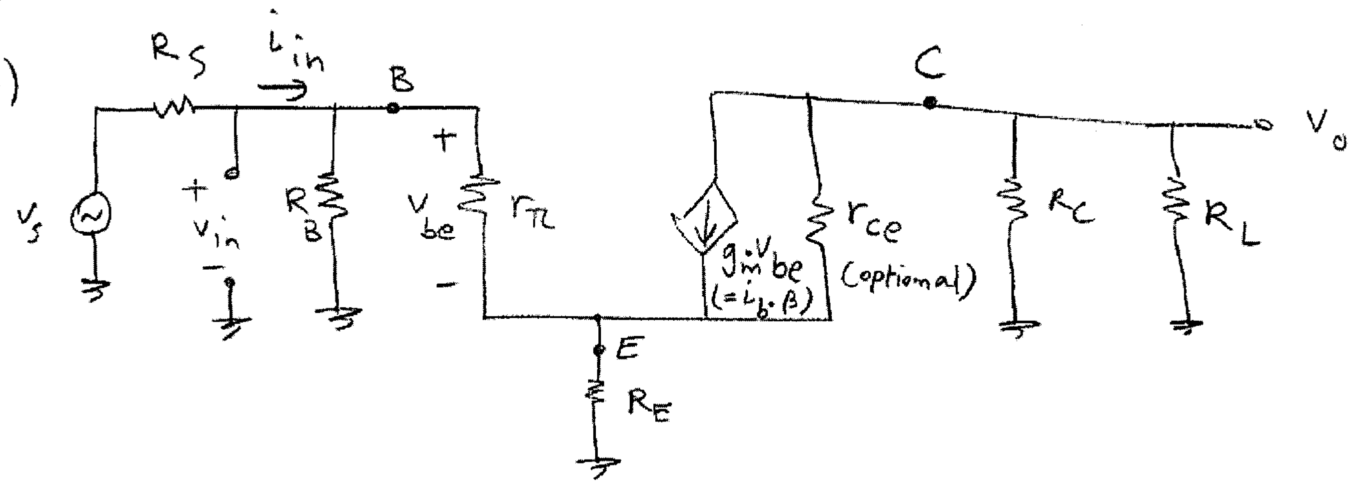
Homework 7 Solutions

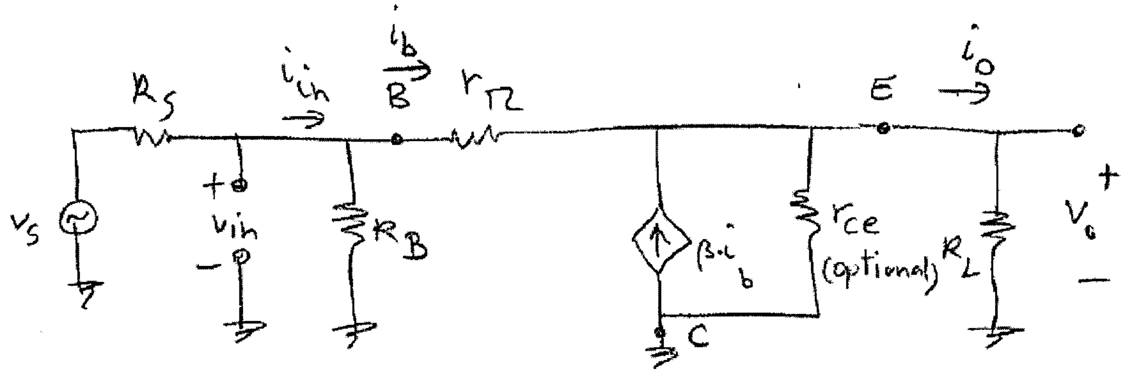
Fall 2018

Prob. 1

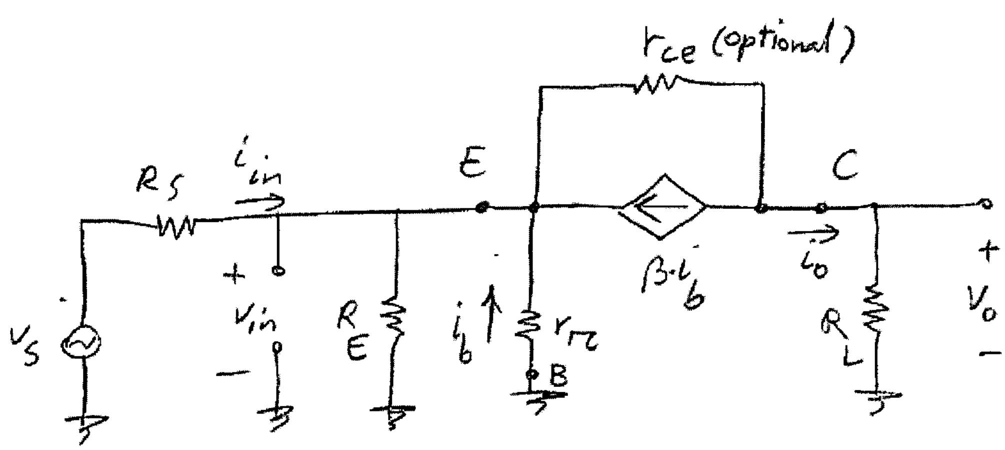
a)



b)

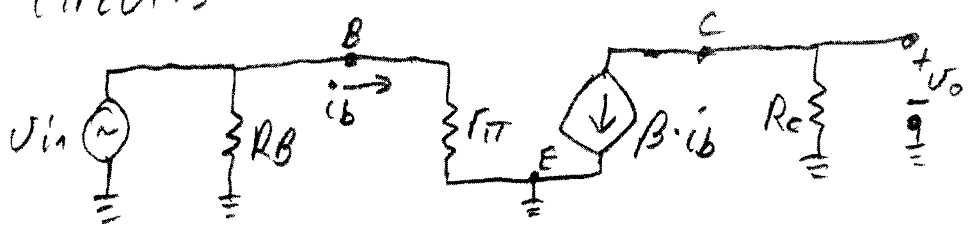


c)



Problem 2

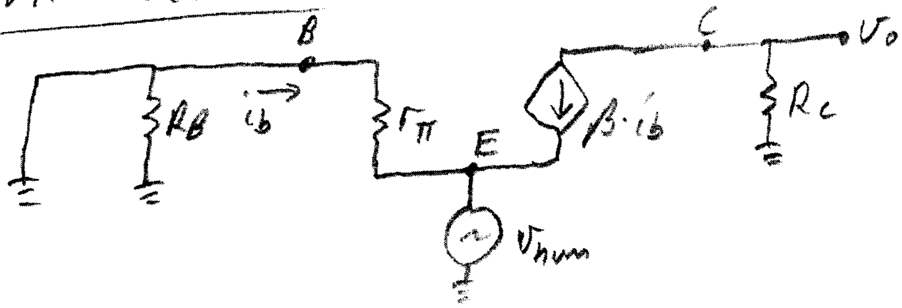
The equivalent circuit is identical for both circuits when $v_{hum} = 0$ during small-signal analysis.



$$i_b = \frac{v_{in}}{r_\pi} \quad \text{Sub.}$$

$$v_o = -\beta \cdot i_b \cdot R_c = -\beta \cdot \frac{v_{in}}{r_\pi} \cdot R_c \rightarrow \boxed{\frac{v_o}{v_{in}} = -\frac{\beta \cdot R_c}{r_\pi}}$$

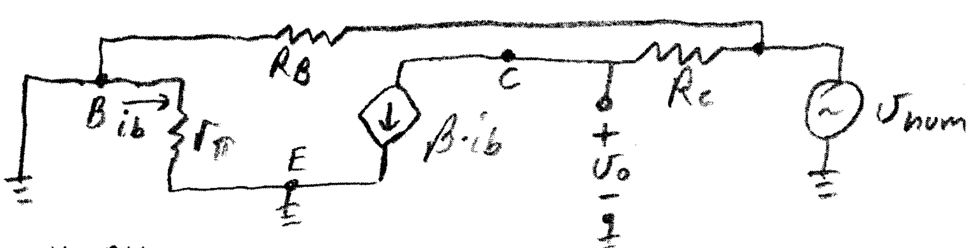
(a) With $v_{in} = 0$:



$$i_b = -\frac{v_{hum}}{r_\pi} \quad \text{Sub.}$$

$$v_o = -\beta \cdot i_b \cdot R_c = \beta \cdot \frac{v_{hum}}{r_\pi} \cdot R_c \rightarrow \boxed{\frac{v_o}{v_{hum}} = \frac{\beta \cdot R_c}{r_\pi}}$$

(b) With $v_{in} = 0$:



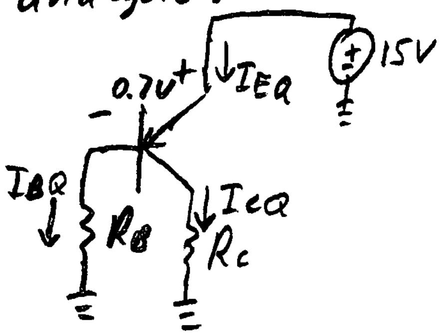
$$i_b = \frac{0V - 0V}{r_\pi} = 0 \rightarrow \beta \cdot i_b = 0$$

KVL: $0 = -v_o - \beta \cdot \cancel{i_b}^{\,0} + v_{hum} \rightarrow v_o = v_{hum} \rightarrow \boxed{\frac{v_o}{v_{hum}} = 1}$

... Prob. 2 cont.:

DC analysis:

(a)



$0 = -15V + 0.7V + I_{BQ} \cdot R_B$

$\hookrightarrow I_{BQ} = \frac{14.3V}{1\times10^6 \Omega} = 14.3\mu A$

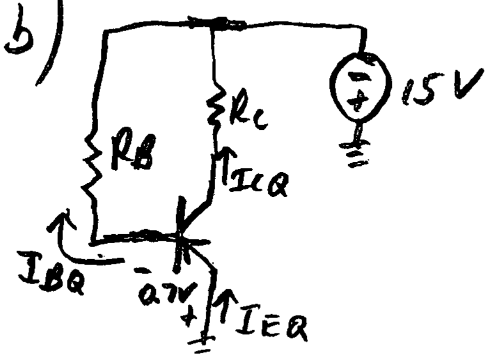
$I_{CQ} = \beta \cdot I_{BQ} = 1.43 mA$

$r_\pi = \frac{\beta \cdot V_T}{I_{CQ}} = \frac{100 \cdot 26mV}{1.43mA} = 1.82k\Omega$

$$\frac{v_o}{v_{in}} = \frac{-\beta \cdot R_c}{r_\pi} = -258.5$$

$$\frac{v_o}{v_{hum}} = \frac{\beta R_c}{r_\pi} = 258.5$$

(b)



$0 = 0.7V + I_{BQ} \cdot R_B - 15V$

$\hookrightarrow I_{BQ} = \frac{14.3V}{R_B} = 14.3\mu A$

$I_{CQ} = \beta \cdot I_{BQ} = 1.43mA$

$r_\pi = \frac{\beta \cdot V_T}{I_{CQ}} = 1.82k\Omega$

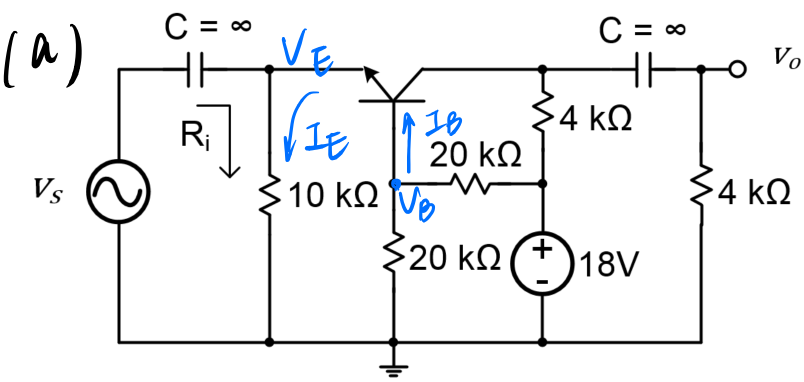
$$\frac{v_o}{v_{in}} = \frac{-\beta \cdot R_c}{r_\pi} = -258.5$$

$$\frac{v_o}{v_{hum}} = 1$$

Circuit (b) is better because it has a lower gain from v_{hum} to v_o, while having the same gain from v_{in} to v_o.

Problem (3)

(a)



DC analysis:

Capacitors → open circuit

$$\begin{cases} I_E = \dfrac{V_E}{10k\Omega}, \quad V_E = V_B - 0.7V \\ I_B = \dfrac{18 - V_B}{20k\Omega} - \dfrac{V_B}{20k\Omega} \\ I_E = (1+\beta) I_B, \quad \beta = 250 \end{cases}$$

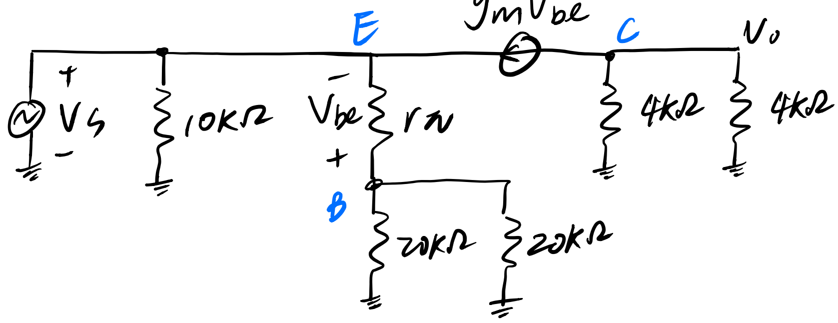
$V_B = 8.9671$, $V_E = 8.2671$

$I_E = 0.82671\,mA$, $I_C = \frac{\beta}{1+\beta} I_E = 0.82342\,mA$

$g_m = \frac{I_C}{V_T} = \frac{0.82342}{26} = \boxed{0.03167\Omega^{-1}}$ $r_\pi = \frac{\beta}{g_m} = \frac{250}{0.03167} = \boxed{7.893k\Omega}$

Small signal analysis

Capacitors → short circuit



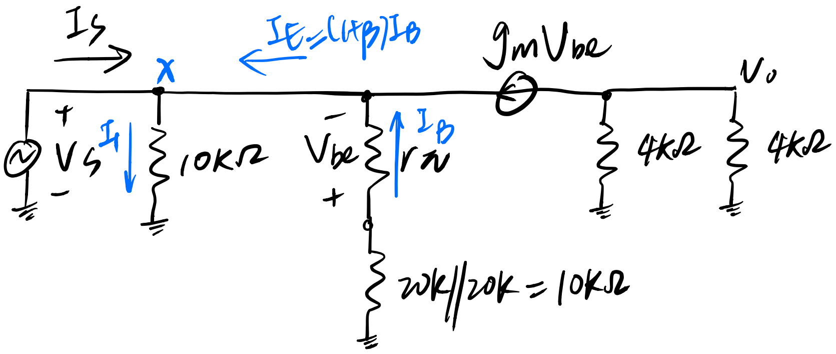
$-V_{be} = V_s \cdot \frac{r_\pi}{r_\pi + 20k \| 20k}$

$V_s = -2.272 V_{be}$

$V_o = -g_m V_{be} \cdot (4k \| 4k) = -63.6 V_{be}$

$A_{vs} = \frac{V_o}{V_s} = \frac{-63.6 V_{be}}{-2.272 V_{be}} = \boxed{27.99 = A_{vs}}$

(b)



$$\begin{cases} I_B = \dfrac{V_{be}}{r_\pi} = -\dfrac{V_s}{r_\pi + 10k\Omega} \\ I_E = (1+\beta) I_B = -V_s \dfrac{1+\beta}{r_\pi + 10k\Omega} \\ I_1 = V_s \cdot \dfrac{1}{10k\Omega} \end{cases}$$

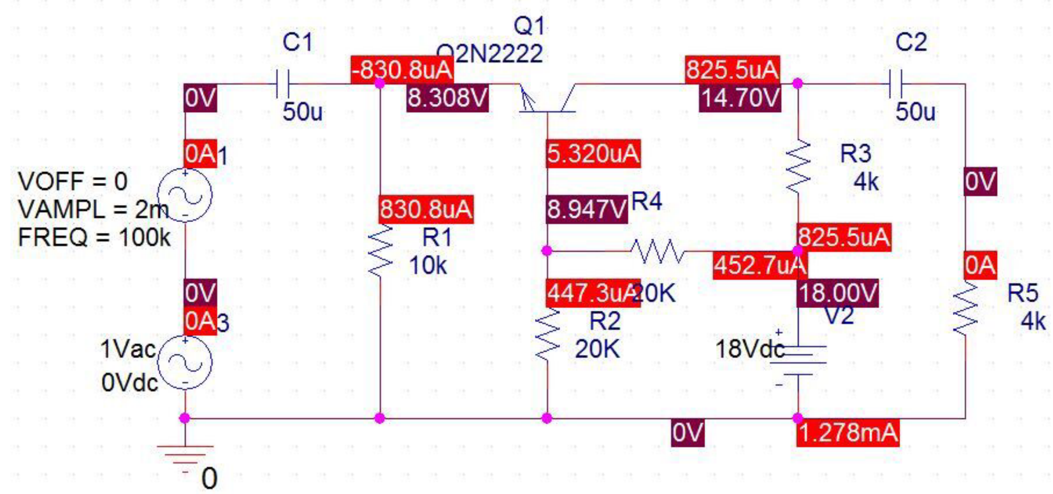
KCL @ node X:

$I_s = I_1 - I_E = V_s \left[\frac{1}{10k\Omega} + \frac{1+\beta}{r_\pi + 10k\Omega} \right]$

$R_i = \frac{V_s}{I_s} = \frac{1}{\frac{1}{10k\Omega} + \frac{1+\beta}{r_\pi + 10k\Omega}} = \boxed{70.66\Omega = R_i}$

Problem (4)

(a) Biasing point



	Q2N2222
	NPN
IS	14.340000E-15
BF	255.9
NF	1
VAF	74.03
IKF	.2847
ISE	14.340000E-15
NE	1.307
BR	6.092
NR	1
RB	10
RC	1
CJE	22.010000E-12
MJE	.377
CJC	7.306000E-12
MJC	.3416
TF	411.100000E-12
XTF	3
VTF	1.7
ITF	.6
TR	46.910000E-09
XTB	1.5
CN	2.42
D	.87

**** BIPOLAR JUNCTION TRANSISTORS

NAME	Q_Q1
MODEL	Q2N2222
IB	5.32E-06
IC	8.25E-04
VBE	6.39E-01
VBC	-5.75E+00
VCE	6.39E+00
BETADC	1.55E+02
GM	3.18E-02
RPI	5.42E+03
RX	1.00E+01
RO	9.67E+04
CBE	4.93E-11
CBC	3.49E-12
CJS	0.00E+00
BETAAC	1.72E+02
CBX/CBX2	0.00E+00
FT/FT2	9.60E+07

→ $I_B > 0$ (active/saturation region)

→ $V_{CE} > 0.2V$ (active region) ✓

$$A_{v_s} = \frac{V_o}{V_s} = \frac{-g_m V_{be}(4k\Omega // 4k\Omega)}{-V_{be}\,\frac{r_\pi + 20k\Omega // 20k\Omega}{r_\pi}} = 22.355 = A_{v_s}$$

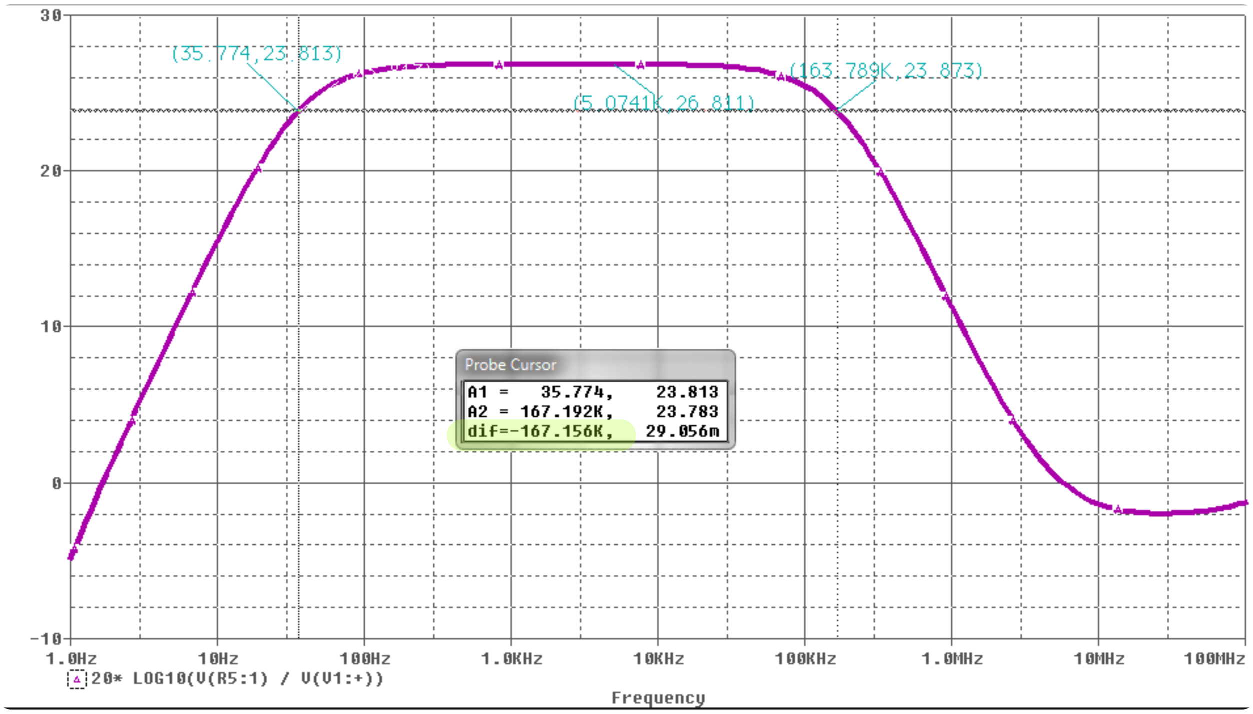
A_{v_s} is smaller compared to hand calculation, the reasons are:

① $\beta_{ac} = 155$ instead of 250, which makes $r_\pi = \frac{\beta}{g_m}$ smaller;

② $V_A = 74.03V$ instead of ∞, which makes $r_{ce} = \frac{V_A}{I_C}$ finite. → R_{out} becomes smaller as well.

⑥

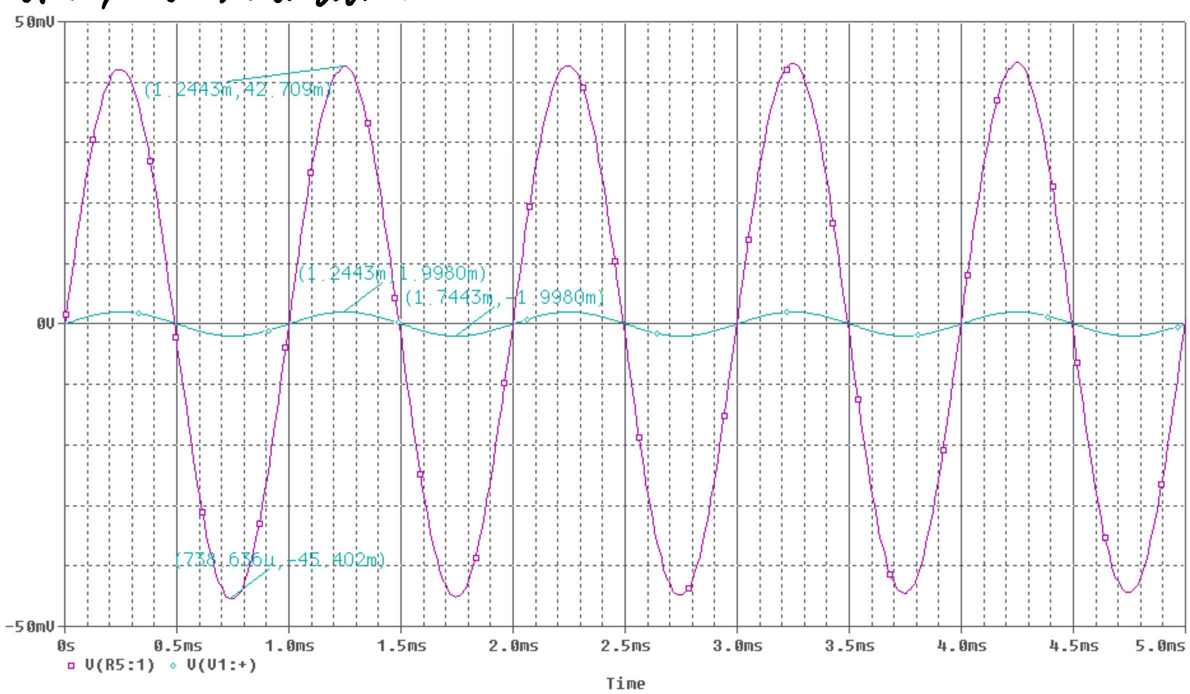
(b) AC simulation:



$A_{vs} = 26.811\ dB \rightarrow A_{vs} = 10^{\left(\frac{26.811}{20}\right)} = \boxed{21.9}$

$\boxed{BW = 167.156\ KHz}$

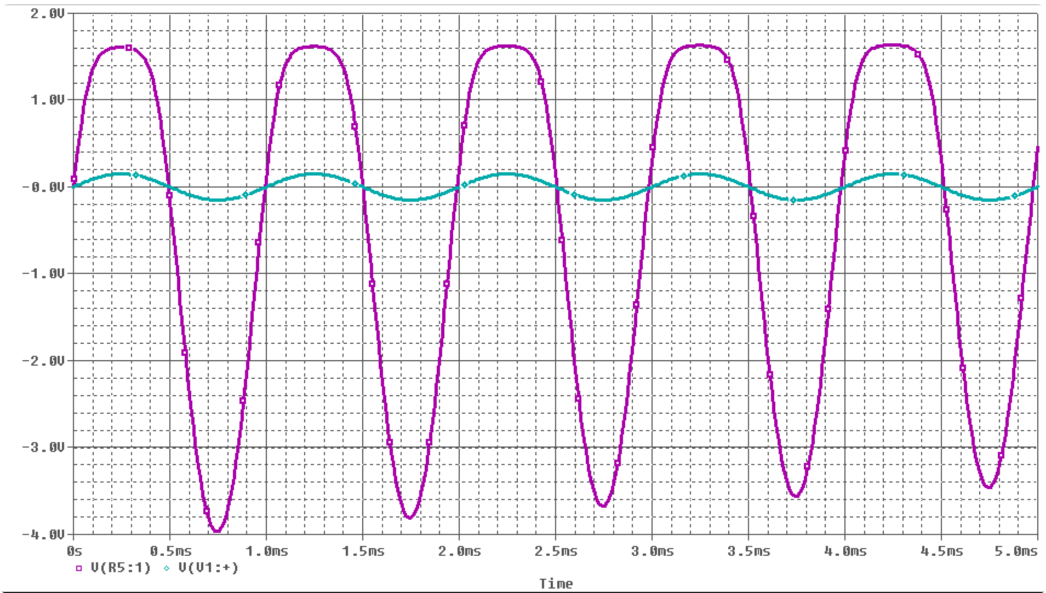
(c) transient simulation



$$A_{vs} = \frac{42.709mV - (-45.402mV)}{1.998mV - (-1.998mV)} = \boxed{22.04}$$

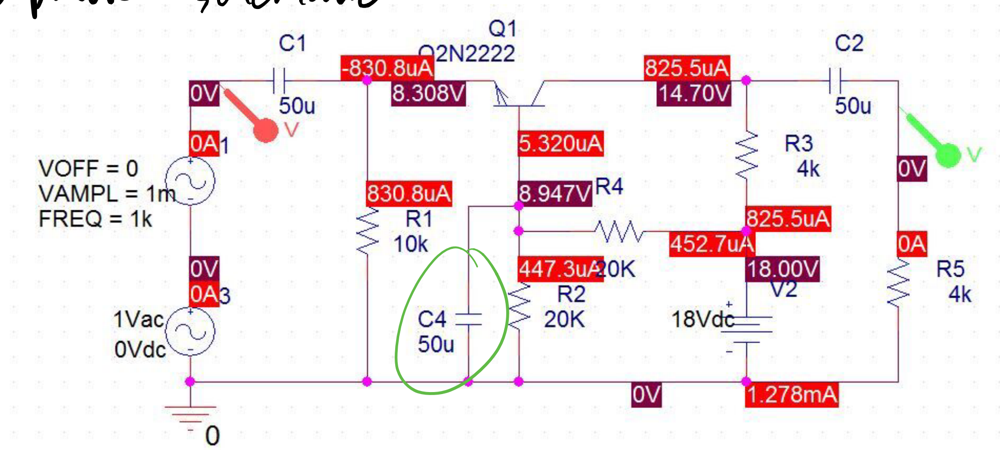
agrees with the gain at 1KHz in part (c) → 26.811dB

(d)

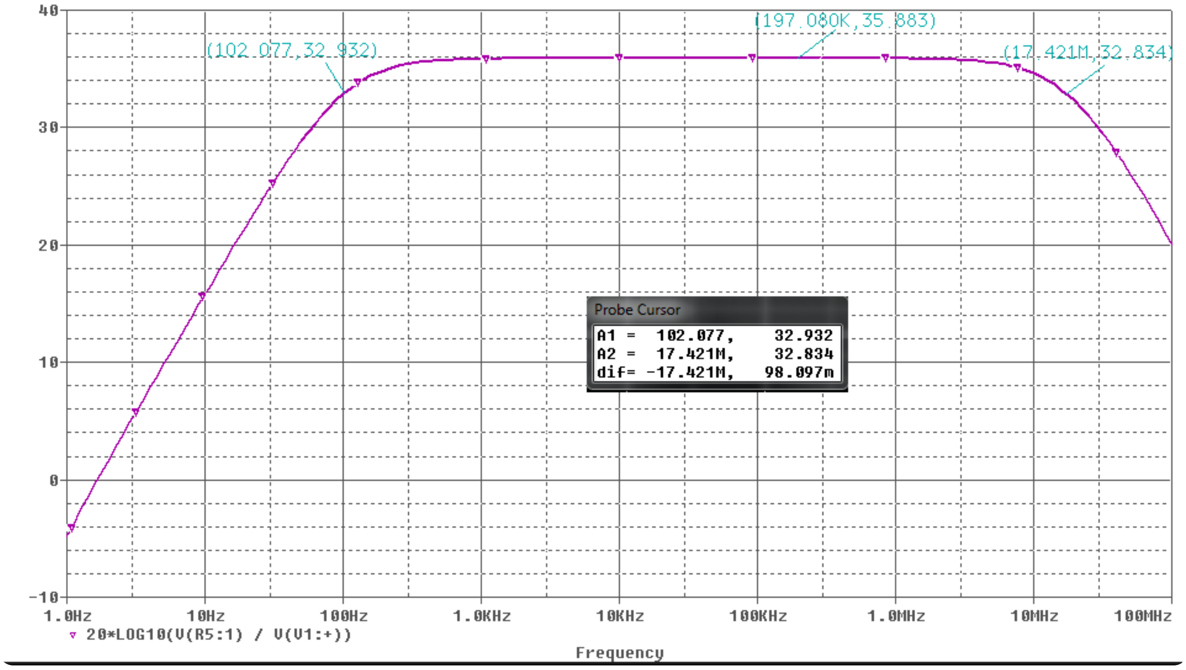


From DC analysis in part (a), we observe that headroom between collector and supply is $\Delta V = 18V - 14.7V = 3.3V$. With the gain of 22, by increasing V_s, V_o gets closer to 3.3V but can never exceed 3.3V (otherwise peak value of V_o will go beyond the supply voltage). Therefore, V_o gets saturated when V_s is close to $\frac{3.3V}{22} = 150mV$.

(e) Updated schematic:



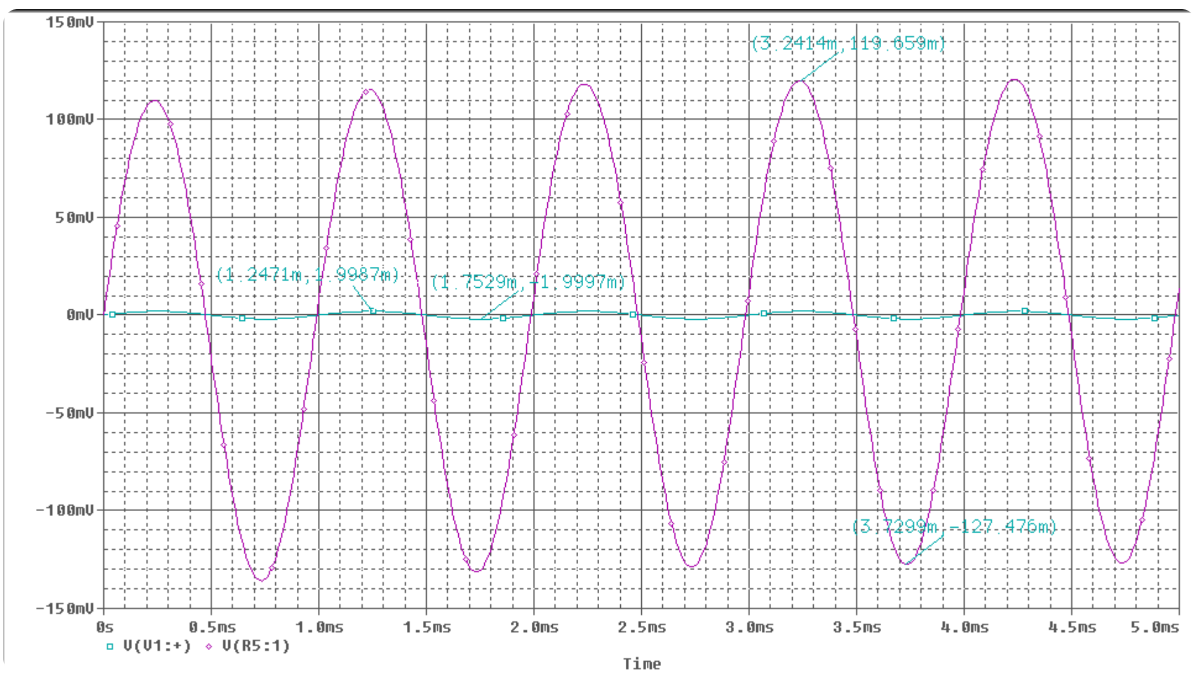
AC simulation:



$A_{vs} = 35.883\,dB \rightarrow A_{vs} = 10^{\left(\frac{35.883}{20}\right)} = 62.25 = A_{vs}$

$BW = 17.412\,MHz$

transient simulation



$$A_{vs} = \frac{119.650\,mV - (-127.476\,mV)}{1.9987\,mV - (-1.9997\,mV)} = 61.8 = A_{vs}$$