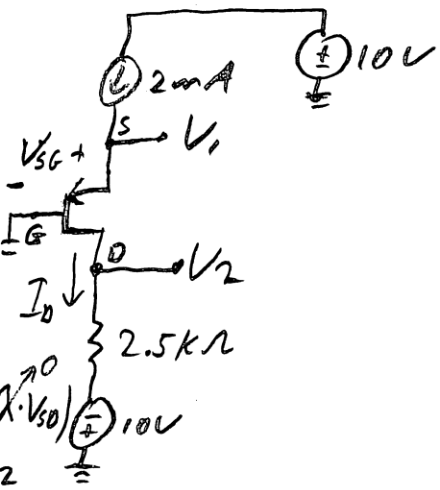
(p.1)

EECE 2412: HW 9 Solutions



Problem 1

a) Assuming that the PMOS transistor is operating in the saturation region:

$$I_D = 2mA = \frac{1}{2}\left(\frac{W}{L}\right)\mu_p \cdot C_{ox}\left(V_{SG}-|V_{t0}|\right)^2\cdot\left(1+\lambda\cdot V_{SD}\right) \quad (\lambda \to 0)$$

$$2mA = \frac{1}{2}\left(1\times10^{-3}\frac{A}{V^2}\right)\cdot\left(V_{SG}-|-2V|\right)^2$$

↳ $V_{SG} = 4\ V$ (or $V_{SG} = 0V$ → ignore)

KVL: $0 = -V_{SG} + V_1 \rightarrow V_1 = V_{SG} = \boxed{4V = V_1}$

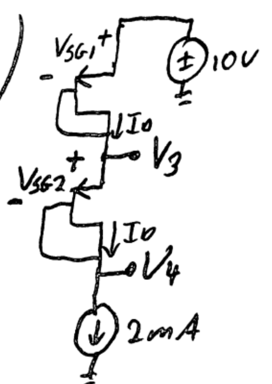
KVL: $0 = 10V - I_D\cdot 2.5k\Omega + V_2$

↳ $V_2 = (2mA)\cdot(2.5k\Omega) - 10V = \boxed{-5V = V_2}$

check: ①: $V_{SG} = 4V > |V_{t0}| = 2V$

②: $V_{SD} = V_1 - V_2 = 9V > V_{SG} - |V_{t0}| = 2V$ } The PMOS is in saturation

b)



Both transistors are diode-connected ($V_G = V_D$)

↳ both transistors are in saturation

$$I_D = 2mA = \frac{1}{2}\left(1\ \frac{mA}{V^2}\right)\cdot\left(V_{SG} - |-V_{t0}|\right)^2$$

$V_{SG1} = V_{SG2} = V_{SG} = 4V$ [as in part a)]

KVL: $0 = -10V + V_{SG1} + V_3$

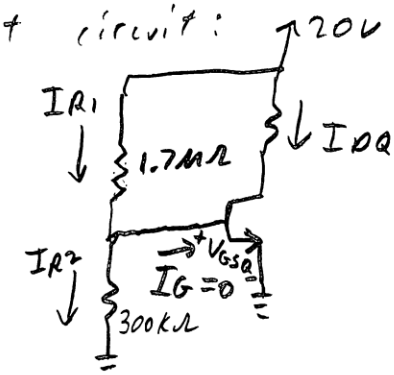
→ $V_3 = 10V - 4V = \boxed{6V = V_3}$

KVL: $0 = -10V + V_{SG1} + V_{SG2} + V_4$

↳ $V_4 = 10V - 2\cdot 4V = \boxed{2V = V_4}$

(p.2) **Problem 2**

DC equivalent circuit:



a) $I_{R1} = I_{R2}$ because $I_G = 0$ (NMOS gate current ≈ 0)

$$V_{GSQ} = 20V \cdot \frac{300k\Omega}{1.7M\Omega + 300k\Omega} = 3V$$

The AC input signal is coupled to the gate with a capacitor (assumed to be a short-circuit in this AC analysis). Applying the superposition principle, the total voltage $v_{GS}(t)$ is:

$$\boxed{v_{GS}(t) = V_{GSQ} + v_{in}(t) = 3V + \sin(2000\pi t) \cdot V}$$

b) – c)

For the saturation region, plot the following equation vs. V_{DS} for the given V_{GS} values:

① : $I_D = K \cdot (V_{GS} - V_{to})^2 = 0.5 \frac{mA}{V^2} \cdot (V_{GS} - 1V)^2$

For the triode region, plot the following equation vs. V_{DS}: ② : $I_D = K \cdot [2 \cdot (V_{GS} - V_{to}) \cdot V_{DS} - V_{DS}^2]$

The load line equation from KVL is:

$$0 = -20V + I_D \cdot 2k\Omega + V_{DS}$$

↳ Note: $I_D = 0 \rightarrow V_{DS} = 20V$

$I_D = 5mA \rightarrow V_{DS} = 10V$

Also notice from equation ① that:

$V_{GS} = 4V \rightarrow I_D = 4.5mA \rightarrow V_{DS} = 20V - 4.5mA \cdot 2k\Omega = 11V$

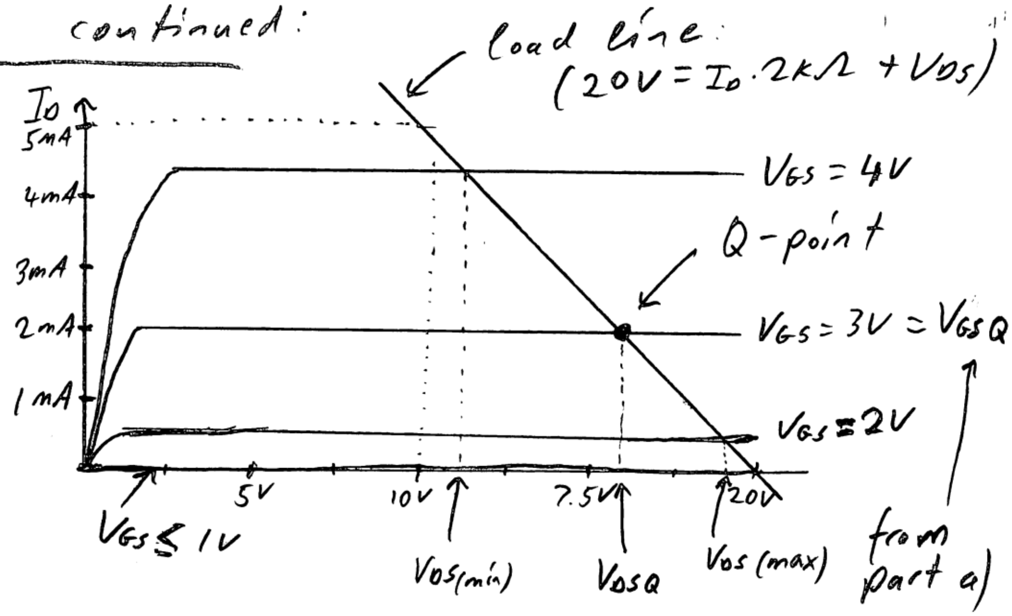
$V_{GS} = 3V \rightarrow I_D = 2mA \rightarrow V_{DS} = 20V - 2mA \cdot 2k\Omega = 16V$

$V_{GS} = 2V \rightarrow I_D = 0.5mA \rightarrow V_{DS} = 20V - 0.5mA \cdot 2k\Omega = 19V$

in cutoff: $V_{GS} = 1V \rightarrow V_{GS} > V_{to} = 1V$ is not satisfied $\rightarrow I_D = 0$

... Prob. 2 continued:

b) – c)



d) From the above plot:

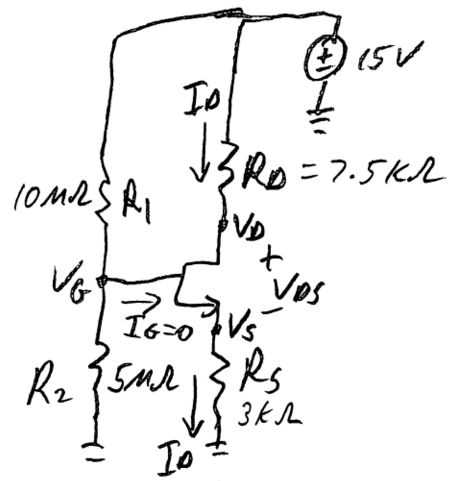
$V_{DS(min)} \approx 11V$

$V_{DSQ} \approx 16V$

$V_{DS(max)} \approx 19V$

Prob. 3

DC equivalent circuit:



a)

$$V_G = 15V \cdot \left(\frac{R_2}{R_1 + R_2}\right) = 5V$$

$$V_S = I_D \cdot R_S = I_D \cdot 3000$$

Assuming that the NMOS operates in the saturation region:

$$I_D = K \cdot (V_{GS} - V_{to})^2 = 1 \times 10^{-3} \frac{A}{V^2} \cdot \left(5V - 3000 \tfrac{V}{A} \cdot I_D - 1V\right)^2$$

↳ Solving for I_D: $I_{D_1} = 1.778 mA$, $I_{D2} = 1 mA$

with I_{D1}: $V_{GS} = 5V - 3000 \cdot I_{D1} = -0.33 V < V_{to}$
(the NMOS would not be in saturation)

with I_{D2}: $V_{GS} = 5V - 3000 \cdot I_{D2} = 2V > V_{to} = 1V$

KVL: $0 = -15V + I_{D2} \cdot R_D + V_{DS} + I_{D2} \cdot R_S$

↳ $V_{DS} = 15V - I_{D2}(7500\Omega + 3000\Omega) = 4.5V$

$V_{DS} = 4.5V > V_{GS} - V_t = 1V$

↳ The transistor is in saturation with $I_{D2} = 1mA$

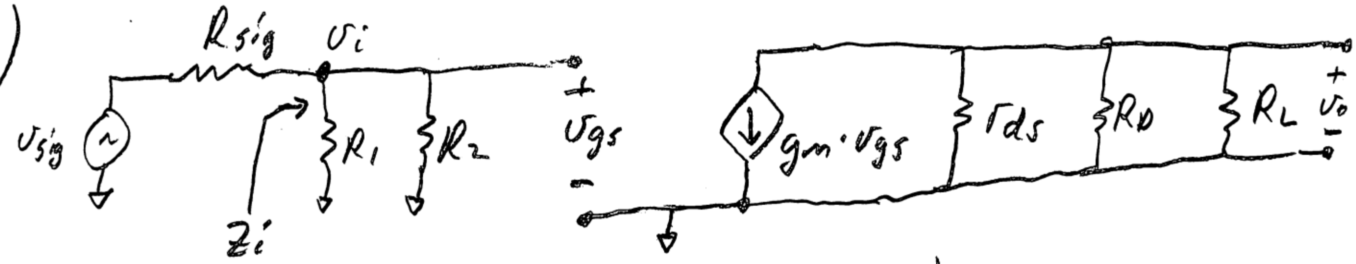
$$\boxed{I_{DQ} = I_{D2} = 1 mA}$$

$$g_m = \frac{2 \cdot I_{DQ}}{V_{GSQ} - V_{to}} = \frac{2 \cdot (1mA)}{2V - 1V} = \boxed{2 \frac{mA}{V} = g_m}$$

$$r_{ds} = \frac{V_A}{I_{DQ}} = \frac{100V}{1mA} = \boxed{100k\Omega = r_{ds}}$$

Prob. 3 cont.: small-signal equivalent circuit:

b)



$v_i = v_{gs}$, $v_o = -g_m \cdot v_{gs} \cdot (r_{ds} \| R_D \| R_L)$

$v_o = -g_m \cdot v_i \cdot (r_{ds} \| R_D \| R_L)$

$$\boxed{A_v = \frac{v_o}{v_i} = -g_m\,(r_{ds} \| R_D \| R_L)}$$

$$A_v = -2\times10^{-3} \cdot \left(\frac{1}{100000} + \frac{1}{7500} + \frac{1}{10000}\right)^{-1} = \boxed{-8.22 = A_v}$$

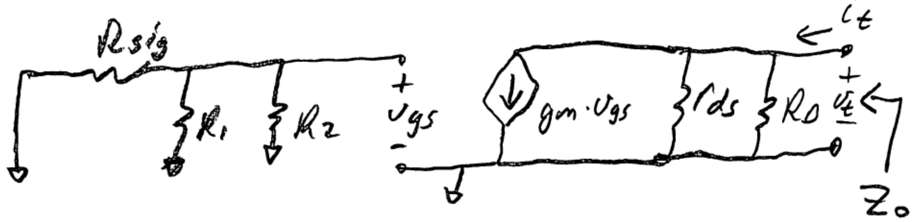
$$v_i = v_{sig} \cdot \left(\frac{R_1 \| R_2}{R_{sig} + R_1 \| R_2}\right) \rightarrow \frac{v_i}{v_{sig}} = \frac{R_1 \| R_2}{R_{sig} + R_1 \| R_2}$$

$$A_{vs} = \frac{v_o}{v_{sig}} = \frac{v_i}{v_{sig}} \cdot \frac{v_o}{v_i} = \frac{R_1 \| R_2}{R_{sig} + R_1 \| R_2} \cdot A_v = \frac{\left(\frac{1}{10\times10^6} + \frac{1}{5\times10^6}\right)^{-1}}{100\times10^3 + \left(\frac{1}{10\times10^6} + \frac{1}{5\times10^6}\right)^{-1}} \cdot (-8.22)$$

$$\boxed{A_{vs} = -7.98}$$

c) $Z_i = R_1 \| R_2 = \boxed{3.33\ M\Omega = Z_i}$

d) Equivalent circuit to demtermine Z_o:

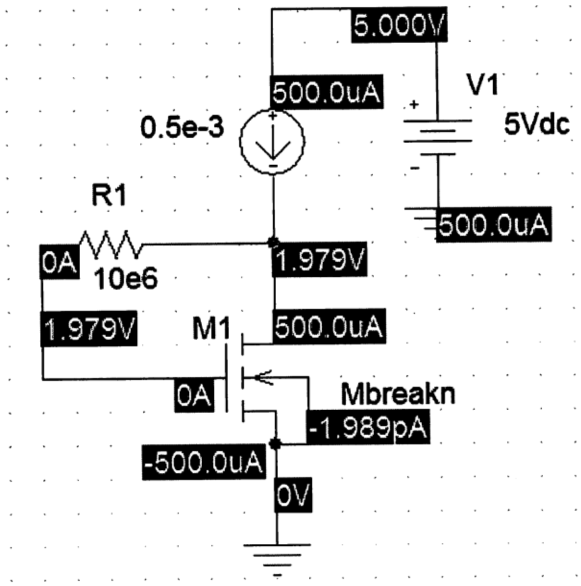


$v_{gs} = 0 \rightarrow g_m \cdot v_{gs} = 0$

$$i_t = g_m v_{gs} + \frac{v_t}{r_{ds}} + \frac{v_t}{R_D} = \frac{v_t}{r_{ds}} + \frac{v_t}{R_D} = v_t\left(\frac{1}{r_{ds}} + \frac{1}{R_D}\right)$$

$$Z_o = \frac{v_t}{i_t} = \frac{1}{\frac{1}{r_{ds}} + \frac{1}{R_D}} = \frac{1}{\frac{1}{100\times10^3} + \frac{1}{7500}} = \boxed{6977\ k\Omega = Z_o}$$

Problem 4, Part a)



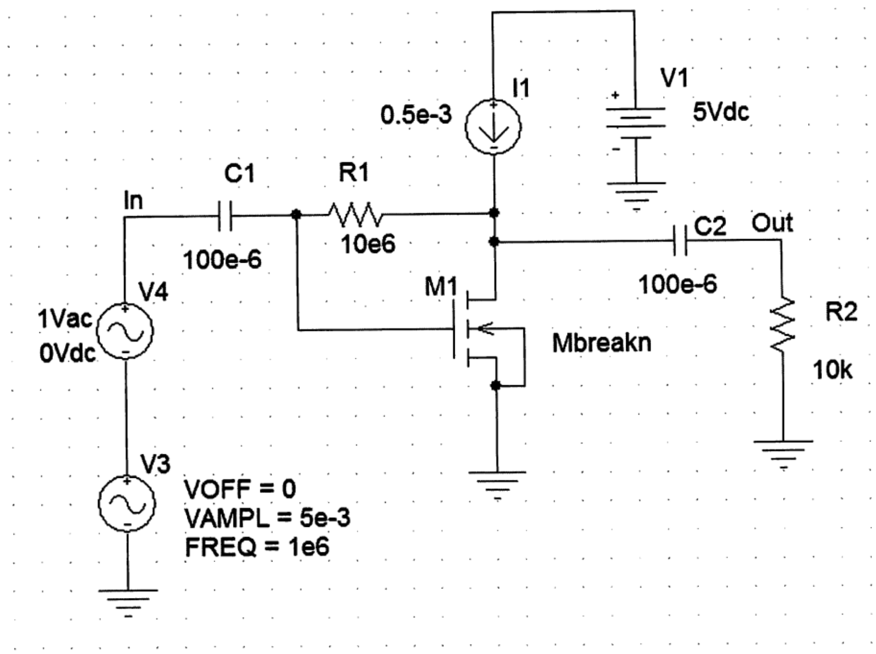
Part b)

DC operating point info for the NMOS transistor:

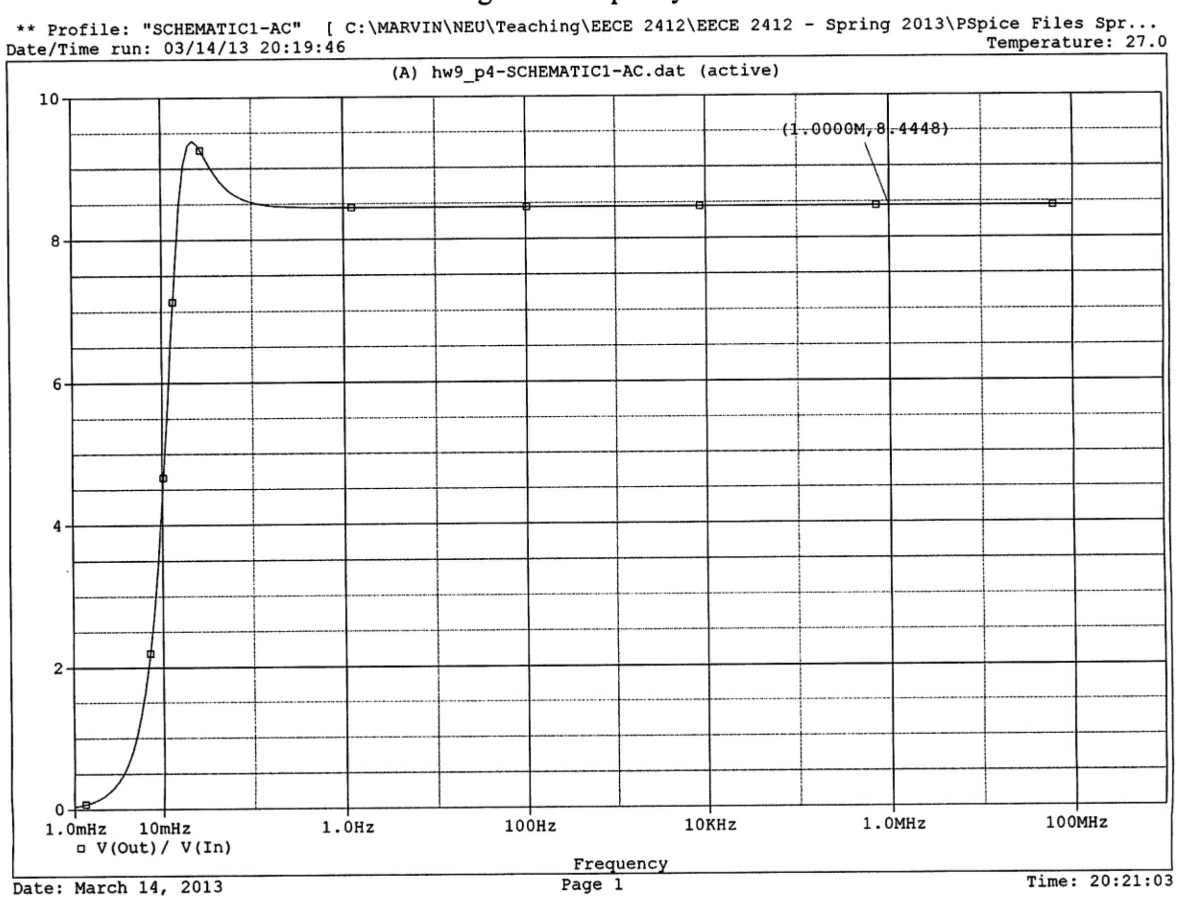
AME	M_M1
MODEL	Mbreakn
ID	5.00E-04
VGS	1.98E+00
VDS	1.98E+00
VBS	0.00E+00
VTH	9.00E-01
VDSAT	1.08E+00
Lin0/Sat1	-1.00E+00
if	-1.00E+00
ir	-1.00E+00
TAU	-1.00E+00
GM	9.27E-04
GDS	**9.62E-06**
GMB	0.00E+00
CBD	0.00E+00
CBS	0.00E+00
CGSOV	0.00E+00
CGDOV	0.00E+00
CGBOV	0.00E+00
CGS	0.00E+00
CGD	0.00E+00
CGB	0.00E+00

drain-source resistance: $r_{ds} = 1/GDS = 103.95k\Omega$

Part c)

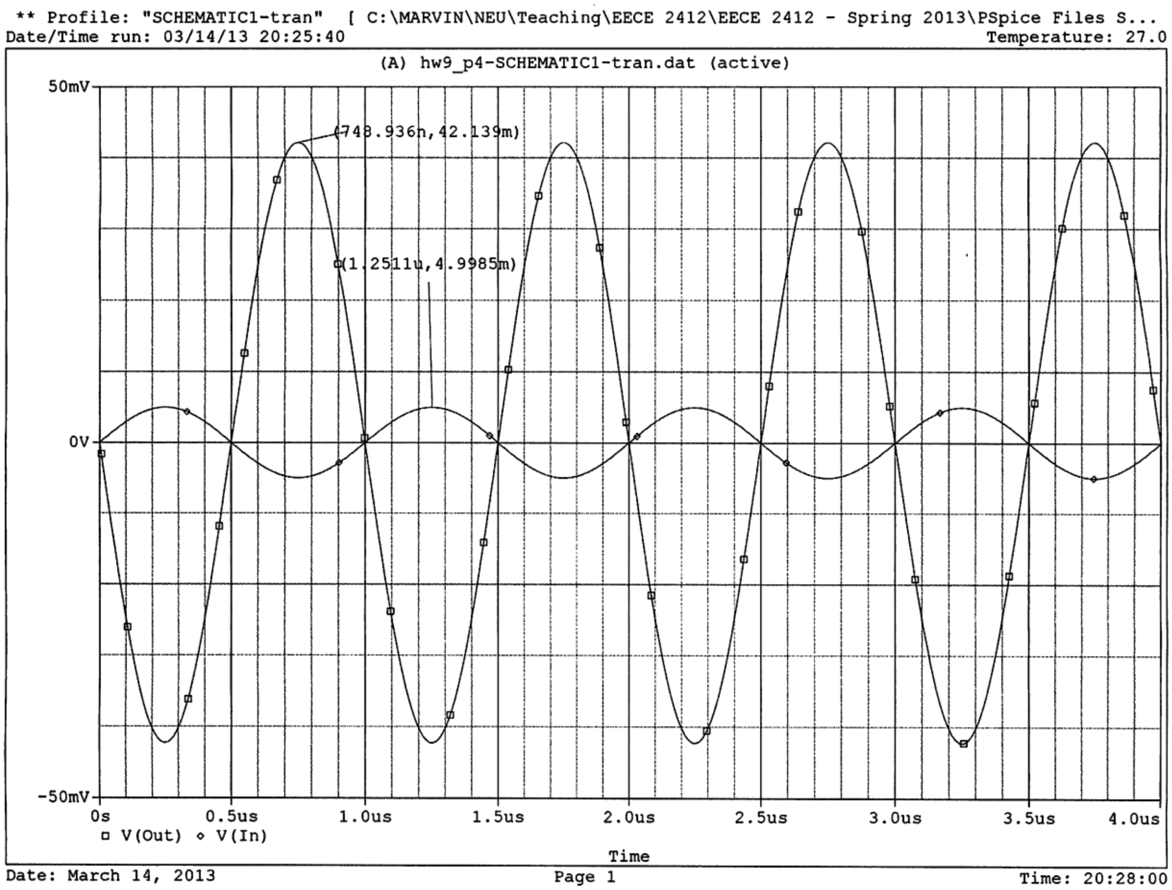


gain vs. frequency:



midband gain: $|A_v| \approx$ **8.44** (close to $|A_v| = |-8.26|$ from the hand calculation)

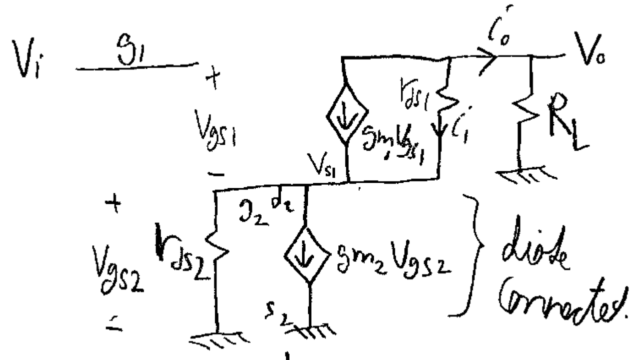
Part d)



The **voltage gain** is:

$-V_{o\text{-}peak} / V_{in\text{-}peak} = -42.14mV / 4.999mV = \mathbf{-8.43}$ (close to the hand calculation result)

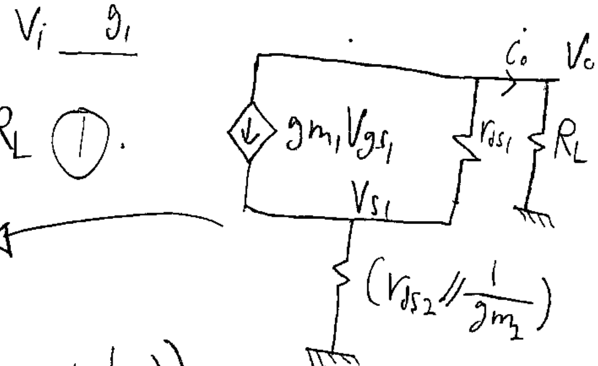
Problem 5



$$V_{in} = V_{gs1} + V_{s1}$$

$$V_o = \left(-g_{m1}V_{gs1} - \frac{V_o - V_{s1}}{r_{ds1}}\right)R_L$$

$$V_o\left(1 + \frac{R_L}{R_{ds1}}\right) = \left(\frac{V_{s1}}{r_{ds1}} - g_{m1}V_{in} + g_{m1}V_{s1}\right)R_L$$



$$V_o\left(1 + \frac{R_L}{r_{ds1}}\right) = V_{in}\left(-g_{m1}R_L\right) + V_{s1}\left(\frac{1}{r_{ds1}} + g_{m1}\right)R_L \quad (1).$$

$$V_{s1} = \left(\frac{1}{g_{m2}} /\!/ r_{ds2}\right)\left(g_{m1}V_{gs1} + \frac{V_o - V_{s1}}{r_{ds1}}\right) \longleftarrow$$

$$V_{s1} = \left(\frac{1}{g_{m2}} /\!/ r_{ds2}\right)\left(g_{m1}V_{in} + \frac{V_o}{r_{ds1}} - V_{s1}\left(g_{m1} + \frac{1}{r_{ds1}}\right)\right) \longrightarrow \text{Note that } V_{gs1} = V_{in} - V_{s1}$$

$$V_{s1}\left(1 + \left(\frac{1}{g_{m2}} /\!/ r_{ds2}\right)\left(g_{m1} + \frac{1}{r_{ds1}}\right)\right) = \left(\frac{1}{g_{m2}} /\!/ r_{ds2}\right)\left(g_{m1}V_{in} + \frac{V_o}{r_{ds1}}\right)$$

$$V_{s1} = \frac{\left(\frac{1}{g_{m2}} /\!/ r_{ds2}\right)\left(g_{m1}V_{in} + \frac{V_o}{r_{ds1}}\right)}{1 + \left(\frac{1}{g_{m2}} /\!/ r_{ds2}\right)\left(g_{m1} + \frac{1}{r_{ds1}}\right)}$$

Substitute in (1)

$$V_o\left(1 + \frac{R_L}{r_{ds1}}\right) = V_{in}\left(-g_{m1}R_L\right) + \frac{R_L\overbrace{\left(\frac{1}{r_{ds1}} + g_{m1}\right)\left(\frac{1}{g_{m2}} /\!/ r_{ds2}\right)}^{X}\left(g_{m1}V_{in} + \frac{V_o}{r_{ds1}}\right)}{1 + \left(\frac{1}{g_{m2}} /\!/ r_{ds2}\right)\left(g_{m1} + \frac{1}{r_{ds1}}\right)}$$

$$V_o\left(1 + \frac{R_L}{r_{ds1}} - \frac{R_L}{r_{ds1}}\frac{X}{1+X}\right) = V_{in}\left(-g_{m1}R_L + g_{m1}R_L\frac{X}{X+1}\right)$$

$$V_o\left(1 + \frac{R_L}{r_{ds1}(1+X)}\right) = V_{in}\left(\frac{-g_{m1}R_L}{1+X}\right)$$

$$V_o\left(1 + X + \frac{R_L}{r_{ds1}}\right) = -V_{in}g_{m1}R_L$$

$$\frac{V_o}{V_{in}} = \frac{-g_{m1}R_L}{1 + X + \frac{R_L}{r_{ds1}}} = \boxed{\frac{-g_{m1}r_{ds1}R_L\left(1 + g_{m2}r_{ds2}\right)}{r_{ds1} + r_{ds2} + R_L + g_{m1}r_{ds1}r_{ds2} + g_{m2}r_{ds2}\left(r_{ds1} + R_L\right)}}$$