

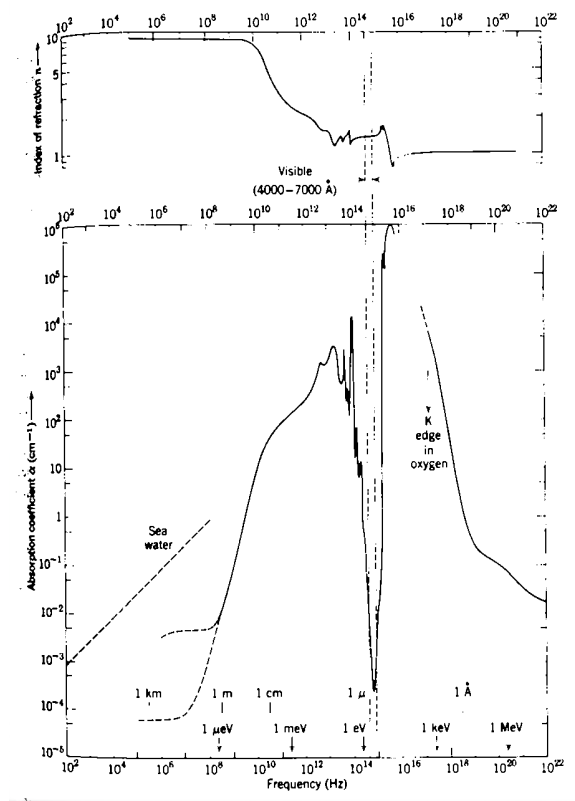
# Optics for Engineers

## Chapter 1

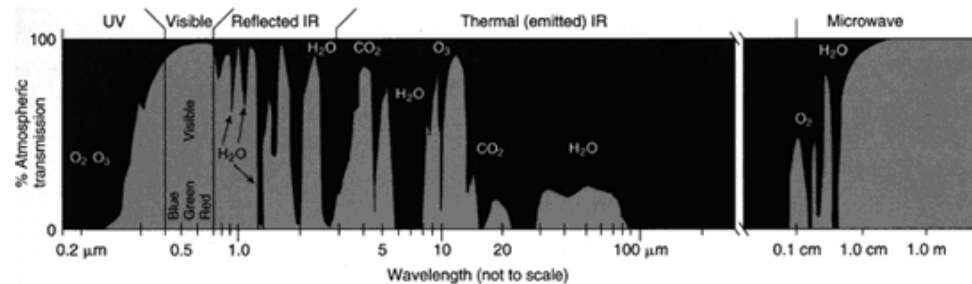
Charles A. DiMarzio  
Northeastern University

Dec. 2013

# Why Optics?



A. Liquid Water

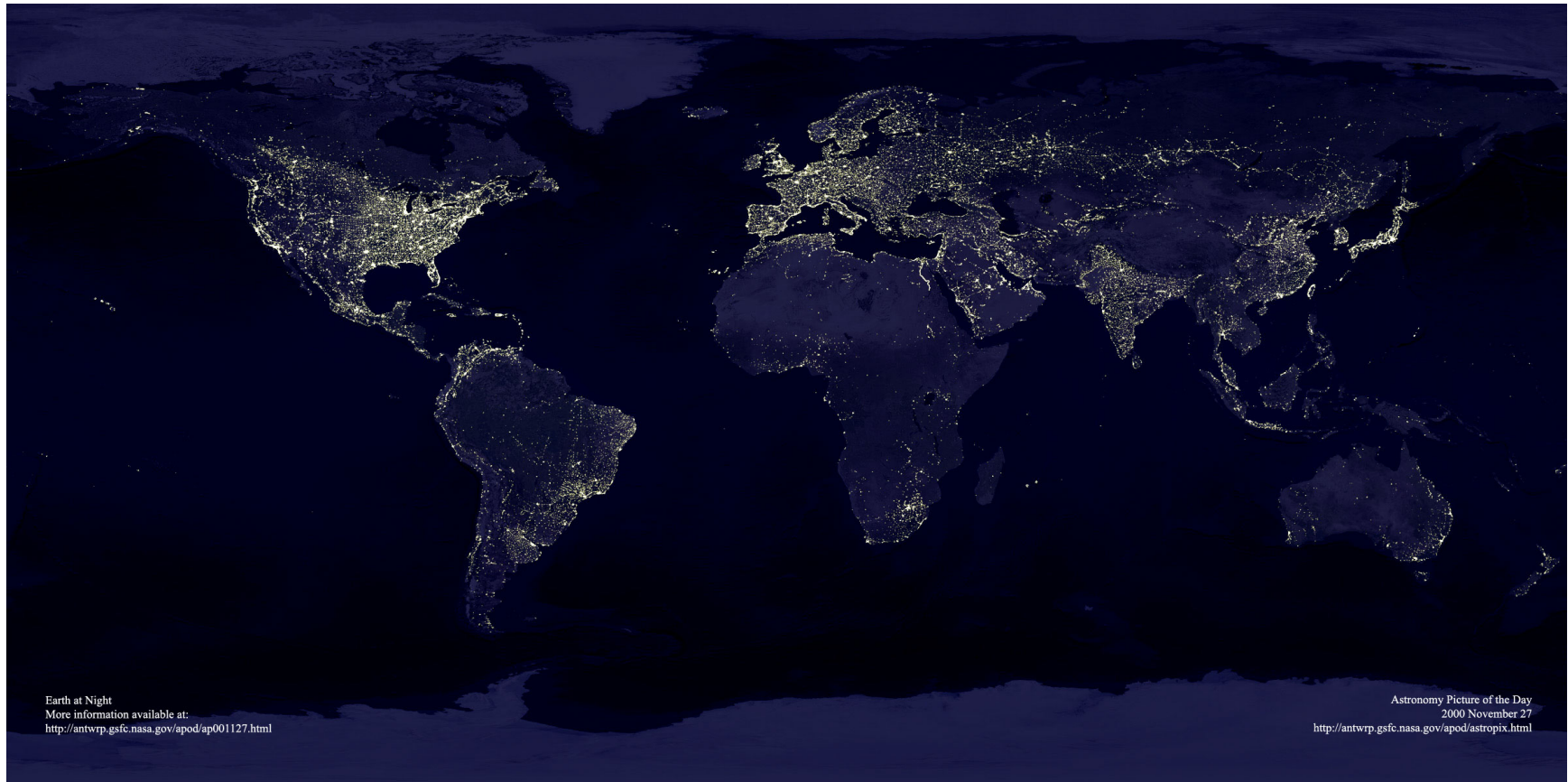


## B. Water Vapor

### ELECTROMAGNETIC TRANSMISSION.

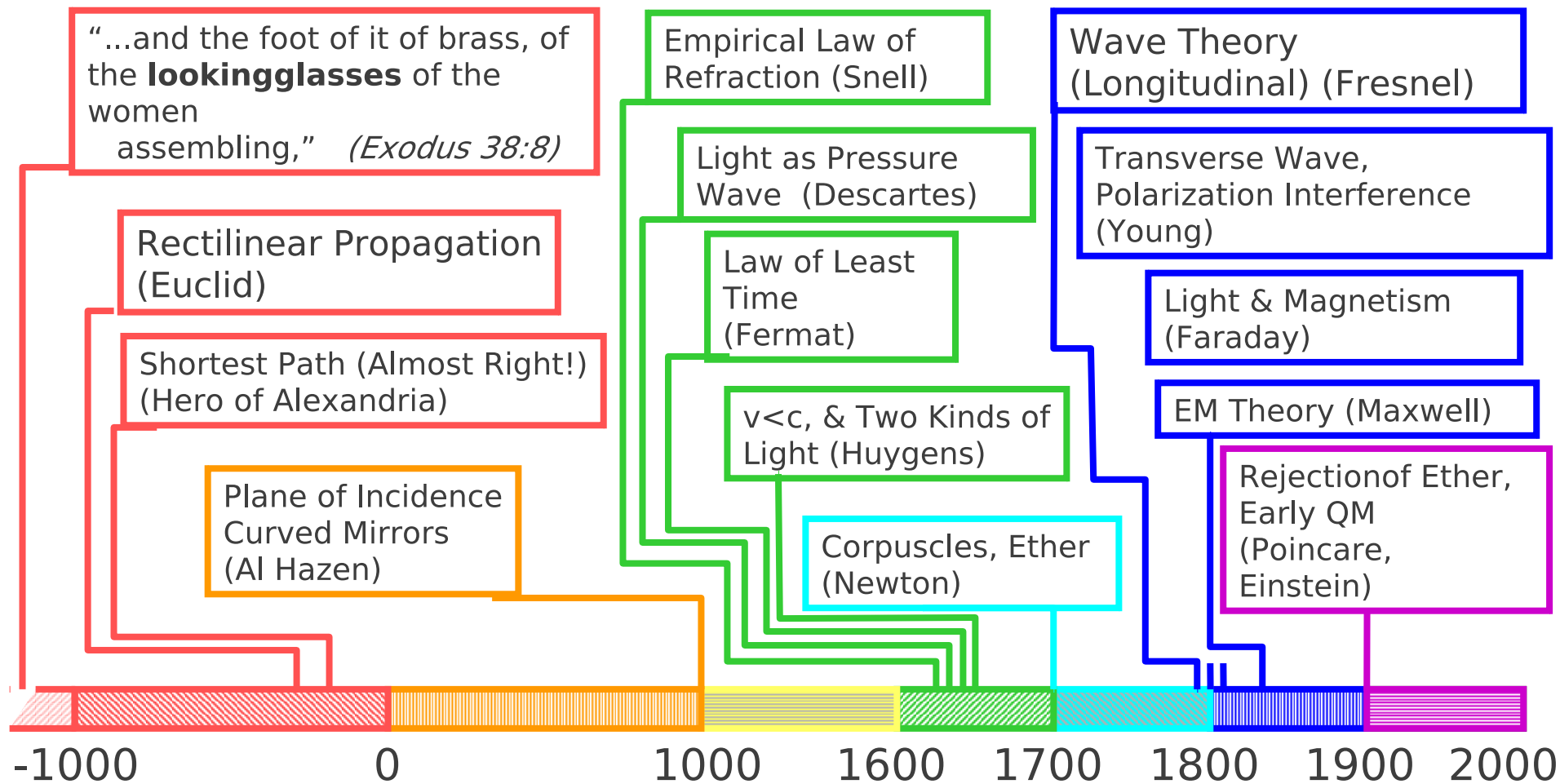
Water strongly absorbs most electromagnetic waves, with the exception of wavelengths near the visible spectrum (A, From Jackson *Classical Electrodynamics*, ©1975). The atmosphere also absorbs most wavelengths, except for very long wavelengths and a few transmission bands (B, NASA's Earth Observatory).

# Why Optics?



C. Mayhew & R. Simmon (NASA/GSFC), NOAA/ NGDC, DMSP Digital Archive).

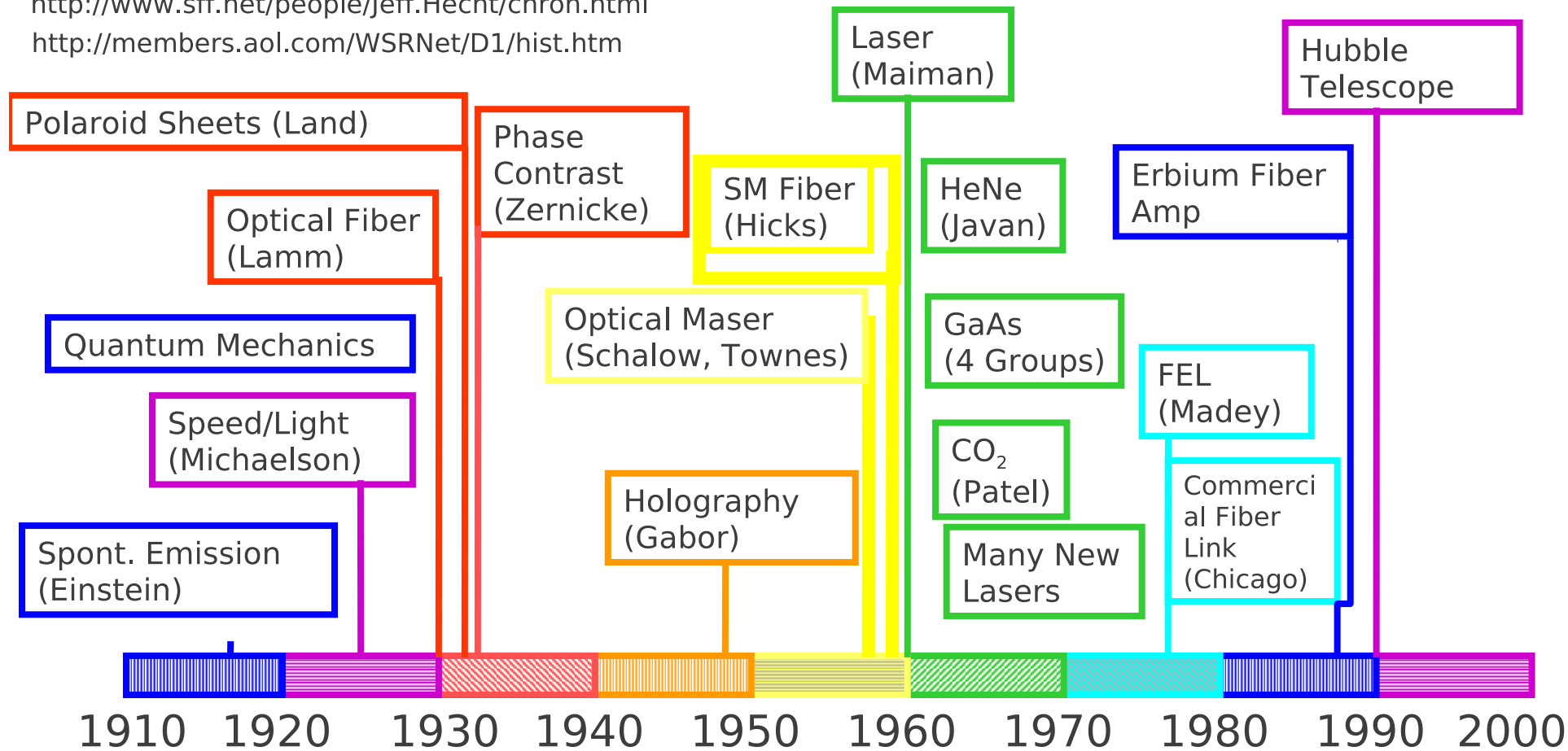
# History of Optics (1)





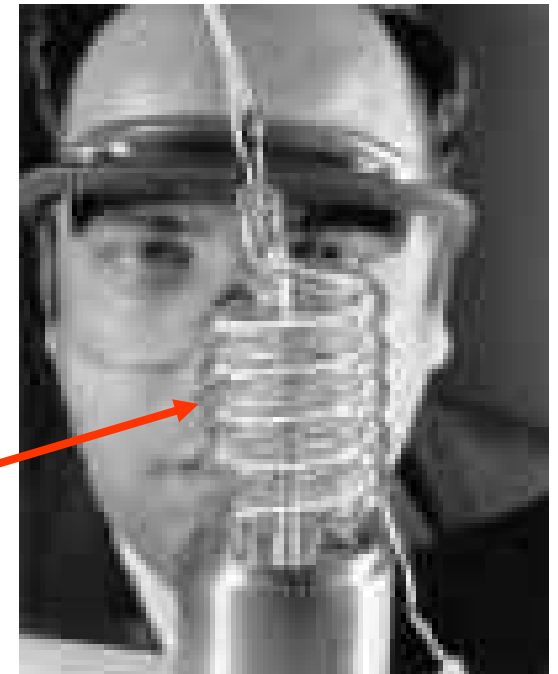
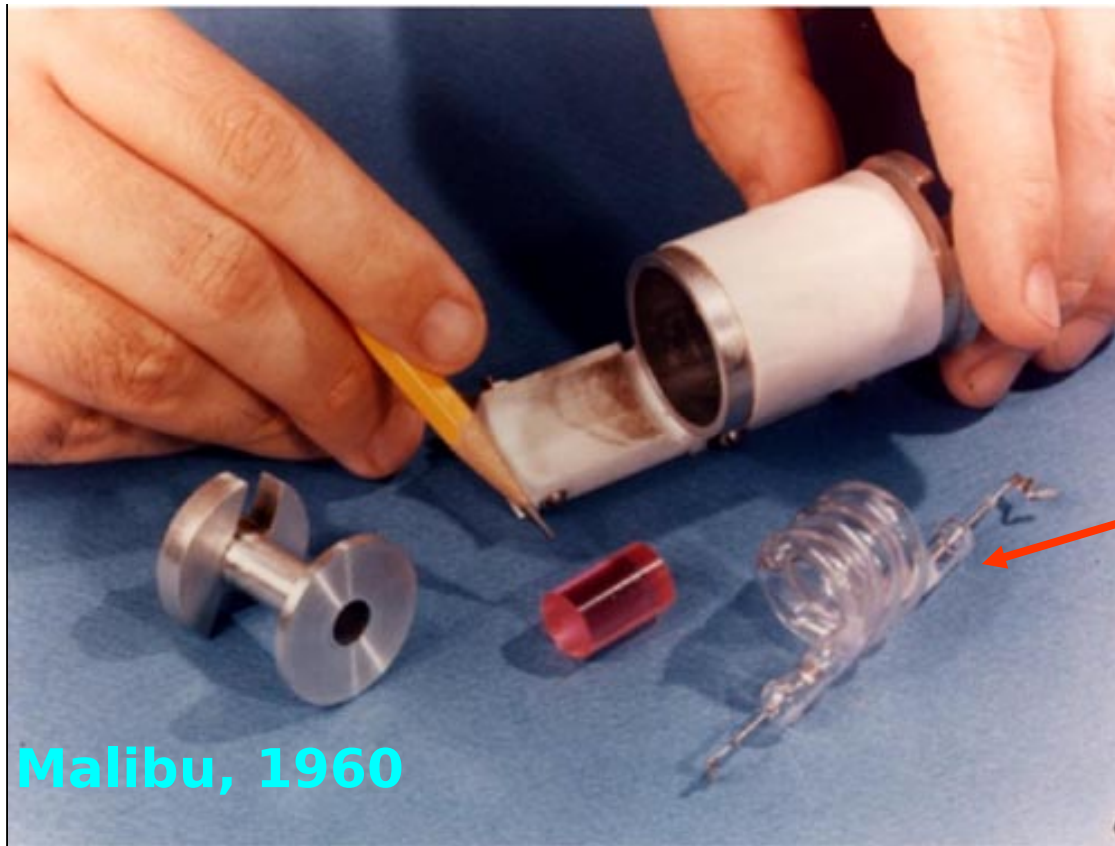
# History of Optics (2)

<http://www.sff.net/people/Jeff.Hecht/chron.html>  
<http://members.aol.com/WSRNet/D1/hist.htm>



2000–present? Too Early to Decide

# The First Laser?

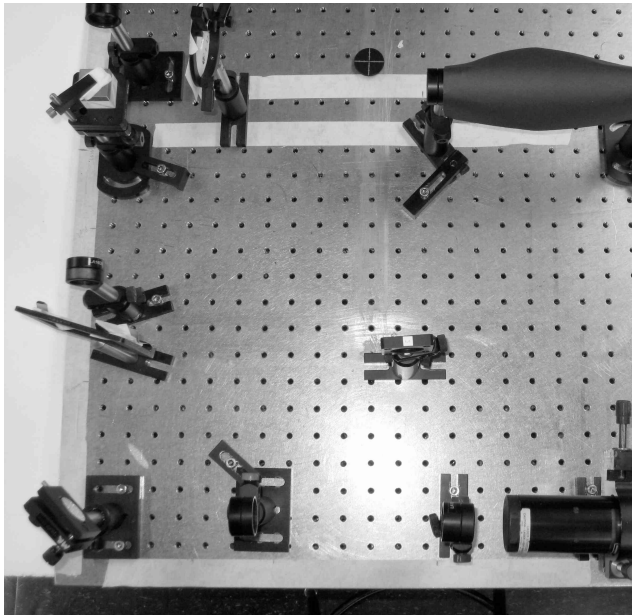


News Photo of the  
First Laser

“Laser, inter eximia naturae dona numeratum plurimis compositionibus inseritur\*”

\* “The laser is numbered among the most miraculous gifts of nature and lends itself to a variety of applications.” Pliny, *Natural History XXII*, 49.

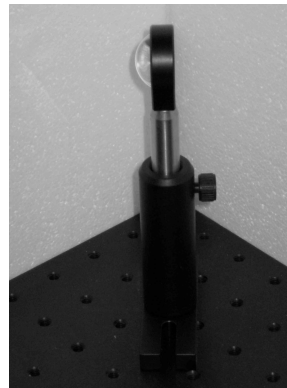
# Laboratory Systems



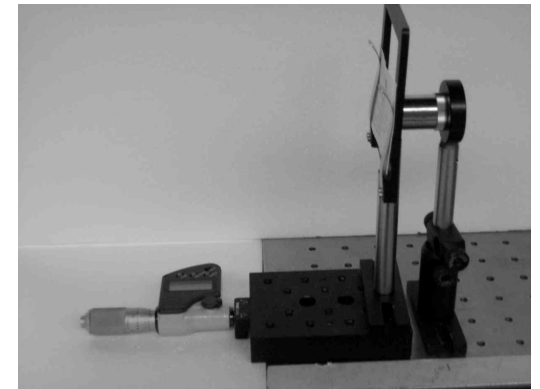
A. Breadboard



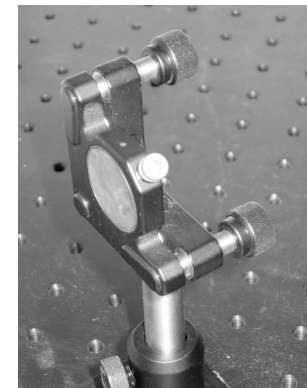
B. Mounting Hardware



C. Lens Mount

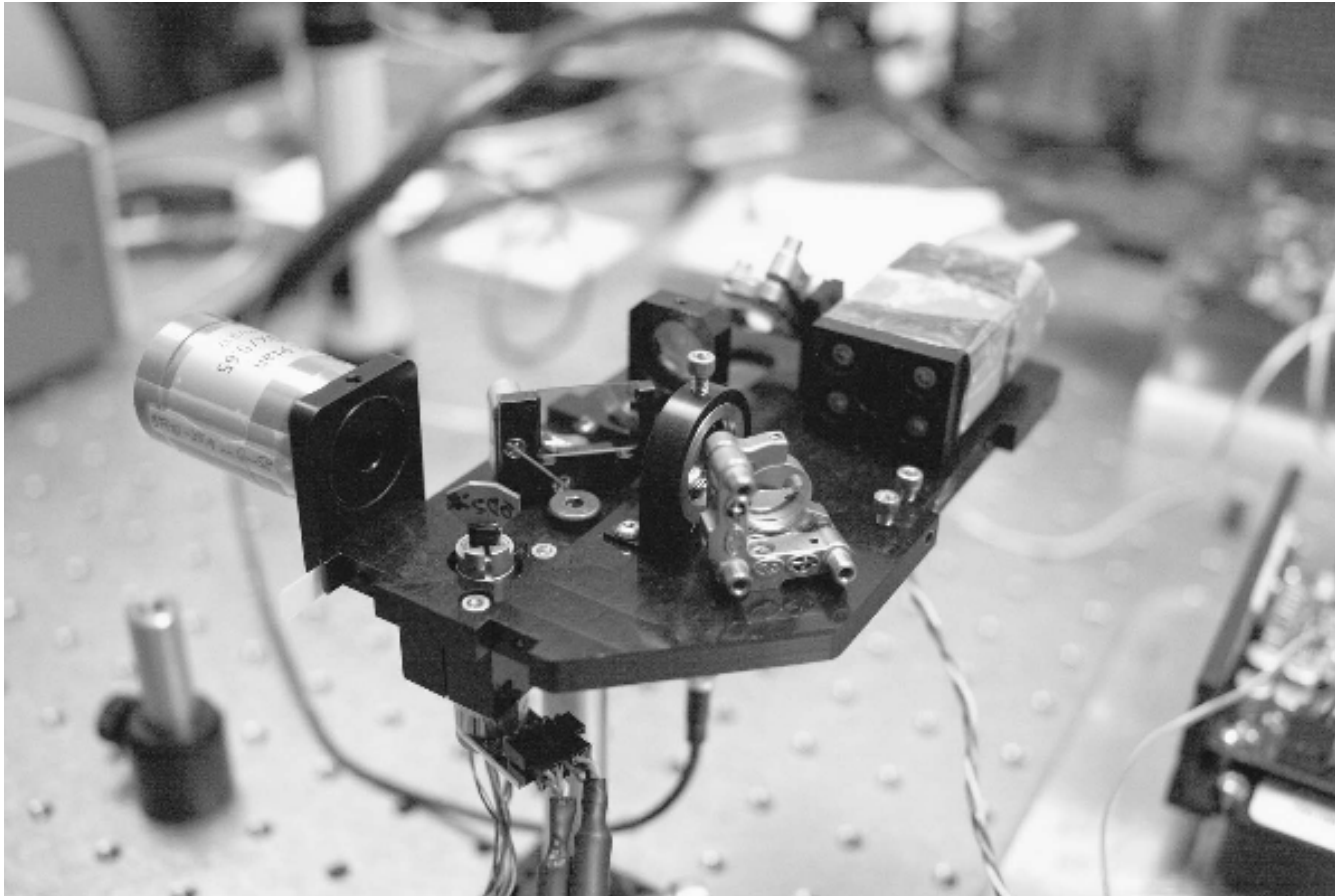


D. Micro-positioning



E. Angle Mount

# Custom Design



Reflectance Confocal Microscope. This 405-nm, line-scanning reflectance confocal microscope includes a mixture of custom-designed components, commercial off-the-shelf mounting hardware, and a commercial microscope objective. (Photo by Gary Peterson of Memorial Sloan Kettering Cancer Center)

# Maxwell's Equations

Faraday's Equation

$$\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t}$$

Ampere's Equation  
(No Currents)

$$\nabla \times \mathbf{H} = \frac{\partial \mathbf{D}}{\partial t}$$

Gauss' Equations

$$\nabla \cdot \mathbf{D} = \rho = 0$$

$$\nabla \cdot \mathbf{B} = 0$$

Constitutive Parameters

$$\mathbf{D} = \epsilon \mathbf{E}$$

$$\mathbf{B} = \mu \mathbf{H}$$

# Susceptibilities, $\chi$

$$\mathbf{D} = \epsilon_0 (1 + \chi) \mathbf{E} \quad \mathbf{B} = \mu_0 (1 + \chi_m) \mathbf{H} = \mu_0 \mathbf{H}$$

$$\mathbf{D} = \epsilon_0 \mathbf{E} + \mathbf{P} \quad \mathbf{B} = \mu_0 \mathbf{H} + \mathbf{M} = \mu_0 \mathbf{H}$$

with Polarizations defined by...

$$\mathbf{P} = \epsilon_0 \chi \mathbf{E} \quad \mathbf{M} = \mu_0 \chi_m \mathbf{H} = 0$$

At optical frequencies ...

$$\chi_m = 0 \quad \text{so} \quad \mu = \mu_0$$

Even magneto-optical effects are found in  $\epsilon_0 \chi$

# Getting to the Wave Equation

$$\begin{array}{ccccc}
 & & \downarrow \text{Material Properties} & & \\
 & & \downarrow & & \\
 & & \mathbf{D} = \epsilon \mathbf{E} & & \\
 \mathbf{D} & \leftarrow & & \rightarrow & \mathbf{E} \\
 \uparrow & & & & \uparrow \\
 \nabla \times \mathbf{H} = \frac{\partial \mathbf{D}}{\partial t} & & & & \nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t} \\
 \downarrow & & & & \downarrow \\
 \mathbf{H} & \leftarrow & & \rightarrow & \mathbf{B} \\
 & & \mathbf{B} = \mu_0 \mathbf{H} & & 
 \end{array}$$

In the case that  $\epsilon$  is a scalar constant,  $\epsilon$ ,

$$\nabla^2 \mathbf{E} = \mu \epsilon \frac{\partial^2 \mathbf{E}}{\partial t^2}$$

# The Wave Equation

$$\nabla^2 \mathbf{E} = \mu\epsilon \frac{\partial^2 \mathbf{E}}{\partial t^2}$$

## One Solution

$$\mathbf{E} = \hat{x} E_0 e^{j(\omega t - nkz)},$$

- Electric Field Direction:  $\hat{x}$
- Propagation Direction:  $\hat{z}$
- Angular Frequency:  $\omega = 2\pi\nu = 2\pi f$
- Wave vector Magnitude:  $|nk|$  (Other Authors Differ)
- Speed of a Constant-Phase Point:  $v = \frac{\omega}{nk}$

Substitute in Wave Equation

$$-n^2 k^2 \mathbf{E} = -\mu\epsilon \omega^2 \mathbf{E} \quad \text{so} \quad n^2 k^2 = \mu\epsilon \omega^2$$



# The Scalar Wave Equation

- Vector Wave Equation and a Solution

$$\nabla^2 \mathbf{E} = \mu\epsilon \frac{\partial^2 \mathbf{E}}{\partial t^2} \quad \mathbf{E} = \hat{x} E_0 e^{j(\omega t - nkz)}$$

- Vector Direction of  $\mathbf{E}$ 
  - May Change
  - But Won't Until Chapter 6
  - So Work With One Component

- Scalar Wave Equation (Often Useful)

$$\nabla^2 E = \mu\epsilon \frac{\partial^2 E}{\partial t^2} \quad E = E_0 e^{j(\omega t - nkz)}$$

# Waves in Space and Time

- Plane Wave

$$n = 1$$

$$E = E_0 e^{j(\omega t - kz)}$$

- Wavelength

$$\lambda = 2\pi/k$$

Vertical Line

- Period

$$T = 2\pi/\omega$$

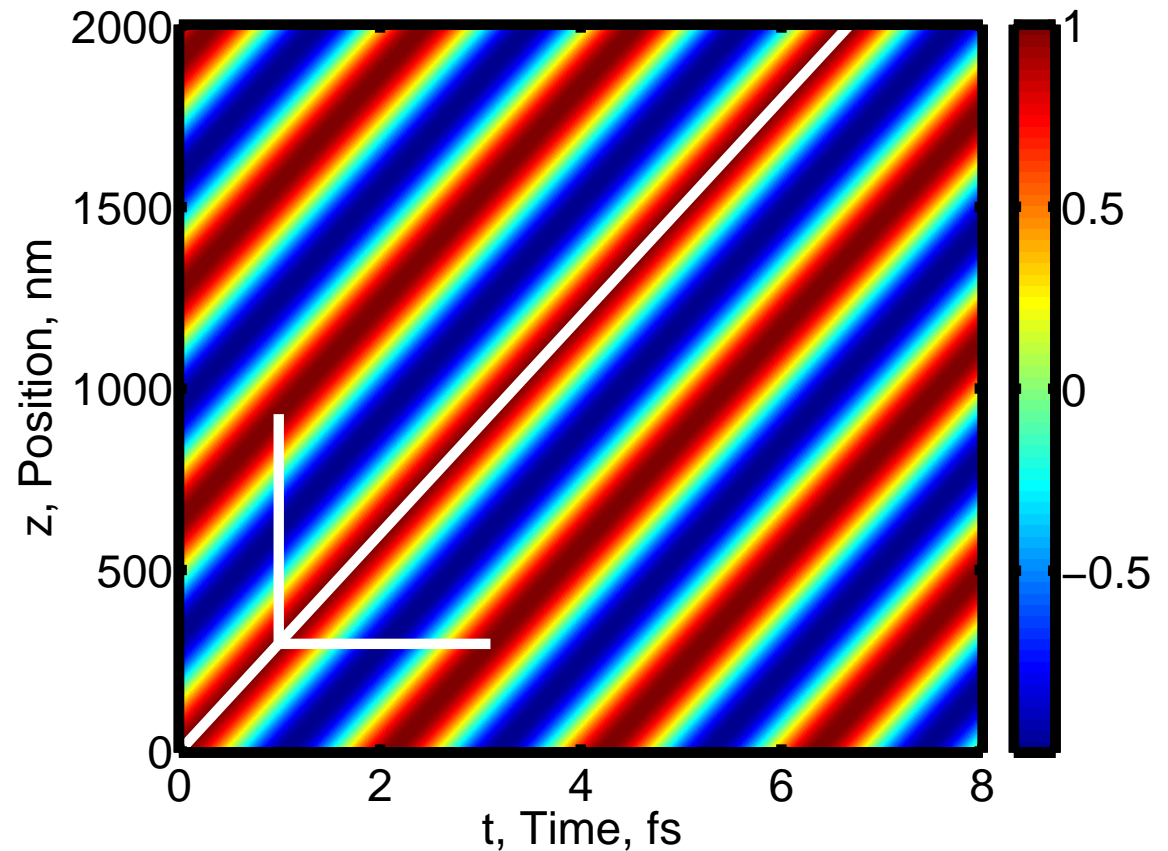
Horizontal Line

- Speed

$$v = dz/dt$$

Constant Phase

Diagonal Line



# Wave Speed

- Speed of a Constant-Phase Point:  $v = \frac{\omega}{nk}$
- Solution of Wave Equation

$$n^2 k^2 = \mu \epsilon \omega^2$$

- Wave Speed

$$v = \frac{1}{\sqrt{\epsilon \mu}}$$

- Vacuum Wave Speed

$$c = \frac{1}{\sqrt{\epsilon_0 \mu_0}} = 2.99792458 \times 10^8 m/s$$

# Index of Refraction

- Wave Speed

$$v = \frac{1}{\sqrt{\epsilon\mu}}$$

- Vacuum Wave Speed

$$c = \frac{1}{\sqrt{\epsilon_0\mu_0}}$$

- Index of Refraction

$$n = \frac{c}{v}$$

$$n = \sqrt{\frac{\epsilon\mu}{\epsilon_0\mu_0}} = \sqrt{\frac{\epsilon}{\epsilon_0}}$$

Material	Approximate Index
Vacuum (also close for air)	1.00
Water (visible to NIR)	1.33
Glass	1.5
ZnSe (Infrared)	2.4
Germanium	4.0

# Wavelength Convention

- Vacuum Wavelength:  $\lambda = \frac{2\pi}{k}$
- Wavelength in Medium:  $\lambda_{material} = \frac{\lambda}{n} = \frac{2\pi}{nk}$
- Vacuum Wave Vector Magnitude:  $|k| = \frac{2\pi}{\lambda}$
- Wave Vector in Medium:  $k_{material} = nk$
- Frequency:  $\nu = \frac{c}{\lambda} = \frac{v}{\lambda/n}$  (Remember that  $n = \frac{c}{v}$ )
- Frequency Does Not Change in Medium
- Many authors define  $\lambda$  and  $k$  in the medium and they often use  $\lambda_0$  for vacuum wavelength

# Dispersion for Visible Light

- Dispersion

$$\frac{dn}{d\lambda} \quad \text{or} \quad \frac{dn}{d\nu}$$

- Abbe Number

$$V_d = \frac{n_d - 1}{n_F - n_C},$$

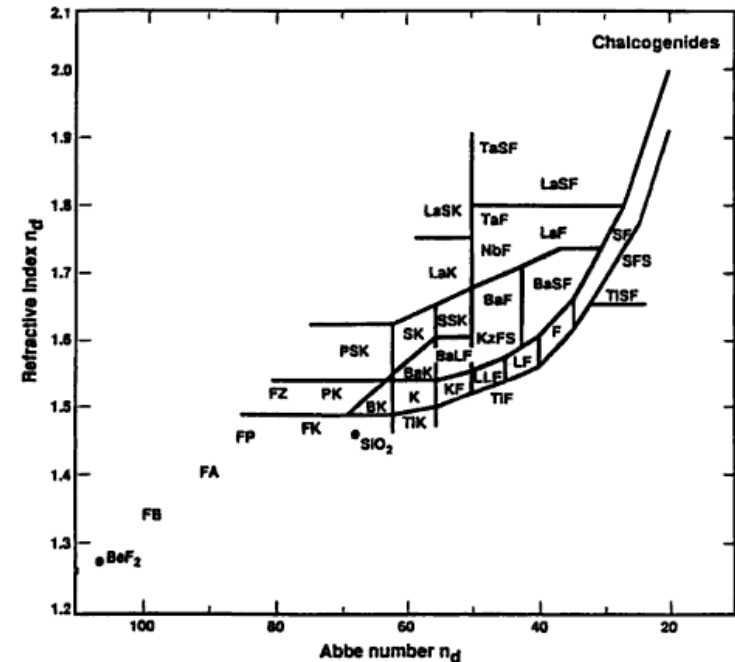
$n_d$  at  $\lambda_d = 587.6\text{nm}$ , yellow

$n_F$  at  $\lambda_F = 486.1\text{nm}$ , blue

$n_C$  at  $\lambda_C = 656.3\text{nm}$ , red

- Low  $V_d \rightarrow$  High Dispersion

- Crown ( $V_d > 50$ )
- Flint ( $V_d < 50$ )



(Reprinted from Weber's CRC Handbook of Laser Science and Technology, Supplement 2)

# Index of Refraction Summary

## Take-Away Messsge

- $n$  Depends on Material
- Small Values, Limited Choices, Slight Variations
  - $n \approx 1$  for Air
  - $n \approx 1.33$  for Water
  - $n \approx 1.5$  for Glass

# Other Fields

$$\mathbf{D} = \hat{x}D_0e^{j(\omega t - nkz)}$$

$$\mathbf{H} = \hat{y}H_0e^{j(\omega t - nkz)}$$

$$D_0 = \epsilon E_0.$$

$$\mathbf{B} = \hat{y}B_0e^{j(\omega t - nkz)}$$

More generally:

$$\mathbf{D} = \epsilon \mathbf{E}$$

Harmonic Functions:

$$\frac{\partial \mathbf{H}}{\partial t} = j\omega \mathbf{H}$$

$$-\mu j\omega \mathbf{H} = \nabla \times \mathbf{E} = jnkE_0\hat{y}$$

$$H_0 = \frac{n}{\mu c}E_0 = n\sqrt{\frac{\epsilon_0}{\mu_0}}E_0$$

Impedance:  $Z$  or  $\eta$

$$E_0 = H_0Z$$



# Poynting Vector

- Equation

$$\mathbf{S} = \mathbf{E} \times \mathbf{H}$$

- Magnitude of  $\mathbf{S}$ : Power per Unit Area
  - Irradiance:

$$I = \frac{dP}{dA_{\perp}} = |\mathbf{S}|$$

- Direction of  $\mathbf{S}$ : Propagation of Energy
- Notation Confusion
  - Here  $I$  is used for Irradiance,  $\mathbf{E}$  for Field
  - In radiometry (Ch. 12)  $E$  is used for Irradiance,  
 $I$  for intensity (W/sr)
  - The word “Intensity” is often misused for “irradiance,” particularly in older literature

# Circuits Analogy

$$V = IR$$

$$|\mathbf{E}| = |\mathbf{H}| Z$$

$$|\mathbf{E}| = |\mathbf{H}| \eta$$

$$P = IV$$

$$\mathbf{S} = \mathbf{E} \times \mathbf{H}$$

$$|\mathbf{S}| = |\mathbf{E}| |\mathbf{H}|$$

For Plane Wave ( $\mathbf{E} \perp \mathbf{H}$ )

$$P = \frac{V^2}{R}$$

$$|\mathbf{S}| = \frac{|\mathbf{E}|^2}{Z}$$

$$P = I^2 R$$

$$|\mathbf{S}| = |\mathbf{H}|^2 Z$$

Not Often Used in Optics

$V$  in Volts

$\mathbf{E}$  in  $\frac{\text{Volts}}{\text{meter}}$

$I$  in Amperes

$\mathbf{H}$  in  $\frac{\text{Amperes}}{\text{meter}}$

$R$  in Ohms

$Z$  or  $\eta$  in Ohms

$P$  in Watts

$\mathbf{S}$  in  $\frac{\text{Watts}}{\text{meter}^2}$

$|\mathbf{S}|$  is Irradiance

# Irradiance

Equation:  $I = |\mathbf{S}| = \frac{|\mathbf{E}|^2}{Z}$

Often Used:  $I = |\mathbf{E}|^2$

Assumes  $Z$  is Constant

Is OK for Ratios under that Assumption

Provides Incorrect Relationship Between  $I$  and  $\mathbf{E}$ , but ...

Fields can Never be Directly Measured

And Fields are Seldom of Interest Anyway

So Strange Units for Fields Seldom Cause Problems ...

But, When in Doubt, Do it the Right Way.

# Wave Notation Conventions (1)

- Components of a Wave Traveling in  $+z$  direction

$$\cos(\omega t - kz) = \frac{1}{2}e^{\sqrt{-1}(\omega t - kz)} + \frac{1}{2}e^{-\sqrt{-1}(\omega t - kz)},$$

$$\sin(\omega t - kz) = \frac{1}{2\sqrt{-1}}e^{\sqrt{-1}(\omega t - kz)} - \frac{1}{2\sqrt{-1}}e^{-\sqrt{-1}(\omega t - kz)}.$$

- Complex Representation of a Real Wave\*

$$\mathbf{E}_r = \hat{\mathbf{x}}E_0e^{\sqrt{-1}(\omega t - kz)} + \hat{\mathbf{x}}E_0^*e^{-\sqrt{-1}(\omega t - kz)}$$

$$\mathbf{E}_r = \mathbf{E}e^{\sqrt{-1}\omega t} + \mathbf{E}^*e^{-\sqrt{-1}\omega t}$$

- Positive Frequency Term Used in Linear Calculations

$$\mathbf{E}e^{+j\omega t}$$

# Wave Notation Conventions (2)

- Positive Frequency Field Term:  $\mathbf{E}$
- Linear Calculations Using  $\mathbf{E}$
- Negative Frequency Term Assumed:  $\mathbf{E}^*$
- Real Field (As in Maxwell's Equations):

$$\mathbf{E}e^{j\omega t} + \mathbf{E}^*e^{-j\omega t}$$

- Use Real Field for Non-Linear Calculations
  - Detectors
  - Non-Linear Optics
  - Any Time You Are Not Sure

# Irradiance from Complex Field

From the Real Field

$$|S| = \frac{(\mathbf{E}e^{j\omega t} + \mathbf{E}^*e^{-j\omega t})^2}{Z} = 2\frac{|\mathbf{E}|^2}{Z}$$

The average irradiance over a cycle is half this value, so

$$\langle |S| \rangle = \frac{|\mathbf{E}|^2}{Z}.$$

\* Error in Text

# Wavelength, Frequency, Photons

Quantity	Equation	Typical Units	Example
Vacuum Wavelength	$\lambda$	nm or $\mu\text{m}$	Green: 500nm
Frequency	$f = \nu = \frac{c}{\lambda}$	THz.	600THz
Wave vector	$ k  = \frac{2\pi}{\lambda}$		
Wavenumber*	$\tilde{\nu} = \frac{1}{\lambda}$	$\text{cm}^{-1}$	$20,000\text{cm}^{-1}$
Photon Energy	$h\nu$	J	$4 \times 10^{-19}\text{J}$
**	$\frac{h\nu}{e}$	eV	2.5eV
Photon Momentum	$\mathbf{p} = \frac{h\mathbf{k}}{2\pi}$		$1.3 \times 10^{-27} \frac{\text{kg } m}{s}$

\* Used in Spectroscopy (proportional to energy)

\*\* Electron Volts; Energy Units

# Spectral Regions

Band	Low $\lambda$	High $\lambda$	Characteristics
Vacuum Ultraviolet	100nm	300nm	Requires vacuum for propagation.
Ultraviolet C (UVC)	100nm	280nm	
Oxygen Absorption		280nm	
Ultraviolet B (UVB)	280nm	320nm	Causes sunburn. Is partially blocked by glass.
Glass Transmission	350nm	2.5 $\mu$ m	
Ultraviolet A (UVA)	320nm	400nm	Is used in a “black light.” Transmits through glass.
Visible Light	400nm	710nm	Is visible to humans, transmitted through glass, detected by silicon.
Near-Infrared (NIR)	750nm	1.1 $\mu$ m	Is transmitted through glass and biological tissue, is detected by silicon.
Si Band Edge	1.2 $\mu$ m		Is not a sharp edge.
Water Absorption	1.4 $\mu$ m		Is not a sharp edge.
Mid-Infrared	3 $\mu$ m	5 $\mu$ m	Is used for thermal imaging. Is transmitted by Zinc Selenide and Germanium.
Far-Infrared (FIR)	8 $\mu$ m	$\approx 14\mu$ m	Is used for thermal imaging. Is transmitted through ZnSe Ge, detected by HgCdTe etc.



# Spectral Regions

## In–Practice

- Glass is Transparent in UVA, Visible, NIR
  - $n \approx 1.5$
  - Useful for Windows, Lenses, Prisms, *etc.*
- Silicon Absorbs at these Wavelengths
  - Useful for Detectors
- Different Materials Required in UVC, UVB, Mid– and Far–IR
  - Normally Expensive Ones!

# Photon Example

- Pulsed Laser
  - Average Power:  $P_{av} = 1\text{W}$
  - Pulse Repetition Frequency:  $PRF = 80\text{MHz}$
  - Pulse Width:  $\tau = 100\text{fs}$
  - Wavelength:  $800\text{nm}$
- Questions
  - Pulse Energy?
  - Photons per Pulse?
  - Peak Power?

# Photon Example

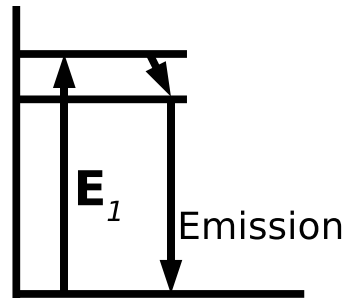
- Pulsed Laser

- Average Power:  $P_{av} = 1\text{W}$
- Pulse Repetition Frequency:  $PRF = 80\text{MHz}$
- Pulse Width:  $\tau = 100\text{fs}$
- Wavelength:  $800\text{nm}$

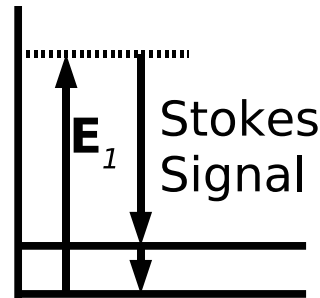
- Answers

- Pulse Energy:  $Q = \frac{P_{av}}{PRF} = 12.5\text{nJ}$
- Photons per Pulse:  $N = \frac{Q}{h\nu} = \frac{Q\lambda}{hc} = 5 \times 10^{10}$
- Peak Power:  $P_{pk} = \frac{Q}{\tau} = \frac{P_{av}}{\tau PRF} = 125\text{kW}$

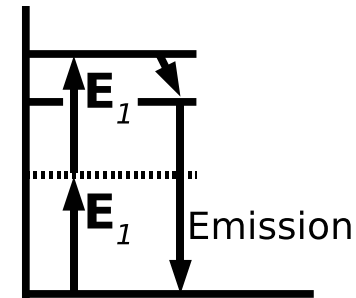
# Energy Levels in Materials



Fluorescence



Raman Scattering

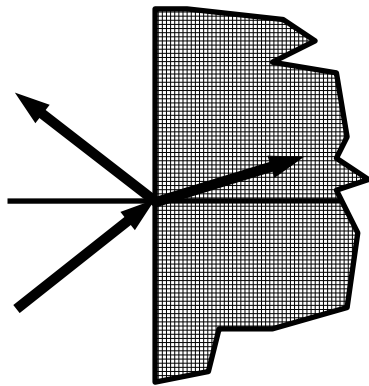


2-Photon-Excited  
Fluorescence

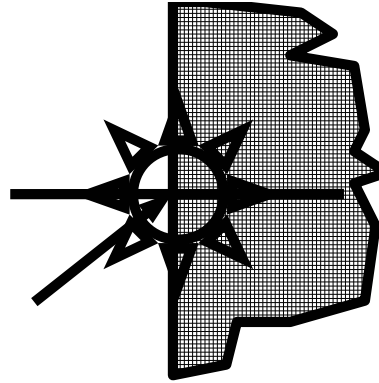
These show three different types of interaction of light with material. Solid horizontal lines are real states, and dashed lines are called “virtual states.” More on these subjects later.

Q: Why do spectroscopists like energy units ( $\text{cm}^{-1}$  or eV) more than wavelength units?

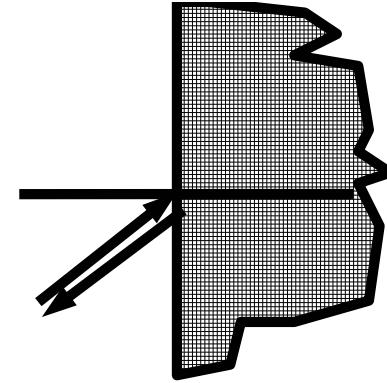
# Light at an Interface



Specular Behavior



Scattering and  
Diffuse Behavior



Retroreflection

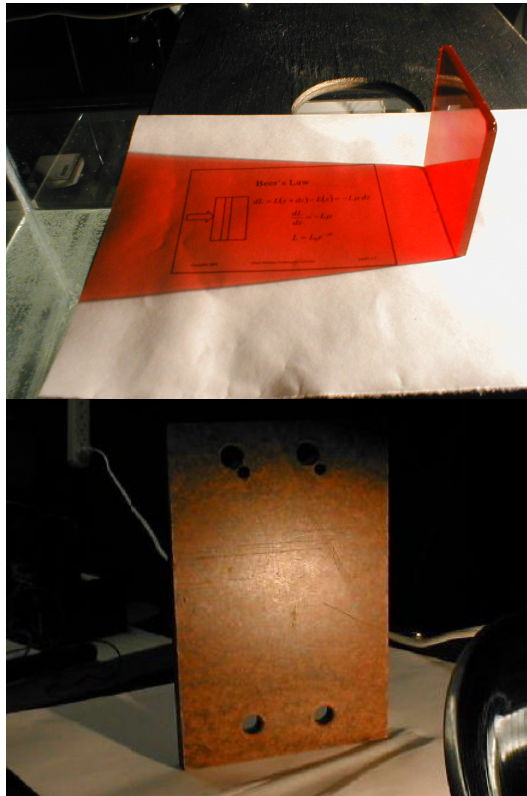
In these interactions, energy of the photon is conserved.

## Take-Away Messsge

Light at an interface has specular, diffuse, and retro-reflective behavior. We will mostly consider specular behavior until much later.

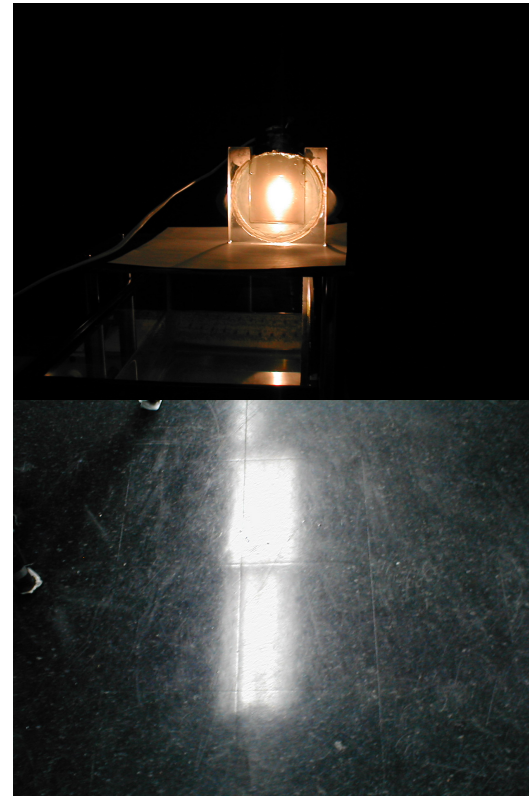
# Light in a Material

Transmission  
& Absorption  
(Colored  
Glass)



Diffuse  
Reflection  
(Rusty Iron)

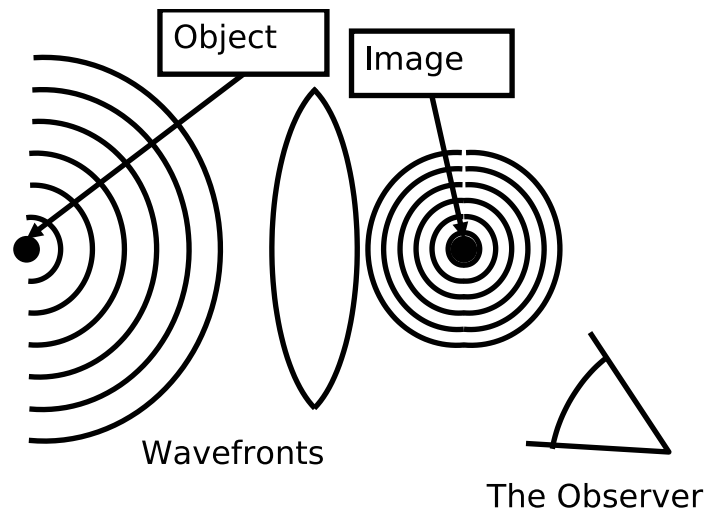
Transmission  
and Scattering  
(Milk and  
Water)



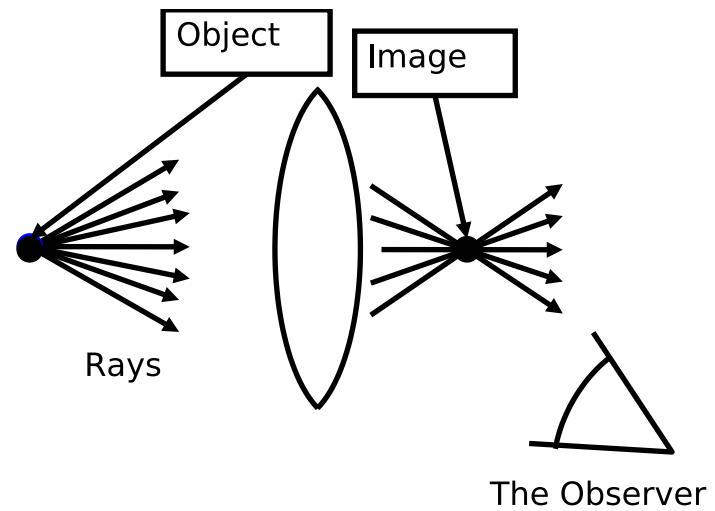
Diffuse and  
Specular  
Reflection  
(Floor)

Photon energy is still conserved. In absorption, a fraction of the photons is absorbed, but the photon energy is still unchanged.

# Imaging Concepts



Wave Picture



Ray Picture

# Eikonal Equation (1)

- A General Solution to the Wave Equation

$$E = e^{a(\mathbf{r})} e^{jk[\ell(\mathbf{r}) - \omega t]} = e^{a(\mathbf{r})} e^{jk[\ell(\mathbf{r}) - ct]}$$

- $a$  Is Related to Amplitude
- $\ell$  is related to Phase

- Scalar Wave Equation

$$\nabla^2 E = -\omega^2 \frac{n^2}{c^2} E \quad \omega = kc$$

- Substitute:

$$\left\{ \nabla^2 (a + jk\ell) + [\nabla (a + jk\ell)]^2 \right\} E = -n^2 k^2 E$$

$$\nabla^2 a + jk \nabla^2 \ell + (\nabla a)^2 + 2jk \nabla a \nabla \ell - k^2 (\nabla \ell)^2 = -n^2 k^2$$



# Eikonal Equation (2)

- Equation from Previous Page

$$\nabla^2 a + jk \nabla^2 \ell + (\nabla a)^2 + 2jk \nabla a \nabla \ell - k^2 (\nabla \ell)^2 = -n^2 k^2$$

- Divide by  $k^2$

$$\frac{\nabla^2 a}{k^2} + j \frac{\nabla^2 \ell}{k} + \frac{(\nabla a)^2}{k^2} + 2j \frac{\nabla a}{k} \nabla \ell - (\nabla \ell)^2 = -n^2$$

- Assume  $\lambda \rightarrow 0$  (How Small?)

$$\frac{\nabla^2 a}{k^2} = \frac{\lambda^2 \nabla^2 a}{4\pi^2} \rightarrow 0 \quad \frac{(\nabla a)^2}{k^2} = \frac{(\lambda \nabla a)^2}{4\pi^2} \rightarrow 0$$

$$\frac{\nabla^2 \ell}{k} = \frac{\lambda \nabla^2 \ell}{2\pi} \rightarrow 0 \quad \frac{\nabla a}{k} \nabla \ell = \frac{\lambda \nabla a \nabla \ell}{2\pi} \rightarrow 0$$

- Both  $a$  and  $\ell$  vary on a size scale much larger than  $\lambda$ .

# Eikonal Equation (3)

- Result: The Eikonal Equation

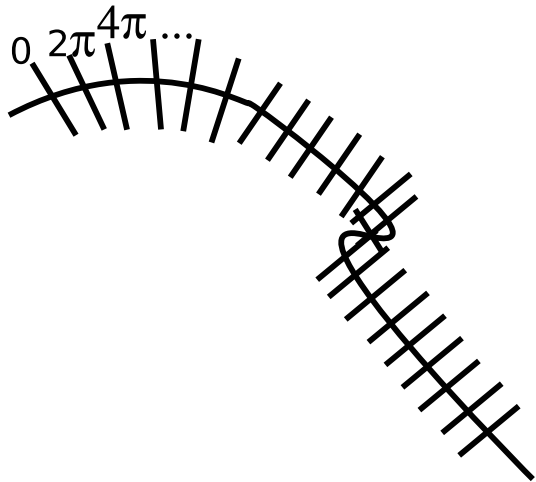
$$|\nabla \ell| = n.$$

- Optical Path Length:  $\ell$ 
  - Physical Path Length:  $\ell_p$
  - Index of Refraction:  $n$

$$\ell = OPL = \int_A^B n d\ell_p$$

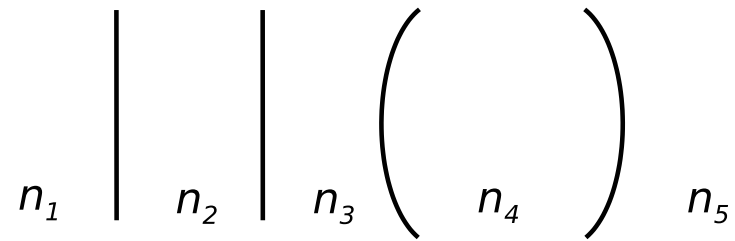
- Phase:  $\Delta\phi = k\Delta\ell = 2\pi\frac{OPL}{\lambda}$

# Optical Path Length



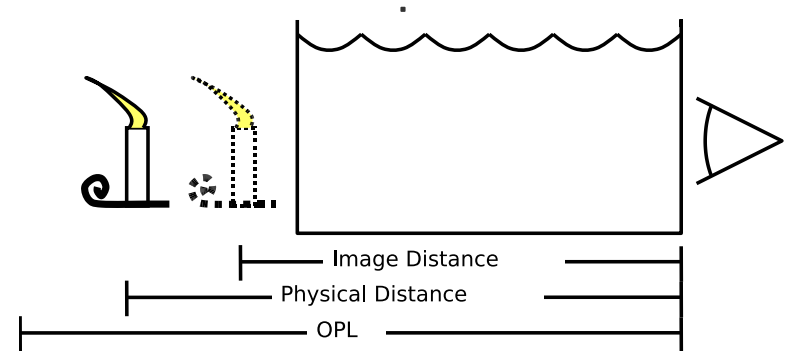
Optical Path Length. The OPL between any two points is obtained by integrating along a path between those two points. The eikonal equation ensures that the integral does not depend on the path chosen. The phase difference between the two points can be computed from the OPL.

200min



Optical Path in Discrete Materials:

$$OPL = \ell = \sum_m n_m \ell_m$$



Optical Path in Water. The optical path is longer than the physical path, but the geometric path is shorter.

# Optical Path Length

## Take–Away Messsge

The Eikonal Equation is very general and powerful. It can be used for complicated problems like gradient–index fibers and mirages and others beyond our scope here.

Most important here is the concept of Optical Path Length in discrete materials.

$$OPL = \ell = \sum_m n_m \ell_m.$$

We will use it to understand refraction and thus prisms and lenses, and later we will use it in interferometry (Ch. 10).

# Gradient Index

- The Eikonal Equation

$$|\nabla \ell| = n$$

- Differentiate: Ray Equation ( $\mathbf{r}$  Is Path of Light)

$$\frac{d}{d\ell} \left[ n(\mathbf{r}) \frac{d\mathbf{r}}{d\ell} \right] = \nabla n(\mathbf{r})$$

- Changing the Ray Direction

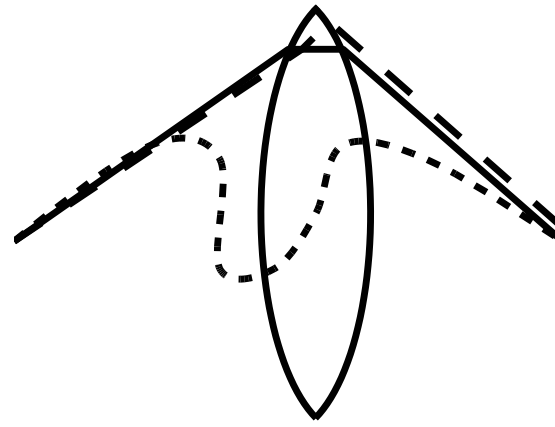
$$n(\mathbf{r}) \frac{d^2 \mathbf{r}}{d\ell^2} = \nabla n(\mathbf{r}) - \frac{d\mathbf{r}}{d\ell} \frac{dn(\mathbf{r})}{d\ell}$$

- Useful for looming, mirage, gradient-index fiber, *etc.*

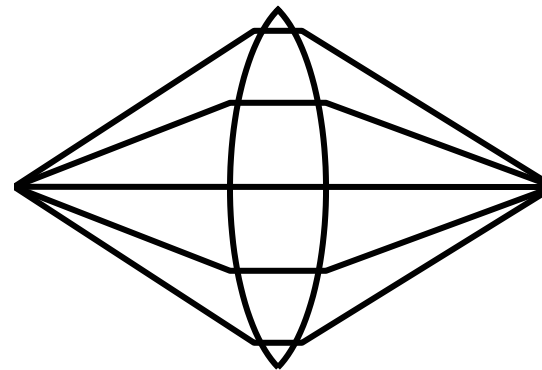
# A Lens, Simply

$$\sum_{m=1}^3 n_m \ell_m = \ell_1 + n_{\text{glass}} \ell_2 + \ell_3,$$

- Center of Lens
  - Thick Glass
  - $OPL > PPL$
  - Less Air, Low Index
- Edge of Lens
  - Less Glass
  - More Air
- All Paths Equal?
  - Rays Arrive in Phase
  - Point Is Imaged
  - Object and Image are Said to be Conjugates



Fermat's Principle. Light travels the shortest optical path.



Imaging. All paths are Minimal, Points are conjugate; One is the Image of the Other.

# What's to Come?

- Geometric Optics
  - Ray Tracing (2)
  - Matrix Optics (3)
  - Stops & Windows (4)
  - Aberrations (5)
- Wave Phenomena
  - Polarization (6)
  - Interference (7)
  - Diffraction (8)
    - \* Fraunhofer
    - \* Fresnel
    - \* Gaussian Beams (9)
- Coherence (10)
  - Spatial
  - Temporal
- Fourier Optics (11)
  - Coherent
  - Incoherent
- Radiometry (12)
  - Radiometry
  - Photometry
  - Color
- Detectors (13)
- Non-Linear Optics (14)