Electrical Engineering
Week 8

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EECE–2210
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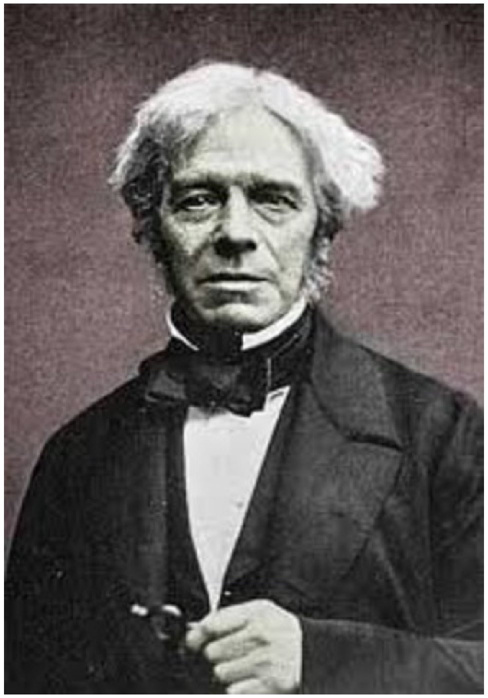
Week 8 Agenda: Inductors

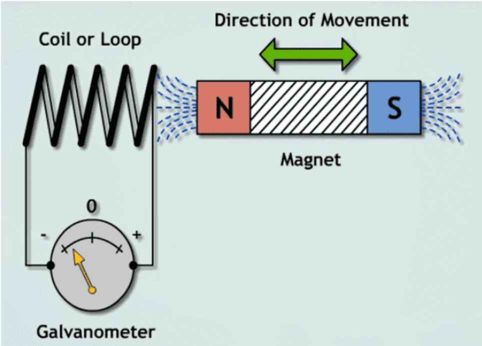
- Physical Concepts
- Symbols
- i–v Behavior
- Fabrication
- Power and Energy
- Parallel and Series Combinations
- Steady–State Solutions
- "Instantaneous" Current Change

The Inductor

- Coil of Wire
- Air or Ferromagnetic Core
- Current $\rightarrow$ Magnetic Field (Electromagnet)
- Changing Field $\rightarrow$ Voltage (Faraday's Law)
- Voltage Opposes Change in Current

$$v(t) = L\frac{di(t)}{dt}$$

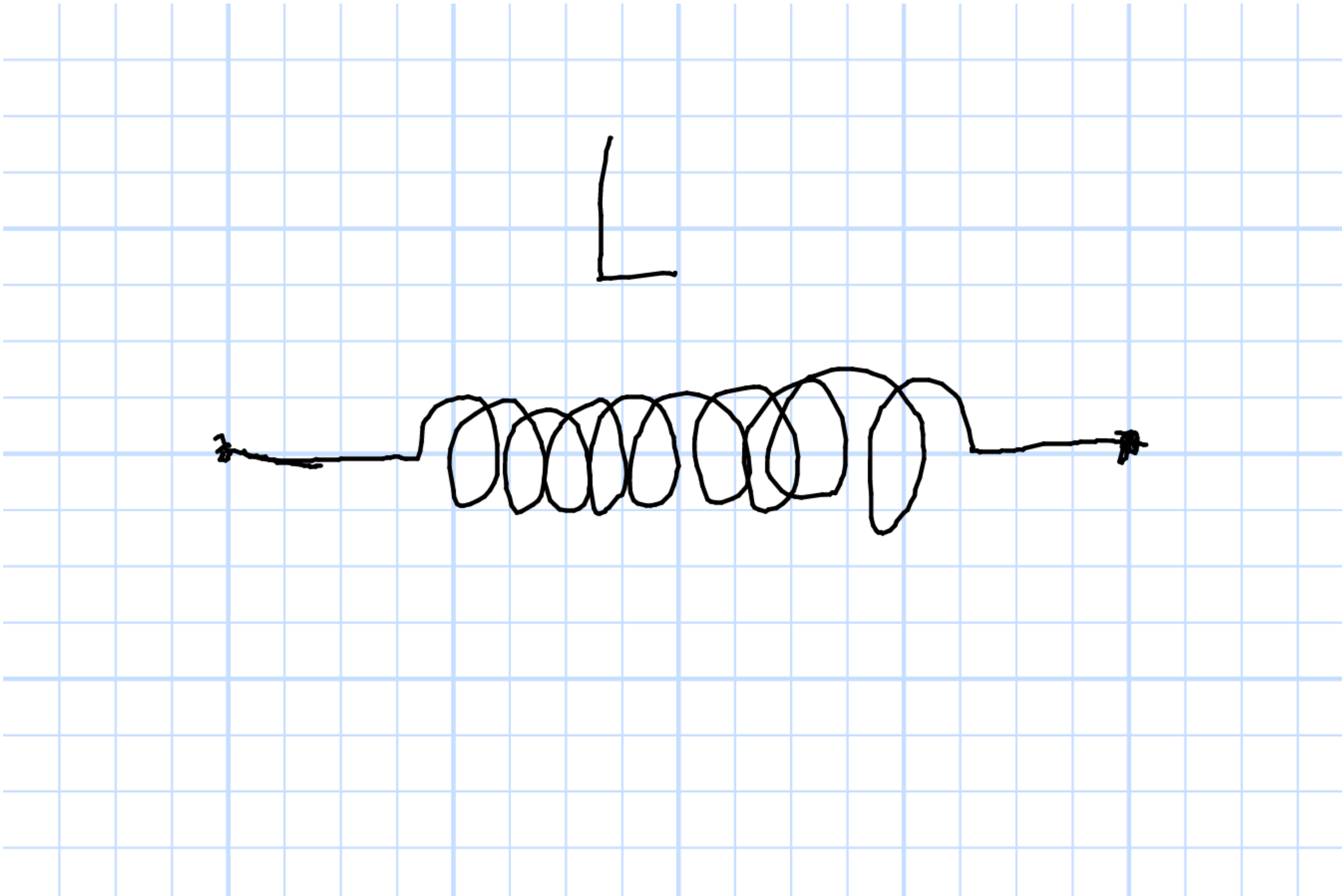




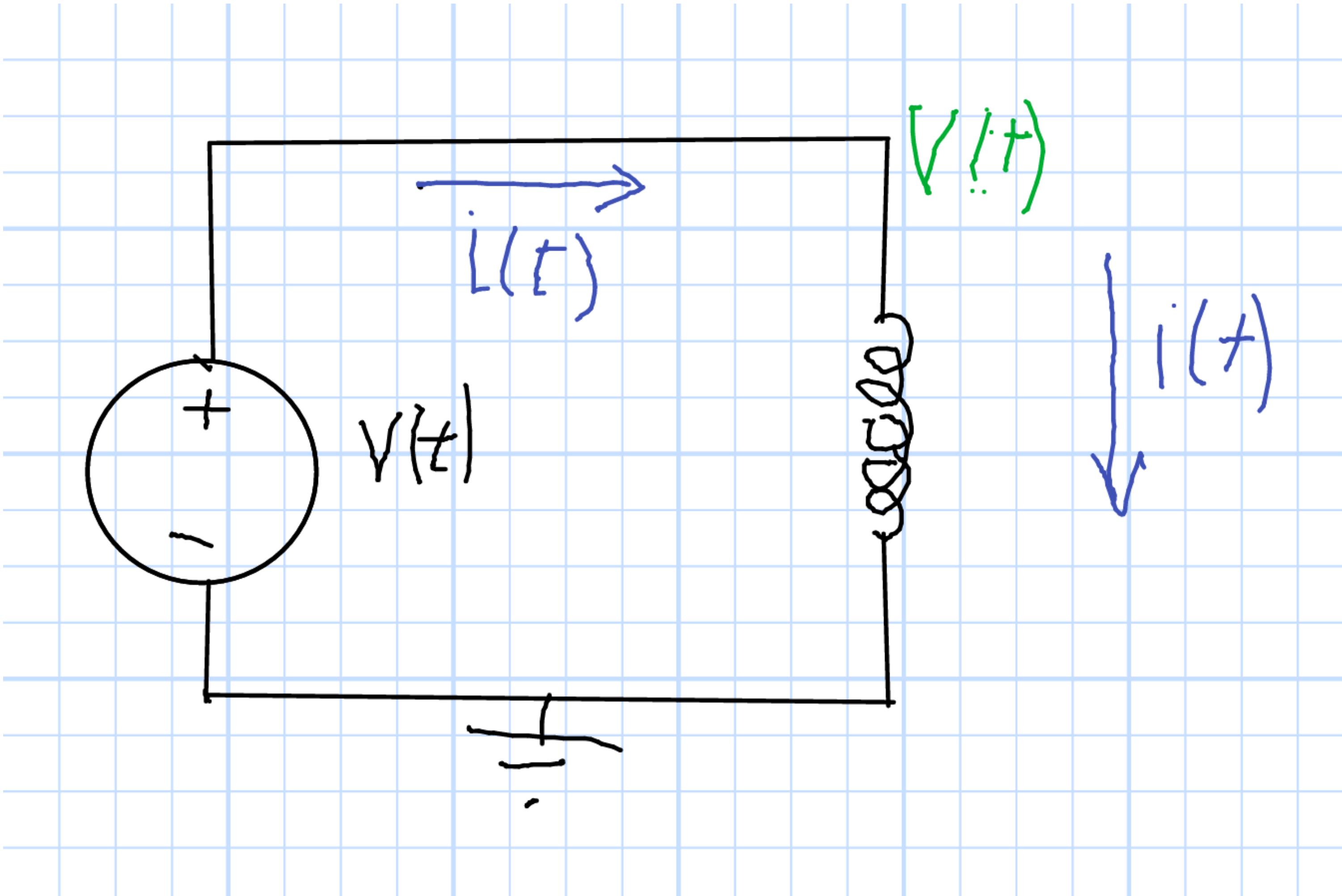
timetoast.com

www.electrical4u.net

Symbol

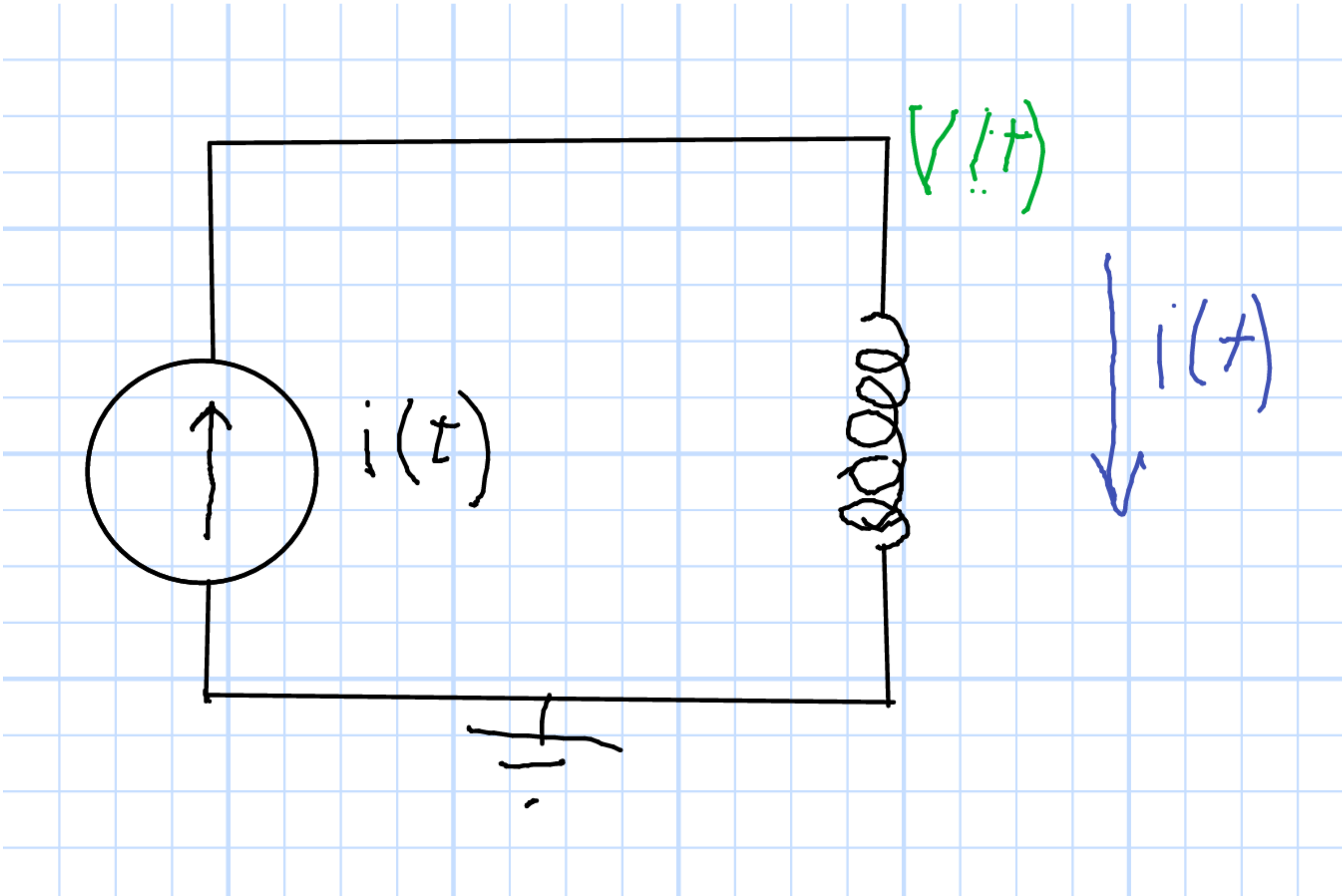


Voltage Source



$$i = \int v \; dt$$

Current Source

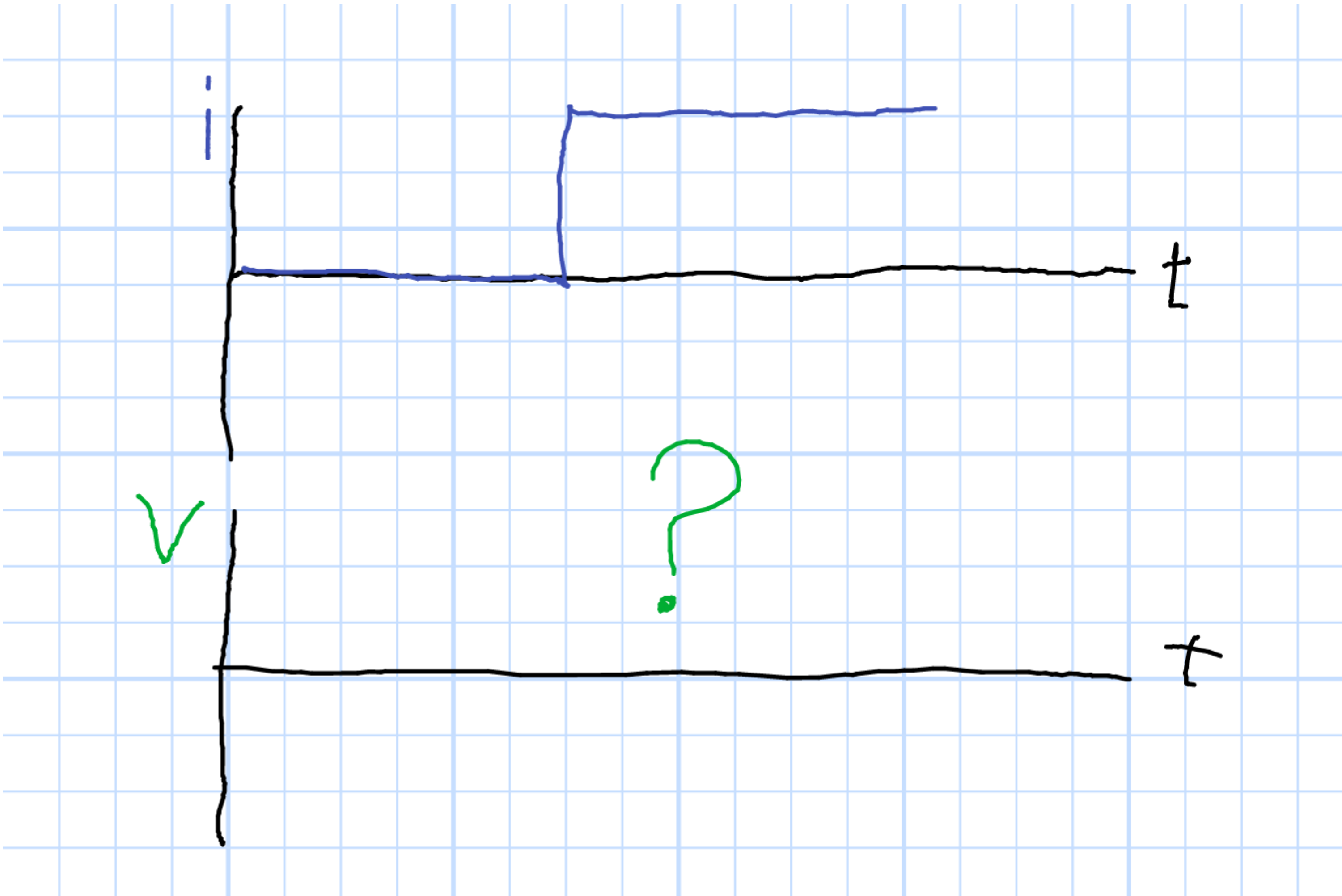


$$v = L\frac{di}{dt}$$

i–v Behavior

$$v = L\frac{di}{dt}$$

Step Current



Current is Continuous

Values

$$v = L\frac{di}{dt}$$

Typical Values:

$$\text{Volts} = L\frac{\text{mA}}{\text{ms}}$$

$$L \text{ in } \frac{\text{Vs}}{\text{A}} = \text{Henries} = \text{H}$$

mH, μH Common in RF.
kH Do Exist.

Fabrication

- Coil of Wire (Many Turns)
- Field of a solenoid

$$B(t) = \frac{\mu N}{\ell} i(t)$$

- Inductance of a solenoid

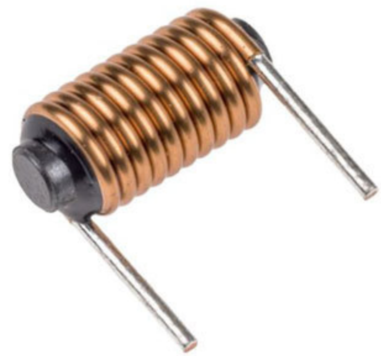
$$v(t) = \frac{\mu A N^2}{\ell} \frac{di(t)}{dt} \qquad L = \frac{\mu A N^2}{\ell}$$

- Air, Iron, Ferrite Core (Increased Field)
- Solenoid, Toroid, Helmholz Coils *etc.*
- Many Options

$\mu = \mu_r \times 1.26 \times 10^{-6}\mathrm{H/m}$

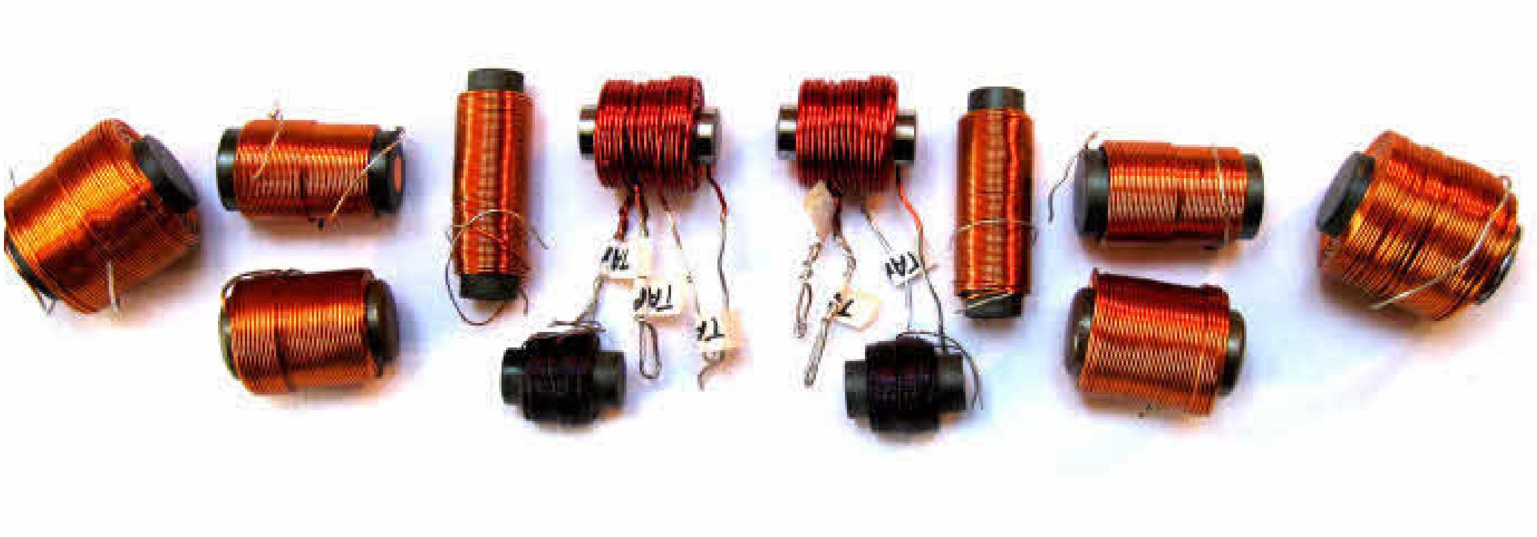
Inductors

indiamart.com



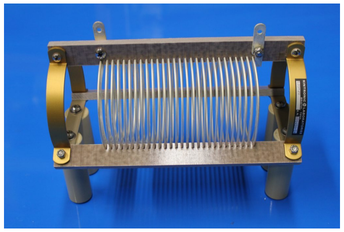
components101.com

toroids.com

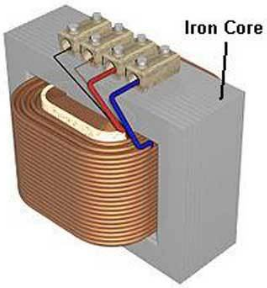


falconacoustics.co.uk

More Inductors & Transformers

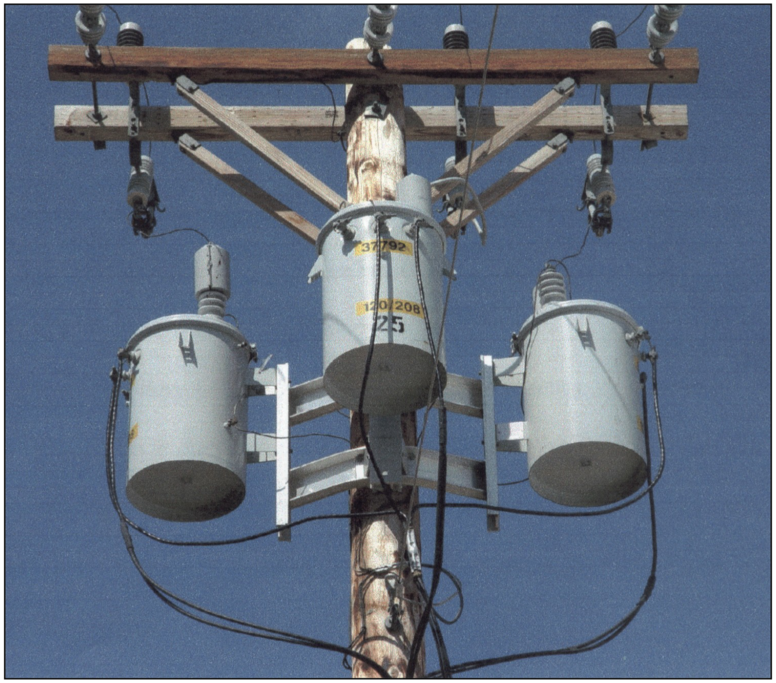


kintronic.com



polytechnichub.com

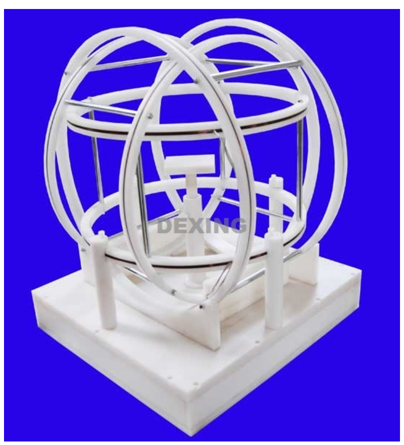
globalspec.com



coloradocountrylife.coop

electricianinperth.com.au

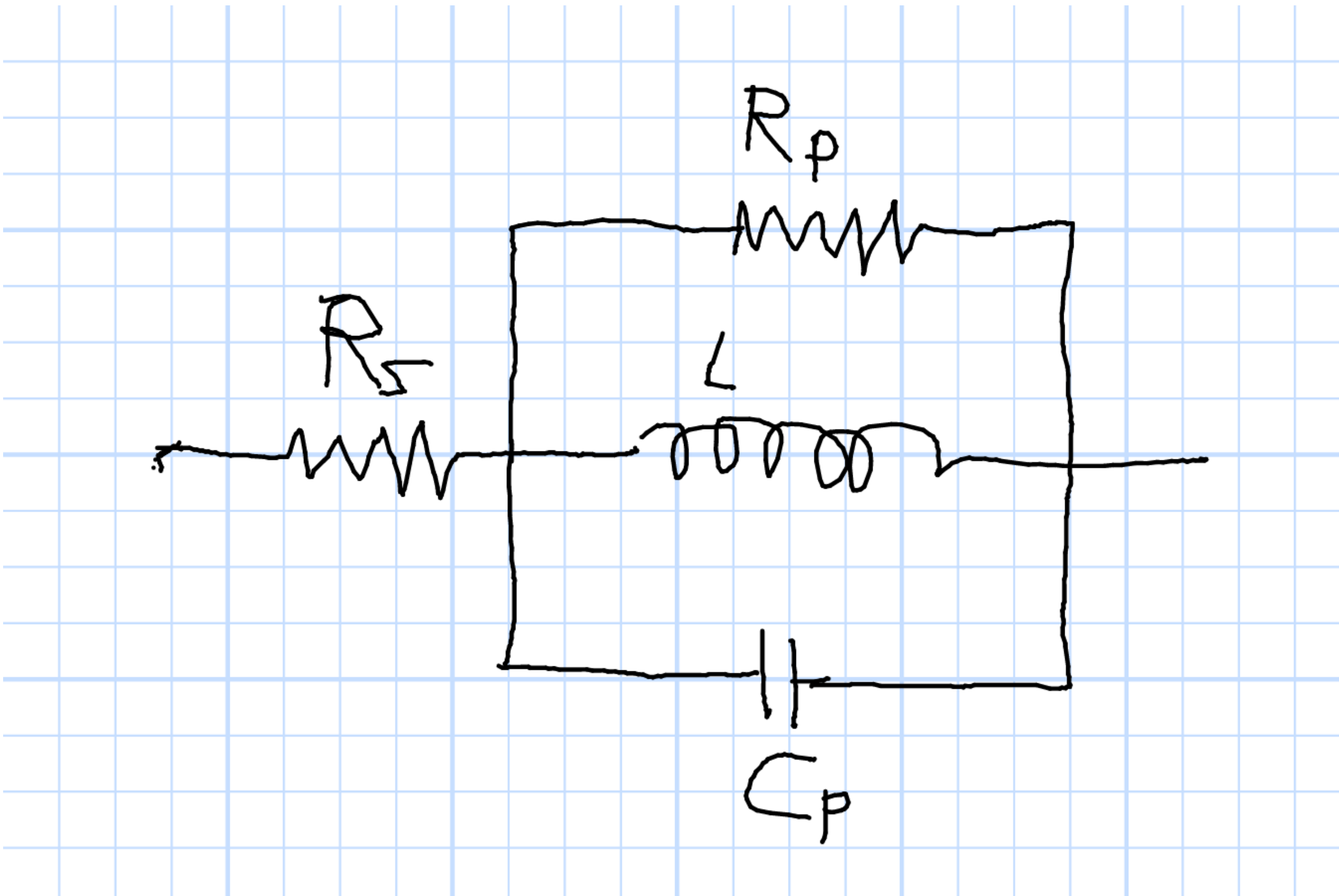
Helmholz Coils



3bscientific.com

magnetic-instrument.com

Real Inductors



R_s, C_p Small. R_p Large

Power and Energy

$$v(t) = L\frac{di(t)}{dt}$$

$$p(t) = i(t)\,v(t) = i(t)\,L\frac{di(t)}{dt}$$

$$w = \int p(t)\,dt = \frac{i^2 L}{2}$$

Example: Still Another Cup of Coffee

$$w = 42\text{kJ} \qquad i^2 L = 84\text{kJ}$$

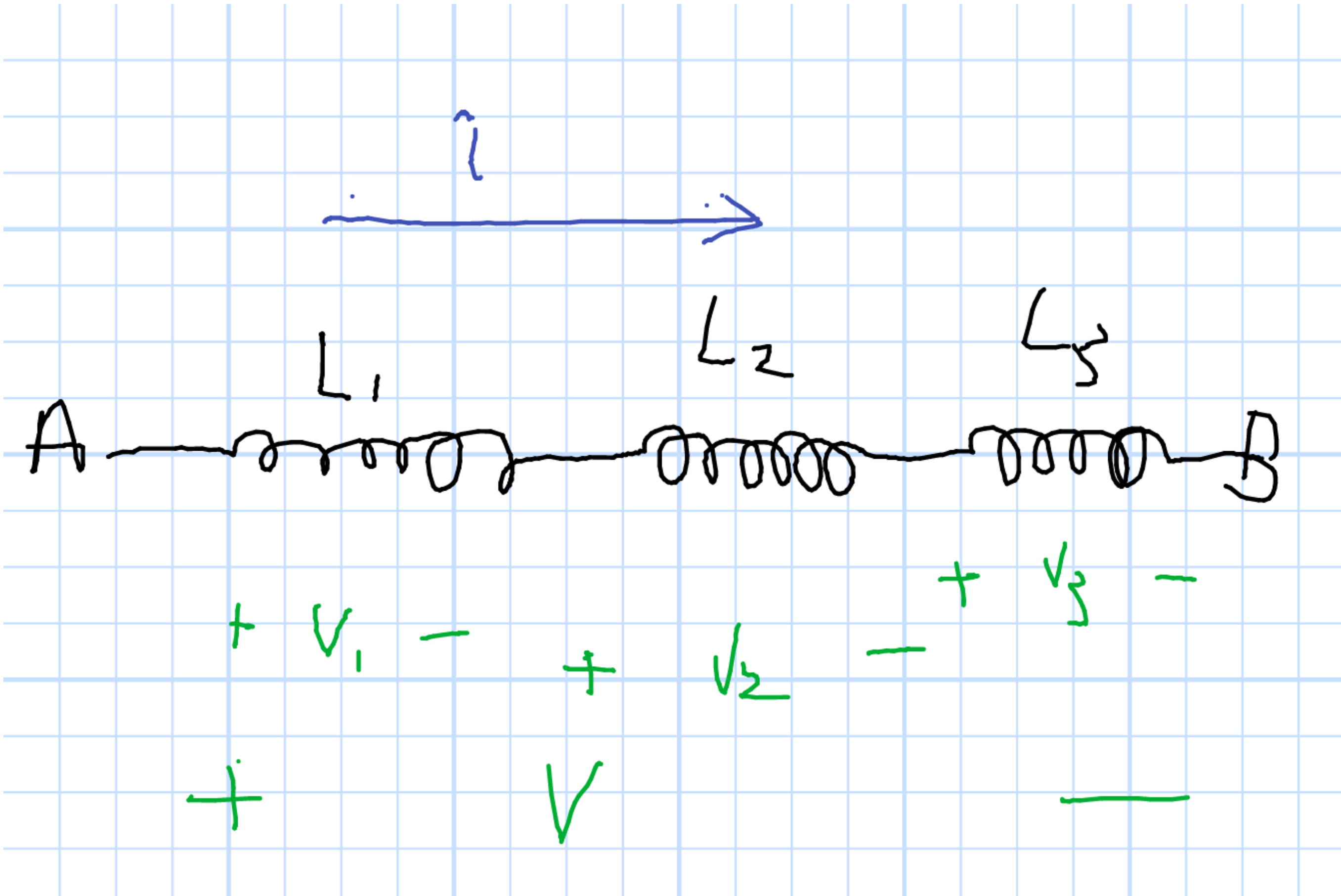
$$L = 6\text{H} \qquad i = 118\text{A}$$

Note: $v \approx 0$, so $p \approx 0$, except during turn–on and turn–off. These times can be exciting!

MRI Magnet Quench

fickr.com Superconducting magnet in use, Low T, R_s, v, High i.
In quench, $-di/dt\uparrow$, $T\uparrow$, $R_s\uparrow$, High v, i, p.

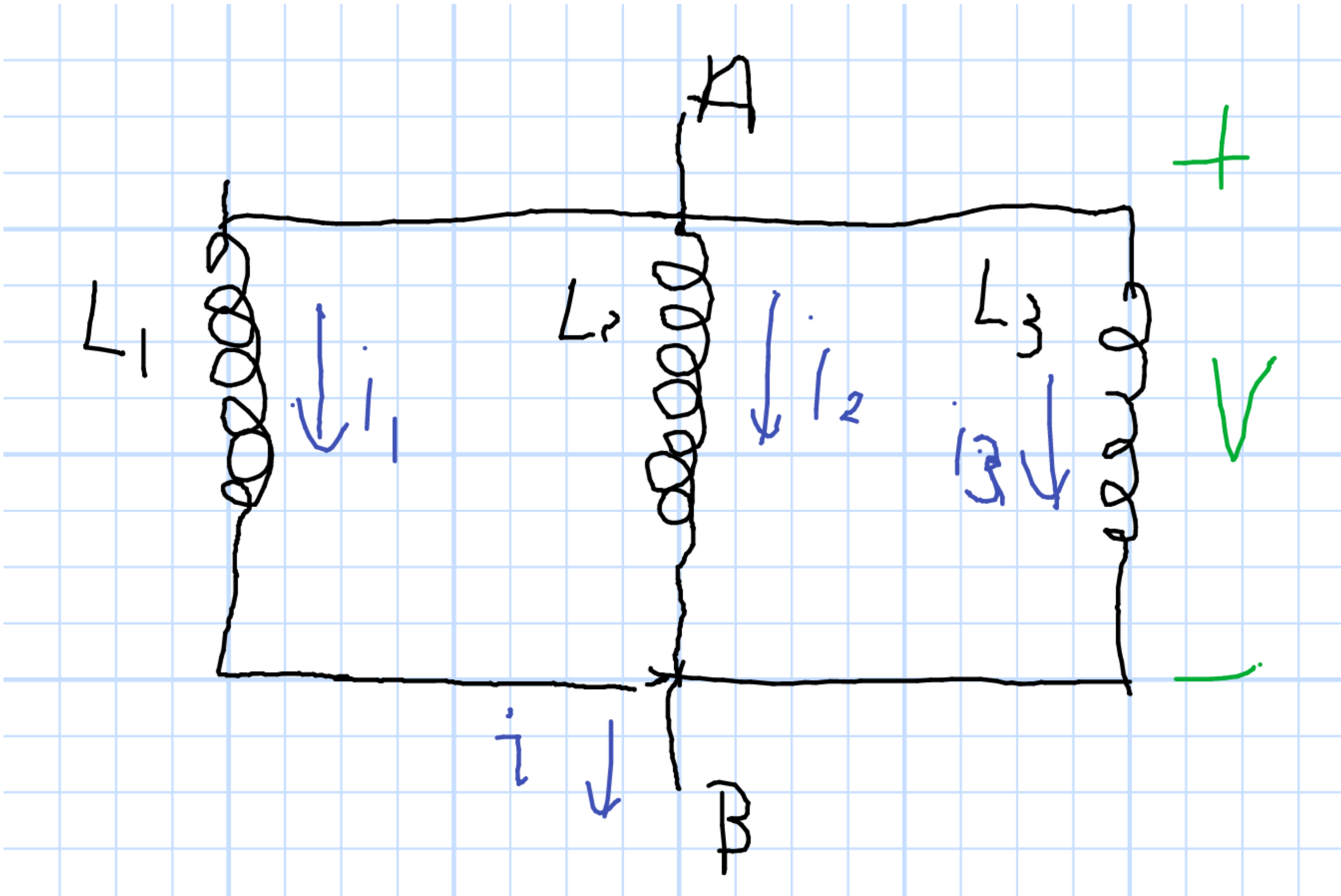
Inductors in Series



$$v_n = L_n \frac{di}{dt} \qquad v = \sum v_n \qquad L = \sum L_n$$

Just Like Resistors

Parallel Inductors



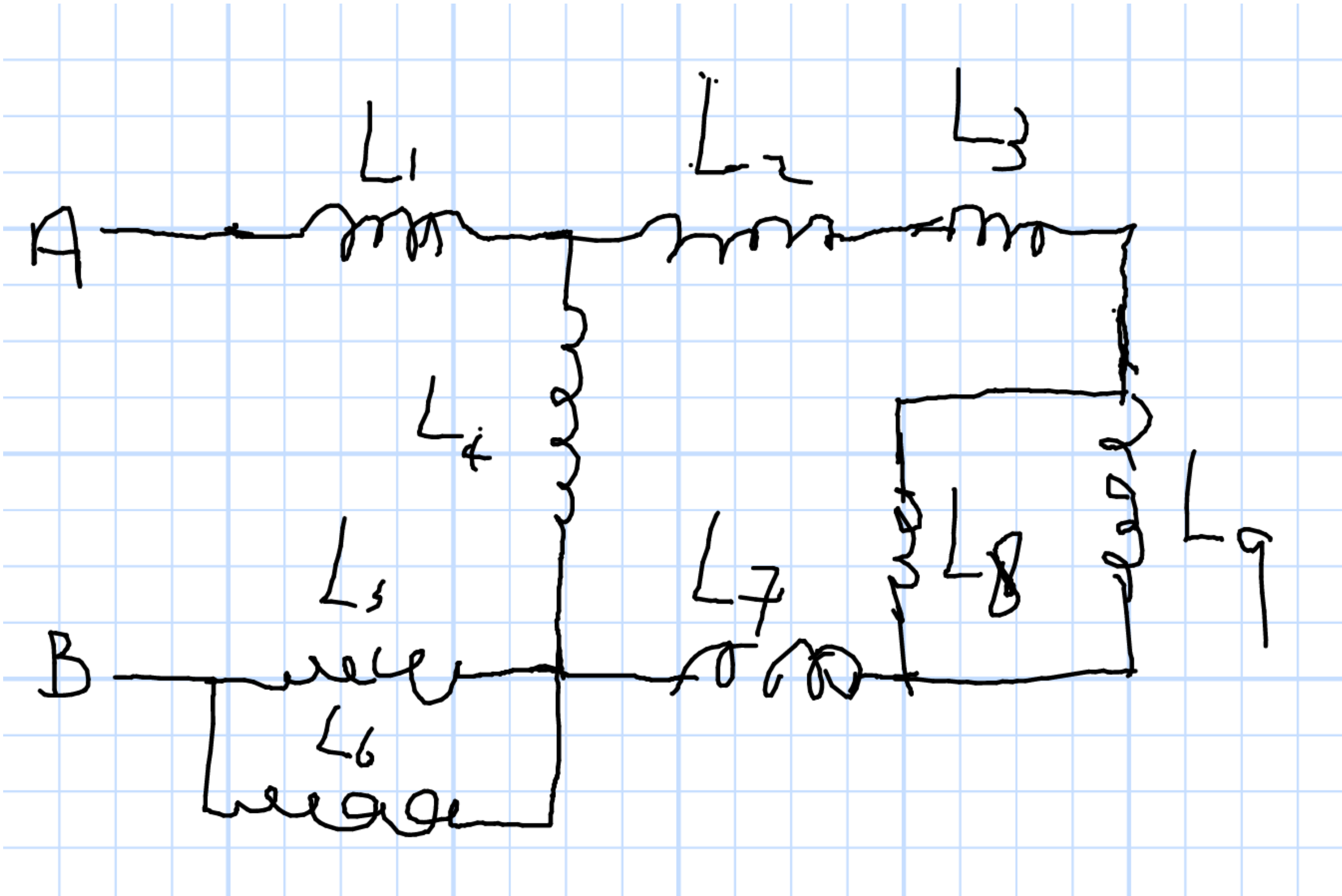
$$\frac{1}{L_n}\frac{di_n}{dt} = v \qquad \frac{di}{dt} = \sum \frac{di_n}{dt} \qquad \frac{L}{=}\sum \frac{1}{L_n}$$

Just Like Resistors Again

Parallel/Series Summary

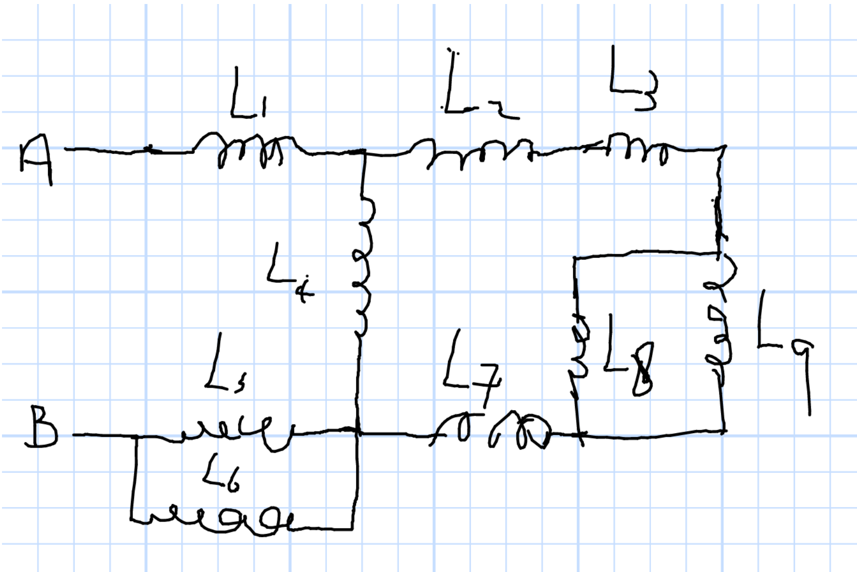
	Series	Parallel
Voltage Sources	$v = \sum v_n$	Contradictory
Current Sources	Contradictory	$i = \sum i_n$
Resistors	$R = \sum R_n$	$\frac{1}{R} = \sum \frac{1}{R_n}$
Inductors	$L = \sum L_n$	$\frac{1}{L} = \sum \frac{1}{L_n}$
Capacitors	$\frac{1}{C} = \sum \frac{1}{C_n}$	$C = \sum C_n$

Parallel/Series Example (1)



$$L_{AB} = L_1 + \{L_4 \parallel [L_2 + L_3 + (L_8 \parallel L_9) + L_7] + [L_5 \parallel L_6]\}$$

Parallel/Series Example (2)



$$L_{1:9} = 1\text{mH}$$

$$L_{23897} = 1 + 1 + \frac{1}{2} + 1 = 3.5\text{mH}$$

$$L_{423897} = 1 \parallel 3.5 = 778\mu\text{H}$$

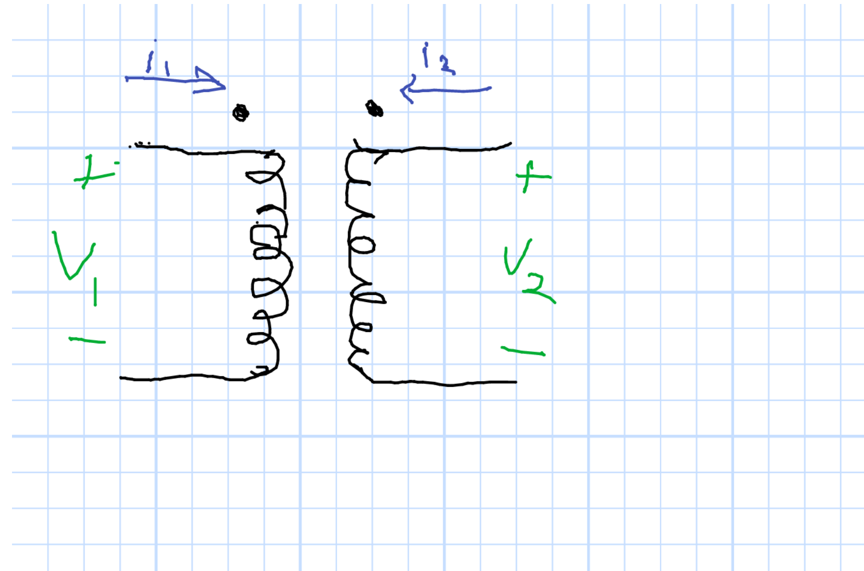
$$L_{AB} = 1 + 0.778 + \frac{1}{2} = 2.28\text{mH}$$

Mutual Inductance

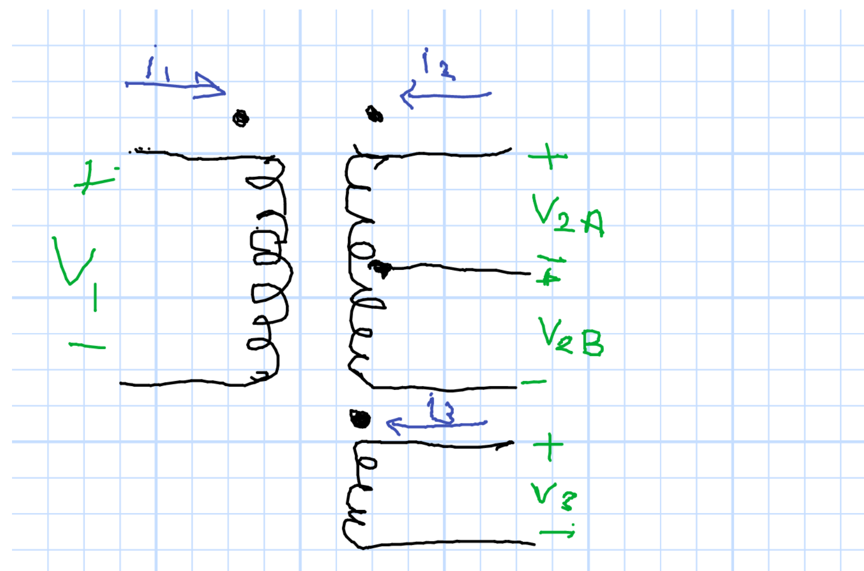
- Two or More Coils
- Same Core

$$v_1(t) = L_1 \frac{di_1(t)}{dt} + M \frac{di_2(t)}{dt}$$

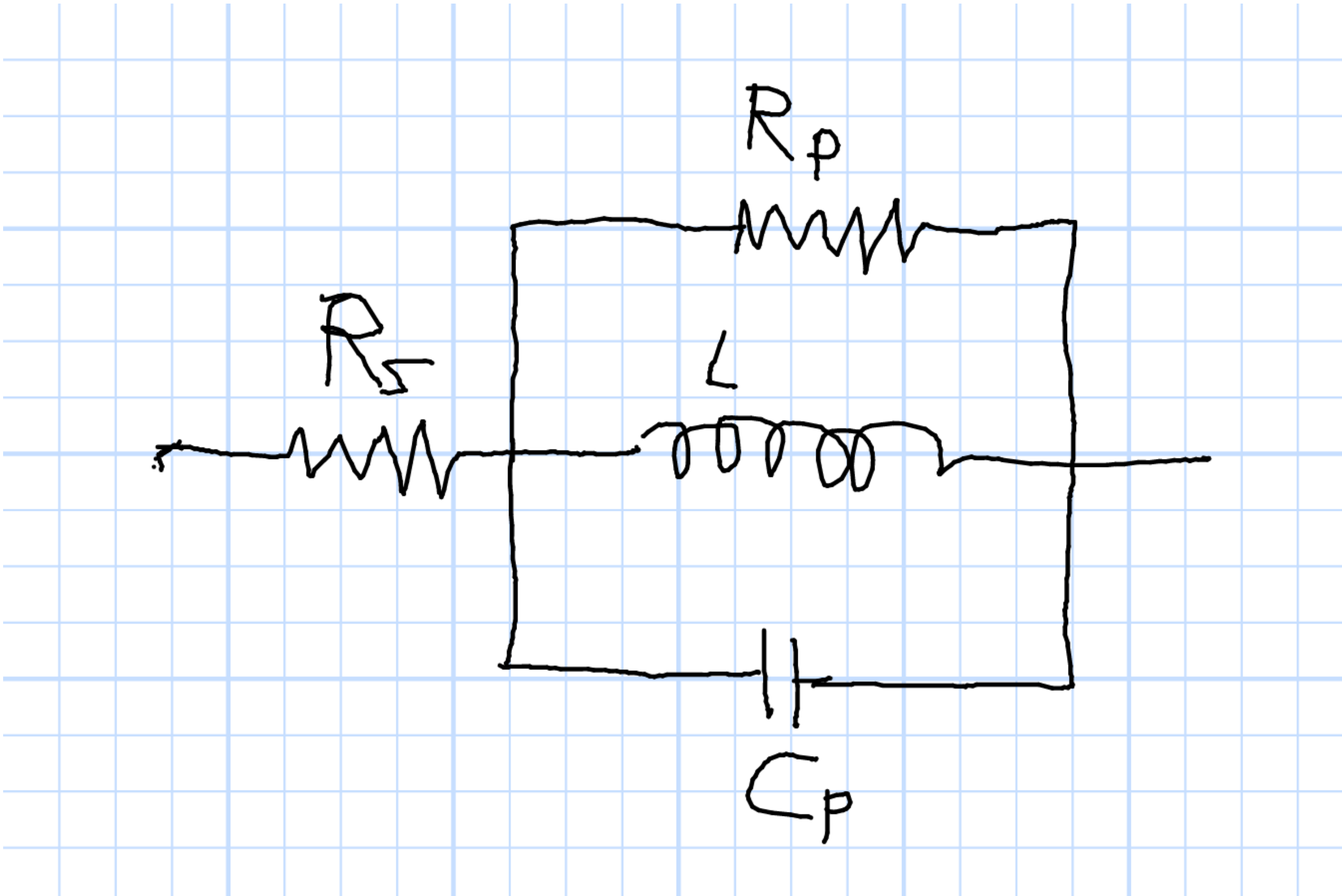
$$v_2(t) = L_2 \frac{di_2(t)}{dt} + M \frac{di_1(t)}{dt}$$



- M Same Units as L
- Transformers
  - AC Only
  - Higher Frequency $\rightarrow$ Smaller

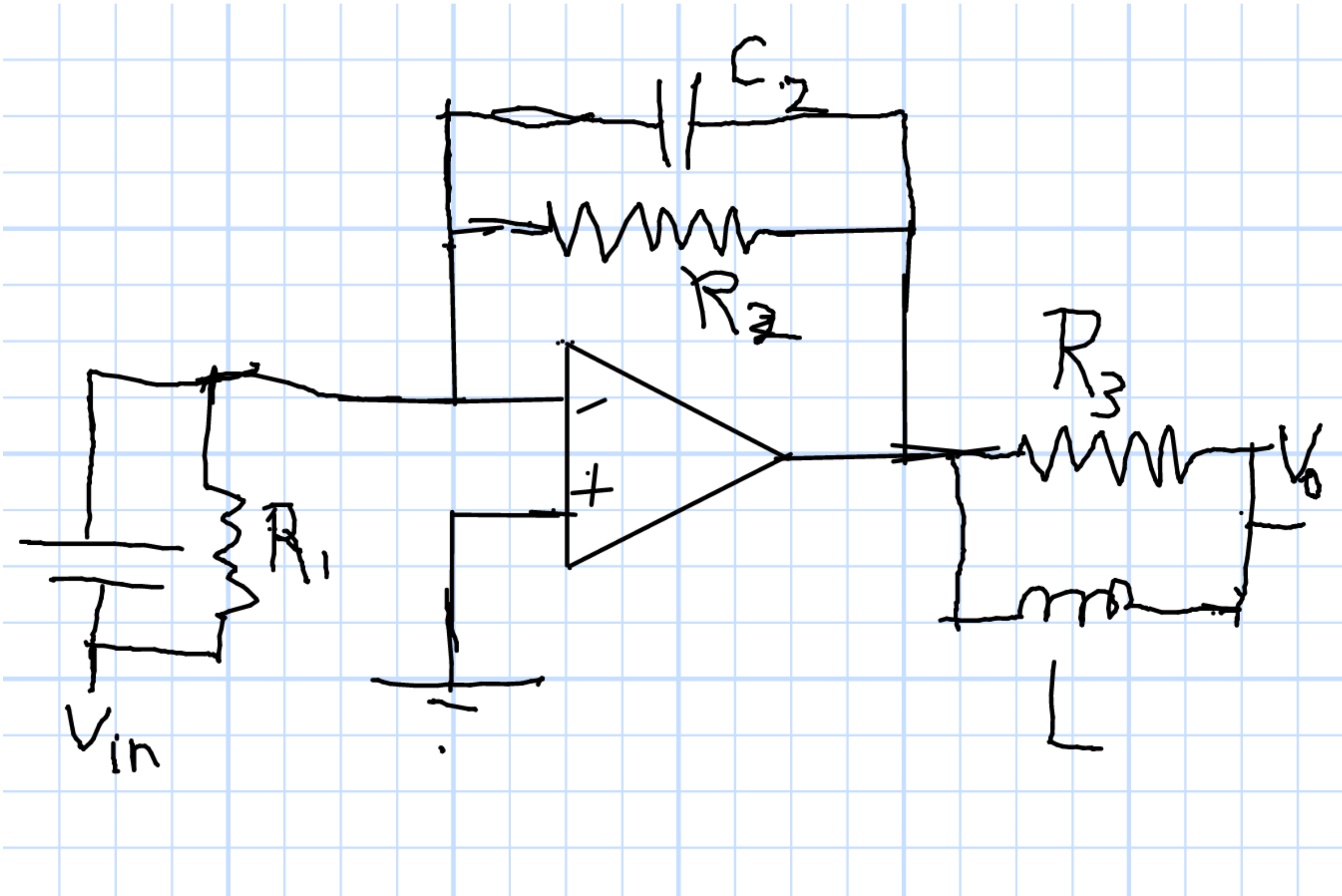


Inductors at DC (Steady State)



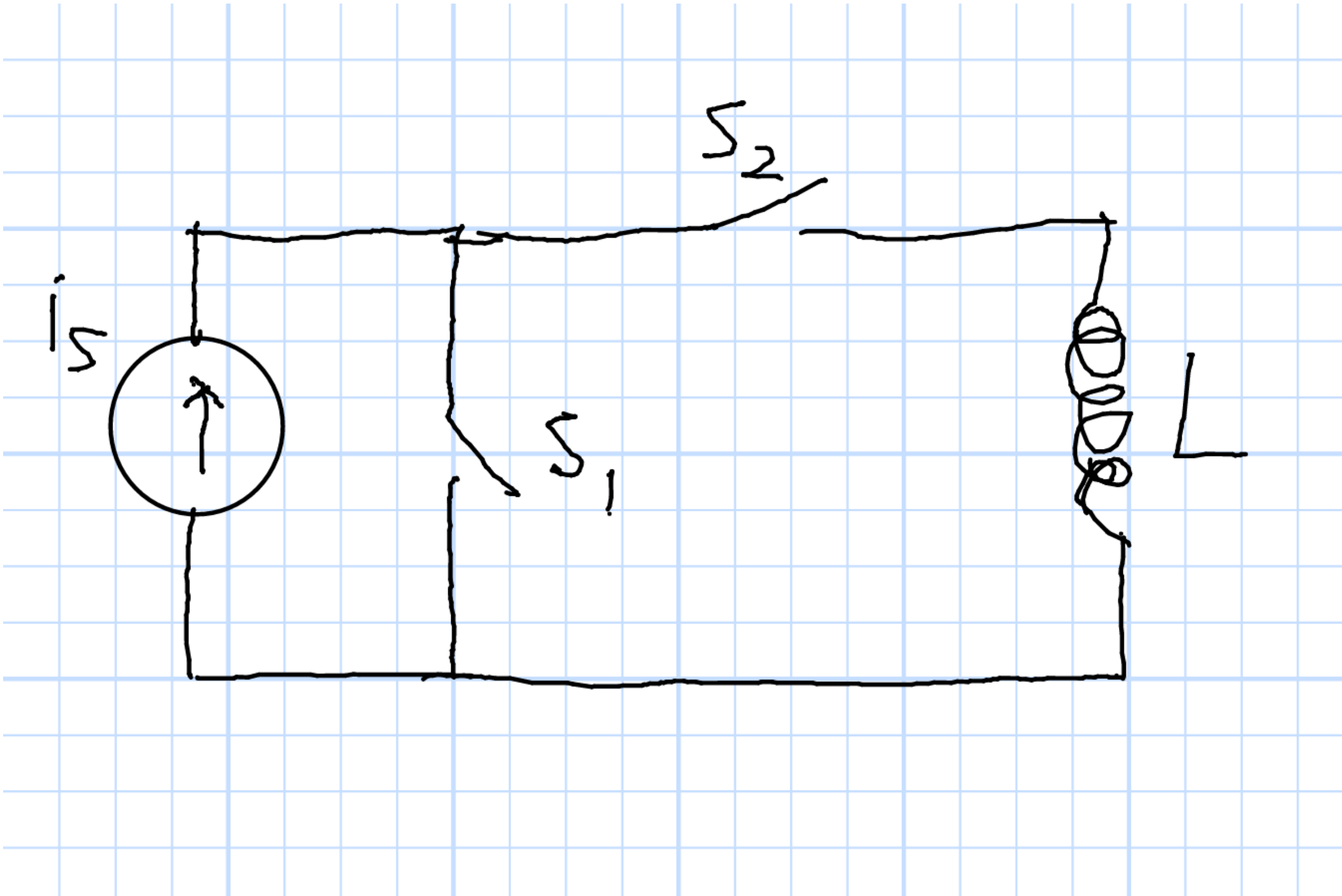
C_p Open, L Short, R_p Large (ignore). All that's left is R_s (just the resistance of the wire). $v = iR_s \to 0$

Steady State



Steady State (Short L, Open C): $v_o = -v_{in}R_2/R_1$ and $R_{out} = 0$

What Happens?



S_1, S_2 Closed. Open S_1. Open S_2

Jacob's Ladder

https://www.youtube.com/watch?v=PXiOQCRiSp0