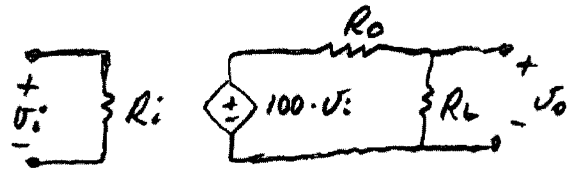
EECE 2412 : HW 1 Solutions

Fall 2018

Problem 1



$R_L = 10\,k\Omega$

$A_v = \frac{v_o}{v_i} = 90$

$$v_o = 100 \cdot v_i \cdot \left(\frac{R_L}{R_o + R_L}\right)$$

$$A_v = \frac{v_o}{v_i} = 100 \cdot \left(\frac{10000}{R_o + 10000}\right) = 90 \xrightarrow{\text{solve}} R_o = 1111\,\Omega$$

$$\boxed{R_o = 1.11\,k\Omega}$$

Problem 2

$$P_{in} = \frac{(V_{in-RMS})^2}{R_{in}} = \frac{(0.1V)^2}{100 \times 10^3\,\Omega} = 100 \times 10^{-9}\,W = 0.1\,\mu W$$

$$P_{supply} = V_{supply} \cdot I_{supply} = 15V \cdot 2A = 30W$$

$$P_{out} = \frac{(V_{o-RMS})^2}{R_L} = \frac{(10V)^2}{8} = 12.5\,W$$

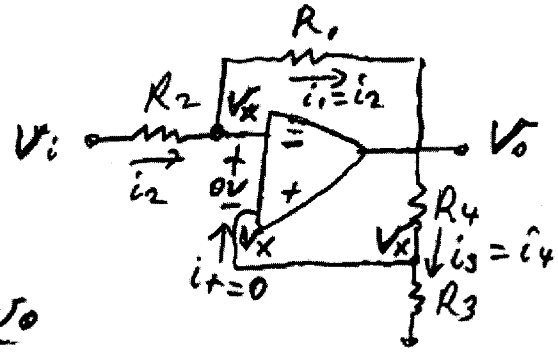
dissipated power: $P_d = P_{in} + P_{supply} - P_{out} = 17.5000001\,W$

$\hookrightarrow \boxed{P_d \approx 17.5W}$

efficiency: $\eta = \frac{P_{out}}{P_{supply}} \times 100\%$

$$\eta = \frac{12.5W}{30W} \times 100\% = \boxed{41.67\% = \eta}$$

Problem 3



①: $V_x = \left(\frac{R_3}{R_3+R_4}\right)\cdot V_o$

$$i_2 = \frac{V_i - V_x}{R_2} = \frac{V_x - V_o}{R_1}$$

algebraic rearrangement ↳ $\frac{V_i}{R_2} = \left(\frac{1}{R_1} + \frac{1}{R_2}\right)\cdot V_x - \frac{V_o}{R_1}$

Substituting equ. ① into the above equation yields:

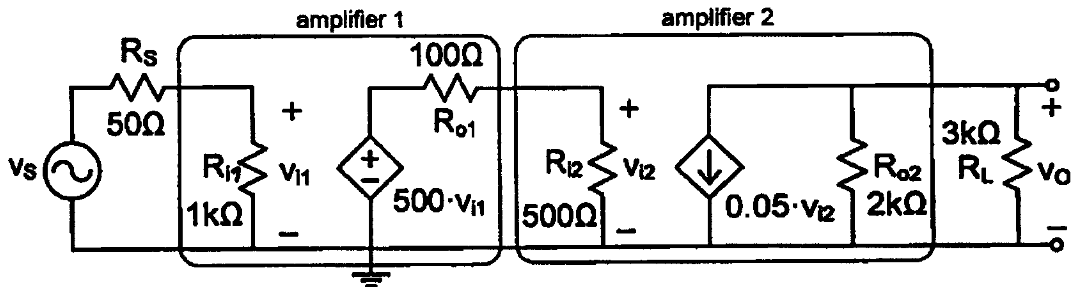
$$\frac{V_i}{R_2} = \left(\frac{1}{R_1} + \frac{1}{R_2}\right)\cdot\left(\frac{R_3}{R_3+R_4}\right)\cdot V_o - \frac{V_o}{R_1}$$

$$\frac{V_i}{R_2} = \left[\left(\frac{1}{R_1} + \frac{1}{R_2}\right)\cdot\left(\frac{R_3}{R_3+R_4}\right) - \frac{1}{R_1}\right]\cdot V_o$$

$$\boxed{\frac{V_o}{V_i} = \frac{1}{R_2\cdot\left[\left(\frac{1}{R_1}+\frac{1}{R_2}\right)\cdot\left(\frac{R_3}{R_3+R_4}\right) - \frac{1}{R_1}\right]}}$$

(p.3) **Problem 4**

a)



$$v_{i1} = v_s \cdot \left(\frac{R_{i1}}{R_{i1}+R_s}\right) \xrightarrow{\text{sub.}}$$

$$v_{i2} = 500 \cdot v_{i1} \cdot \left(\frac{R_{i2}}{R_{i2}+R_{o1}}\right) = 500 \cdot v_s \cdot \left(\frac{R_{i1}}{R_{i1}+R_s}\right)\cdot\left(\frac{R_{i2}}{R_{i2}+R_{o1}}\right)$$

sub.

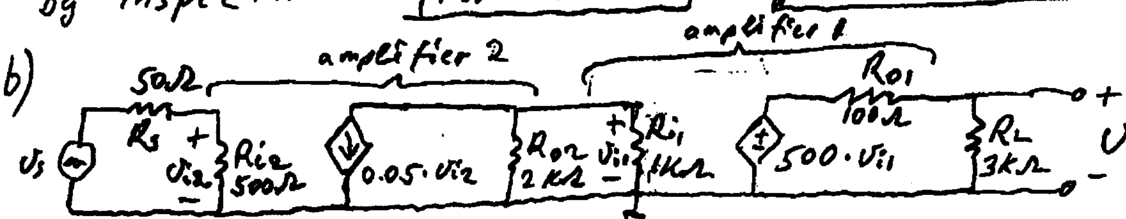
$$v_o = -0.005 \cdot v_{i2} \cdot (R_{o2} \| R_L) = -0.05 \cdot 500 \cdot v_s \cdot \left(\frac{R_{i1}}{R_{i1}+R_s}\right)\cdot\left(\frac{R_{i2}}{R_{i2}+R_{o1}}\right)\cdot(R_{o2}\|R_L)$$

$$A_{vs} = \frac{v_o}{v_s} = 0.05 \cdot 500 \cdot \left(\frac{1k\Omega}{1k\Omega+50\Omega}\right)\left(\frac{500\Omega}{500\Omega+100\Omega}\right)\cdot\left(\frac{1}{\frac{1}{2k\Omega}+\frac{1}{3k\Omega}}\right)$$

$\boxed{A_{vs} = -2.38 \times 10^4}$ or $A_{vs(dB)} = 87.54\ dB = 20 \cdot \log(|A_{vs}|)$

by inspection: $\boxed{R_i = 1k\Omega}$ $\boxed{R_o = 2k\Omega}$

b)



$$v_{i2} = v_s \cdot \left(\frac{R_{i2}}{R_{i2}+R_s}\right)$$

$$v_{i1} = -0.05 \cdot v_{i2} \cdot (R_{o2}\|R_{i1}) = -0.05 \cdot v_s \cdot \left(\frac{R_{i2}}{R_{i2}+R_s}\right)\cdot(R_{o2}\|R_{i1})$$

sub.

$$v_o = 500 \cdot v_{i1} \cdot \left(\frac{R_L}{R_L+R_{o1}}\right) = 500 \cdot (-0.05) \cdot v_s \cdot \left(\frac{R_{i2}}{R_{i2}+R_s}\right)\cdot(R_{o2}\|R_{i1})\cdot\left(\frac{R_L}{R_L+R_{o1}}\right)$$

$$A_{vs} = \frac{v_o}{v_s} = -500 \cdot 0.05 \cdot \left(\frac{500\Omega}{500\Omega+50\Omega}\right)\cdot\left(\frac{1}{\frac{1}{1k\Omega}+\frac{1}{2k\Omega}}\right)\cdot\left(\frac{3k\Omega}{3k\Omega+100\Omega}\right)$$

$\boxed{A_{vs} = -1.47 \times 10^4}$ or $A_{vs(dB)} = 20 \cdot \log(|A_{vs}|) = 83.32\ dB$

$\boxed{R_i = 500\ \Omega}$ $\boxed{R_o = 100\ \Omega}$

Prob. 5

a) ①: $A_{vs}(s) = \frac{V_o(s)}{V_s(s)} = \frac{V_i(s)}{V_s(s)} \times \frac{V_o(s)}{V_i(s)}$, where $s = j\omega$

$V_i(s) = \frac{R_i}{R_i + R_s} \cdot V_s(s) \longrightarrow$ ②: $\frac{V_i(s)}{V_s(s)} = \frac{150}{150+50} = 0.75$

$V_o(s) = -0.1 \cdot V_i(s) \cdot (R_o \| \frac{1}{sC_L} \| R_L) = -0.1 \cdot V_i(s) \cdot \left(\frac{1}{\frac{1}{R_o} + sC_L + \frac{1}{R_L}} \right)$

$V_o(s) = -0.1 \cdot V_i(s) \times \left(\frac{\frac{1}{\frac{1}{R_o} + \frac{1}{R_L}}}{1 + \frac{s \cdot C_L}{\frac{1}{R_o} + \frac{1}{R_L}}} \right) = -0.1 \cdot V_i(s) \cdot \left(\frac{R_{eq}}{1 + sC_L R_{eq}} \right)$

where: $R_{eq} = \frac{1}{\frac{1}{R_o} + \frac{1}{R_L}} = R_o \| R_L$

$R_{eq} = 3.75 k\Omega$

③: $\frac{V_o(s)}{V_i(s)} = -0.1 \cdot \left(\frac{R_{eq}}{1 + sC_L R_{eq}} \right)$

Substituting ② and ③ in to ①, and $s = j\omega$:

$A_{vs}(j\omega) = 0.75 \cdot (-0.1) \cdot \left(\frac{R_{eq}}{1 + j\omega C_L R_{eq}} \right) = -\frac{0.075 \cdot R_{eq}}{1 + j\frac{\omega}{\omega_{3dB}}}$

$$\boxed{A_{vs}(j\omega) = \frac{-281.25}{1 + j\left(\frac{\omega}{2667}\right)}}$$

where: $\omega_{3dB} = \frac{1}{C_L R_{eq}}$

$\omega_{3dB} = 2667$ rad.

b) $f_{3dB} = \frac{\omega_{3dB}}{2\pi} = \frac{1}{2\pi C_L R_{eq}} = \frac{2667}{2\pi} = \boxed{424 \text{ Hz} = f_{3dB}}$ (see part a)

c) $GBW = f_t \approx |A_{vs}(\omega=0)| \cdot f_{3dB}$

where: $|A_{vs}(\omega=0)| = \left| \frac{-281.25}{1 + j\left(\frac{0}{2667}\right)} \right| = 281.25$

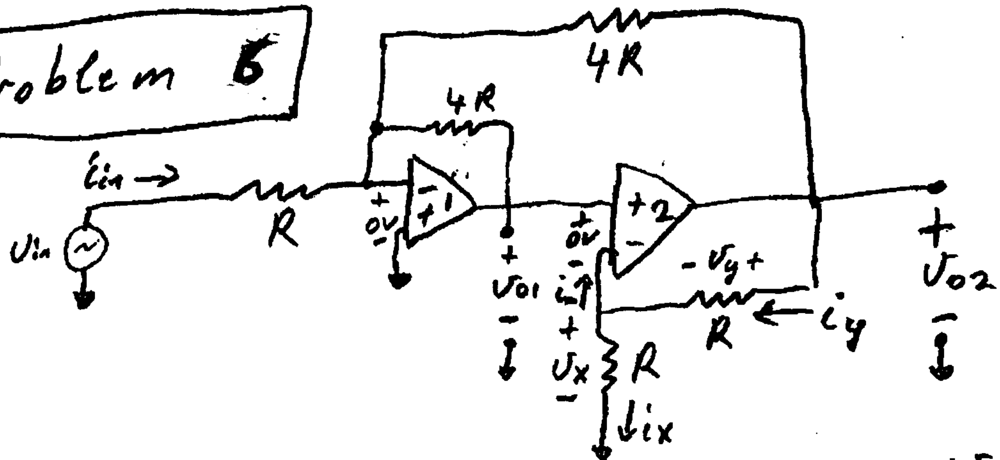
Thus,

$GBW \approx (281.25) \cdot (424 \text{ Hz})$

$$\boxed{GBW \approx 119 \text{ kHz}}$$

(15)

Problem 5



Summing-point constraint for amplifier 1: $v_- = v_+ = 0V$

$$\hookrightarrow (1): i_{in} = \frac{v_{in} - 0V}{R} = \frac{v_{in}}{R}$$

Summing-point constraint for amplifier 2:

$$0 = -v_{01} + 0V + v_x \longrightarrow (2): v_x = v_{01}$$

$$i_- = 0 \rightarrow i_x = i_y = \frac{v_x}{R} \overset{\text{sub.}}{=} \frac{v_{01}}{R} \rightarrow (3): i_x = i_y = \frac{v_{01}}{R}$$

KVL: $0 = -v_{02} + v_y + v_x$

$$\hookrightarrow v_{02} = v_y + v_x = i_x R + i_y R \overset{i_y = i_x}{=} 2 i_x R = 2 v_x$$

$$(4): v_{02} = 2 \cdot v_x \underset{\text{sub. (2)}}{=} 2 \cdot v_{01}$$

KCL at the inverting input of amplifier 1:

$$0 = i_{in} + \frac{v_{01}}{4R} + \frac{v_{02}}{4R} \underset{\text{sub. (1) and (4)}}{=} \frac{v_{in}}{R} + \frac{v_{01}}{4R} + \frac{2 v_{01}}{4R}$$

Rearranging the above equation:

$$-v_{in} \cdot 4 = 3 v_{01} \longrightarrow \boxed{A_1 = \frac{v_{01}}{v_{in}} = -\frac{4}{3}}$$

From equ. (4): $v_{02} = 2 \cdot v_{01} \overset{\text{sub.}}{=} 2 \cdot \left(-\frac{4}{3} \cdot v_{in}\right) = -\frac{8}{3} \cdot v_{in}$

$$\boxed{A_2 = \frac{v_{02}}{v_{in}} = -\frac{8}{3}}$$