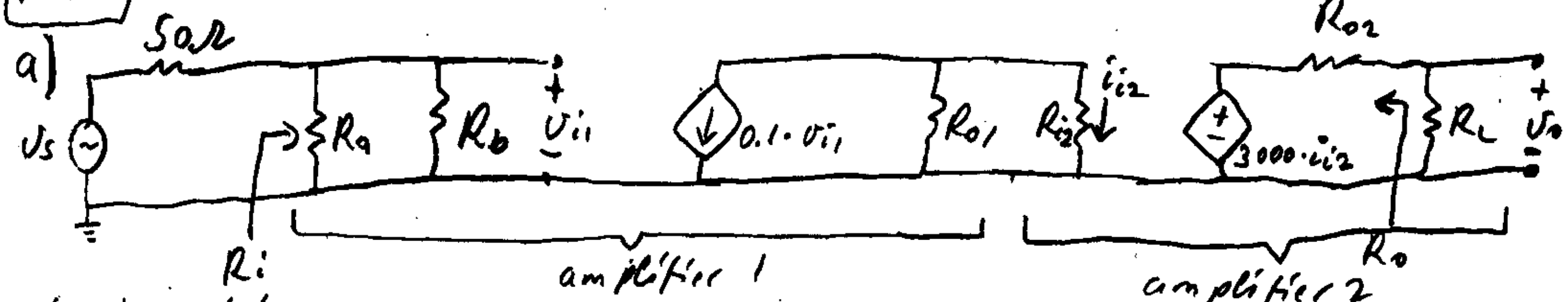
EECE 2412 : HW1 Solutions

Fall 2016

Prob. 1

a)



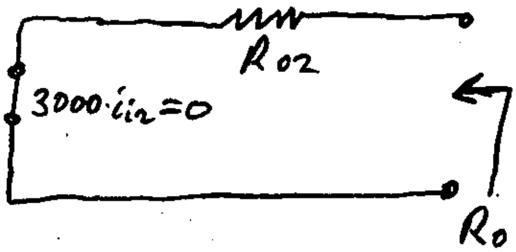
input resistance:

$$R_i = R_a \| R_b = \frac{1}{\frac{1}{1000} + \frac{1}{2000}} = \boxed{666.67\,\Omega = R_i}$$

output resistance (with R_L disconnected):

↳ set $v_s = 0 \rightarrow v_{i1} = 0 \rightarrow 0.1 \cdot v_{i1} = 0 \rightarrow 3000 \cdot i_{i2} = 0$

↳ output stage with $v_s = 0$:



$$\boxed{R_o = R_{o2} = 200\,\Omega}$$

voltage gain:

$$\frac{v_{i1}}{v_s} = \frac{R_i}{50\Omega + R_i} = \frac{R_a \| R_b}{50\Omega + (R_a \| R_b)} = \frac{666.67\,\Omega}{50\Omega + 666.67\,\Omega} = 0.9302$$

$$i_{i2} = -0.1 \cdot v_{i1} \cdot \left(\frac{R_{o1}}{R_{o1} + R_{i2}}\right)$$

← note: negative sign because i_{i2} flows into the opposite direction compared to $(0.1 \cdot v_{i1})$ in the divid

$$↳ \frac{i_{i2}}{v_{i1}} = -0.1 \cdot \left(\frac{R_{o1}}{R_{o1} + R_{i2}}\right) = -0.1 \cdot \left(\frac{3000}{3000 + 100}\right) = -96.77 \times 10^{-3}\ \frac{A}{V}$$

$$v_o = 3000 \cdot i_{i2} \cdot \left(\frac{R_L}{R_L + R_{o2}}\right)$$

$$↳ \frac{v_o}{i_{i2}} = 3000\left(\frac{4000}{4000 + 200}\right) = 2857\ \frac{V}{A}$$

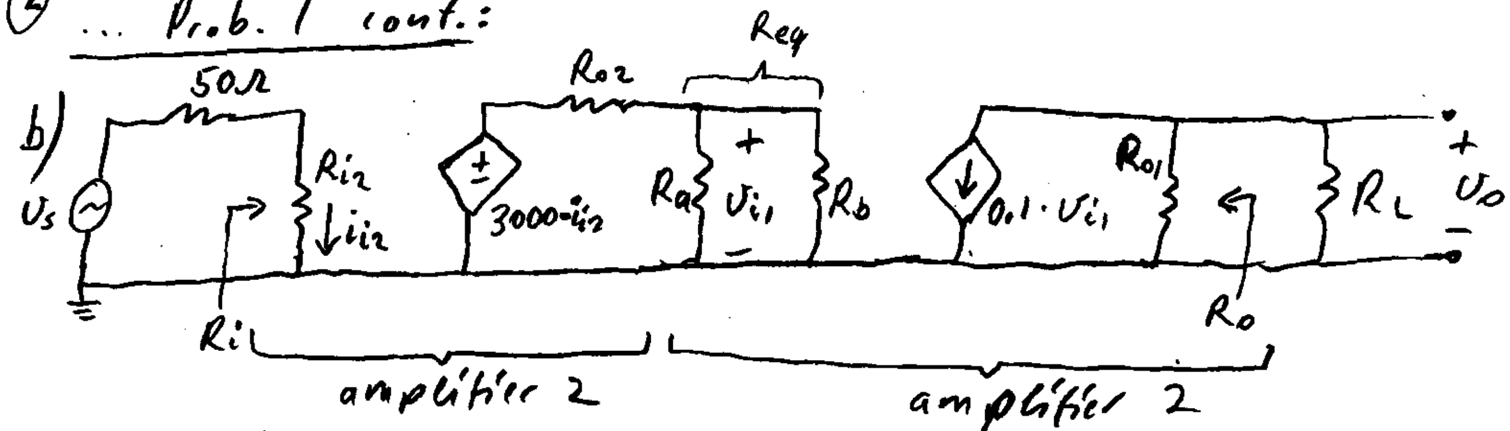
$$A_{vs} = \frac{v_o}{v_s} = \frac{v_{i1}}{v_s} \times \frac{i_{i2}}{v_{i1}} \times \frac{v_o}{i_{i2}} = (0.9302) \times \left(-96.77 \times 10^{-3}\,\frac{A}{V}\right) \times \left(2857\,\frac{V}{A}\right)$$

$$\boxed{A_{vs} = -257.2}$$

$$A_{vs}|_{dB} = 20 \log(|A_{vs}|) = \boxed{48.2\ dB = A_{vs}|_{dB}}$$

(2) ... Prob. 1 cont.:

b)



input resistance:

$$\boxed{R_i = R_{i2} = 100\,\Omega}$$

output resistance:

$$\boxed{R_o = R_{o1} = 3\,k\Omega}$$

voltage gain:

$$i_{i2} = \frac{v_s}{50\Omega + R_{i2}} \longrightarrow \frac{i_{i2}}{v_s} = \frac{1}{50\Omega + 100\Omega} = 6.6667\times 10^{-3}\ \frac{A}{V}$$

$$R_{eq} = R_a // R_b = 1k\Omega // 2k\Omega = 666.67\,\Omega$$

$$v_{i1} = 3000 \cdot i_{i2} \cdot \left(\frac{R_{eq}}{R_{eq} + R_{o2}}\right)$$

$$\hookrightarrow \frac{v_{i1}}{i_{i2}} = 3000 \cdot \left(\frac{666.67}{666.67 + 200}\right) = 2308\ \frac{V}{A}$$

$$v_o = -0.1 \cdot v_{i1} \times (R_{o1} // R_L)$$

$$\hookrightarrow \frac{v_o}{v_{i1}} = -0.1 \times \left(\frac{1}{\frac{1}{3000} + \frac{1}{4000}}\right) = -171.4$$

$$A_{vs} = \frac{v_o}{v_s} = \frac{i_{i2}}{v_s} \times \frac{v_{i1}}{i_{i2}} \times \frac{v_o}{v_{i1}} = \left(6.6667\times 10^{-3}\frac{A}{V}\right)\times\left(2308\frac{V}{A}\right)\times(-171.4)$$

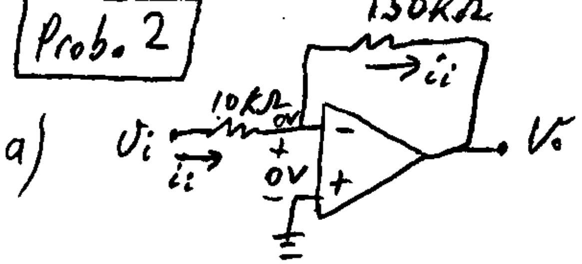
$$\boxed{A_{vs} = -2637}$$

$$A_{vs}|_{dB} = 20\log(|A_{vs}|) = \boxed{68.4\ dB = A_{vs}|_{dB}}$$

→ The second configuration results in higher voltage gain from the source to the load.

③ **Prob. 2**

a)

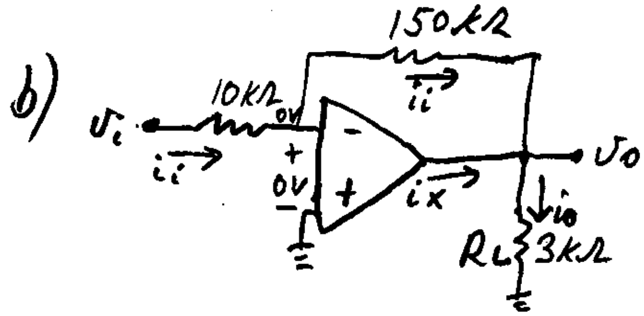


$$i_i = \frac{v_i - 0V}{10k\Omega} = \frac{0V - v_o}{150k\Omega}$$

$$\hookrightarrow \boxed{\frac{v_o}{v_i} = -\frac{150k\Omega}{10k\Omega} = -15}$$

$$\boxed{R_i = \frac{v_i}{i_i} = 10k\Omega}$$

b)



The extra 3kΩ resistor at the output does not affect the KCL at the inverting terminal.

$$\hookrightarrow i_i = \frac{v_i - 0V}{10k\Omega} = \frac{0V - v_o}{150k\Omega}$$

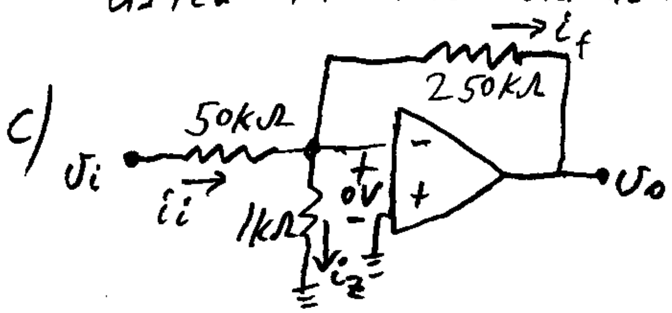
$$\hookrightarrow \boxed{\frac{v_o}{v_i} = -15}$$

$$\boxed{R_i = \frac{v_i}{i_i} = 10k\Omega}$$

Comments:

* An ideal op-amp supplies/sinks any value of i_x such that $i_o = i_x + i_i$ and $\frac{v_o}{v_i} = -15$ are satisfied
* Practical op-amps have output current limitations listed in the datasheets → in reality: A small value of R_L reduces the gain

c)



$i_z = \frac{0V}{1k\Omega} = 0 \rightarrow i_z \cdot 1k\Omega = 0 \rightarrow$ no voltage drop across the 1kΩ resistor

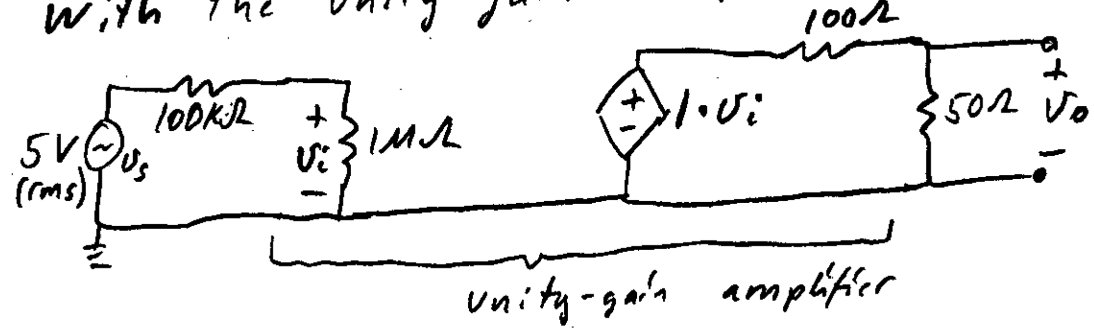
$$i_i = \cancel{i_z} + i_f = i_f$$

$$\frac{v_i - 0V}{50k\Omega} = \frac{0V - v_o}{250k\Omega} \longrightarrow \boxed{\frac{v_o}{v_i} = -5}$$

$$\boxed{R_i = \frac{v_i}{i_i} = 50k\Omega}$$

④ **Prob. 3**

With the unity-gain amplifier:



$$\frac{v_i}{v_s} = \frac{1M\Omega}{1M\Omega + 100K\Omega} = 0.90909$$

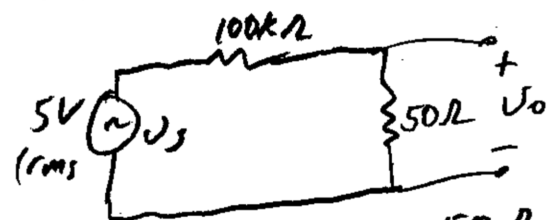
$$\frac{v_o}{v_i} = \frac{50}{50+100} = 0.3333$$

$$\frac{v_o}{v_s} = \frac{v_i}{v_s} \times \frac{v_o}{v_i} = (0.90909) \times (0.3333) = 0.30303$$

$$v_o = 0.30303 \times 5V = \boxed{1.5151 \; V_{rms} = v_o}$$

$$P_o = \frac{v_{o(rms)}^2}{R_L} = \frac{(1.5151)^2}{50} = \boxed{45.91 \; mW = P_o}$$ with amp.

without amplifier:



$$v_o = 5V_{rms} \cdot \frac{50\Omega}{50 + 100K\Omega} = \boxed{2.499 \times 10^{-3} \; V_{rms} = V_o}$$

$$P_o = \frac{v_{o(rms)}^2}{R_L} = \frac{(2.499 \times 10^{-3})^2}{50} = \boxed{124.88 \; nW = P_o}$$

The unity-gain amplifier helps to deliver a significantly higher output power to the load.

(5)

Problem 4

$$P_{in} = \frac{(V_{in\_RMS})^2}{R_{in}} = \frac{(0.1V)^2}{100\times10^3\Omega} = 100\times10^{-9} W = 0.1\mu W$$

$$P_{supply} = V_{supply} \cdot I_{supply} = 15V \cdot 2A = 30W$$

$$P_{out} = \frac{(V_{o\_RMS})^2}{R_L} = \frac{(10V)^2}{8} = 12.5W$$

dissipated power: $P_d = P_{in} + P_{supply} - P_{out} = 17.5000001\,W$

$\hookrightarrow \boxed{P_d \approx 17.5W}$

efficiency: $\eta = \frac{P_{out}}{P_{supply}} \times 100\%$

$$\eta = \frac{12.5W}{30W} \times 100\% = \boxed{41.67\% = \eta}$$

Problem 5

a) Frequency-dependent impedance of the capacitor: $Z_C = \frac{1}{j\omega C}$

$$v_i(j\omega) = i_s(j\omega)\cdot(Z_C \| R_i) = \hat{i}_s(j\omega) \cdot \frac{1}{\frac{1}{Z_C} + \frac{1}{R_i}} = i_s(j\omega) \cdot \frac{1}{j\omega C + \frac{1}{R_i}}$$

(1): $v_i(j\omega) = i_s(j\omega) \cdot \frac{R_i}{1 + j\omega C R_i}$

(2): $v_o(j\omega) = 100 \cdot v_i(j\omega) \cdot \left(\frac{R_L}{R_o + R_L}\right)$

substituting (1) into (2):

$$v_o(j\omega) = 100 \cdot \hat{i}_s(j\omega) \cdot \left(\frac{R_i}{1 + j\omega C R_i}\right) \cdot \left(\frac{R_L}{R_o + R_L}\right)$$

$$T(j\omega) = \frac{v_o(j\omega)}{i_s(j\omega)} = \frac{100\cdot(1000)\cdot(500)}{[1 + j\omega\cdot(10\times10^{-6})\cdot(1000)]\cdot(20+500)}$$

$$\boxed{T(j\omega) = \frac{9.615\times10^4}{1 + j\omega\cdot(0.01)} = \frac{9.615\times10^4}{1 + j\frac{\omega}{100}}}$$

Note: The 3dB frequency of this single-pole amplifier is at $\omega_{3dB} = 100$ rad/s.

... Prob. 5 continued:

b) $\omega_{in} = 10 \text{ rad/s} \quad \left(f_{in} = \frac{10}{2\pi} = 1.59 \text{ Hz}\right)$ $\quad \omega = \omega_{in} = 10$

$\hookrightarrow i_s(t) = 2\,mA \cdot \sin(10 \cdot t) \quad \Leftrightarrow \quad \hat{i}_s(j\cdot 10) = (2 \angle 0°)\, mA$

Tranfer function at $\omega = \omega_{in} = 10$:

$$T(j\cdot 10) = \frac{9.615\times 10^4}{1 + j\cdot\left(\frac{10}{100}\right)} = (9.568\times 10^4 \angle -5.71°)$$

$$\hookrightarrow V_o(j\cdot 10) = T(j\cdot 10)\cdot \hat{i}_s(j\cdot 10) = (9.568\times 10^4 \angle -5.71°)\cdot(2\times 10^{-3}\angle 0°)$$

$$V_o(j\cdot 10) = 191.4 \angle -5.71° \; [V]$$

$$\hookrightarrow \boxed{V_o(t) = 191.4\,V \times \sin(10t - 5.71°)}$$

c) $\omega_{in} = 1000 \text{ rad/s} \rightarrow \hat{i}_s(t) = 2\,mA \cdot \sin(1000t)$

$$T(j1000) = \frac{9.615\times 10^4}{1 + j\left(\frac{1000}{100}\right)} = (9.568\times 10^3 \angle -84.29°)$$

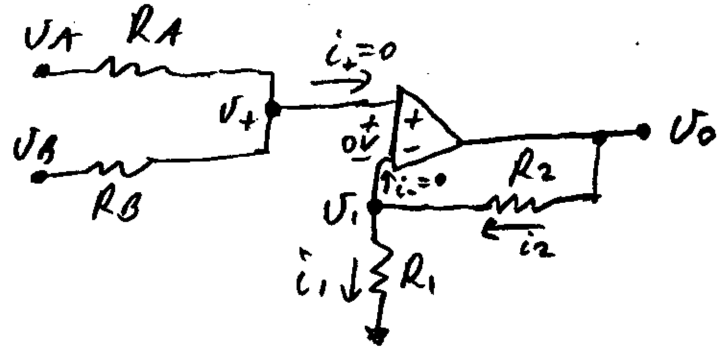
$$V_o(j\cdot 1000) = T(j1000)\cdot \hat{i}_s(j1000)$$, where: $\hat{i}_s(j1000) = 2\angle 0°\, mA \updownarrow 2\,mA\cdot\sin(1000t)$

$$V_o(j1000) = (9.568\times 10^3 \angle -84.29°)\cdot(2\angle 0° \times 10^{-3})$$

$$V_o(j1000) = 19.14 \angle -84.29° \; [V]$$

$$\hookrightarrow \boxed{V_o(t) = 19.14\,V \times \sin(1000t - 84.29°)}$$

(7) **Prob. 6**



$v_+ - 0V = v_1 \rightarrow v_+ = v_1$

$i_1 = \frac{v_1}{R_1}$

$i_2 = i_1 + \cancel{i_-} = i_1 = \frac{v_1}{R_1}$

KCL at the non-inverting terminal:

$0 = \frac{v_A - v_+}{R_A} + \frac{v_B - v_+}{R_B} \rightarrow 0 = \frac{v_A - v_1}{R_A} + \frac{v_B - v_1}{R_B}$

$\rightarrow \frac{v_A}{R_A} + \frac{v_B}{R_B} = \frac{v_1}{R_A} + \frac{v_1}{R_B} \rightarrow v_1 \cdot \left(\frac{R_A + R_B}{R_A \cdot R_B}\right) = \frac{v_A}{R_A} + \frac{v_B}{R_B}$

(1): $v_1 = \frac{R_B \cdot v_A + R_A \cdot v_B}{R_A + R_B}$

KVL at the output:

$0 = -v_o + i_2 R_2 + v_1 \xrightarrow{\text{sub. } i_1 = i_2} v_o = i_1 R_2 + v_1$

(2): $v_o \underset{i_1 = \frac{v_1}{R_1}}{=} \frac{R_2}{R_1} \cdot v_1 + v_1 = v_1 \left(1 + \frac{R_2}{R_1}\right)$

Sub. (1) into (2):

$$\boxed{v_o = \left(1 + \frac{R_2}{R_1}\right) \times \left(\frac{R_B \cdot v_A + R_A \cdot v_B}{R_A + R_B}\right) = \left(\frac{R_1 + R_2}{R_1}\right) \times \left(\frac{R_B \cdot v_A + R_A \cdot v_B}{R_A + R_B}\right)}$$