PROBLEM A

Observations: \( I_{D1} = I_{D2} \), \( V_{DS1} + V_{DS2} = V_{DD} \)

\( V_{DS1} = V_{in} \), \( V_{DS2} = 0 \), \( V_{DS1} = V_{o} \)

Based on the characteristics, for \( V_{in} \leq 2V \), where \( V_{hi} = 2V \), Q1 is cutoff so \( I_{D1} = 0 \)

Thus, \( I_{D2} = 0 \). On the \( Q2 \) characteristics, for \( V_{DS2} = 0 \),
the only place \( I_{D2} = 0 \) is at the origin. Then \( V_{DS2} = 0 \)
and \( V_{DS1} = V_{DD} \). As \( V_{in} \) increases slightly above 2V,
Q1 begins to conduct slightly but is still less than
800\( \mu \)A. Thus, Q2 operates “near” the origin and
\( V_{DS2} \) is small so \( V_{DS1} = V_{o} = \) a bit less than \( V_{DD} \).

When \( V_{in} > 4V \), Q1 is capable of conducting at > 800\( \mu \)A
However, Q2 limits the current to 800\( \mu \)A and \( V_{DS1} \)
becomes “small” and \( V_{DS2} \) begins to approach \( V_{DD} \).
This process continues as \( V_{in} \) increases beyond 4V.
The transfer characteristics
**Problem 8**

\[ I_D = 0.8(V_{GS} - 2)^2 \text{ mA} \quad V_A = 60\Omega \]

The dc circuit is

\[ V_{TH} = \frac{6R}{6R + 12R} = 4V \quad R_{TH} = 6R \parallel 12R = 4R \cdot \]

\[ V_GS = 4 - 1R \times I_D \text{ (mA)} = 4 - 1 \times I_D \]

Substitute this for \( V_{GS} \) in \( I_D \) equation and get

\[ 0.8 (4 - I_D - 2)^2 = I_D \]

All currents in mA

\[ 0.8 (2 - I_D)^2 = I_D \quad \text{or} \quad 0.8 (4 - 4I_D + I_D^2) = I_D \]

and \( 4 - 4I_D + I_D^2 = 12.5I_D \) \quad \text{and} \quad I_D = 5.25I_D + 4 = 0 \]

\[ I_D = 5.25 \pm \sqrt{(5.25)^2 - 16} \quad \text{and} \quad I_D = 0.925 \text{ mA} \text{ or } I_D = 4.325 \text{ mA} \]

\( I_D = 4.325 \text{ mA is not physically possible as } 8 \times 4.325 = 12.975 \text{ which is higher than the supply voltage. Also, } \)

\( I_D \times 1R = 4.325V \text{ makes } V_{GS} = 4 - 4.325 = -0.325V, \text{ well below the threshold voltage putting the MOSFET off! Must select } I_D = 0.925 \text{ mA} \)

KVL for the drain loop is

\[ -12 + I_D \times 3R + V_{CE} + I_D \times 1R = 0 \quad \text{or} \quad V_{CE} = 12 - 4I_D \]

\[ V_{CE} = 12 - 4 \times 0.925 = 8.3V \]
PROBLEM C

The small-signal model of the common-source stage in

Best way to proceed is to convert $g_m V_{gs}$ in parallel with $V_o$
to its voltage-source series $R$.

$$V_{gs} = V_G - V_S$$

$$V_G = \frac{R_G N_{D}^2}{R_G + R_x}$$

$$V_S = -I_D R_S$$

KVL Drain loop: $r_o g_m V_{gs} + I_D (V_o + R_D + R_s) = 0$ and

$$-\frac{g_m r_o x N_D}{V_o + R_D + R_s (1+g_m r_o)}$$

$$I_D = \frac{g_m r_o x N_D}{V_o + R_D + R_s (1+g_m r_o)}$$

$N_{o1} = -I_D R_o = -\frac{g_m r_o x R_D N_D}{V_o + R_D + R_s (1+g_m r_o)}$

There are the literal answers to (4) and

$N_{o2} = I_D R_S = \frac{g_m r_o x N_D R_S}{V_o + R_D + R_s (1+g_m r_o)}$

(5) and (6) are both yes.

2. $A_v = \frac{N_{o1}}{N_{D}} |_{R_S=0} = -\frac{g_m r_o x R_D}{V_o + R_D}$

3. $A_v = \frac{N_{o2}}{N_{D}} |_{R_D=0} = \frac{g_m r_o x R_S}{V_o + R_S (1+g_m r_o)}$
The numerical answers to Problem 2 are:

\[ g_m = 2\sqrt{\frac{k}{ID_m}} = 2\sqrt{\frac{0.8 \times 0.925}{\frac{1}{2}}} = 1.172 \quad R_o = 60k \Omega \]

\[ \alpha = \frac{R_o}{R_o + R_L} = \frac{4k}{4k + 0.57k} = 0.845 \]

2) \[ A_v1 = \frac{-1.172 \times 60 \times 0.875 \times 3}{60 + 8 + 1} = -4.9 \]

3) \[ A_v2 = \frac{1.172 \times 60 \times 0.875 \times 1}{60 + 1 (1 + 1.72 \times 60)} = 0.55 \]

4) \[ A_v3 = \frac{-1.172 \times 60 \times 0.875 \times 3}{60 + 8 + 1 (1 + 1.72 \times 60)} = -1.62 \]

5) \[ A_v4 = \frac{1.172 \times 60 \times 0.875 \times 1}{60 + 3 + 1 (1 + 1.72 \times 60)} = 0.54 \]

**Problem D**

To obtain \( R_{D1} \) and \( R_{D2} \), we can use Thévenin's theorem. The Thévenin resistance is \( V_{TH}/I_{SC} \). We already have \( V_{TH} \) for both. To obtain \( I_{SC} \), make \( R_o = 0 \) and find \( I_D \big|_{R_o = 0} \) to get \( R_{D1} \); for \( R_{D2} \), make \( R_S = 0 \) and get \( I_D \big|_{R_S = 0} \). Then take the appropriate ratio:

\[ V_{TH1} = \frac{-g_m R_o \times R_D N_o}{[R_0 + R_D + R_S (1 + g_m R_o)]} \]

\[ I_D \big|_{R_o = 0} = -\frac{g_m R_o \times N_o}{[R_0 + R_S (1 + g_m R_o)]} \]

\[ R_{D1} = \frac{V_{TH1}}{I_D \big|_{R_o = 0}} = \frac{R_{D}}{[R_0 + R_D + R_S (1 + g_m R_o)]} \]

Thus \( R_{D} \) simplifies to:

\[ R_{D1} \parallel [R_0 + R_S (1 + g_m R_o)] \]

\[ V_{TH2} = \frac{g_m R_o \times R_S N_o}{[R_0 + R_D + R_S (1 + g_m R_o)]} \]

\[ I_D \big|_{R_S = 0} = \frac{g_m R_o \times N_o}{[R_0 + R_D]} \]

\[ R_{D2} = \frac{V_{TH2}}{I_D \big|_{R_S = 0}} = \frac{R_S (R_0 + R_D)}{[R_0 + R_D + R_S (1 + g_m R_o)]} \]
Rewriting \( R_{02} \) as:

\[
R_{02} = \frac{R_s (V_o + R_d)}{V_o + R_d + R_s + g_m R_o R_s} = \frac{R_s (V_o + R_d)}{V_o + R_d + R_s + g_m R_o R_s} \times \frac{1}{1 + \frac{g_m R_o R_s}{V_o + R_d + R_s}}
\]

The first term is \( R_s \parallel (V_o + R_d) \), and the second illustrates the impedance reduction of the feedback provided by \( R_s \). Note that \( R_s \) is part of the input loop affecting \( V_{gs} \) and \( R_s \) is part of the output loop indicating feedback.

Numerically, the values are:

\[
R_{01} = 3 \parallel \frac{[60 + 3 + 1 \times (1 + 1.72 \times 60)]}{311167.2} = 3 \parallel 167.2 = 2.95 \text{ kΩ}
\]

\[
R_{02} = \left[ \frac{1}{1 \parallel (60 + 3)} \right] \times \frac{1}{1 + \frac{1.72 \times 60}{60 + 3}} = 0.984 \times \frac{1}{2.61} = 0.376 \text{ kΩ}
\]

Observe: For \( R_0 \), the effect of the feedback makes the term in the \( \parallel \) larger so that the parallel combination is more closely \( 3 \text{ kΩ} = R_0 \). For \( R_{02} \), the effect of feedback reduces the output resistance. The resistance seen by \( R_d \) in \( R_{01} \) increases because of the feedback provided by \( R_s \).