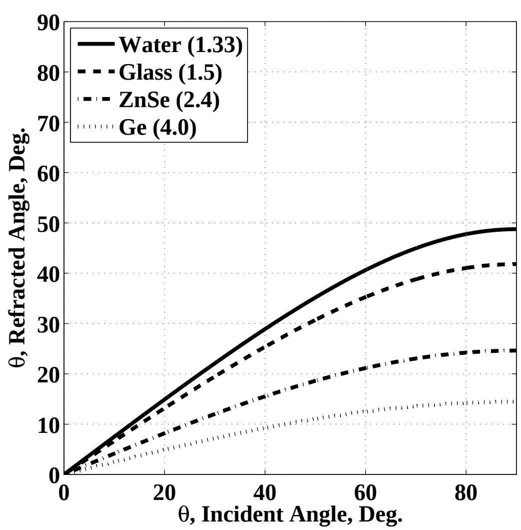
Snell's Law: Examples



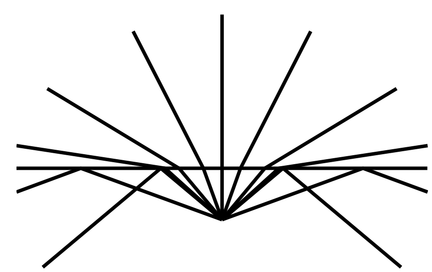
Air to Glass



Material to Air



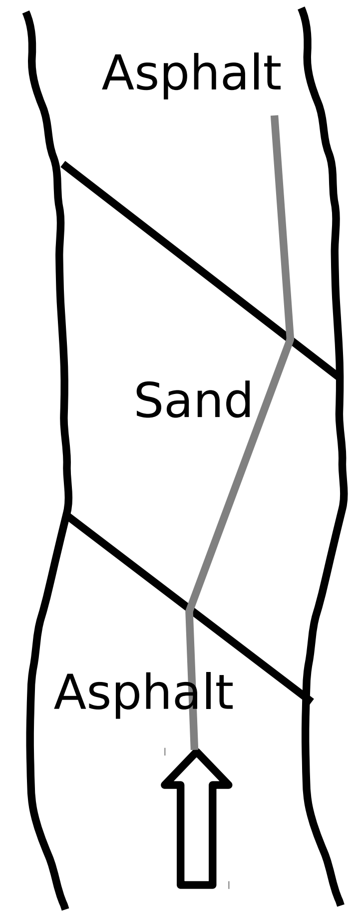
Air to Material



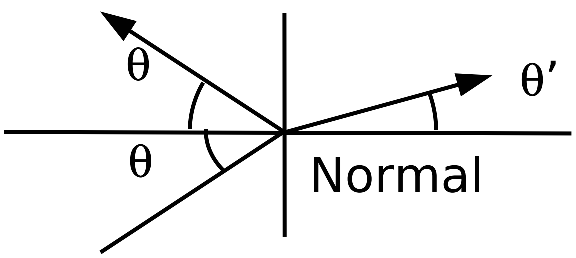
Total Internal Reflection

Snell's Law: Analogy

- Driving a car from an asphalt road onto a dirt road
  - Asphalt–to–dirt line is diagonal.
  - Dirt slows speed.
- Car tries to turn to the right.
- Dirt–to–Asphalt: Car turns back to the left.
- This is just an analogy to remember the direction.



Reflection and Refraction



Reflection:

$$\theta_r = \theta.$$

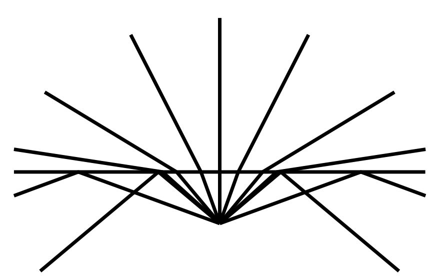
Refraction

Total Internal Reflection

- Critical Angle (No Solution for θ')

$$\boxed{n \sin\theta_c = 1}$$

- For $\theta < \theta_c$ Reflection and Refraction
- For $\theta > \theta_c$ 100% Reflection

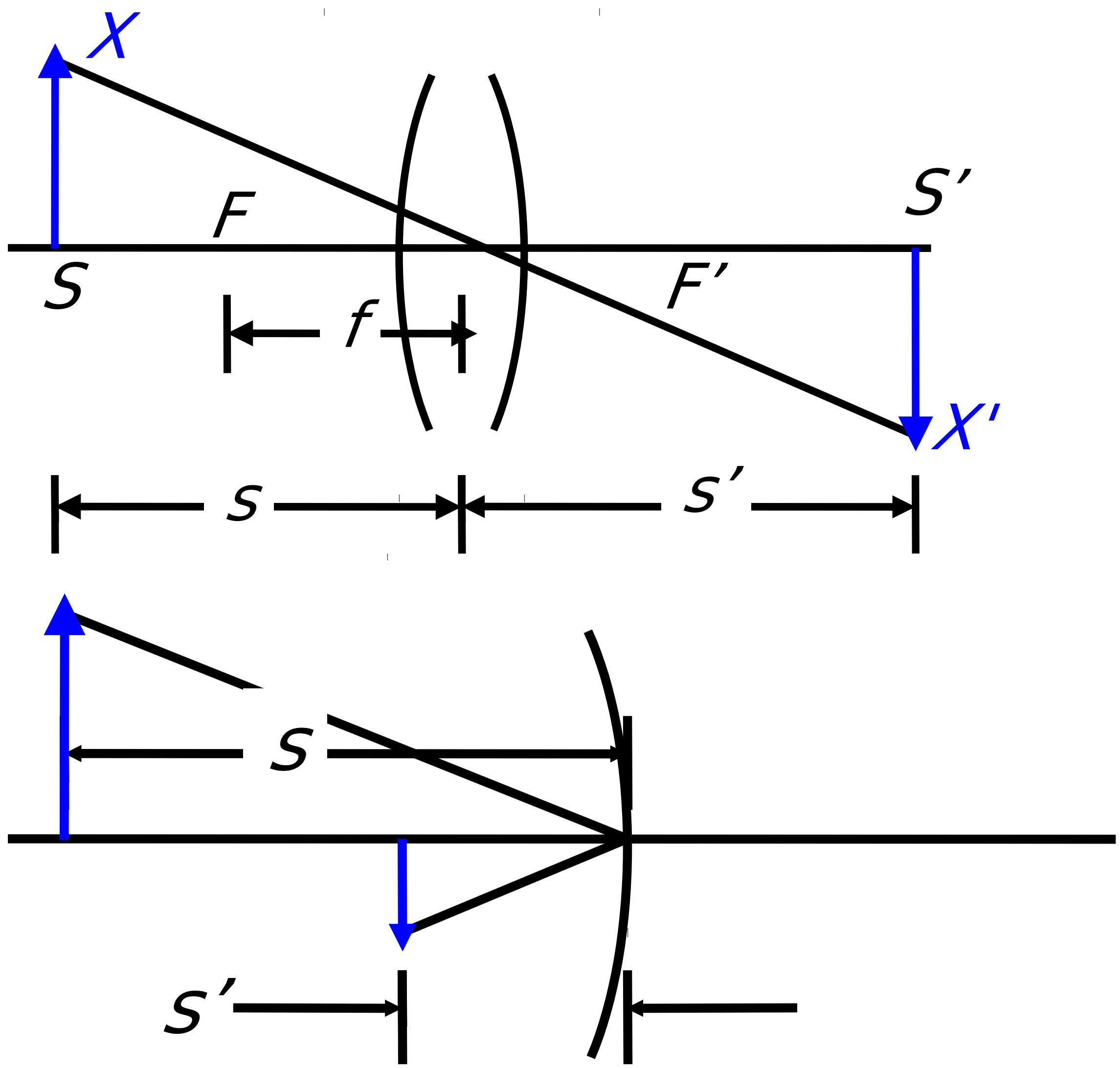


Snell's Window

Carol Grant

Q: How would you reconstruct the original scene?

Imaging Sign Conventions and Coordinates



- z Coordinates
  - s for Object
  - s' for Image
- Lens
  - $s > 0$ to Left
  - $s' > 0$ to Right
  - $f > 0$ for Converging
- Mirror
  - $s > 0$ to Left
  - $s' > 0$ to Left
  - $f > 0$ for Concave

Upper Case for points, Lower for lengths ($x, x' > 0$ Upright)

Imaging Terms

We will discuss these in detail later.
The important issues now are the definitions.

Quantity	**Definition**	**Equation**	**Notes**
Object distance	s		Positive to the left
Image distance	s'	$\frac{1}{s}+\frac{1}{s'}=\frac{1}{f}$	Positive to the right for interface or lens. Positive to the left for mirror.
Magnification	$m=\frac{x'}{x}$	$m=-\frac{x'}{x}$	
Angular magnification	$m_\alpha=\frac{\partial\alpha'}{\partial\alpha}$	$\left\|m_\alpha\right\|=\frac{1}{\left\|m\right\|}$	
Axial Magnification	$m_z=\frac{\partial s'}{\partial s}$	$\left\|m_z\right\|=\left\|m\right\|^2$	

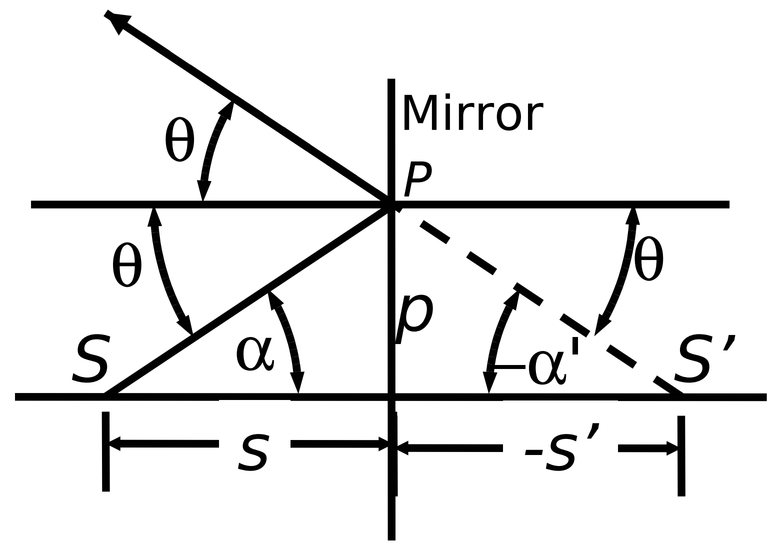
Pause for Reflection

Take–Away Messsge

- All of Geometric Optics is Based on
  - Law of Reflection
$$\theta' = \theta$$
  - Snell's Law of Refraction
$$n \sin\theta = n' \sin\theta'$$
- But Imaging Equations are Useful Simpifications
  - We Turn to Those Next (Plane mirror, curved mirror, plane dielectric interface, curved dielectric interface, simple lens, compound lens).

Reflection at a Plane Mirror (1)

- Narcissus
- "...the looking glasses of the women..." Exodus 38:8
  - **Image Location:**
    - Similar Triangles
    - $s' = -s$
      (Planar reflector)
    - Virtual Image as Shown
      (Dotted Lines)



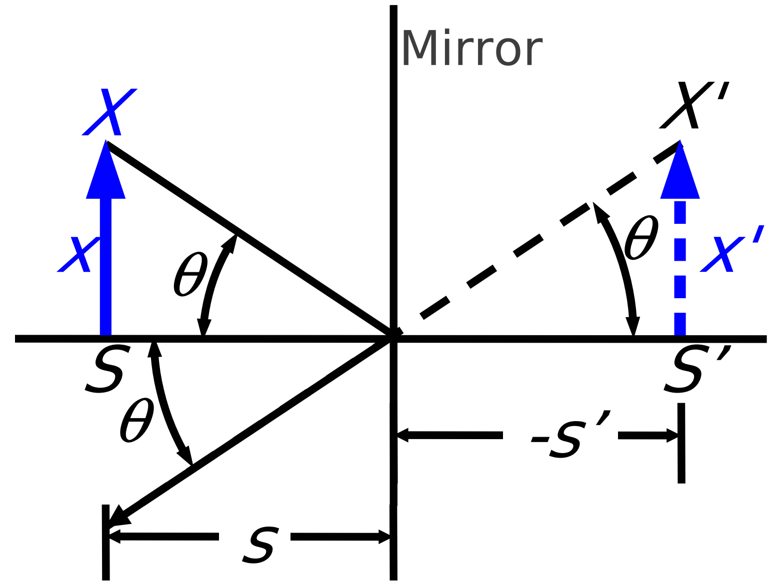
- Q: Could we have a virtual object? How?

Reflection at a Plane Mirror (2)

- **Magnification (Transverse)**
  - More Similar Triangles
  - Result: $x' = x \quad m = 1$

$$m = \frac{x'}{x} = \frac{-s'}{s} = 1$$

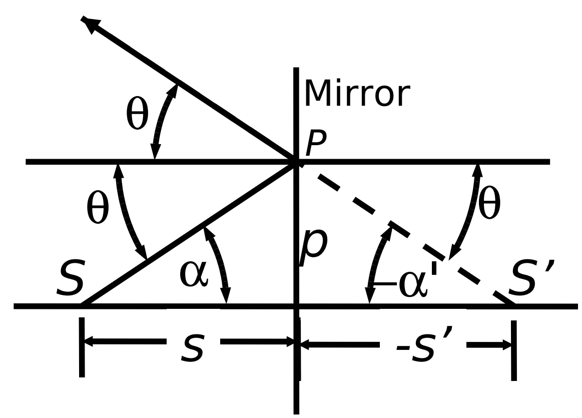
(Planar reflector)



Upright ($m > 0$) & Virtual (Dotted Lines)

- **Angular Magnification**

$$m_\alpha = \frac{d\alpha'}{d\alpha} = -1 \qquad \text{(Planar reflector)}$$



Reflection at a Plane Mirror (3)

Axial Magnification

$$m_z = \frac{ds'}{ds} = \frac{s'}{s} = -1 \qquad \text{(Planar reflector)}$$

Summary of Imaging Parameters

$$s = -s' \qquad m = 1 \qquad m_z = -1$$

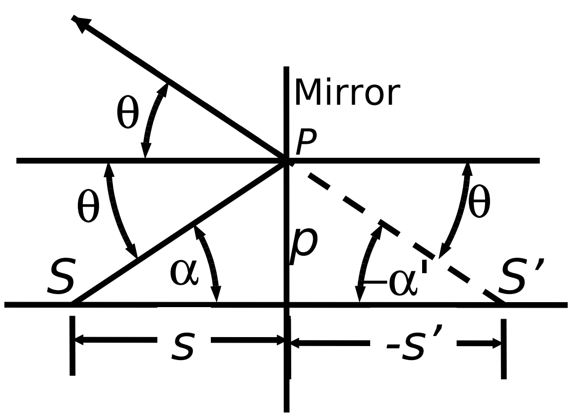
Upright, Virtual, Perverted*, but Not Distorted**
*Right–Handed Coordinate System Imaged to Left–Handed
**Distorted Means $m_z \neq m$.

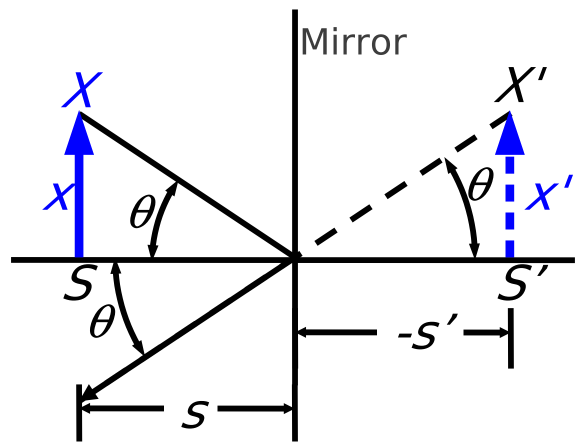
Misconception: Mirror Does Not Reverse Left and Right
Left is Left, Right is Right, but Front is Back

Imaging Equations

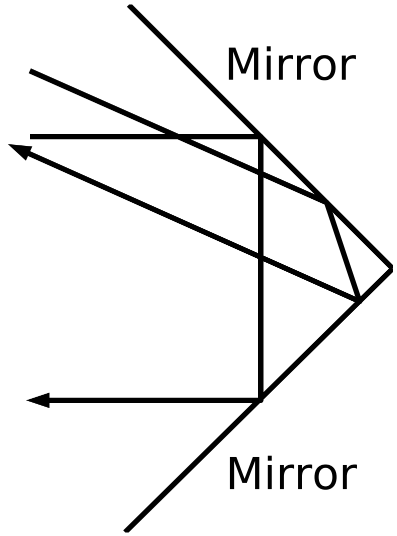
Surface	s'	m	m_α	m_z	**Image****	**O***	**D***	**P***
Planar Mirror***	$s' = -s$	1	-1	-1	Virtual	Upright	No	Yes
Concave Mirror $s > f$	$\frac{1}{s'} + \frac{1}{s} = \frac{1}{f}$	$-s'/s$	$-m^2$	$-1/m$	Real	Inverted	Yes	No
Convex Mirror	$\frac{1}{s'} + \frac{1}{s} = \frac{1}{f}$	$-s'/s$	$-m^2$	$-1/m$	Virtual	Upright	Yes	Yes
Planar Refractor	$\frac{s}{n} = \frac{s'}{n'}$	1	$\frac{n}{n'}$	$\frac{n'}{n}$	Virtual	Upright	Yes	No
Curved Refractor $s > f$	$\frac{n}{s} + \frac{n'}{s'} = \frac{n'-n}{r}$	$-\frac{ns'}{n's}$	-1	$-\frac{n}{n'}m^2$	Real	Inverted	Yes	Yes

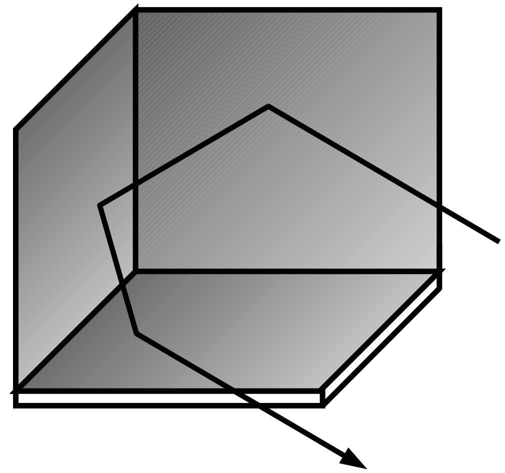
* "O" mean Orientation, "D" means Distortion, and "P" means Perversion of coordinates.
** The Image is Defined as Real or Virtual for a Real Object
*** Complete Analysis in green text. The rest is coming.

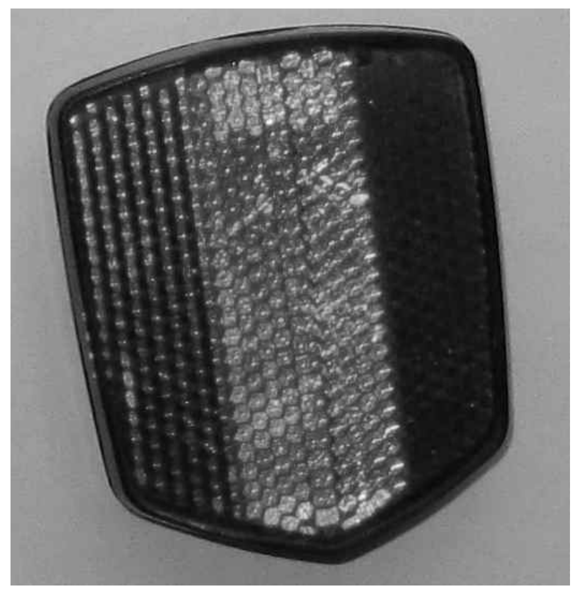




The Retroreflector







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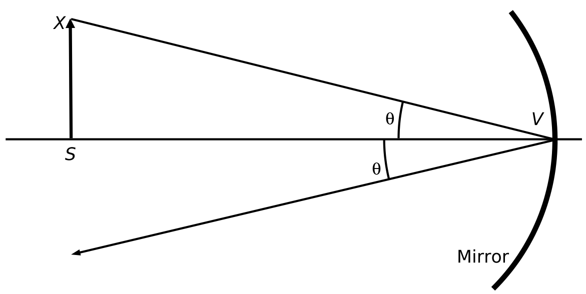
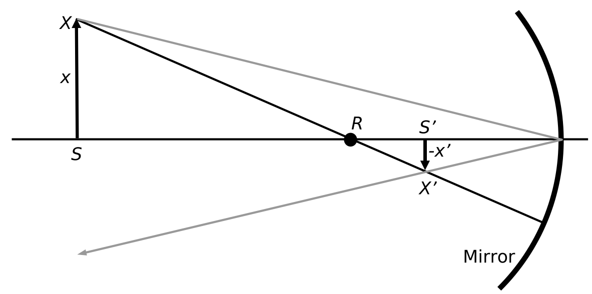
Curved (Spherical) Mirror (1)

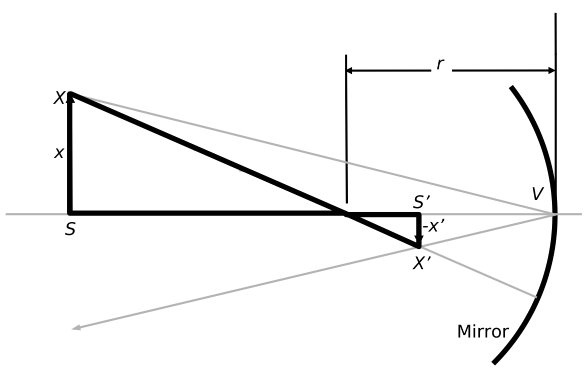
All Rays from the Object Go Through the Image (No Aberrations).
Work with the Easy Ones.



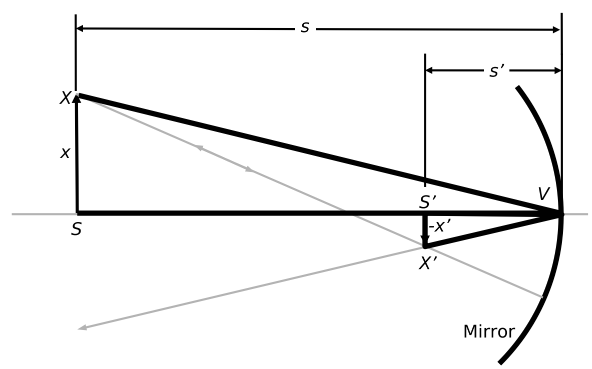
A. Vertex Ray



B. Radial Ray



C. Ray Intersection



D. Similar Triangles

$$\frac{x}{s-r} = \frac{-x'}{r-s'} \qquad (C)$$

$$\frac{x}{s} = \frac{-x'}{s'} \qquad (D)$$

Image Location

$$\frac{1}{s'} + \frac{1}{s} = \frac{2}{r}$$

Magnification

$$m = \frac{x'}{x} = -\frac{s'}{s}$$

Curved (Spherical) Mirror (2)

- Focal Length Defined in General

$$\boxed{\frac{1}{s'}+\frac{1}{s}=\frac{1}{f}}$$

- Specific Result for Spherical Mirror

$$\boxed{f=\frac{r}{2} \qquad \text{(Spherical reflector)}}$$

- Physical Signficance and Definition of f

$$s' \rightarrow f \qquad s \rightarrow \infty \qquad \text{or} \qquad s \rightarrow f \qquad s' \rightarrow \infty.$$

Curved (Spherical) Mirror (3)

- Angular Magnification

$$m_\alpha = \frac{s}{s'} \qquad |m_\alpha| = |1/m| \qquad \text{(Spherical reflector)}$$

- Axial Magnification

$$\frac{1}{s} + \frac{1}{s'} = \frac{2}{r}$$

$$-\frac{ds}{s^2} - \frac{ds'}{(s')^2} = 0$$

$$m_z = \frac{ds'}{ds} = -\left(\frac{s'}{s}\right)^2 \qquad m_z = -m^2 \qquad |m_z| = |m|^2$$

Curved (Spherical) Mirror (4)

- Imaging Equation

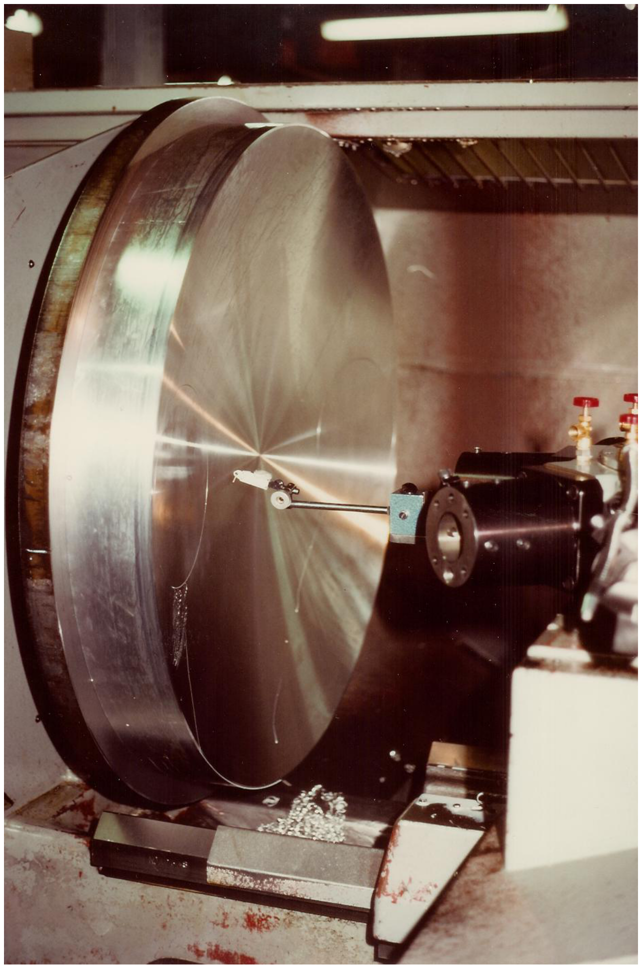
$$\frac{1}{s'} + \frac{1}{s} = \frac{1}{f} \qquad f = \frac{r}{2}$$

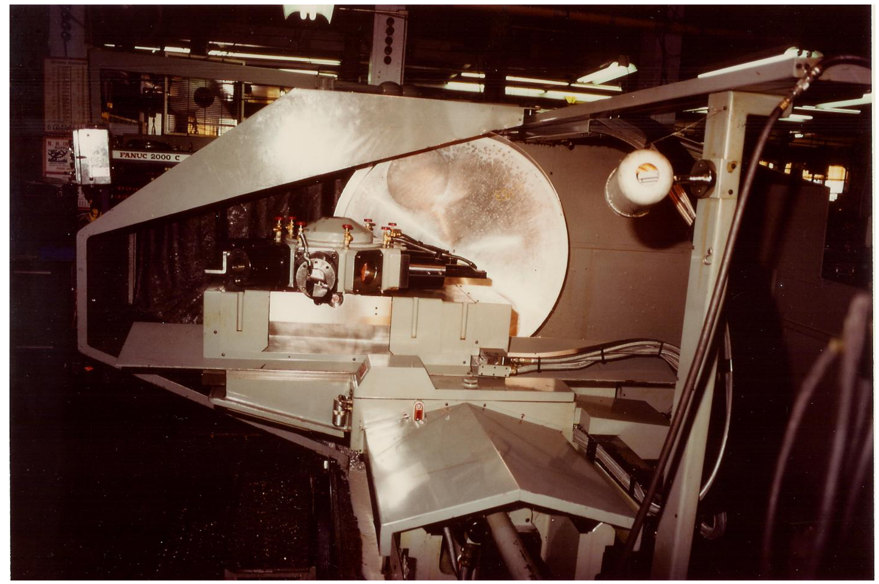
- Magnification

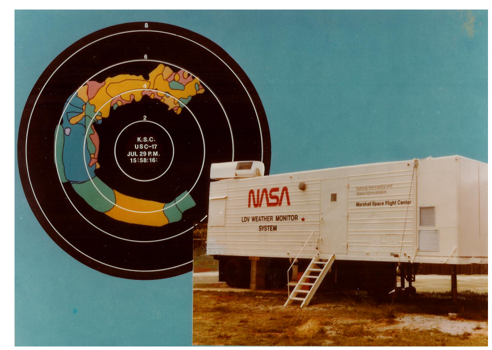
$$m = -\frac{s'}{s} \qquad m_\alpha = \frac{1}{m} \qquad m_z = -m^2$$

- Summary: The Image in this Case is...
  - Real
  - Inverted
  - Distorted (Unless $s = s'$)
  - Handedness–Preserved
- Q: Can a Concave Mirror Ever Produce a Virtual Image of a Real Object? (Hint: What if $s' = 0$?)

Large Reflective Optics







"Every Material that Transmits 10μm Light is Expensive."
Not Completely True, but Close.

Reflective or Refractive

In–Practice

- Reflective optical elements are common for large diameters, *e.g.* tens of cm. and up.

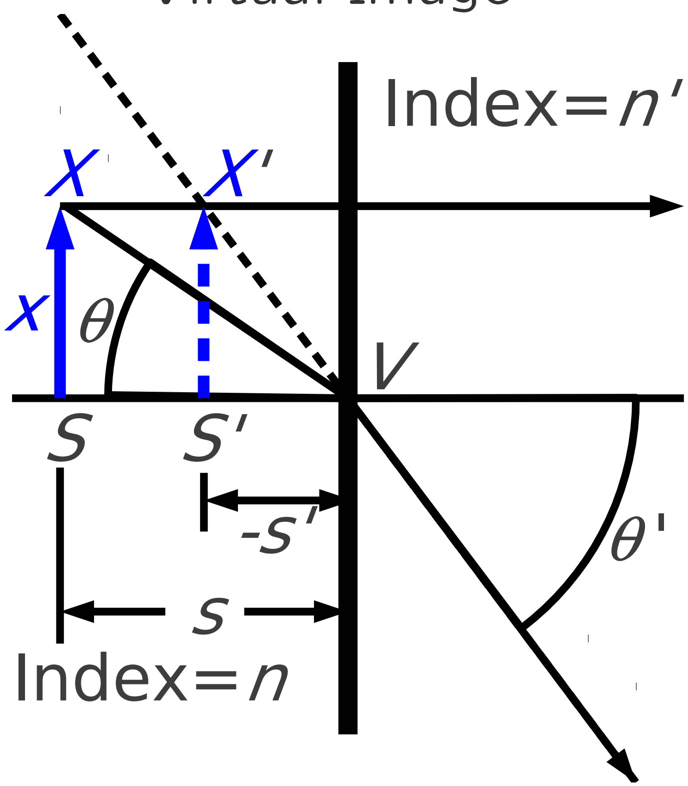
- The decision often changes a bit in the infrared where optical materials are more expensive.

- Reflective optics are also common for wide ranges of wave-length.

  – n varies with wavelength, which complicates design of re-fractive systems.

Refraction at a Plane Surface: The Fishtank Problem (1)

- Fishtank Setup
  - Object Inside
  - Viewer Outside
  - Virtual Image



- Geometry

$$\tan\theta = \frac{x}{s} \qquad \tan\theta' = \frac{x'}{s'} = \frac{x}{s'}$$

- Snell's Law (Small Angles)

$$n \sin\theta \approx n\frac{x}{s} \qquad n' \sin\theta' \approx n'\frac{x}{s'}$$

- Refraction at a planar Interface

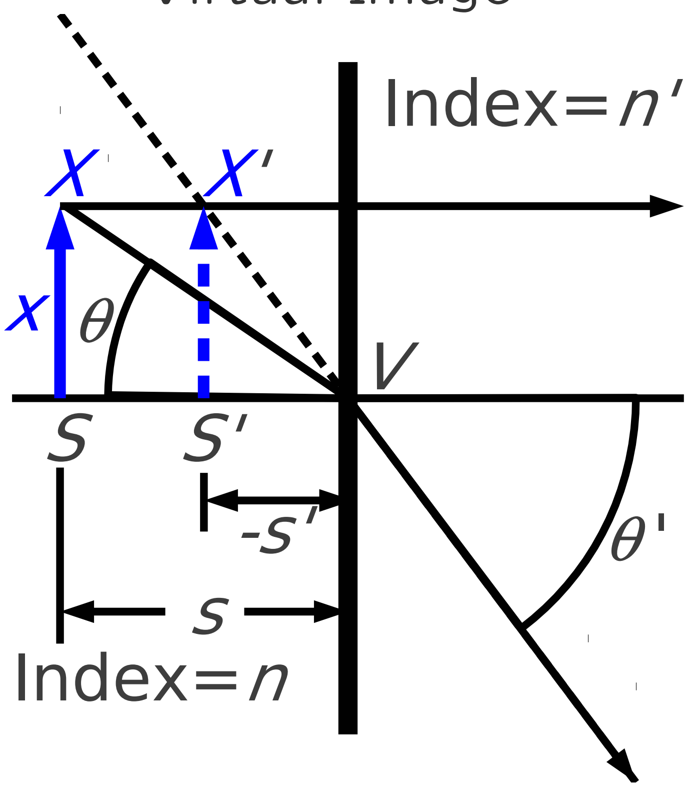
$$\boxed{\frac{n}{s} = \frac{n'}{s'}}$$

- Fishtank from Outside

$$n = 1.33 \qquad n' = 1 \qquad s' = \frac{1}{1.33}s$$

The Fishtank Problem (2)

- Fishtank Setup
  - Object Inside
  - Viewer Outside
  - Virtual Image



- Fishtank Paradox
  - Physical Thickness $z = s$
  - Geometric Thickness

$$\ell_g = \frac{z}{n}$$

  - Optical Pathlength

$$OPL = zn$$

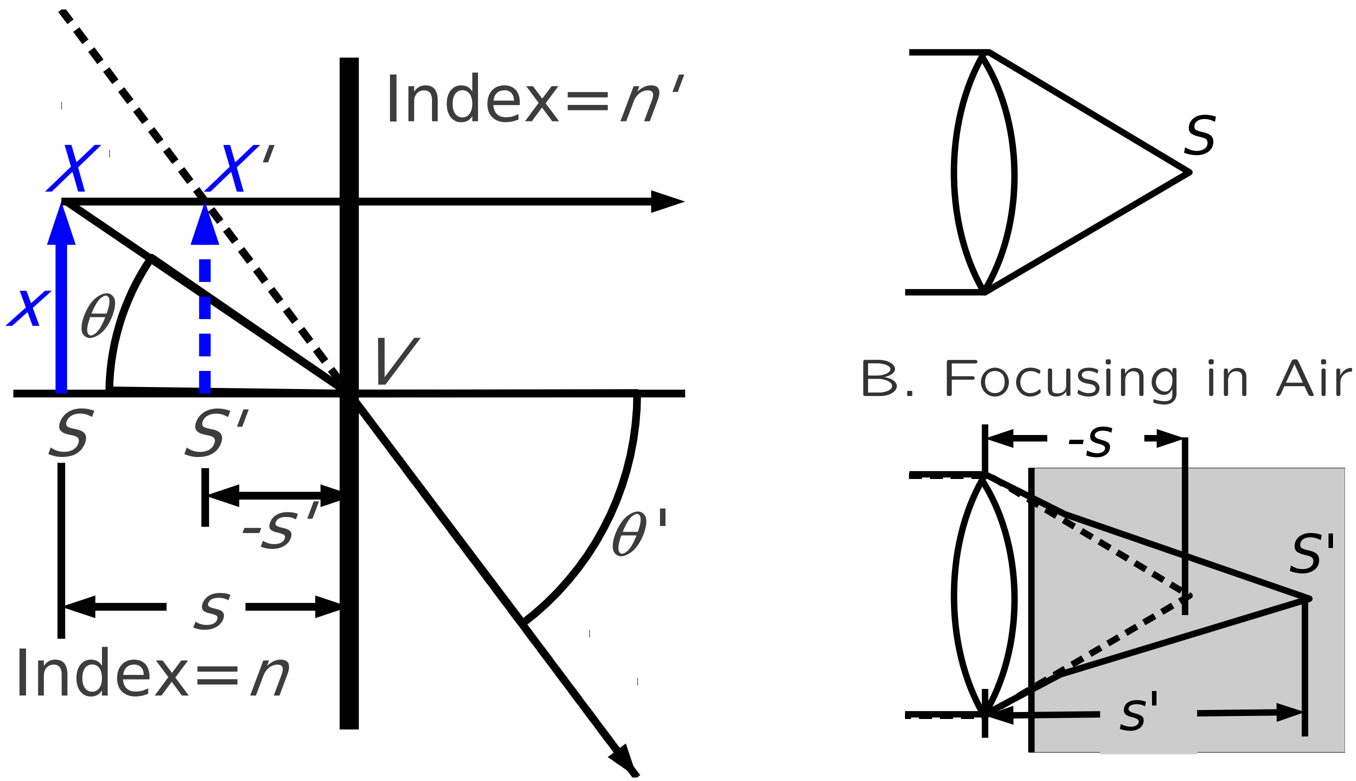
- Magnifications

$$m = \frac{x'}{x} = 1 \qquad m_\alpha = \frac{n}{n'}$$

$$m_z = \frac{ds'}{ds} = \frac{n'}{n}$$

- Virtual, Upright, Distorted

Practical Example



A. Planar Interface

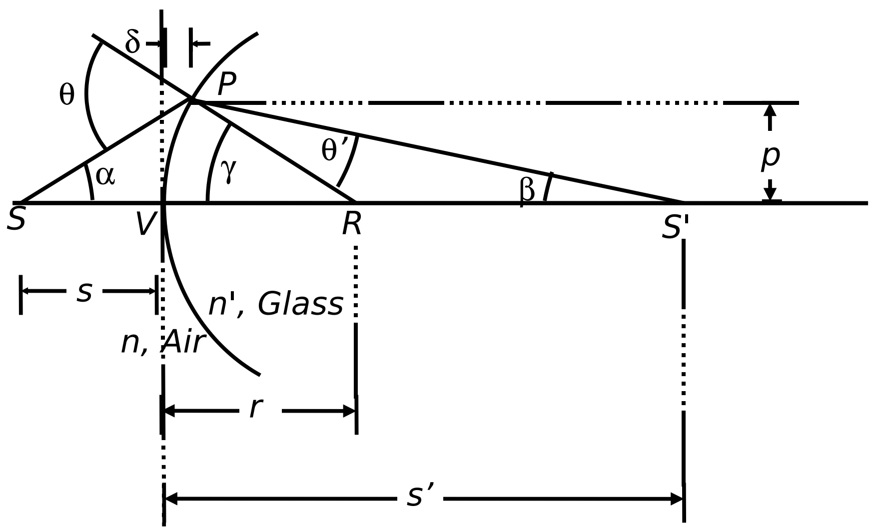
B. Focusing in Air

C. Focusing in Skin

Focusing Depth Decreases, but OPL Increases. *e.g.* Focus to 100μm and image 75μm. Time Gate at 133μm (Optical Coherence Tomography) Together, measure index and depth?

300min

Refraction: Curved Interface (1)



$\theta = \alpha + \gamma$ from $\triangle\ S, P, R,$

and

$\gamma = \theta' + \beta$ from $\triangle\ S', P, R$

$$\tan\alpha = \frac{p}{s+\delta} \qquad \tan\beta = \frac{p}{s'-\delta} \qquad \tan\gamma = \frac{p}{r-\delta}$$

For Small Angles tan ? = sin ? =? and $\delta \to 0$

$$\alpha = \frac{p}{s} \qquad \beta = \frac{p}{s'} \qquad \gamma = \frac{p}{r}$$

$$\theta = \frac{p}{s} + \frac{p}{r} \qquad \theta' = \frac{p}{r} - \frac{p}{s'}$$

Refraction: Curved Interface (2)

- Previous Page...

$$\theta = \frac{p}{s} + \frac{p}{r} \qquad\qquad \theta' = \frac{p}{r} - \frac{p}{s'}$$

- Snell's Law (Small Angles sin ? =?)

$$n\theta = n'\theta'$$

$$\frac{np}{s} + \frac{np}{r} = \frac{n'p}{r} - \frac{n'p}{s'}$$

$$\frac{n}{s} + \frac{n'}{s'} = \frac{n'-n}{r}$$ (Refraction at a curved surface)

Refraction: Curved Interface (3)

- Focal Lengths (More Complicated Now)
  - Back Focal Length (Refraction at a curved surface)

$$s \rightarrow \infty \qquad BFL = f' = s' = \frac{n'r}{n' - n}$$

  - Front Focal Length

$$s' \rightarrow \infty \qquad FFL = f = s = \frac{nr}{n' - n}$$

- Ratio (Calculated for this Example, but Much More General)

$$\frac{f'}{f} = \frac{n'}{n}$$

Refracting Power

- Why? Because Refracting Power is Often Additive (Later)
- Definition

$$P = \frac{n}{f} = \frac{n'}{f'}$$

- Units

$$\text{Diopter} = \text{m}^{-1}$$

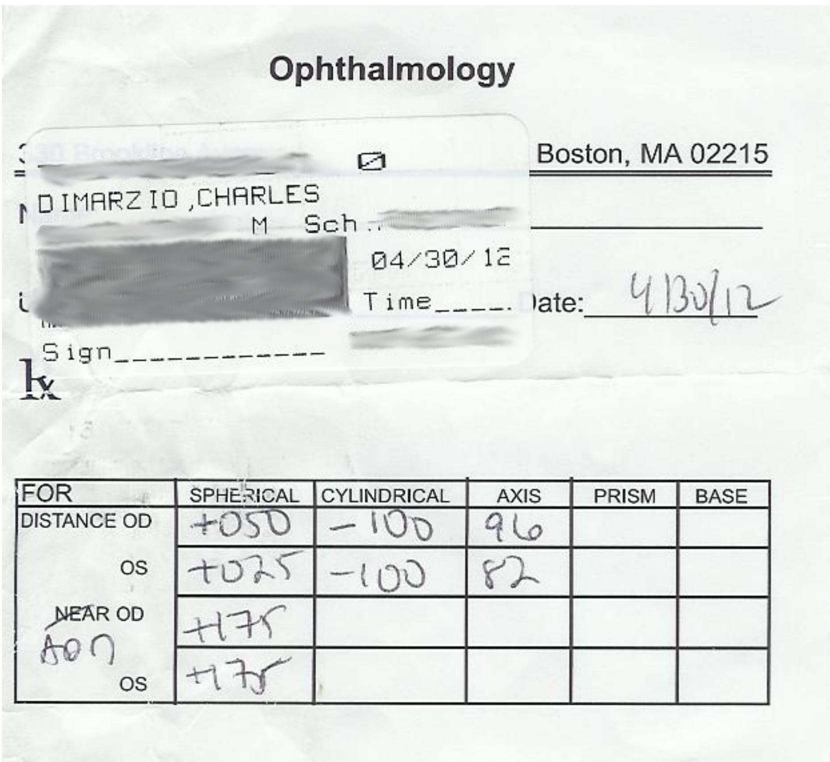
- Refraction at a Curved Interface

$$\boxed{P = \frac{n' - n}{r}}$$

Q: What combinations of n, n', and r yield positive (or negative) refracting power?

400min 17 Jan 2014

Eyeglass Prescription



Ophthalmology

[redacted] Boston, MA 02215

DIMARZIO ,CHARLES

M Sch [redacted]

[redacted] 04/30/12

Time____ Date: 4/30/12

Sign___________

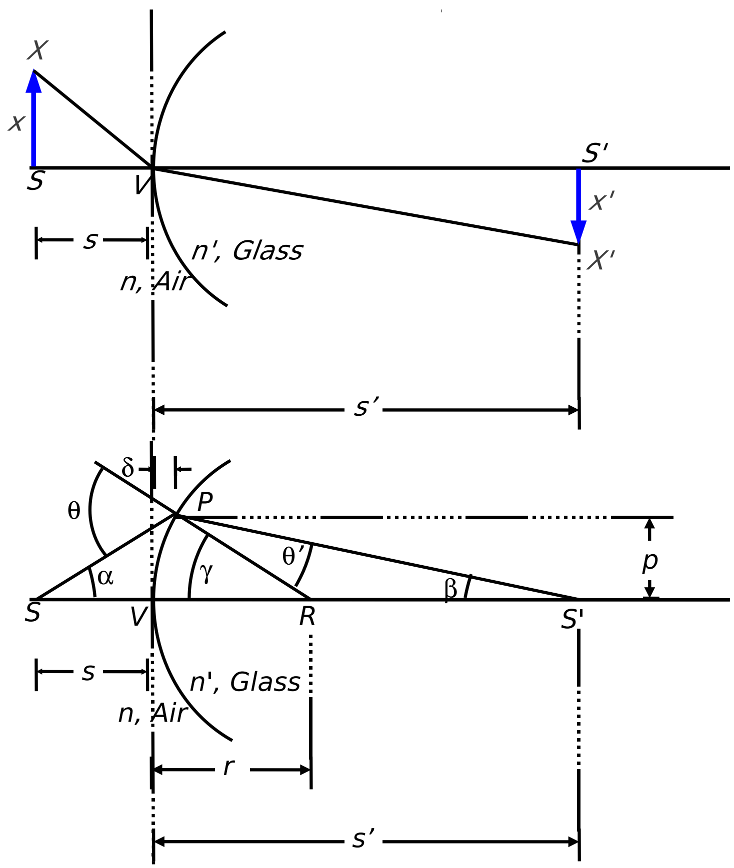
Rx

FOR	SPHERICAL	CYLINDRICAL	AXIS	PRISM	BASE
DISTANCE OD	+050	−100	96		
OS	+025	−100	82		
NEAR OD (ADD)	+175				
OS	+175				

- In Hundreths of Diopters
- Near Values Add to Far

- Left Eye (Oculus Dexter) Far:
  - $+0.50$ diopter 4° from Horizontal
  - -0.50 diopter 96°
- Left Eye Near (Add 1.75):
  - 2.25 diopter 4°
  - -1.25 diopter 96°
- Right Eye (Oculus Sinister) Far:
  - 0.25 diopter -8°
  - -0.75 diopter 82°
- Right Eye Near (Add 1.75):
  - 2.00 diopter -8°
  - -1.00 diopter 82°

Magnifications



Snell's Law at the Vertex

$$m = -\frac{ns'}{n's}$$

$$m_\alpha = \frac{-d\beta}{d\alpha} = -\frac{s'}{s} = -\frac{n}{n'}\frac{1}{m}$$

$$\frac{ds'}{ds} = -\frac{n}{n'}\left(\frac{s'}{s^2}\right)^2$$

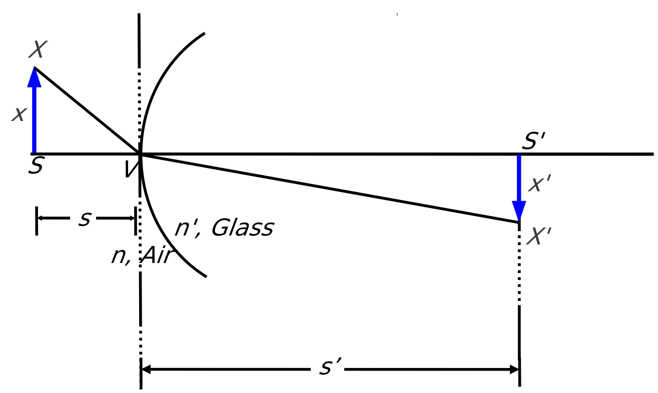
$$\frac{n}{s} + \frac{n'}{s'} = \frac{n'-n}{r} \qquad -\frac{n\ ds}{s^2} - \frac{n'\ ds'}{(s')^2} = 0$$

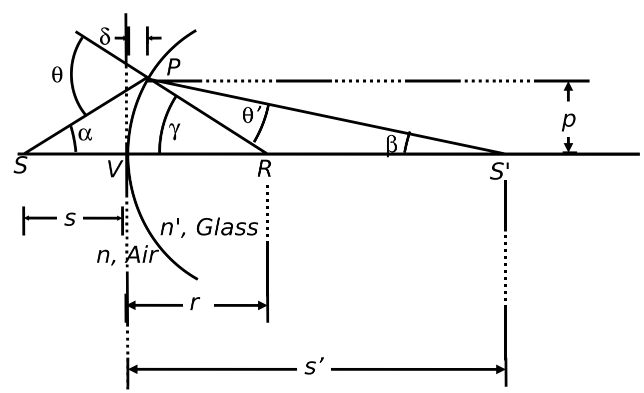
$$m_z = -\frac{n}{n'}m^2$$

Imaging Equations

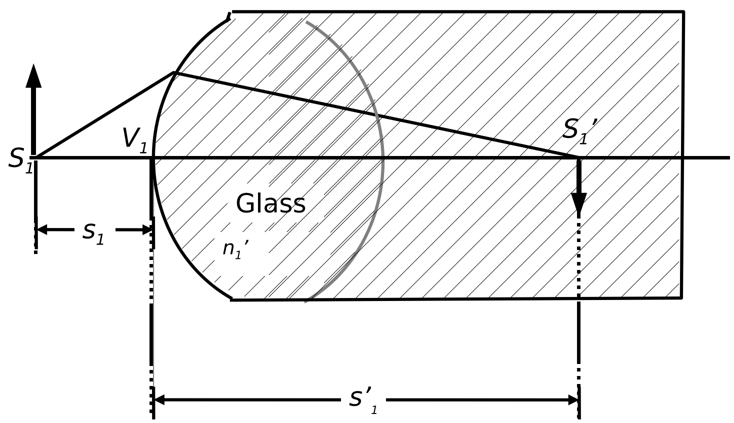
Surface	s'	m	m_α	m_z	**Image****	**O***	**D***	**P***
Planar Mirror	$s' = -s$	1	-1	-1	Virtual	Upright	No	Yes
Concave Mirror $s > f$	$\frac{1}{s'} + \frac{1}{s} = \frac{1}{f}$	$-s'/s$	$-m^2$	$-1/m$	Real	Inverted	Yes	No
Convex Mirror	$\frac{1}{s'} + \frac{1}{s} = \frac{1}{f}$	$-s'/s$	$-m^2$	$-1/m$	Virtual	Upright	Yes	Yes
Planar Refractor	$\frac{s}{n} = \frac{s'}{n'}$	1	$\frac{n}{n'}$	$\frac{n'}{n}$	Virtual	Upright	Yes	No
Curved Refractor $s > f$	$\frac{n}{s} + \frac{n'}{s'} = \frac{n'-n}{r}$	$-\frac{ns'}{n's}$	-1	$-\frac{n}{n'}m^2$	Real	Inverted	Yes	Yes

* "O" mean Orientation, "D" means Distortion, and "P" means Perversion of coordinates.
** The Image is Defined as Real or Virtual for a Real Object

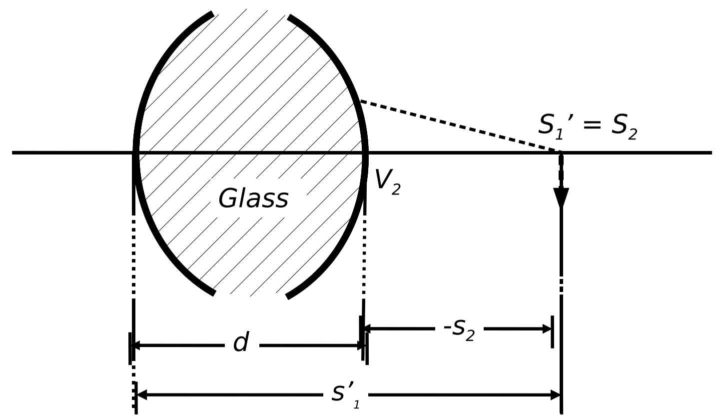




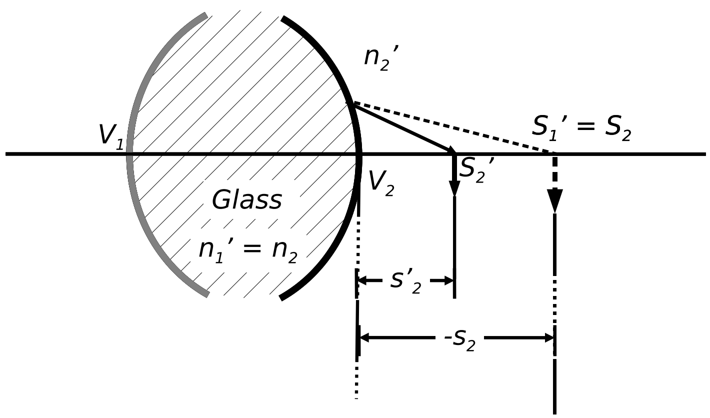
The Simple Lens



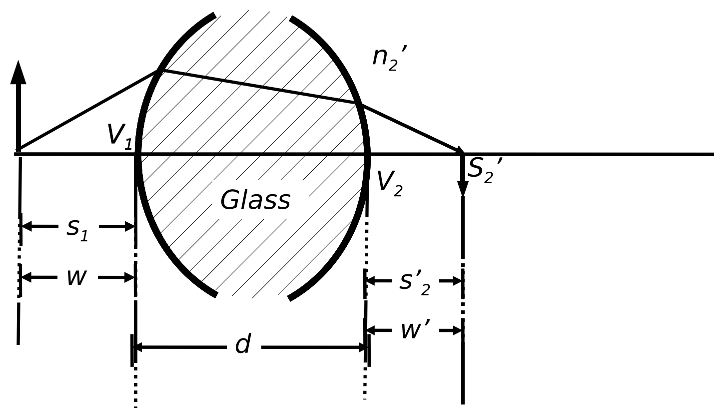
First Surface Object and Image



Second Surface Object

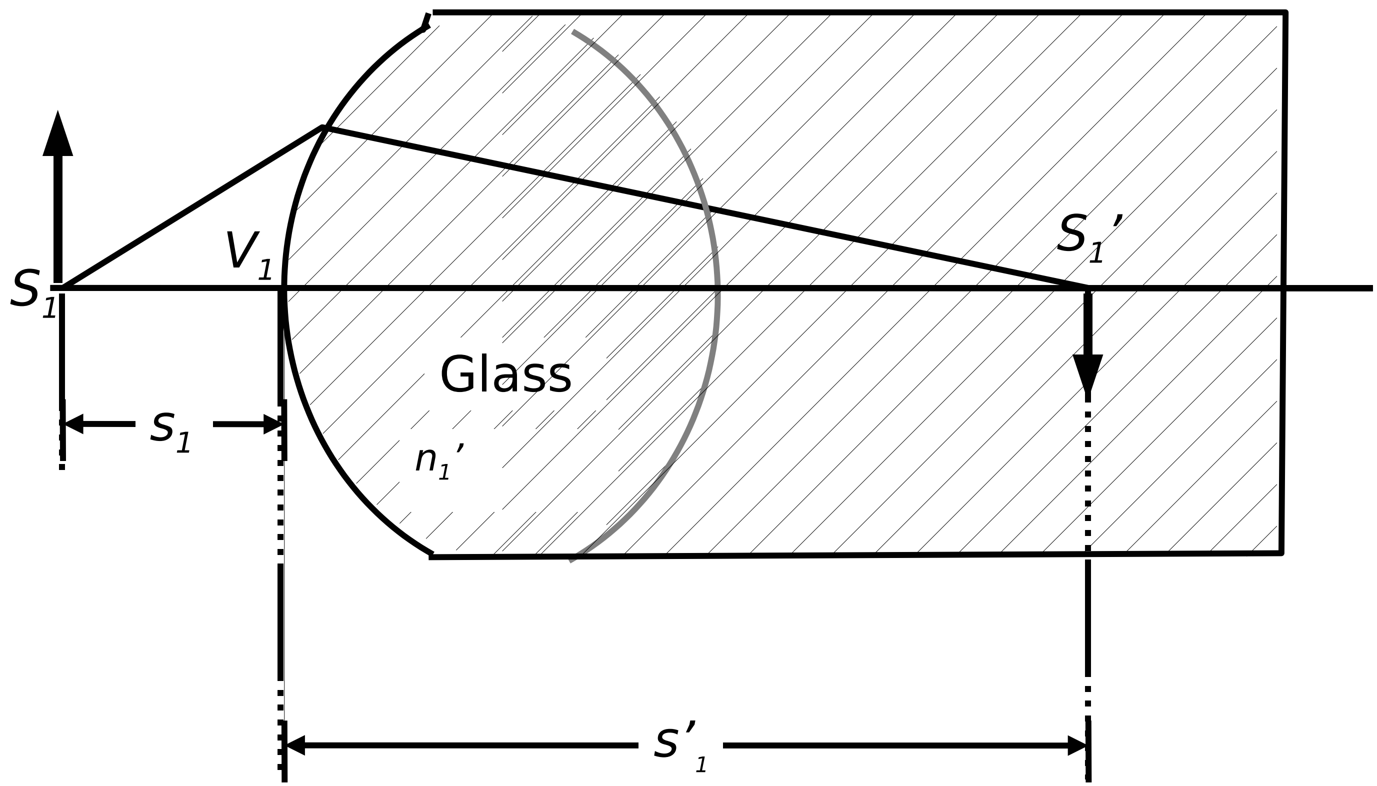


Second Surface Image



Complete Lens

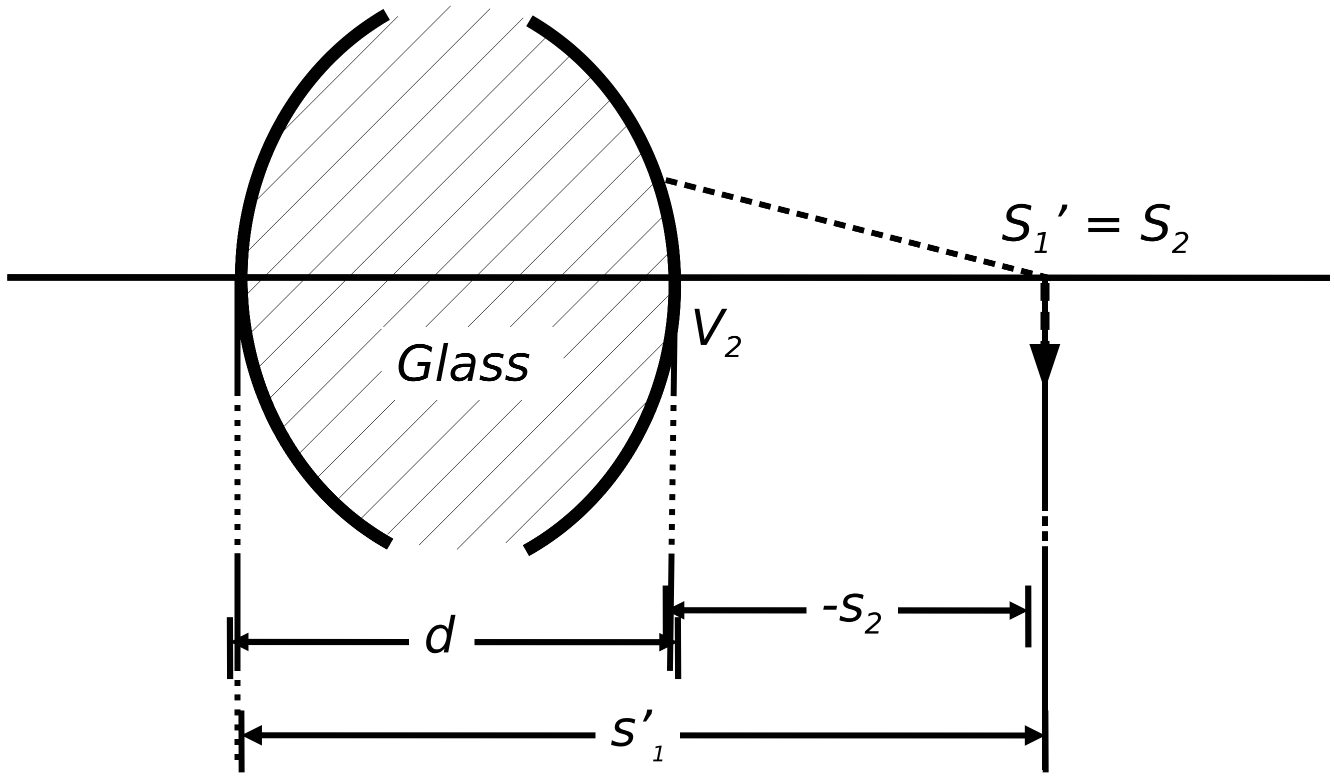
First Surface Solution



$$\frac{n_1}{s_1}+\frac{n_1'}{s_1'}=\frac{n_1'-n_1}{r_1}$$

Note Subscript $_1$

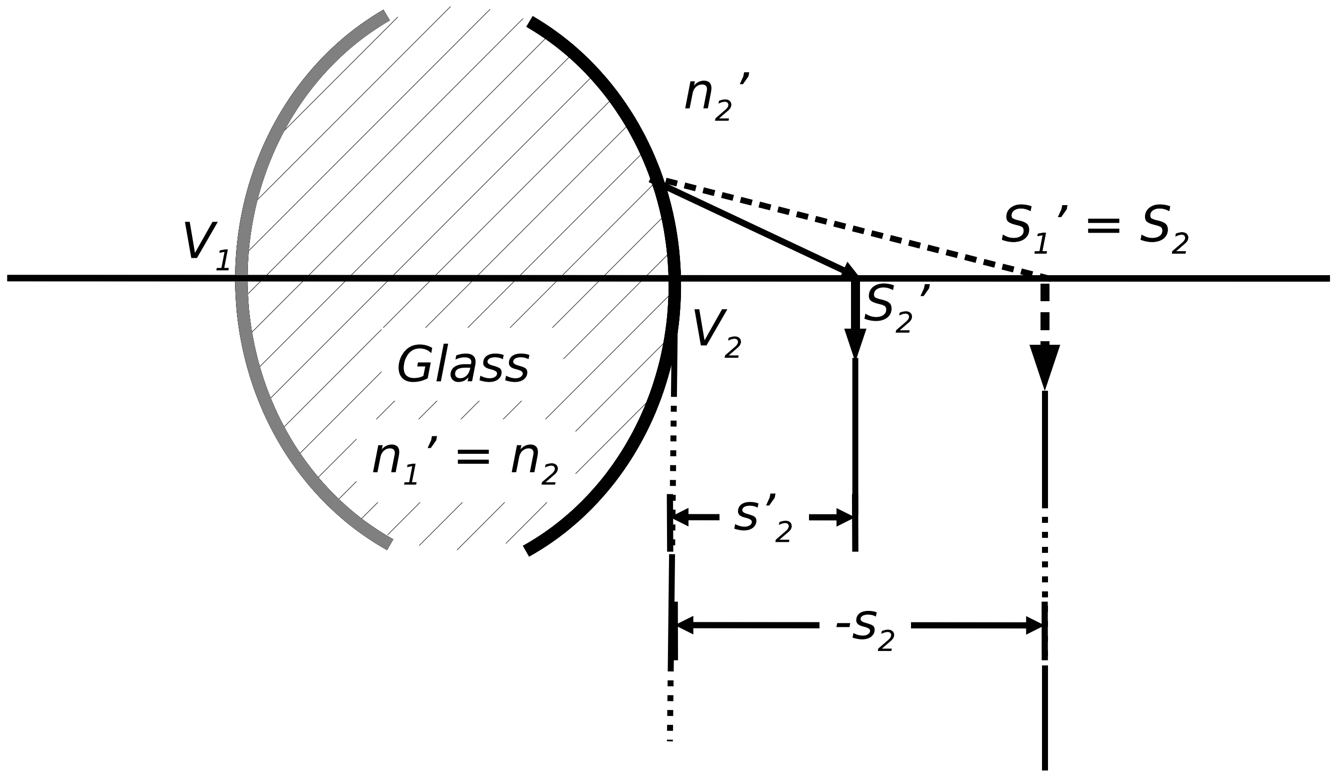
Second Surface Object



$$s_2 = -\left(s_1' - d\right) \qquad n_2 = n_1'$$

Virtual Object in this Case (Often but not Always)

Second Surface Solution

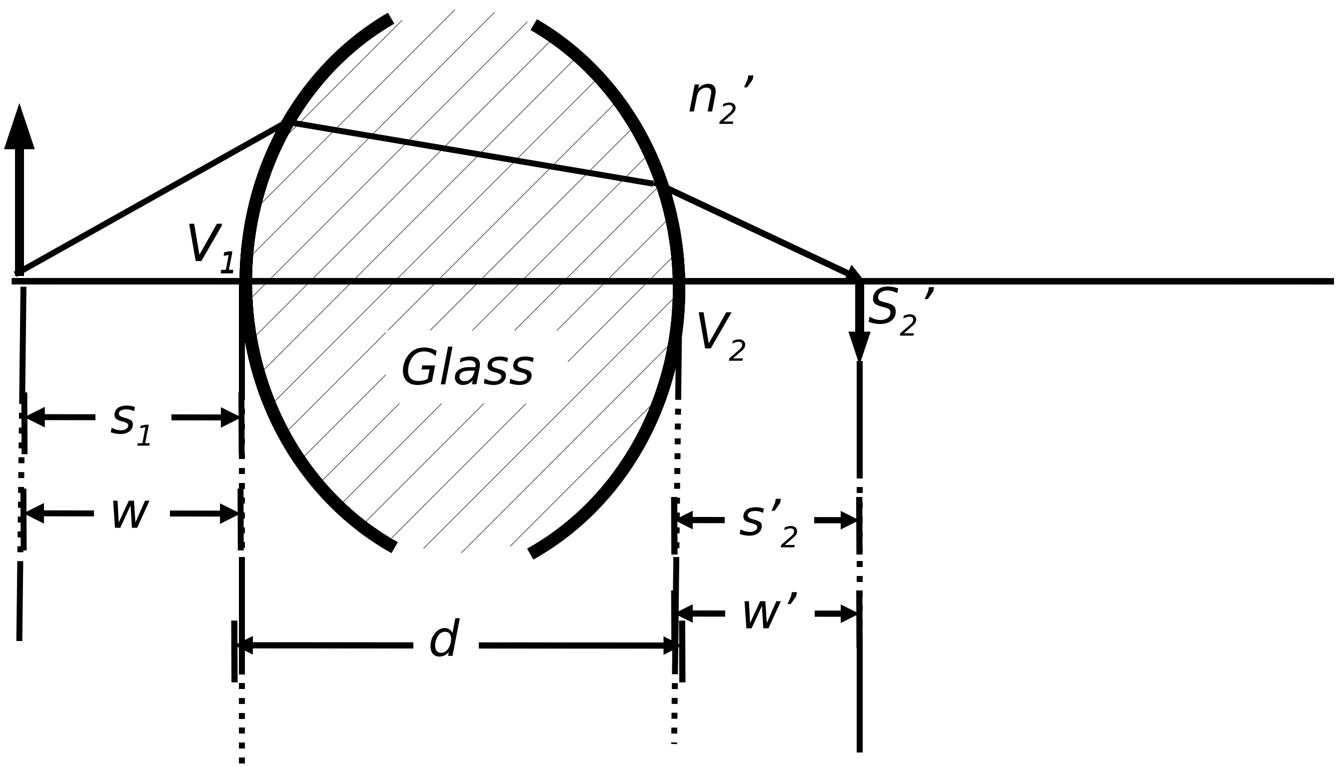


$$\frac{n_2}{s_2}+\frac{n_2'}{s_2'}=\frac{n_2'-n_2}{r_2} \qquad \frac{n_1'}{d-s_1'}+\frac{n_2'}{s_2'}=\frac{n_2'-n_1'}{r_2}$$

Sign Convention for Radius

- Our Convention
  - $r > 0$ for Convex Viewed from Source
  - Direction Consistent within Drawings
  - Easier for Computation
- Other Convention
  - $r > 0$ for Convex Viewed from Lower Index
  - r Defines the Tool for Manufacture
  - Sign of r Consistent with Optical Power
- **In–Practice**
  - Check the Equations Carefully
  - Provide Drawings to Vendors

Complete Simple Lens (1)



$$\frac{n_2'}{s_2'} = \frac{n_2' - n_1'}{r_2} - n_1' \frac{n_1' - n_1 - r_1 \frac{n_1}{s_1}}{d\left(n_1' - n_1\right) - n_1' r_1 - d n_1' r_1 \frac{n_1}{s_1}}$$

Complete Simple Lens (2)

$$\frac{n_2'}{s_2'} = \frac{n_2' - n_1'}{r_2} - n_1' \frac{n_1' - n_1 - r_1 \frac{n_1}{s_1}}{d\left(n_1' - n_1\right) - n_1' r_1 - d n_1' r_1 \frac{n_1}{s_1}}$$

That's Ugly! Let's Define Some New Notation:
w from Object to First Vertex
w' from Last Vertex to Image

$$w = s_1 \qquad w' = s_2'$$

$$n = n_1 \qquad n' = n_2' \qquad n_\ell = n_1' = n_2$$

$$\frac{n'}{w'} = \frac{n' - n_\ell}{r_2} - n_\ell \frac{n_\ell - n - r_1 \frac{n}{w}}{d\left(n_\ell - n\right) - n_\ell r_1 - d n_\ell r_1 \frac{n}{w}}$$

Complete Simple Lens (3)

$$\frac{n'}{w'} = \frac{n' - n_\ell}{r_2} - n_\ell \frac{n_\ell - n - r_1 \frac{n}{w}}{d\left(n_\ell - n\right) - n_\ell r_1 - d n_\ell r_1 \frac{n}{w}}$$

That's Still Ugly. Set $n = n' = 1$. Not General, but Useful.

$$\frac{1}{w'} = \frac{1 - n_\ell}{r_2} - n_\ell \frac{n_\ell - 1 - r_1 \frac{1}{w}}{d\left(n_\ell - 1\right) - n_\ell r_1 - d n_\ell \frac{r_1}{w}} \qquad \text{(Thick Lens in Air)}$$

Or Even Simpler, Set $d = 0$.

$$\frac{n}{w} + \frac{n'}{w'} = \frac{n' - n_\ell}{r_2} + \frac{n_\ell - n}{r_1} \qquad \text{(Thin Lens)}$$

The Thin Lens (1)

$$\frac{n}{w}+\frac{n'}{w'}=\frac{n'-n_\ell}{r_2}+\frac{n_\ell-n}{r_1}$$

Now The s *vs.* w Distinction Doesn't Matter.

$$\frac{n}{s}+\frac{n'}{s'}=\frac{n'-n_\ell}{r_2}+\frac{n_\ell-n}{r_1}$$

$$\frac{n}{s}+\frac{n'}{s'}=P_1+P_2=P$$

Back and Front Focal Lengths

$$BFL=f'=\frac{n'}{P_1+P_2} \qquad FFL=f=\frac{n}{P_1+P_2}$$

where $$P_1=\frac{n_\ell-n}{r_1} \qquad P_2=\frac{n'-n_\ell}{r_2}$$

The Thin Lens (2)

$$BFL = f' = \frac{n'}{P_1 + P_2} \qquad FFL = f = \frac{n}{P_1 + P_2}$$

Focal–Length Relationship (Generally True)

$$\frac{f'}{f} = \frac{n'}{n}$$

Specifically

$$f = f' \qquad \text{if} \qquad n = n'$$

And In Air (Probably the Most–Used Equation in Optics)

$$\boxed{\frac{1}{s} + \frac{1}{s'} = \frac{1}{f}}$$

The Thin Lens in Air

- The Lensmaker's Equation

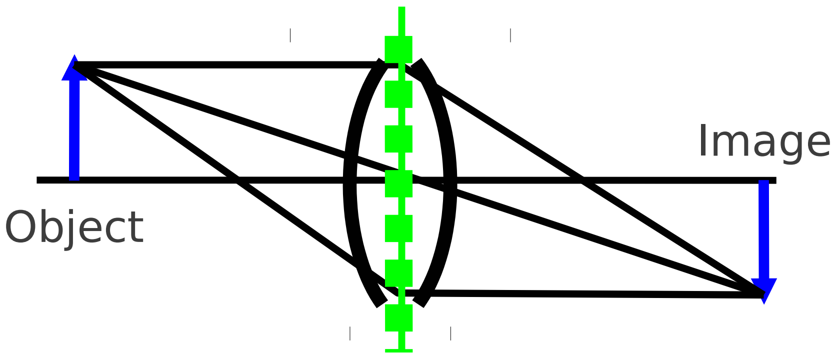
$$\boxed{\frac{1}{f} = \frac{1}{f'} = P_1 + P_2 = (n_\ell - 1)\left(\frac{1}{r_1} - \frac{1}{r_2}\right)}$$

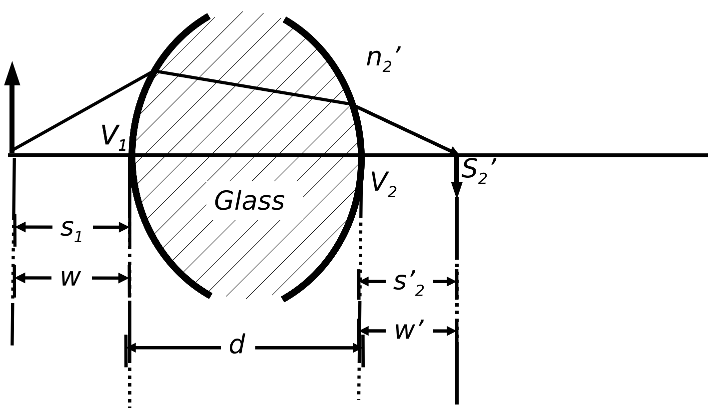
- Be Careful About Signs (Biconvex Means $r_1 > 0$ and $r_2 < 0$)

$$P_1 = \frac{n_\ell - 1}{r_1} \qquad P_2 = \frac{n_\ell - 1}{-r_2}$$

- Powers Add for Thin Lenses

The Thin Lens Magnification





For $n = n'$ and $d = 0$

$$\frac{x'}{x} = \frac{-s'}{s}$$

$$m = \frac{-s'}{s} \qquad \text{(Lens in Air)}$$

$$m = \frac{-ns'}{n's} \qquad \text{(General)}$$

Axial Magnification

$$m_z = \frac{ds'}{ds} = \frac{n}{n'}\left(\frac{s'}{s}\right)^2 = \frac{n'}{n}m^2$$

Thin Lens in Air: Summary

- Making The Lens (We Still Have Some Choices)

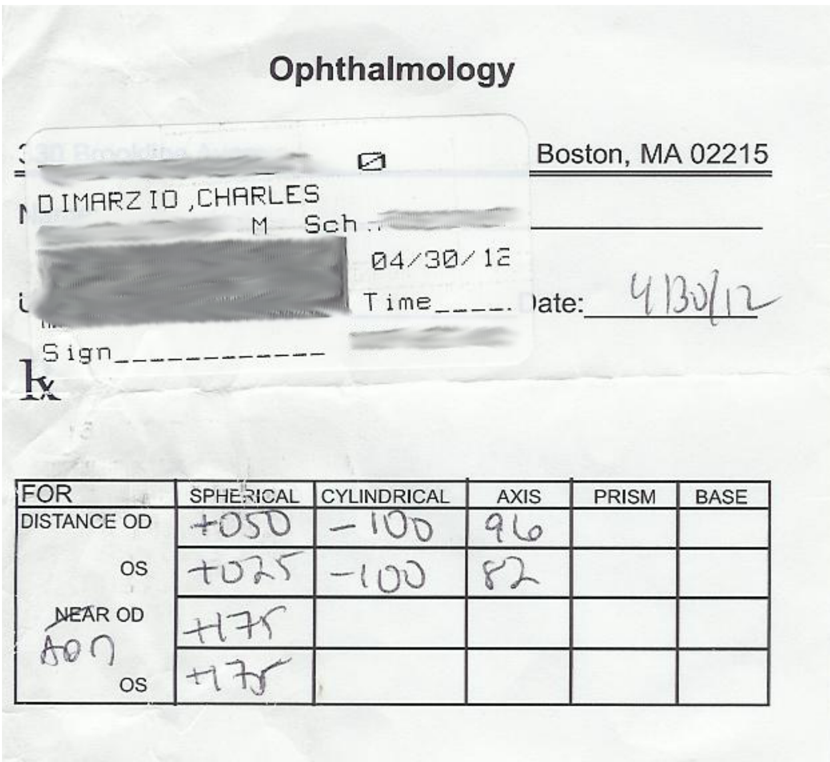
$$\boxed{\frac{1}{f} = \frac{1}{f'} = P_1 + P_2 = (n_\ell - 1)\left(\frac{1}{r_1} - \frac{1}{r_2}\right)}$$

- Using the Lens

$$\boxed{\frac{1}{s} + \frac{1}{s'} = \frac{1}{f} \qquad\qquad m = -\frac{s'}{s}}$$

500min 21 Jan 2014 (JH)

Eyeglass Prescription Revisited


Ophthalmology

[redacted] Boston, MA 02215

DIMARZIO,CHARLES
M Sch [redacted]
[redacted]
04/30/12
Time____ Date: 4/30/12
Sign___________
Rx

FOR	SPHERICAL	CYLINDRICAL	AXIS	PRISM	BASE
DISTANCE OD	+050	-100	96		
OS	+025	-100	82		
NEAR OD Add	+175				
OS	+175				

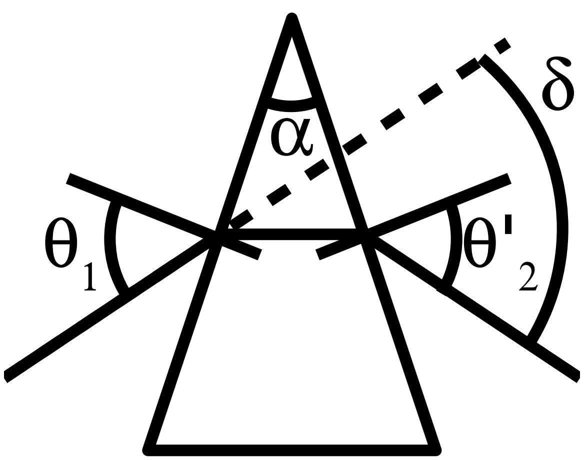
- Adding Powers
- Convex Front
- Concave Back
- Cylinder
- Many Options

Lenses

In–Practice

- If you just want to buy a lens, it may be enough to specify the focal length.
- There are infinite combinations of r_1 and r_2 that give the same focal length.
  - See Ch. 5 to help decide which to use.
- For thin lenses, optical powers are additive.
- Thick lenses are more complicated. See Ch. 3.
- Rigorous use of Snell's Law is often used to undertand the details. See Ch. 5.

Prisms (1)



$$\sin\theta_1' = \frac{\sin\theta_1}{n} \qquad \text{in Air}$$

$$(90^\circ - \theta_2) + \left(90^\circ - \theta_1'\right) + \alpha = 180^\circ$$

$$\theta_2 + \theta_1' = \alpha.$$

Applying Snell's law,

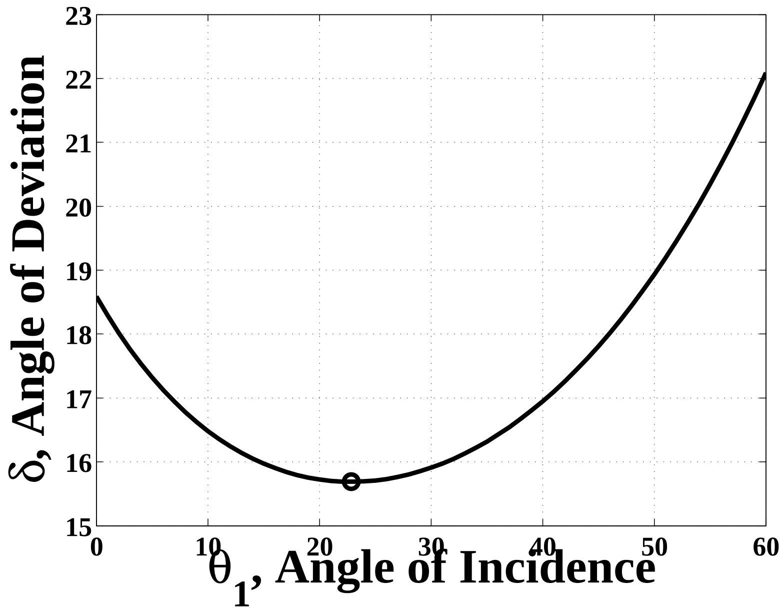
$$\sin\theta_2' = n\sin\theta_2 = n\sin\alpha - \theta_1'$$

$$\sin\theta_2' = n\left(\cos\theta_1'\sin\alpha - n\sin\theta_1'\cos\alpha\right)$$

$$\sin\theta_2' = \sqrt{n^2 - \sin^2\theta_1}\sin\alpha - \sin\theta_1'\cos\alpha$$

Deviation $\qquad \delta = \theta_1 + \theta_2' - \alpha$

Prisms (2)



$\alpha = 30^\circ$ and $n = 1.5$

- Deviation

$$\delta = \theta_1 + \theta_2' - \alpha$$

- Minimum Deviation

$$\delta_{min} = 2\sin^{-1}\left(n \sin\frac{\alpha}{2}\right) - \alpha$$

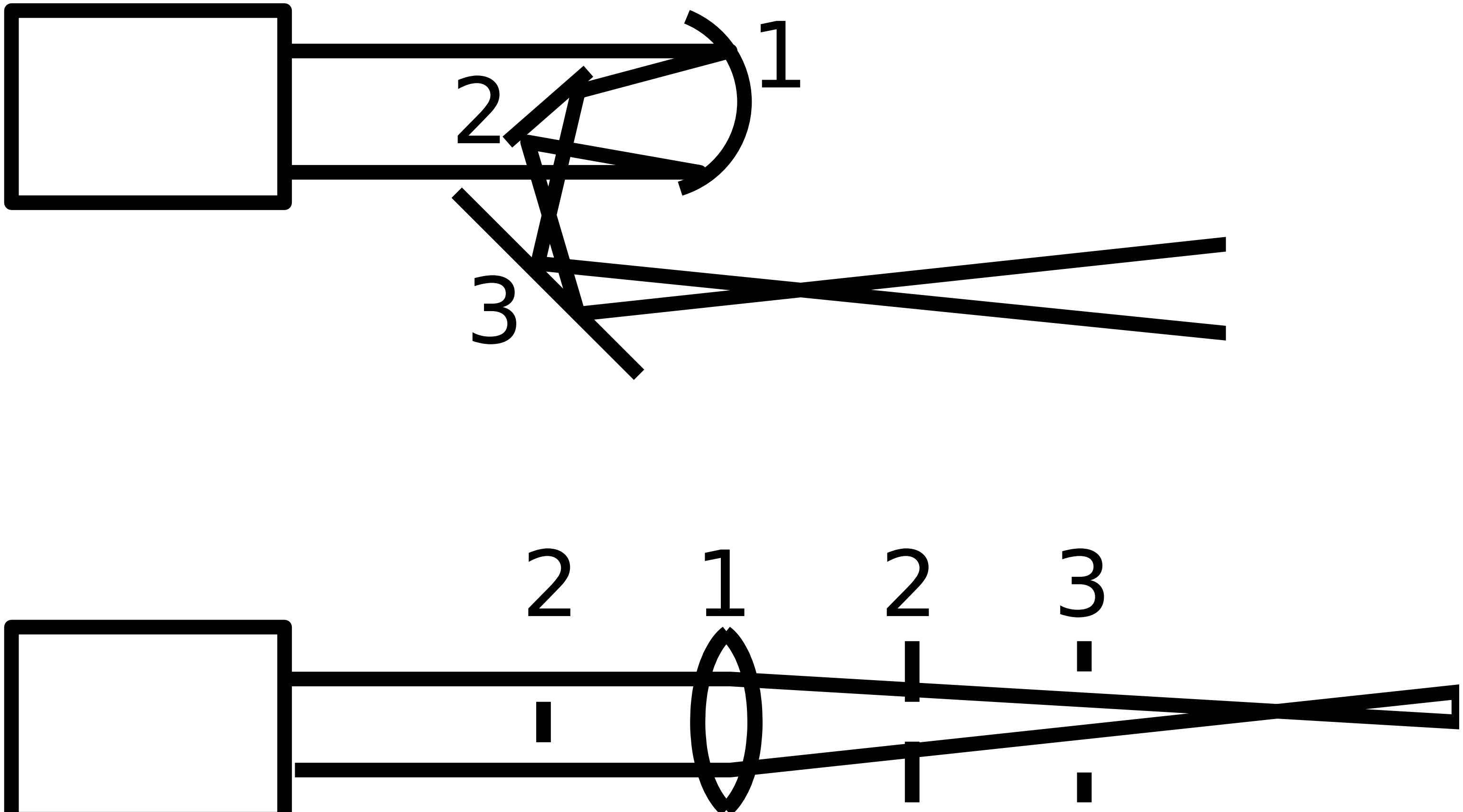
at $$\theta_1 = \sin^{-1}\left(n \sin\frac{\alpha}{2}\right)$$

- Small Prism Angles

$$\delta_{min} \approx (n-1)\,\alpha$$

at $$\theta_1 = \frac{n\alpha}{2}$$

“Unfolding” Reflective Systems



Top Shows Actual System. Bottom Shows it Unfolded for Analysis