1

EECE 2412 – HW 6 Solutions

Prob. 1

For the circuit in Fig. 4.10 on page 224, the equation for the load line at the input is:

$$V_{BB} + v_{in}(t) = R_B \cdot i_B(t) + v_{BE}(t)$$

Substituting the given values:

$$0.8V + 0.2 \cdot \sin(2000\pi t) V = 40 \times 10^3 \cdot i_B(t) + v_{BE}(t)$$

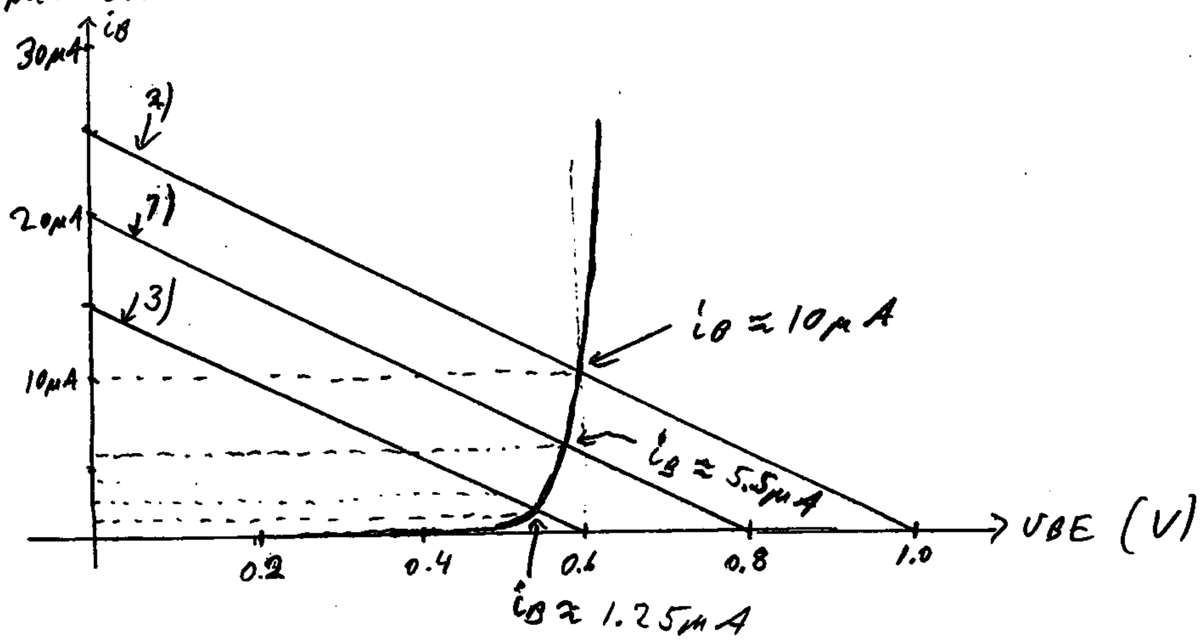
Observing the relevant cases:

1) $v_{in}(t) = 0$: $i_B = 0 \rightarrow v_{BE} = 0.8V$
   $v_{BE} = 0$ when $i_B = \frac{0.8V}{40000} = 20\mu A$

2) $v_{in}(t) = 0.2V$: $i_B = 0 \rightarrow v_{BE} = 1V$
   $v_{BE} = 0$ when $i_B = \frac{1V}{40000} = 25\mu A$

3) $v_{in}(t) = -0.2V$: $i_B = 0 \rightarrow v_B = 0.6V$
   $v_{BE} = 0$ when $i_B = \frac{0.6V}{40000} = 15\mu A$

Plotting the three load line cases on top of the input characteristic curve:

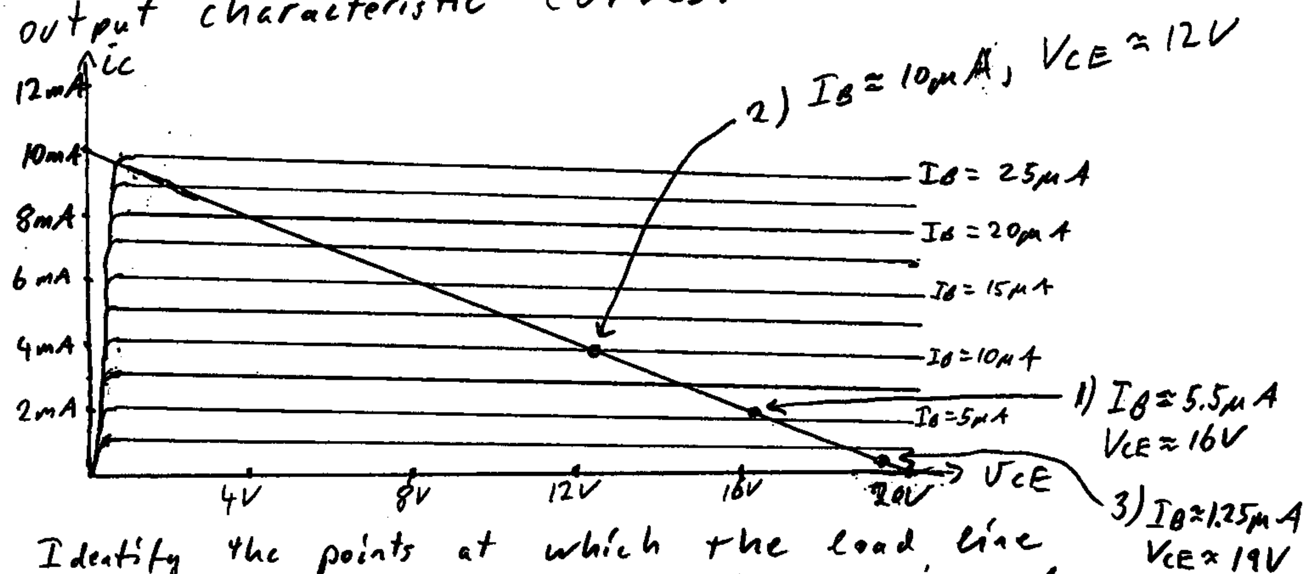


... Problem 1 continued:

Output load line equation: $V_{CC} = R_C \cdot i_C + v_{CE}$

Substituting the given values: $20 = 2000 \cdot i_C + v_{CE}$

Plotting this equation on top of the output characteristic curves:



Identify the points at which the load line intersects with the curves for the i_B values that were found on the previous page:

The min. and max. instantaneous operating points are:

$\boxed{V_{CE} \approx 12\,V,\ I_B \approx 10\mu A}$, $I_C \approx 4mA$

$\boxed{V_{CE} \approx 19V,\ I_B \approx 1.25\mu A}$, $I_C \approx 0.5mA$

The output signal is $\boxed{\underline{\text{not}}\ \text{clipped.}}$

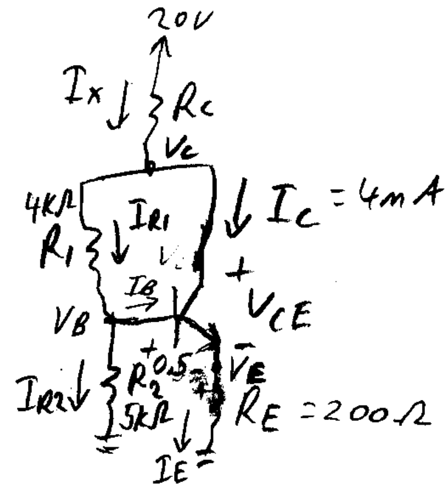
(Clipping would occur if $V_{CE} < 0.2\,V$ ← BJT would enter the saturation region

or $V_{CE} > 20\,V$ for this case

↳ not possible due to the supply voltage limit

3

Prob. 2 $I_c = 4mA$ $\beta = 80$



a)

$$I_E = \frac{1+\beta}{\beta} \cdot I_c = \frac{81}{80} \cdot 4mA = 4.05mA$$

$$V_E = I_E \cdot R_E = (4.05 \times 10^{-3}) \cdot 200 = 0.81V$$

$$V_B = V_E + V_{BE} = 0.81 + 0.5 = 1.31V$$

$$I_{R2} = \frac{V_B}{R_2} = \frac{1.31}{5000} = 262\mu A$$

$$I_B = \frac{I_c}{\beta} = \frac{4mA}{80} = 50\mu A$$

$$I_{R1} = I_{R2} + I_B = 312\mu A$$

$$V_c = V_B + I_{R1} \cdot R_1 = 1.31 + (312 \times 10^{-6}) \cdot 4000 = 2.558V$$

$$I_x = I_c + I_{R1} = 4mA + 312\mu A = 4.312mA$$

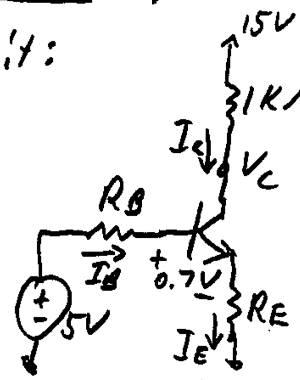
$$R_c = \frac{20V - V_c}{I_x} = \frac{20 - 2.558}{4.312 \times 10^{-3}} = \boxed{4.045 K\Omega = R_c}$$

b) $V_{CE} = V_c - V_E = 2.558V - 0.81V$

$$\boxed{V_{CE} = 1.748V}$$

Problem 3

a) DC circuit:



Assuming active mode (by design):

① : $I_E = \frac{I_c}{\alpha} = \left(\frac{\beta+1}{\beta}\right) \cdot I_c$

② : $I_B = \frac{I_c}{\beta}$

FYI:
for DC analysis:
Capacitor → open-circuit
Inductor → short-circuit
AC volt. source → short-circuit
AC current source → open-circuit

KVL: $0 = -5V + I_B \cdot R_B + 0.7V + I_E \cdot R_E$

Sub. ① and ②: $4.3V = \left(\frac{I_c}{\beta}\right) \cdot R_B + \left(\frac{\beta+1}{\beta}\right) \cdot I_c \cdot R_E$

case 1: Sub. $\beta = 100$, $I_c = 4mA$ into the above equ.:

$$4.3V = \left(\frac{4mA}{100}\right) R_B + \left(\frac{100+1}{100}\right) \cdot (4mA) \cdot R_E$$

③ ; $4.3V = (40 \times 10^{-6}) \cdot R_B + (4.04 \times 10^{-3}) \cdot R_E$

case 2: Sub. $\beta = 300$, $I_c = 5mA$ into the equ.: $(5.01667 \times 10^{-3}) \cdot R_E$

④ : $4.3V = (16.667 \times 10^{-6}) \cdot R_B + (5.01667 \times 10^{-3}) \cdot R_E$

Solving ③ and ④ simultaneously:

$\boxed{R_B = 32.25 k\Omega}$ $\boxed{R_E = 750 \Omega}$

b) equ. 4.42 in the book:

$g_m = \frac{I_{CQ}}{V_T} \longrightarrow g_{m(max)} = \frac{5mA}{26mV} = 0.192 \frac{A}{V} = 192\ mS$ (milli-Siemens)

V_T ↳ ≈ 26mV at room temp. $\left(V_T = \frac{K \cdot T}{q}\right)$

$g_{m(min)} = \frac{4mA}{26mV} = 153.8\ mS$ $\boxed{153.8mS < g_m < 192mS}$

equ. 4.37 in the book: $r_\pi = \frac{\beta \cdot V_T}{I_{CQ}}$

$r_{\pi_1} = \frac{(300) \cdot (26mV)}{5mA} = 1.56k\Omega$

$r_{\pi_2} = \frac{(100) \cdot (26mV)}{4mA} = 650\Omega$

$\boxed{650\Omega < r_\pi < 1560\Omega}$

Lecture slides: $r_{ce} = \frac{V_A}{I_{CQ}}$

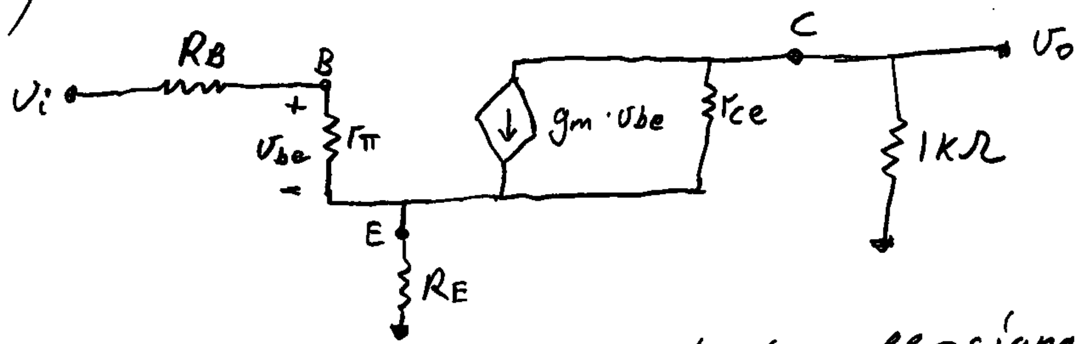
$r_{ce(max)} = \frac{100V}{4mA} = 25k\Omega$

$r_{ce(min.)} = \frac{100V}{5mA} = 20k\Omega$

$\boxed{20k\Omega < r_{ce} < 25k\Omega}$

... Problem 3 continued:

c) small-signal equivalent circuit:



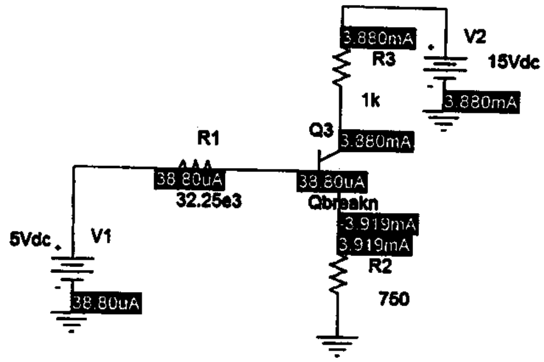
(FYI: In the AC analysis (small-signal equivalent circuit), replace:
DC voltage sources $\xrightarrow{\text{with}}$ short-circuits
DC current sources → open-circuits)

d) PSpice: see next page

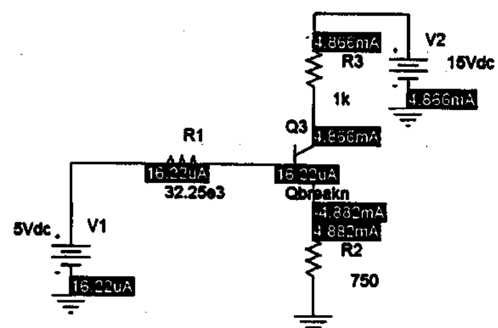
6 *Prob. 3 cont.:*

d)

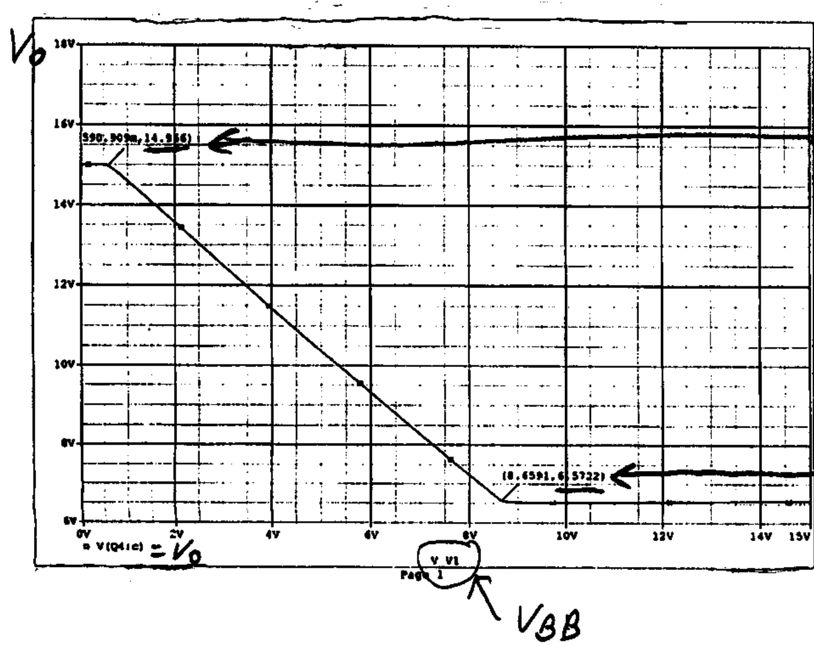
Verification with β = 100 (I_C is close to 4mA):



Verification with β = 300 (I_C is close to 5mA):



DC transfer characteristic curve at the collector (V_C vs. V_{BB}) with the QN222 BJT model:



$V_{o(max)}$

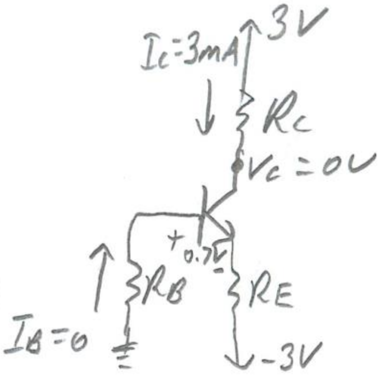
$V_{o(max)} - V_{o(min)}$
$= 14.97 - 6.57V$
$= 9.4V$

$\boxed{V_{o\,peak\text{-}peak(max)} = 9.4V}$

$V_{o(min)}$

when:
$V_{BB} = \dfrac{(0.59V + 8.66V)}{2}$
$= 4.625V$

Problem 4



a) With $\beta = 0 \rightarrow I_B = 0$

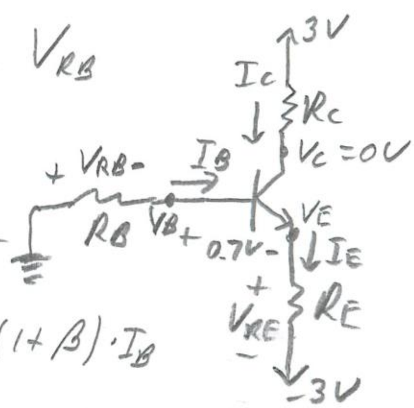
$\hookrightarrow I_E = I_c = 3mA$

KVL: $0 = -3V + I_c \cdot R_c + V_c$

$V_c = 0V \hookrightarrow R_c = \frac{3V}{I_c} = \frac{3V}{3mA} = \boxed{1k\Omega = R_c}$

KVL: $0 = \cancel{I_B}^{\,0} \cdot R_B + 0.7V + I_E \cdot R_E - 3V$

$\hookrightarrow R_E = \frac{(3V - 0.7V)}{3 \times 10^{-3} A} = \boxed{766.67\,\Omega = R_E}$



b) $\beta = 90$, constraint: $\frac{V_{RE}}{10} = V_{RB}$

$\frac{I_E \cdot R_E}{10} = I_B \cdot R_B \rightarrow \frac{I_E R_E}{10} = \frac{I_E \cdot R_B}{\beta + 1}$

using $I_E = (1+\beta) \cdot I_B$

$\rightarrow$ ①: $R_B = \frac{\beta+1}{10} \cdot R_E$

KVL: $0 = V_{RB} + 0.7V + V_{RE} - 3V$

$2.3V = \frac{V_{RE}}{10} + V_{RE} \longrightarrow V_{RE} = 2.0909V$

$I_E = \frac{I_c}{\alpha} = \frac{\beta+1}{\beta} \cdot I_c = \frac{90+1}{90} \cdot 3mA = 3.0333\,mA$

$R_E = \frac{V_{RE}}{I_E} = \frac{2.0909V}{3.0333mA} = \boxed{689.3\,\Omega = R_E}$

Substituting the value of RE into equation ①:

$R_B = \frac{90+1}{10} \cdot 689.3\,\Omega = \boxed{6272\,\Omega = R_B}$

$R_c = \frac{3V - V_c}{I_c} = \frac{3V - 0V}{3mA} = \boxed{1k\Omega = R_c}$

8 ... Problem 4 cont.:

c) Standard 5%-resistor values for the results in part b):

$\boxed{R_C = 1k\Omega}$ $\boxed{R_E = 680\,\Omega}$ $\boxed{R_B = 6.2\,k\Omega}$

d) With $\beta = \infty$: $I_B = 0$, $I_C = I_E$

KVL: $0 = \cancel{I_B}^{0} \cdot R_B + V_B \rightarrow \boxed{V_B = 0}$

KVL: $0 = \cancel{I_B}^{0} \cdot R_B + 0.7V + V_E \rightarrow \boxed{V_E = -0.7V}$

KVL: $0 = \cancel{I_B}^{0} \cdot R_B + 0.7V + I_E \cdot R_E - 3V$

$\hookrightarrow I_E = \frac{3V - 0.7V}{R_E} = \frac{2.3V}{680\Omega} = 3.382\,mA$

$\boxed{I_C = I_E = 3.382\,mA}$

With $\beta = 90$:

KVL: $0 = I_B \cdot R_B + 0.7V + I_E \cdot R_E - 3V$

$0 = \frac{I_E}{\beta+1} \cdot R_B + I_E \cdot R_E - 2.3V$

$2.3V = \frac{I_E}{91} \cdot 6200 + I_E \cdot 680 \rightarrow I_E = 3.074\,mA$

$I_C = \alpha \cdot I_E = \frac{\beta}{\beta+1} \cdot I_E = \frac{90}{91} \cdot I_E = \boxed{3.041\,mA = I_C}$

KVL: $0 = I_B \cdot R_B + V_B \rightarrow V_B = -I_B \cdot R_B = -\frac{I_C}{\beta} \cdot R_B = -\frac{3.041mA}{90} \cdot 6200\Omega$

$\boxed{V_B = -0.209V}$

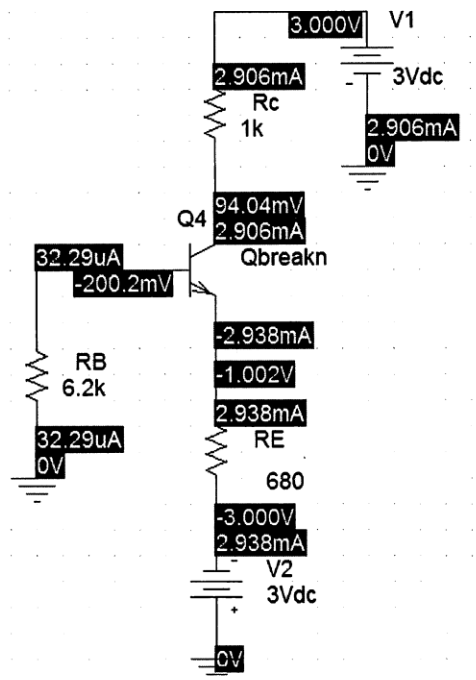
$V_E = V_B - 0.7V = \boxed{-0.909V = V_E}$

$V_C = 3V - I_C \cdot R_C = 3V - 3.041mA \cdot 1k\Omega = \boxed{-0.0405V = V_C}$

9

Problem 4, Part e)

with β=90:



with β=400: